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STABILITY ON POSITIVE ALMOST PERIODIC HIGH-ORDER HOPFIELD NEURAL NETWORKS

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Abstract. This essay explores a class of almost periodic high-order Hopfield neural networks involving time-varying delays. By taking advantage of some novel differential inequality techniques, several assertions are derived to substantiate the positive exponential stability of the addressed neural networks, which refines and extends the corresponding results in some existing references. In particular, a demonstrative experiment is presented to check the effectiveness and validity of the theoretical outcomes.

1. Introduction

In the early 1980s, physicist Hopfield [1] proposed the Hopfield neural network, a model with the associative memory function. He introduced the concept of the activation function and established stability criteria for the model using nonlinear dynamical methods. Liu [2] employed the fixed point theorem and differential inequality techniques to investigate the exponential stability of almost periodic solutions in a class of lower-order Hopfield neural networks with continuously distributed time delays. As is well known, compared to lower-order neural networks, higher-order Hopfield neural networks (HHNNs) possess greater storage capacity, faster convergence speed, and wider tolerance range. These advantages have attracted considerable attention from mathematicians, biologists, and computer scientists alike.

Since the limited speed of information processing and the inherent communication time of neurons, time delays usually occur in neural networks, which include bounded

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time-varying delay, unbounded time-varying delay, and continuously distributed delay [3–5]. Guo et al. [3] investigated the convergence of a class of multi-memory neural networks with a single delay and nonlinear coupling terms. Using the Lyapunov functional method, they derived sufficient conditions for the existence of a unique equilibrium point and the global exponential stability of the system. Aouiti [4] studied the existence and exponential stability of pseudo-almost periodic solutions in HHNNs with mixed time-varying delays and impulsive neutral terms. Zhang and Li [5] investigated the local exponential stability of almost periodic solutions in HHNNs with unbounded activation functions and time-varying delays.

The following HHNNs with time-varying delays

$$\begin{aligned} x_i'(t) =& - c_i(t)(x_i(t)) + \sum_{j=1}^{n} a_{ij}(t)\hat{g}_j(x_j(t-\tau_{ij}(t))) \\ &+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)g_j(x_j(t-\sigma_{ijl}(t)))g_l(x_l(t-\nu_{ijl}(t))) + I_i(t), \end{aligned} \tag{1.1}$$

have been frequently employed to many fields such as optimization, parallel computing, pattern recognition and image processing [6–10]. Here $i, j, l \in S := \{1, 2, \cdots, n\}$, n represents the quantity of the neural network units, $x_i(t)$ denotes the state variable, $c_i(t) > 0$ stands the velocity which the ith unit resets it feasible to the stationary state in isolation while the network is not connected from outer inputs at time t. The biological explanations on the connection weights $a_{ij}(t)$ and $b_{ijl}(t)$, transmission delays $\tau_{ij}(t) \geqslant 0$, $\sigma_{ijl}(t) \geqslant 0$, $\nu_{ijl}(t) \geqslant 0$, and the outer inputs $I_i(t)$ can be found in [11–13]. In particular, $\hat{g}_j$ and g_j are the activation functions of signal transmission of the jth unit.

Recently, significant research outcomes on the positive stability and convergence of various positive nonlinear systems with delays have been acquired in [14–17]. These studies offer valuable insights into understanding and managing their positive dynamics and potential impacts on diverse industries. The exponential stability analysis of positive neural networks, such as periodic solutions, almost periodic solutions, and equilibria, plays a crucial role in describing the behavior of dynamic systems and has received a wide range of interest from many researchers, who obtained many good results [18–20]. Here, the quite a lot outcomes are derived in [14–20], which investigates the positive stability of the addressed HHNNs (1.1) incorporating the monotone nondecreasing activation functions on $(0, +\infty)$.

It should be pointed out that, in the neural networks system involving electronic circuits, oscillators are fundamental components and normally originate almost periodical signals (square waves, sinusoids, etc.). Consequently, almost periodic oscillations undertake important roles in describing the qualitative properties of nonlinear neural networks dynamic systems (see, for example, [1, 3, 4] and the references therein). To analyze the positive almost periodic stability of HHNNs (1.1), it is always assumed that $I_i, c_i, a_{ij}, b_{ijl}, \tau_{ij}, \sigma_{ijl}, \nu_{ijl} : \mathbb{R} \to [0,\ +\infty)$ are almost periodic functions, and there are constants $\tau, \tilde{c}_i, \underline{I}_i$ and $\overline{I}$ obeying that

$$\tau = \max\{\max_{i,j\in S} \sup_{t\in\mathbb{R}} \tau_{ij}(t), \max_{i,j,l\in S} \sup_{t\in\mathbb{R}} \sigma_{ijl}(t), \max_{i,j,l\in S} \sup_{t\in\mathbb{R}} \nu_{ijl}(t)\},$$
$$0 < \tilde{c}_i = \inf_{t\in\mathbb{R}} c_i(t), 0 < \underline{I}_i = \inf_{t\in\mathbb{R}} I_i(t), \overline{I} = \max_{i\in S} \sup_{t\in\mathbb{R}} I_i(t), \tag{1.2}$$

$$\Gamma = \max_{i\in S} \sup_{t\in\mathbb{R}} \{\sum_{j=1}^{n} a_{ij}(T)|\hat{g}_j(0)| + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T) M_j^g |g_l(0)|\}. \tag{1.3}$$

For $X(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$, we label the following norm:

$$\|X(t)\| = \max_{i\in S} |x_i(t)|.$$

Under the above basic assumptions which also hold in what follows, it should be mentioned that the positive almost periodic exponential stability of HHNNs (1.1) were deeply discussed in [6]. More specifically, under the following requirements:

(H_1) For every $j \in S, \hat{g}_j, g_j : \mathbb{R} \to \mathbb{R}$ are non-decreasing function on $[0, +\infty)$, $\hat{g}_j = g_j$ and there exist non-negative constants L_j^g and M_j^g obeying that

$$g_j(0) = 0, |g_j(u) - g_j(v)| \leqslant L_j^g |u - v|, |g_j(u)| \leqslant M_j^g, \text{ for all } u, v \in \mathbb{R}.$$

(H_2) There exist constants $\eta > 0$ and $\lambda > 0$ such that for arbitrary $t > 0$, there holds

$$(\lambda - c_i(t)) + \sum_{j=1}^{n} a_{ij}(t) L_j^g e^{\lambda\tau} + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)(L_j^g M_l^g + M_j^g L_l^g) e^{\lambda\tau} < -\eta. \tag{1.4}$$

Sufficient criteria ensuring the positive almost periodic exponential stability of solutions are gained in [6] for HHNNs (1.1) accompanying the initial value assumptions:

$$x_i(t) = \varphi_i(t), t \in [-\tau, 0], i \in S, \tag{1.5}$$

where $\varphi = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t))^T \in C([-\tau, 0]; \mathbb{R}^n)$.

Clearly, the condition (H_1) involving that the activation functions have monotonicity on the positive half-axis limits the involvement and application range of the positive network system. Therefore, a natural question arises: whether we can find weaker conditions guaranteeing the global exponential stability of the positive almost periodic solutions of HHNNs (1.1) involving initial value assumptions (1.5). This is the purpose of this paper.

This paper is structured as follows. In Section 2,we present two new activation function conditions and some key lemmas. In Section 3, we focus on addressing the global exponential stability of positive almost periodic solution of HHNNs (1.1). A simulation example is provided to support the correctness of the theoretical results in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminary Results

In this section, two new assumptions are introduced to prepare for addressing the problem stated in the introduction. In addition, we present some lemmas that will be employed throughout the remainder of this paper.

Hereafter, we make the following assumptions:

(H_1^*) For each $j \in S$, one can choose nonnegative constants $\hat{L}_j^g$, L_j^g and M_j^g agreeing with

$$\begin{cases} |\hat{g}_j(u) - \hat{g}_j(v)| \leqslant \hat{L}_j^g |u-v|, \ |g_j(u)| \leqslant M_j^g, \\ |g_j(u) - g_j(v)| \leqslant L_j^g |u-v|, \ \text{for all } u, v \in \mathbb{R}, \end{cases} \tag{2.1}$$

and $\hat{g}_j, g_j$ are non-decreasing functions on a bounded interval $(\pi, \Pi) \subset (0, +\infty)$, where $\pi = \frac{\Pi}{\theta} < \Pi = \frac{\bar{I}+\Gamma}{\eta}$, and θ is a sufficiently large positive number satisfying that

$$-I_i(t) < -c_i(t)\pi + \sum_{j=1}^{n} a_{ij}(t)\hat{g}_j(\pi) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t) g_j(\pi) g_l(\pi), \ \text{for all } t > 0. \tag{2.2}$$

(H_2^*) There exist constants $\eta > 0$ and $\lambda > 0$ such that for arbitrary $t > 0$, there holds

$$(\lambda - c_i(t)) + \sum_{j=1}^{n} a_{ij}(t)\hat{L}_j^g e^{\lambda\tau} + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)(L_j^g M_l^g + M_j^g L_l^g) e^{\lambda\tau} < -\eta. \tag{2.3}$$

Remark 2.1. Since (H_1) implies that $\hat{g}_j(0) = g_j(0) = 0$ for all $j \in S$, one can easy discover a sufficiently large positive number θ obeying that $\frac{\Pi}{\theta} < \frac{\bar{I}+\Gamma}{\eta}$ and (2.2) are obeyed. Therefore, (H_1) implies that (H_1^*) holds, and (H_1^*) is a condition that relaxes the restriction on the activation function of condition (H_1) to a greater extent, which may further be expanding the design and application of the neural network model associated with it.

Before illustrating the key theorem to be proved in Section 3, we next state some basic lemmas.

Lemma 2.1. *Let* (H_1^*) *and* (H_2^*) *be satisfied, and assume that* $\widetilde{x}(t) = (\widetilde{x}_1(t), \widetilde{x}_2(t), \cdots, \widetilde{x}_n(t))^T$ *is a solution to HHNNs (1.1) accompanying the initial value assumptions*

$$\widetilde{x}_i(s) = \widetilde{\varphi}_i(s),\ \pi < \widetilde{\varphi}_i(s) < \Pi,\ s \in [-\tau, 0],\ \widetilde{\varphi}_i \in C[-\tau, 0],\ i \in S. \tag{2.4}$$

Then

$$\pi < \widetilde{x}_i(t) < \Pi,\ \text{for all } t > 0 \text{ and } i \in S. \tag{2.5}$$

Proof. Suppose the contrary and choose $T > 0$ and $i \in S$ obeying that one of the following two situations occurs:

$$\widetilde{x}_i(T) = \Pi,\ \text{and } \pi < \widetilde{x}_j(t) < \Pi \text{ for all } t \in [-\tau, T) \text{ and } j \in S, \tag{2.6}$$

$$\widetilde{x}_i(T) = \pi,\ \text{and } \pi < \widetilde{x}_j(t) < \Pi \text{ for all } t \in [-\tau, T) \text{ and } j \in S. \tag{2.7}$$

If (2.6) valid, in combination with conditions (H_1^*) and (H_2^*), using the Dini derivative theory and scaling of inequalities, one can conclude that

$$\begin{aligned}
0 &\leqslant \widetilde{x}_i'(T) \\
&= -c_i(T)\widetilde{x}_i(T) + \sum_{j=1}^{n} a_{ij}(T)\hat{g}_j(\widetilde{x}_j(T - \tau_{ij}(T))) \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T) g_j(\widetilde{x}_j(T - \sigma_{ijl}(T))) g_l(x_l(T - \nu_{ijl}(T))) + I_i(T) \\
&\leqslant -c_i(T)\Pi + \sum_{j=1}^{n} a_{ij}(T)\hat{g}_j(\Pi) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T) g_j(\Pi) g_l(\Pi) + I_i(T) \\
&\leqslant -c_i(T)\Pi + \sum_{j=1}^{n} a_{ij}(T)|\hat{g}_j(\Pi) - \hat{g}_j(0) + \hat{g}_j(0)| \\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T) M_j^g |g_l(\Pi) - g_l(0) + g_l(0)| + I_i(T) \\
&\leqslant -c_i(T)\Pi + \sum_{j=1}^{n} a_{ij}(T)\hat{L}_j^g\Pi + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T) M_j^g L_l^g \Pi
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n} a_{ij}(T)|\hat{g}_j(0)| + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T)M_j^g|g_l(0)| + I_i(T)\\
&= [-c_i(T) + \sum_{j=1}^{n} a_{ij}(T)\hat{L}_j^g + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T)M_j^g L_l^g]\Pi + \Gamma + I_i(T)\\
&\leqslant [-c_i(T) + \sum_{j=1}^{n} a_{ij}(T)\hat{L}_j^g e^{\lambda\tau} + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T)M_j^g L_l^g e^{\lambda\tau}]\Pi + \Gamma + I_i(T)\\
&< -\eta \times \frac{\bar{I}+\Gamma}{\eta} + \Gamma + I_i(T)\\
&\leqslant 0.
\end{aligned}
$$

Obviously, this is a contradiction.

If (2.7) valid, through condition (H_1^*), using the Dini derivative theory and scaling of inequalities, we derive that

$$
\begin{aligned}
0 &\geqslant \bar{x}_i'(T)\\
&= -c_i(T)\bar{x}_i(T) + \sum_{j=1}^{n} a_{ij}(T)\hat{g}_j(\bar{x}_j(T-\tau_{ij}(T)))\\
&\quad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T)g_j(\bar{x}_j(T-\sigma_{ijl}(T)))g_l(x_l(T-\nu_{ijl}(T))) + I_i(T)\\
&\geqslant -c_i(T)\pi + \sum_{j=1}^{n} a_{ij}(T)\hat{g}_j(\pi) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(T)g_j(\pi)g_l(\pi) + I_i(T)\\
&> 0,
\end{aligned}
$$

which is absurd and suggests that (2.5) is right. This accomplishes the proof of Lemma 2.1. □

Exploiting a similar evidence of Lemma 2.3 in [6], we can easily acquire the following finding.

Lemma 2.2. *Under the hypotheses adopted in Lemma 2.1, designate $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ as a solution of HHNNs (1.1) accompanying the initial value assumptions*

$$x_i(s) = \varphi_i(s), \pi < \varphi_i(s) < \Pi,\ s \in [-\tau, 0],\ \varphi_i \in C[-\tau, 0],\ i \in S. \tag{2.8}$$

Then for arbitrary $\varepsilon > 0$, there is $k = k(\varepsilon) > 0$ satisfying that every interval $[\alpha, \alpha+k]$ has at the lowest one number ξ for which one can find $N > 0$ obeying

$$\|x(t+\xi) - x(t)\| \leqslant \varepsilon, \text{ for all } t > N. \tag{2.9}$$

3. Positive Almost Periodic Exponential Stability

The main purpose of this section is to obtain the stability of positive almost periodic solutions of HHNNs (1.1). The assumptions and lemmas introduced in the previous sections are applied to achieve this.

Theorem 3.1. *If* (H_1^*) *and* (H_2^*) *are obeyed. Then HHNNs (1.1) possesses only one positive almost periodic solution, which admits global exponential stability.*

Proof. Denote $y(t) = (y_1(t), y_2(t), \cdots, y_n(t))^T$ as a solution of HHNNs (1.1) accompanying the initial assumptions addressed in Lemma 2.2, by referring to the proof of Theorem 3.1 in [6], we can pick a function sequence $\{y(t+t_k)\}_{k=1}^{+\infty}$ which uniformly tends to a continuous function $Z^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ on any compact interval of $\mathbb{R}$, and $Z^*(t)$ is a positive almost periodic solution of HHNNs (1.1). Furthermore, again applying the similar proof of [6, Theorem 3.1], one can easy reveal that $Z^*(t)$ admits global exponential stability , which completes the proof. □

Remark 3.1. Because (H_1^*) does not satisfy the key assumption adopted in [7–14] which require the monotone nondecreasing activation functions on $(0, +\infty)$, the results in Theorem 3.1 are more extensive than the corresponding ones in the above presented references.

4. Simulation Example

Regard the following almost periodic HHNNs involving time-varying delays:

$$(4.1)\qquad \begin{cases} x_1'(t) = -3x_1(t) + \frac{1}{8}\hat{g}_1(x_1(t-4\sin^2 t)) + \frac{1}{8}\hat{g}_2(x_2(t-5\sin^2 t)) \\ \qquad\qquad + \frac{1}{16}g_2(x_2(t-3\sin^2 t))g_1(x_1(t-\sin^2 t)) + I_1(t), \\ x_2'(t) = -3x_2(t) + \frac{1}{8}\hat{g}_1(x_1(t-2\cos^2 t)) + \frac{1}{8}\hat{g}_2(x_2(t-4\cos^2 t)) \\ \qquad\qquad + \frac{1}{16}g_2(x_2(t-\cos^2 t))g_1(x_1(t-5\cos^2 t)) + I_2(t), \end{cases}$$

where

$$(4.2)\qquad \hat{g}_i(x) = g_i(x) = \begin{cases} \arctan x, \ 0 \leqslant x \leqslant 100, \\ \arctan 100 + \sin(x-100), \ x > 100. \end{cases}$$

and $I_1(t) = 5 + \sin(\sqrt{3}t),\ I_2(t) = 2 + \frac{1}{3}e^{|\cos(t)|},\ i = 1, 2.$ We define the parameters as follows:

$$(4.3)\qquad \begin{aligned} &\theta = 300,\ \eta = 1,\ c_1 = c_2 = 3,\ \hat{L}_1^g = L_2^g = \hat{L}_1^g = L_2^g = 1,\ M_1^g = M_2^g = 3, \\ &a_{ij} = \frac{1}{8},\ b_{121} = b_{221} = \frac{1}{16},\ b_{ijl} = 0,\ i, j, l = 1, 2,\ ijl \neq 121,\ ijl \neq 221. \end{aligned}$$

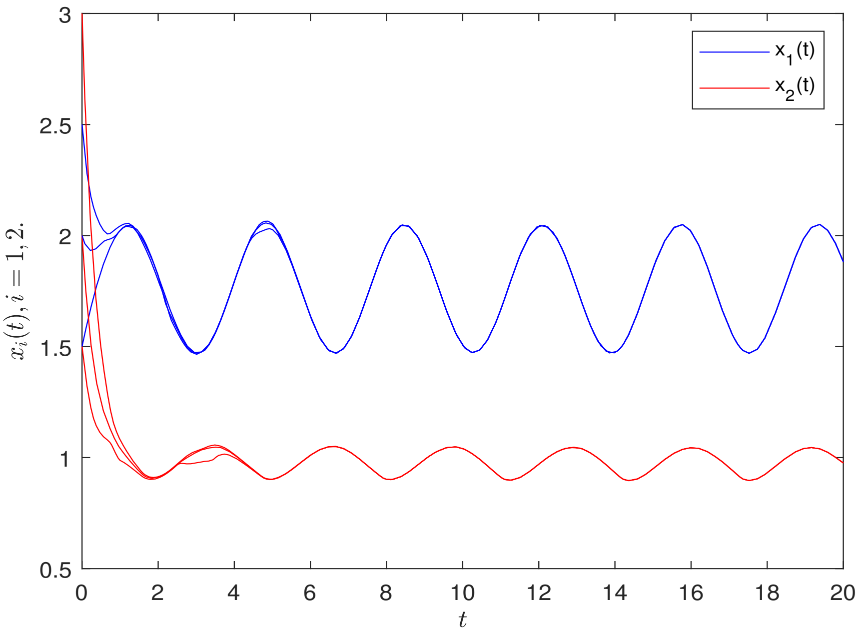

Figure 1. The numerical solution of the HHNNs(4.1) with the initial conditions $(\varphi_1(s), \varphi_2(s)) = (1 + \frac{1}{2}\cos 2t, 2 + \sin 2t)$, $(2 + \frac{1}{2}\cos 2t, \frac{3}{2} + 3\sin 2t)$, $(2 + \frac{3}{2}\sin 2t, 1 + 2\cos 2t)$, $s \in [-5, 0]$.

The calculation gives $\Gamma = 0$, $\overline{I} = \max_{i \in S} \sup_{t \in \mathbb{R}} I_i(t) = 6$, $\pi = \frac{1}{3}$, $\Pi = 100$, $\tau = 5$. Therefore, it is not difficult to verify that HHNNs(4.1) with (4.2)-(4.3) satisfy all the assumptions of Theorem 3.1. By applying the conclusions of Theorem 3.1, we conclude that under the given parameter conditions, the HHNNs(4.1) have a unique almost periodic solution $x^*(t) = (x_1^*(t), x_2^*(t))$, which is global exponential stability. Furthermore, for all $t \in \mathbb{R}$, the solution satisfies $x_i^*(t) \in (\pi, \Pi) = \left(\frac{1}{3}, 100\right)$ for $i = 1, 2$. The numerical trajectories presented in Figure 1 further support the correctness and validity of the theoretical results.

Remark 4.1. It is worth noting that the following assumption:

$$g_i : \mathbb{R} \to \mathbb{R} \text{ is a monotonic increasing function on } (0, +\infty), \ i \in S$$

is a condition for the stability of neural networks discussed in the literature [6,7]. From (4.2), it can be seen that the activation functions of the HHNNs(4.1) are not monotonically increasing on $(0, +\infty)$. Hence, all the findings in [6,7] and the references therein can not be employed to HHNNs (4.1). This implies that the conclusions of Theorem 3.1 in this paper are completely new and fill in the corresponding research gap.

5. Conclusions

The positive high-order Hopfield neural networks involving time-varying delays have been explored in this paper. Without supposing the monotonicity of the activation functions in the positive real half-axis, the globally exponential stability of the positive almost periodic solutions has been built up, which improve and generalize all the results of the literature [6]. The method and strategy adopted in this article can also be exploited to research the positive stability of other neural network systems associated with continuously distributed delays and mixed delays.

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