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NEW RANKING AND NEW ALGORITHM FOR SOLVING DUAL HESITANT FUZZY TRANSPORTATION PROBLEM†

K. HEMALATHA AND VENKATESWARLU. B*

Abstract. In this study, a dual hesitant uncertain setting is employed to study the transportation issue. The dual hesitant fuzzy set handles ambiguous, unreliable, or inaccurate data as well as conditions in real-world practical research queries that are impossible or difficult to solve according to current fuzzy uncertainties. The dual hesitant fuzzy set (DHFS) is composed of a membership hesitant function as well as a non-membership hesitant function. In this investigation, we developed a new scoring formula for converting dual hesitant fuzzy numbers (DHFNs) to crisp values and suggested a novel algorithm called contraharmonic mean for addressing the dual hesitant fuzzy problem of transportation. Excel solver is utilized to find the contraharmonic mean. Additionally, we employed the modified distribution (MODI) method to achieve the best possible result. The recommended approach is then explained using a mathematical instance, and its efficacy can be demonstrated by comparing it to previously used techniques.

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Key words and phrases : Contraharmonic mean, score function, dual hesitant fuzzy transportation problem (DHFTP), initial basic feasible solution (IBFS), MODI method.

1. Introduction

The issue of transportation (TP) is substantially utilized in multiple fields such as investments, manufacturing, job planning, managing inventory, and so on. Today, businesses compete aggressively with one another to develop and deliver better goods to their clients. Reducing transport costs in company and

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government policies is becoming one of the most critical concerns in the modern market environment [1]. The fundamental issue with transportation was initiated by Hitchcock [2]. An initial basic workable solution to the transportation dilemma can be found by using the northwest corner rule, matrix minima, or Vogel's approximation approach [3]. An enhanced Vogel approximation technique for transportation issues has been developed by Korukoglu and Balli [4]. It is necessary to precisely characterize transportation pricing, suppliers, and demands to address these transportation issues [5]. The decision conditions in several real-life situations were difficult to comprehend for various reasons. We must therefore learn to deal with ambiguity to solve real-world problems. In these circumstances, it is possible to view the problem's crisp parameters as fuzzy numbers [6].

The fuzzy set concept was developed by Zadeh [7] to deal numerically with uncertain information when making judgments, and it has been successfully used in several disciplines. Following the significant research performed by Bellman and Zadeh [8], the usage of fuzzy set theory in optimization increased dramatically. The affiliation value of a fuzzy set of elements is between 0 and 1, while that of a crisp set is 1 for a component that is part of it and 0 for a component that doesn't belong [9]. For expressing ambiguity during decision-making, Atanassov [10] put out the idea of an intuitionistic fuzzy set (IFS). IFS differs from FS by the fact it categorizes a component's extent of affiliation and extent of non-membership within the set, which is its primary benefit. A decision maker (DM) may employ IFS to assess the levels of acceptability and non-acceptability [11].

Torra [26] proposed an additional version of fuzzy sets referred to as hesitant fuzzy sets (HFSs), Additionally, Zhu *et al.* [13] put forward dual hesitant fuzzy sets, which are special cases of fuzzy sets, which include intuitionistic fuzzy sets, hesitant fuzzy sets, and fuzzy multisets. The affiliation measures and non-membership measures of the DHFS are expressed by two sets with possible values, and it is a complete set that encompasses multiple pre-existing sets [14]. Because it takes into consideration that more data is provided by decision-makers than conventional fuzzy sets, it looks to be an improved way to be valued in multiple ways in accordance with practical demands [15]. For instance, in a multicriteria choice-making situation, certain decision-makers take into account 0.1, 0.2, and 0.3 as possible outcomes for the degree of inclusion of x within group A, & 0.4, 0.5, and 0.6 as potential measures for the non-membership levels instead of just a single integer or a tuple [13].

First, Maity *et al.* [16] suggested the notion of dual hesitant uncertain transportation issues in addition to a technique for addressing dual-hesitant fuzzy transportation challenges to deal with such real-world transportation issues. It was focused on finding the best solution to the transportation issue under specified constraints using the conventional method, without the use of mathematical tools. Later, Mehar score formula was suggested by Kumar *et al.* [17]. The dual hesitant uncertain transportation issue was initially converted into its corresponding crisp form in the suggested Mehar technique. For resolving dual hesitant issues related to transportation, Jothilakshmi *et al.* [18] presented a heuristic approach. DHFTP was resolved by Prabha *et al.* [19] utilizing the allocation table approach. A non-linear discount cost was used to illustrate the dual hesitant multi-objective fractional transportation issue (DHMOFTP) mathematical framework created by Saranya and Vinotha [20]. Using non-linear discount costs, the authors optimized the proportion of the two objective functions. Intending to improve the goal of improving multi-criteria decision-making, Rodzi *et al.* [21] created Z-Score functionalities of dual-hesitant fuzzy collections. Mo and Huang [22] presented a technique for using numerous attributes in decision-making that integrates the Archimedean t-norm as well as t-conorm with the geometic heronian mean (GHM) operator in a dual-hesitant uncertain setting. For probabilistic dual hesitant fuzzy sets, Garg and Kaur [23] employed a robust correlation coefficient. Yuan and Meng [24] developed innovative similarity measures for the application of a DHFS. A technique to address DHFTP known as Russell's approximation method (RAM) was introduced by Fathima *et al.* [25].

In the present study, dual hesitant uncertain numbers are used for creating the TP framework. The main objective of this study is to lower overall transit costs in a dual hesitant fuzzy context. According to the previous discussions, there are no parallel techniques for ranking and IBFS to solve TP in a dual hesitant uncertain environment. This lack of study prompted the authors to create a unique ranking and IBFS technique for optimizing the transportation issue in a dual-hesitant uncertain situation. We claim that the aforementioned are our paper's significant contributions:

(i) For DHFS, we created a novel score formula.

(ii) With the aid of Excel solver, we employed the contraharmonic approach to obtain IBFS.

(iii) We have taken four numerical examples combining of balanced and unbalanced DHFTP to show the effectiveness of proposed algorithm and then we compared our method with existing methods.

The remaining writing is structured in the following way: Section 2 presents the DHFS preliminary data. A mathematical framework is presented in Section 3. Section 4 describes the strategy for solving the Dual Hesitant Fuzzy Transportation Issue. After that, in section 5, provided numerical examples to demonstrate the utility of the suggested technique. In section 6, offers results and discussion. In section 7, conclusion and future studies are provided.

2. Preliminaries

Below we address the complexity of the TP grounded on the dual-hesitant fuzzy values, and we review the fundamental concepts of the HFS as well as DHFS.

2.1. Intuitionistic fuzzy set. [10] Let $\breve{X}$ signifies a universal set, an IFS $\bar{A}_I$ on $\breve{X}$ is expressed with two functions $\mu_{\bar{A}_I} : \breve{X} \to [0,1], \nu_{\bar{A}_I} : \breve{X} \to [0,1]$ such as $0 \leq \mu_{\bar{A}_I}(\ddot{x}) + \nu_{\bar{A}_I}(\ddot{x}) \leq 1$ for every $\ddot{x} \in \breve{X}$. Where, $\mu_{\bar{A}_I}$ depicts the level of involvement and $\nu_{\bar{A}_I}$ depicts the level of non-involvement of $\ddot{x}$ within the set $\breve{X}$. IFS are typically depicted as follows:

$$\bar{A}_I = \left\langle \ddot{x}, \mu_{\bar{A}_I}, \nu_{\bar{A}_I} \right\rangle \text{for all } \ddot{x} \in \breve{X}.$$

2.2. Hesitant fuzzy set. Let $\breve{X}$ signifies a universal set then HFS, $\bar{A}_{H_F}$ on $\breve{X}$ is defined as [26]:

$$\bar{A}_{H_F} = \left\{ \langle \ddot{x}, \mu_{\bar{A}_{H_F}}(\ddot{x}) \rangle : \ddot{x} \in \breve{X} \right\}$$

where $\mu_{\bar{A}_{H_F}}(\ddot{x})$ is a collection of several distinct values in $[0,1]$, showing the component's possible level involvement $\ddot{x} \in \breve{X}$.

2.3. Dual hesitant fuzzy set. Let $\breve{X}$ signifies a fixed set, a DHFS D_h on $\breve{X}$ is defined as [13]:

$$D_h = \left\{ \langle \ddot{x}, h(\ddot{x}), g(\ddot{x}) \rangle : \ddot{x} \in \breve{X} \right\}$$

where $h(\ddot{x})$ and $g(\ddot{x})$ are two groups of data in $[0,1]$, indicating possible association and non-association levels of $\ddot{x} \in \breve{X}$ to the D_h, accordingly, such as $\mu \in h(\ddot{x}), \nu \in g(\ddot{x}), 0 \leq \mu, \nu \leq 1,\ \mu^+ = \max\{\mu | \mu \in h(\ddot{x})\}, \nu^+ = \max\{\nu | \nu \in g(\ddot{x})\}$ and $0 \leq \mu^+ + \nu^+ \leq 1$.
For ease of use, the pair $d = \{h(\ddot{x}), g(\ddot{x})\}$ is referred to a dual hesitant uncertain element (DHFE) and is specified by $d = \{h, g\}$.

2.4. Proposed ranking of dual hesitant fuzzy set. To convert dual hesitant fuzzy numbers to crisp numbers, the ranking function is a defuzzification tool. Comparisons of fuzzy numbers are made using it. Let $D_h = \left\{ \langle \ddot{x}, h(\ddot{x}), g(\ddot{x}) \rangle : \ddot{x} \in \breve{X} \right\}$ be a DHFS, in which $\breve{X} = \{\ddot{x}_1, \ddot{x}_2, \ddot{x}_3, ...\ddot{x}_n\}$ and $d = \{h, g\}$ be a DHFN. We defined a rank function (s_d) on the DHFS as follows in Eqn. 1:

$$s_d = \frac{1}{l} \sum_{i=1}^{l} c_{ij}^{D_h} \left[\frac{1 + \frac{1}{l}\sum_{i=1}^{l} h(\ddot{x}_i) - \frac{1}{l}\sum_{i=1}^{l} g(\ddot{x}_i)}{2} \right] \tag{1}$$

Based on the new rank function we can analyse two DHFNs. Let d_1 and d_2 are two DHFNs, then the relation between those are given by,
Case (1) $d_1 > d_2$ iff $s_{d_1} > s_{d_2}$
Case (2) $d_1 < d_2$ iff $s_{d_1} < s_{d_2}$
Case (3) $d_1 = d_2$ iff $s_{d_1} = s_{d_2}$

2.5. Contraharmonic mean. The arithmetic mean of the squares of a group with positive numbers divided by the arithmetic mean of the numbers is referred to as the contraharmonic mean of the group of positive numbers is expressed below in Eqn. 2.

$$c\left(x_1, x_2, \cdots x_n\right) = \frac{x_1^2 + x_2^2 + \cdots + x_n^2}{x_1 + x_2 + \ldots + x_n} \tag{2}$$

3. Mathematical expression of dual hesitant fuzzy transportation problem

Assume there are "n" destinations and "m" sources. The distribution system aims to decrease the cost of transporting items from these suppliers to the regions, however, the availability and demand of the commodities are specified with the following assumptions and limitations. Table 1 represents dual-hesitant fuzzy transportation model.
A dual hesitant TP can be mathematically expressed in the following Eqns. 3-6:

$$\text{Minimize } \bar{z}^{D_h} = \sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}_{ij}^{D_h}.x_{ij} \tag{3}$$

Subject to the constraints,

$$\sum_{j=1}^{n} x_{ij} = \bar{a}_i^{D_h},\ i = 1 \ to \ m, \tag{4}$$

$$\sum_{i=1}^{m} x_{ij} = \bar{b}_j^{D_h}, j = 1 \ to \ n, \tag{5}$$

$$x_{ij} \geq 0 \text{ for each } i,\, j \tag{6}$$

where $\bar{c}_{ij}^{D_h}$- dual hesitant expense of moving one unit of a given good supplier i to recipient j,
x_{ij}- transferred quantity from input i to terminal j,
$\bar{a}_i^{D_h}$- units of supply to be carried,
$\bar{b}_j^{D_h}$- number of demand units needed at endpoints.

4. Proposed algorithm for solving dual hesitant fuzzy transportation problem

The following describes the steps of the proposed algorithm.
Step 1: Choose the transport issue where the expenses are represented by dual hesitant fuzzy integers.
Step 2: Use the recommended ranking function, which was devised in Eqn.1, to convert dual hesitant figures into crisp values.
Step 3: Check to see if the chosen issue is balanced or imbalanced, in which situation overall availability, as well as demand, should be equal.

	Destinations				
Sources	$\mathcal{D}_1$	$\mathcal{D}_2$	$\dots$	$\mathcal{D}_n$	Supply
$\mathcal{S}_1$	$c_{11}^{D_h}$	$c_{12}^{D_h}$	$\dots$	$c_{1n}^{D_h}$	$a_1^{D_h}$
$\mathcal{S}_2$	$c_{21}^{D_h}$	$c_{22}^{D_h}$	$\dots$	$c_{2n}^{D_h}$	$a_2^{D_h}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathcal{S}_m$	$c_{m1}^{D_h}$	$c_{m2}^{D_h}$	$\dots$	$c_{mn}^{D_h}$	$a_m^{D_h}$
Demand	$b_1^{D_h}$	$b_2^{D_h}$	$\dots$	$b_n^{D_h}$	

Table 1. Dual hesitant fuzzy transportation problem

(i) Move on to step 5 if the problem is balanced.
(ii) Move on to step 4 when the problem is unbalanced.
Step 4: Adding a dummy column or row at no additional expense will balance the given issue.
Step 5: Employing the contraharmonic technique that was explained in Eqn.2, find the penalty for each row and column through an Excel spreadsheet.
Step 6: Determine the highest penalty among all rows and columns.
Step 7: Select any one of the greatest penalties whenever there is more than one.
Step 8: A minimal amount should be allocated among supply and demand, with the lowest cost of the corresponding greatest penalty determined.
Step 9: If no more allocation is available, remove the entire row or column.
Step 10: Steps 5 to 9 should be repeated until all of the needs and supplies have been met.
Step 11: Proceed to the MODI approach for the ideal solution once determining the IBFS for the dual hesitant uncertain problem of transportation.

5. Numerical examples

This section features DHFTP illustrations.
Example 5.1: Dry fruits, chocolates, cookies, and beverages are the four divisions of a warehouse that provide products to dealers $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ in respective categories. Table 2 provides the monthly requests from dealers as well as the production capacities of these enterprises.

Table 3 displays defuzzified DHFTP data employing the rank function. Since the data set is out of balance, a fake row has been added to bring it balanced. In addition, employing an Excel sheet, we determined the penalty for each row and column as specified in the proposed algorithm. Tables 4-8 use the suffix numbers to represent allocation.

Tables 3-8 use the suffix numbers to represent allocations of the proposed approach is used to find IBFS, and then we used the MODI technique to arrive at the ideal solution, which is 1781.06

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	Supply
Dry fruits	{{0.2,0.5,0.6}, {0.1,0.4,0.3}} (10,15,20)	{{0.4,0.7}, {0.2,0.3}} (45,50)	{{0.1,0.4,0.5}, {0.5,0.4,0.3}} (30,32,35)	27
Chocolates	{{0.2,0.4,0.6,0.1}, {0.3,0.1,0.2,0.4}} (32,34,40,45)	{{0.5,0.1,0.3,0.2}, {0.3,0.4,0.2,0.1}} (80,85,88,90)	{{0.7,0.3}, {0.1,0.3}} (90,95)	23
Cookies	{{0.6,0.5}, {0.1,0.4}} (70,75)	{{0.4,0.3,0.2}, {0.1,0.6,0.5}} (60,62,65)	{{0.2,0.4,0.1,0.5}, {0.3,0.2,0.4,0.1}} (60,75,80,88)	32
Beverages	{{0.4,0.3}, {0.5,0.1}} (20,25)	{{0.6,0.2,0.7}, {0.1,0.2,0.3}} (40,43,47)	{{0.5,0.2}, {0.4,0.3}} (30,36)	5
Demand	37	31	30	

Table 2. Inputs for DHFTP

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	Supply	Penalty
Dry fruits	8.7	30.88	15.19	27	23.01
Chocolates	20.39^{23}	43.73	60.13	23	47.84
Cookies	47.13	28.05	16.5	32	39.93
Beverages	11.81	28.16	16.5	5	21.33
Dummy	0	0	0	11	0
Demand	37	31	30		
Penalty	32.39	33.98	43.32		

Table 3. Defuzzified values with first allocation of DHFTP

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	Supply	Penalty
Dry fruits	8.7	30.88	15.19	27	23.01
Cookies	47.13	28.05^{31}	16.5	32	39.93
Beverages	11.81	28.16	16.5	5	21.33
Dummy	0	0	0	11	0
Demand	14	31	30		
Penalty	36.02	29.09	29.17		

Table 4. Second allocation

Example 5.2: Consider an organization that operates three manufacturing plants at X, Y, and Z that provide goods to warehouses at L_1, L_2, L_3 and L_4 respectively. Table 9 provides information about monthly plant capabilities and warehouse needs.

	$\mathcal{D}_1$	$\mathcal{D}_3$	Supply	Penalty
Dry fruits	8.7	15.19	27	12.83
Cookies	47.13	39.77^{1}	1	43.76
Beverages	11.81	16.5	5	14.54
Dummy	0	0	11	0
Demand	14	30		
Penalty	36.02	29.17		

Table 5. Third allocation

	$\mathcal{D}_1$	$\mathcal{D}_3$	Supply	Penalty
Dry fruits	8.7	15.9	27	12.83
Beverages	11.81	16.5	5	14.54
Dummy	0	0^{11}	11	0
Demand	14	29		
Penalty	10.49	16.20		

Table 6. Fourth allocation

	$\mathcal{D}_1$	$\mathcal{D}_3$	Supply	Penalty
Dry fruits	8.7	15.9^{18}	27	12.83
Beverages	11.81	16.5	5	14.54
Demand	14	18		
Penalty	10.49	15.87		

Table 7. Fifth allocation

	$\mathcal{D}_1$	supply
Dry fruits	8.7^{9}	9
Beverages	11.81^{5}	5
Demand	14	

Table 8. Final allocations

We computed IBFS for the aforementioned table following the suggested approach, and then we used the MODI technique to arrive at the ideal solution, which is 1006.02

Example 5.3: Suppose ABC Limited, which has three production units that deliver goods to three warehouses. According to Table 10, each shop has a particular production capacity, and each warehouse needs a specific quantity.

We computed IBFS for the aforementioned table following the suggested approach, and then we used the MODI technique to arrive at the ideal solution,

	L_1	L_2	L_3	L_4	Supply
X	{{0.2,0.5,0.1}, {0.3,0.4,0.5}} (100,105,107)	{{0.2,0.1,0.3}, {0.5,0.7,0.4}} (52,55,57)	{{0.6,0.3,0.2}, {0.2,0.1,0.3}} (46,48,50)	{{0.4,0.3}, {0.5,0.1}} (20,25)	30
Y	{{0.2,0.4}, {0.5,0.2}} (17,30)	{{0.8,0.5}, {0.1,0.2}} (35,42)	{{0.6,0.2}, {0.3,0.2}} (40,43)	{{0.6,0.2,0.7}, {0.1,0.2,0.3}} (40,43,47)	17
Z	{{0.5,0.3,0.4,0.6}, {0.1,0.3,0.2,0.4}} (60,62,65,67)	{{0.4,0.5,0.2,0.3}, {0.3,0.4,0.5,0.2}} (55,59,63,66)	{{0.3,0.2,0.4,0.1}, {0.6,0.3,0.2,0.1}} (40,50,55,60)	{{0.5,0.2}, {0.4,0.3}} (30,36)	25
Demand	26	10	16	12	

Table 9. Transportation cost of dual hesitant formulation

	I	II	III	Supply
A	{{0.1,0.4}, {0.1,0.3}} (20,30)	{{0.2,0.4,0.6}, {0.1,0.3,0.2}} (40,43,46)	{{0.4,0.5,0.6}, {0.1,0.4,0.3}} (10,15,20)	50
B	{{0.3,0.2,0.4}, {0.5,0.4,0.1}} (30,33,36)	{{0.7,0.6,0.5}, {0.3,0.1,0.2}} (20,24,28)	{{0.2,0.4,0.6}, {0.4,0.1,0.3}} (38,40,42)	35
C	{{0.1,0.3,0.6}, {0.4,0.3,0.2}} (50,51,53)	{{0.5,0.3,0.2}, {0.4,0.5,0.3}} (46,49,52)	{{0.8,0.1}, {0.1,0.2}} (90,98)	40
Demand	28	64	33	

Table 10. Formulating DHFTP

which is 2020.94.

Example 5.4: The firm has distribution centres in Kolkata, Agra, Bombay, and Hyderabad. Products are available in 12,43,15, and28 unit quantities at these centres. A, B, C, and D retail locations need 39, 26, 19, and 14 units, respectively. Table 11 lists the transit costs (in rupees) for each unit between each centre outlet.

We computed IBFS for the aforementioned table following the suggested approach, and then we used the MODI technique to arrive at the ideal solution, which is 2492.72.

Example 5.5 [16, 18]**:** With three sources A, B, and C and three destinations X, Y, and Z the Dual-Hesitant Fuzzy Problem of Transportation is shown in the following Table 12.

We computed IBFS for the aforementioned table following the suggested approach, and then we used the MODI technique to arrive at the ideal solution, which is 705.03.

	A	B	C	D	Supply
Kolkata	{{0.5,0.1,0.3,0.2}, {0.3,0.4,0.2,0.1}} (80,85,88,90)	{{0.6,0.4}, {0.3,0.4}} (80,85)	{{0.7,0.4}, {0.3,0.1}} (45,55)	{{0.2,0.4,0.6}, {0.1,0.3,0.2}} (40,43,46)	12
Agra	{{0.6,0.2,0.7}, {0.1,0.2,0.3}} (40,43,47)	{{0.3,0.2,0.4,0.1}, {0.6,0.3,0.2,0.1}} (40,50,55,60)	{{0.2,0.4,0.1,0.5}, {0.3,0.2,0.4,0.1}} (60,75,80,88)	{{0.1,0.3,0.6}, {0.4,0.3,0.2}} (50,51,53)	43
Bombay	{{0.5,0.3,0.4}, {0.1,0.2,0.3}} (70,80,90)	{{0.7,0.3}, {0.1,0.3}} (90,95)	{{0.6,0.3,0.2}, {0.2,0.1,0.3}} (46,48,50)	{{0.6,0.2,0.7}, {0.1,0.2,0.3}} (40,43,47)	15
Hyderabad	{{0.6,0.2}, {0.3,0.2}} (40,43)	{{0.6,0.8}, {0.1,0.2}} (65,67)	{{0.2,0.5,0.7}, {0.1,0.3,0.2}} (24,28,36)	{{0.2,0.4,0.6}, {0.4,0.3,0.1}} (38,40,42)	15
Demand	39	26	19	14	

Table 11. Formulating DHTP

	X	Y	Z	Supply
A	{{0.5,0.4,0.1}, {0.4,0.5,0.9}} (20,25,30)	{{0.7,0.6,0.5,0.2}, {0.1,0.3,0.4,0.5}} (14,16,20,35)	{{0.6,0.4,0.3}, {0.2,0.6,0.7}} (22,25,36)	20
B	{{0.4,0.2}, {0.3,0.5}} (12,15)	{{0.7,0.6,0.3}, {0.1,0.3,0.6}} (30,35,40)	{{0.6,0.5,0.3}, {0.2,0.3,0.5}} (22,27,30)	24
C	{{0.3,0.2,0.1}, {0.2,0.6,0.7}} (30,40,45)	{{0.2,0.1}, {0.5,0.9}} (25,32)	{{0.6,0.5,0.3,0.2}, {0.2,0.3,0.5,0.7}} (32,35,40,50)	35
Demand	35	24	20	

Table 12. Formulating DHFTP of existing data

6. Results and Discussion

Researchers from diverse fields have focused on a substantial number of studies that tackle TP across different uncertain contexts. Real-world challenges constantly present challenging and sophisticated handling of unclear information. Dealing with uncertainty in various problems has been made successful by fuzzy sets and extensions of them. Table 13 demonstrates that, in comparison to other techniques, our recommended approach achieves better results for solving DHFTP. The dual hesitant transportation issue was addressed in the literature in two distinct ways, one utilizing the highest-cost scenario while the other utilizing the lowest-cost scenario. In practice, however, those scenarios, where the transportation expense is at its maximum and minimum at each node are not possible. Therefore, transportation expenses are a blend of the greatest, average, and lowest levels that could be incurred. It will be easier for us to take into account all potential expenses at each node if we use dual hesitant fuzzy

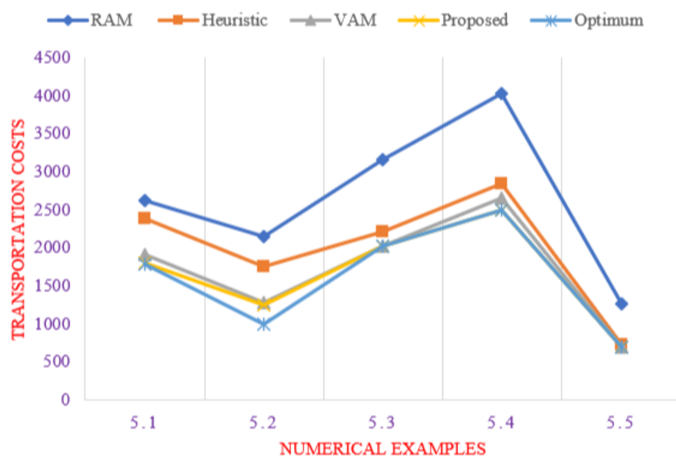

FIGURE 1. Comparison chart

to transportation issues. Additionally, we used four numerical issues in addition to an already-existing problem to demonstrate the usefulness of the suggested score function as well as the algorithm. Out of the five numerical examples, we took two balanced and three imbalanced situations. As a result, the output of our strategy met the transportation objective. Additionally, the proposed solution requires less computing time. Therefore, our research on DHFTP can potentially be informative and helpful in locating an appropriate solution to a variety of real-world, ambiguous decision-making challenges. The efficiency of the suggested approach is demonstrated in Figure 1.

Examples	RAM [25]	Heuristic [18]	VAM	Proposed
1	2630.93	2387.56	1908.98	1780.06
2	2152.98	1752.33	1285.64	1255.88
3	3161.3	2222.58	2020.935	2020.935
4	4030.05	2842.6	2653.07	2492.72
5	1270.95	726.81	705.03	705.03

TABLE 13. Comparison of the proposed technique with existing methods

7. Conclusion

In this study, a different approaching procedure has been suggested to handle the DHFTP. Many enterprises desire to provide their goods to consumers in an affordable manner in the current extremely competitive marketplace to keep it as such. The model of transport offers a strong framework to decide the most effective methods to carry items to the client to deal with this problem. By offering various affiliation and non-affiliation levels to an element, a dual hesitant fuzzy set thus presents an enhanced method of overcoming these ambiguities. In this study, we first proposed a ranking function for DHFS that is better than the existing score formulas. Furthermore, the ranking function plays a significant part in cost reduction. The second claim argues that the suggested technique might offer an impressive IBFS that addresses the dual-hesitant transportation issue by guaranteeing the lowest possible expenses for transportation. The justification of the suggested strategies and the fundamental framework was supported by numerical evidence, and their applicability to current decision-making issues was discussed. The goal of individuals who wish to maximize their profit by reducing transportation expenses will be accomplished with the aid of our suggested approach. This suggested rank function can also be applied to dual hesitant transshipment problems. The suggested ranking technique is unable to solve Pythagorean dual hesitant fuzzy sets, on which will focus in the future.

Conflicts of interest : The authors of this paper declare that there are no competing interests.

Data availability : Data will be made available upon request.

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K. Hemalatha received M.Sc. from Muthurangam Govt. Arts College (Affiliated to Thiruvalluvar University) and pursuing a Ph.D. at Vellore Institute of Technology, Vellore. Her research interest includes Operations Research and Fuzzy optimization.

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.
e-mail: hemakarunakaran321@gmail.com

Venkateswarlu. B received M.Sc. and a Ph.D. from Sri Venkateswara University, Tirupathi. He is currently working as an Associate professor in Mathematics at Vellore Institute of Technology, Vellore. His areas of research are Operations Research, Fuzzy set theory, and its extension.

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.
e-mail: venkatesh.reddy@vit.ac.in