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RESEARCH ARTICLE

Purposeful integration of 3D modeling and printing

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Abstract

New technological advancements that are available in price and usability for K-12 classrooms result in new content areas to be explored and advancement of previous content area struggles. Visualizations of two-dimensional (2D) representations of three-dimensional (3D) figures and the actual 3D figure is a struggle not only limited to mathematics teaching and learning. However, if this struggle can be rectified and potentially improved through mathematics teaching and learning, the broader impacts of this extends beyond classroom mathematics. New 3D modeling software and 3D printers allow users to easily create and share models or download 3D models from online resources and print them to manipulate in their hand. There is plenty of literature now on classroom use of 3D modeling and printing. This article serves to build onto Ball and Stacey's (2005) suggestions for judicious use of calculators and computer software to address the judicious use of 3D modeling and printing technology for teaching mathematics for student learning. We discuss the following teaching strategies: promote careful decision making about 3D modeling and/or printing use, integrate 3D modeling and or printing into the curriculum, tactically restrict use of 3D modeling and or printing, and promote habits of spatial visualization.

Keywords: 3D printing, digital fabrication, spatial visualization, technology

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I. INTRODUCTION

New technological advancements that are available in price and usability for K-12 classrooms result in new content areas to be explored and advancement of previous content area struggles. Visualizations of two-dimensional (2D) representations of three-dimensional (3D) figures and the actual 3D figure is a struggle not only limited to mathematics teaching and learning. However, if this struggle can be rectified and potentially improved through mathematics teaching and learning, the broader impacts of this extends beyond classroom mathematics. New 3D modeling software and 3D printers allow users to easily create and share models or download 3D models from online resources and print them to manipulate in their hand. There is plenty of literature now on classroom use of 3D modeling and printing. This article serves to build onto Ball and Stacey's (2005) suggestions for judicious use of calculators and computer software to address the judicious use of 3D modeling and printing technology for teaching mathematics for student learning. We discuss the following teaching strategies: promote careful decision making about 3D modeling and/or printing use, integrate 3D modeling and or printing into the curriculum, tactically restrict use of 3D modeling and or printing, and promote habits of spatial visualization.

II. STRATEGIES TO SUPPORT JUDICIOUS INTEGRATION OF 3D MODELING AND PRINTING

Promote Careful Decision Making about 3D Modeling and Printing

Aligning to NCTM's technology principle of careful decision making about technology use, we provide examples of meaningful use of 3D modeling and printing to access teaching and learning mathematics in a way we cannot without technology. With easy to use 3D modeling, students and teachers can create and communicate about 3D models unlike before. Teachers can still facilitate learning for students with mental models and paper and pencil, but also now with 3D modeling and 3D printing. There are limitations with physical models sometimes that inaccurately or incompletely represents the theoretical concept like intersections of planes. Assessing student 3D mental models can be as quick as screenshots to capture a specific perspective of the model, or as involved a link for interactive feedback. Being mindful of when technology produces extra benefits in communication about concepts can enhance teaching and learning of mathematics.

Type of activities vary depending on teacher and student familiarity with 3D modeling and printing. Wan and Ivy (2021b) proposed three types of activities: machine function, teacher created models, and student created models. Machine function referred to aspects of 3D modeling or 3D printing at a surface level enough to use the concepts to discuss mathematics. An example of machine function would be comparing different 3D printers for the volume of their build area.

Teacher created models are models that the teacher either found, modified, or created for visualization in the 3D modeling environment or printed for tactile student engagement. An example of a teacher created model while teaching geometry is the visualization of the intersection of planes. Students usually do not have problems visualizing that the intersection of two planes is a line, but some have difficulty visualizing that the intersection of three planes is a point. Since this representation is interactive students can view these figures from different angles and the time spent in class would focus on the mathematics content and not technology use. Figure 1 shows the three rectangular prisms representing the three planes in three different colors. For verification for understanding for an asynchronous classroom, the teacher can assign the students the task of changing the positions of the planes and submitting screenshots of different perspectives for verification. If the teacher creates the model for students to modify to verify that the intersection of three planes is a point, then it lowers the technical difficulty for the students to still be able to access the mathematical concepts and enhance the learning of those concepts.

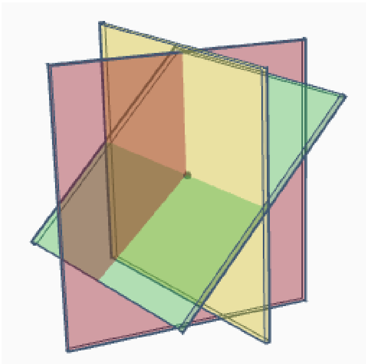

Figure 1. 3D Model representing intersecting planes

Student created models can either be found, modified from teacher models, or created. An example student created model from teacher models would be a triangular prism model that the teacher shares with the students or they can pull out of "basic shapes" in Tinkercad and label the faces to make sure that they visualize all the faces to solve for surface area. Requiring that student work for a problem to include screenshots of all the faces would ensure students calculated the area for each of the faces. An example of a triangular prism with the faces numbered is in Figure 2. An example of *student created model* that is *found* of 3D printing would be where the teacher assigns students to look for 3D printable models of a specific model for the net of a pyramid and the teacher prints the one that is the best fit.

In the interest of time spent on content enhanced by technology, teachers can use a machine function activity or problem to introduce or gauge student interest in 3D modeling and printing. For both teacher and student created models, the level of technological complexity would need to be determined through careful decision making. If students are not yet familiar with placing numbers on faces of the triangular prism model like the example above, but are familiar enough to turn the figure around to get screenshots of the different faces, then the teacher can provide the model for students to use in the interest of classroom time. Additionally, teachers can provide prompts for students to reflect on whether or not these visualizations were helpful in learning mathematics to help guide further learning.

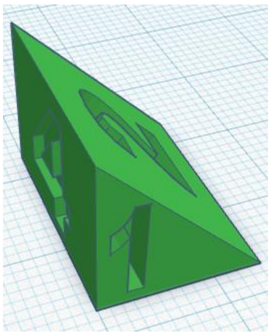

Figure 2. Triangular prism with faces labeled

Integrate 3D Modeling and/or Printing into the Curriculum

There are ways to integrate technology simultaneously with content with minimal loss of content focused class time. A task like the sinusoid activity in Wan and Ivy (2021a) used 3D modeling and printing to illustrate connections between sinusoids and cylinders that otherwise would not be possible with limitations of physical materials. Although the task of graphing the sinusoid to a specific cylinder could be accomplished with materials like a cut dowel, the task of creating a cylinder connected to a specific sinusoid is accessible via 3D modeling and printing. We will use this task as the example to show alignment to curriculum through the Substitution Augmentation Modification and Redefinition (SAMR) Levels for technology integration. The activity aligns to the trigonometry curriculum of graphing sine and cosine functions and the parts of the expression that connect to amplitude, wavelength, and horizontal and vertical shifts.

Traditionally, this topic can be taught without technology where students draw the graphs by hand. With graphing technology, students can see the movements with families of graphs more rapidly than graphing by hand. With 3D modeling and printing, students have a connection between these graphs and a concrete manipulative they can hold and create. Substitution level for this activity would be purchased 3D models or printing from existing online models that represent different oblique cylinders for students to graph – this is substituting for the traditional cut dowel that would have been used for this activity. Augmentation model would be where the teacher creates or makes specific models that graph to specific expressions to illustrate the connections further. Modification would be where students create the models on their own to match specific expressions. At this level, if there is limitation to student knowledge of technology or lack of 1-1 device settings, the teacher and students can create the model together on a projection. Redefinition would be where this activity would not be able to occur without 3D printing and modeling and for this activity it would be where students create models of their own and print them to test how well they modeled the expression.

Activities at all these levels still align to the curriculum, their required levels of technology expertise for the teacher and the students vary. At Substitution Level the teacher can just print the cylinder needed without knowledge of 3D modeling or printing. At Augmentation Level, the teacher needed to know some 3D modeling to create the models needed. For both Substitution and Augmentation Levels the students do not need to interact too much with 3D modeling and printing. At Modification and Redefinition levels the activity requires more technological knowledge of the teacher and the students. If these technological skills could be embedded earlier in the curriculum, then this activity could use higher levels of more technology integration for deeper learning of the mathematics content.

Tactically Restrict the Use of Technology

Ball and Stacey (2005) advocate for students' use of paper and pencil models prior to exploring with technology. When students are working toward a digitally fabricated solution, they may be tempted to begin their work using the software (Tinkercad, SketchUp, etc.). Requiring students to first create a rough model with connecting blocks, such as Unifix cubes, allows students to design more thoughtfully. In these quickly-produced models, students can rethink design issues and repeatedly revise their work prior to investing in the intricacy of a more detailed and complex design.

The thoughtful restriction of the use of technology helps provide space for intentionality in design, communication, and decision making. Planning for the delayed access to technology can be connected to Polya's Problem Solving Process (Polya, 1945). Consider the classic task of designing a box to hold the most popcorn. The teacher must first ensure that students understand the problem at hand - what are the dimensions and shape of a box that will hold the most popcorn? This is important to understand prior to beginning to construct the box. Next, students need to consider which approaches or strategies are appropriate for addressing this problem. It is only after these two steps are complete that students should (possibly with the use of technology) carry out their selected strategy. In this case we delayed the use of technology until the third phase of the process. Of course, students should look back at their work and possibly select an alternative strategy. In this case, technology may need to be tactfully restricted again to ensure students are intentionally reflecting and selecting a new approach rather than using technology to make repeated guesses.

Promote Habits of Spatial Visualization

Spatial visualization is "knowing how to build and manipulate objects mentally, including composing and decomposing objects" (National Council of Teachers of Mathematics [NCTM], 2010). Tasks which integrate digital fabrication tools enhance spatial visualization skills by allowing students to efficiently manipulate virtual objects without the physical limitations of concrete manipulatives. For example, students can drag the edges of objects to change their size and shape, intersect one plane with another, and create animations to simulate events occurring between objects. Spatial skills are connected to mathematical skills through fluid reasoning, working memory, and verbal skills (Atit et al., 2022). With 3D modeling and 3D printing, mental visualization can be communicated and reasoned physically with a virtual or concrete model. Visualization is fundamental to teaching and learning of geometric concepts. Van Hiele levels have been used to help students develop geometric concepts: Level 0 visualization, Level 1 analysis, Level 2 abstraction, Level 3 deduction, and Level 4 Rigor (Burger & Shaughnessy, 1986). The following examples show how these models help move students from visualization to analysis and further through reasoning and varied communication venues that exist outside of the mental model.

A common question in Algebra is the question to determine the size of the box necessary to mail an umbrella given a certain length for the umbrella. Students may or may not be given a diagram – the diagram being a 2D representation of a 3D object. They often do not visualize the two different triangles that contain the two different diagonals. With 3D modeling or the 3D printed object, teachers can easily communicate with the highlighted portions. Figure 3 to the right shows the highlighted right triangle that is difficult to see in the non-dynamic two dimensional black and white text. Although this also can be accomplished with a shoebox and yarn, this 3D model or printed 3D figure has visibility from all angles with respect to the triangle and diagonal in question. Students can verify for themselves and communicate with screenshots if necessary for the areas that are in question.

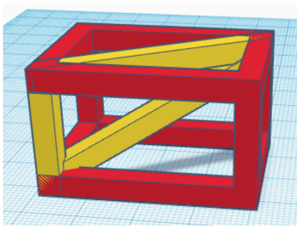

Figure 3. Figure for Pythagorean Theorem problem

Another example for improving spatial visualization is creating models that realistically cannot be created in person in the case of skew lines in geometry. Figure 4 shows a 3D model where rectangular prisms represent skew lines. Although it is a theoretical concept that geometry teachers have gestured through the years, with 3D modeling, it is still a 2D representation of the 3D concept but it is dynamically presented so that students can pan the view around to see where from certain angles it may appear to be parallel. Figure 5 shows the skew lines from Figure 4 from a different angle where they appear to be parallel.

Similarly, the questions about intersections of planes can be quickly demonstrated for the case of two planes and three. Figure 1 showed the intersection of three planes. Students who struggle with mental visualizations struggle to answer those questions about the resulting figure of the intersection of two or three planes, but with 3D modeling, they can quickly see that the intersection of two planes is a line and the intersection of three planes is a point. Once students understand the problem and can visualize it, students then can move beyond visualization to analysis and to additional levels as the problem or project intends.

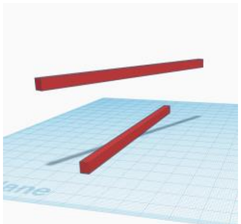

Figure 4. Representation of skew lines

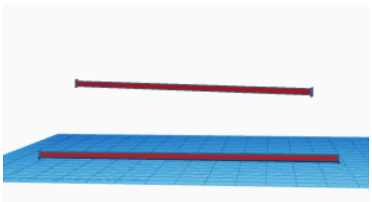

Figure 5. Appearance of "parallel" skew lines

III. CONCLUSION

Ball and Stacey's (2005) original suggestions for integrating calculators and computer software judiciously provide a flexible framework for planning and implementing 3D modeling and printing in today's classrooms. First, teachers should model and promote careful decision making about 3D modeling and printing either through found, modified, or created models. Secondly, teachers should integrate 3D modeling and printing concepts into their curricula in such a way that digital fabrication tasks are experienced with regularity rather than as a supplement to daily instruction. Additionally, teachers should tactfully restrict students' access to digital fabrication tools, including both software and 3D printers. This allows for intentionality in the use and function of these tools rather than quick and shallow use or integration solely as play. Finally, digital fabrication tools should be integrated into mathematics instruction in ways which promote spatial visualization skills and simultaneously build connections between concrete (physical), semi-concrete (drawings or on-screen graphics), and abstract (symbols, measurements, and mathematical equations) representations.

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