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ORTHOGONAL GENERALIZED SYMMETRIC REVERSE BIDERIVATIONS IN SEMI PRIME RINGS

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Abstract. Let R be a semi-prime ring. Let $[\delta_1, D_1]$ and $[\delta_2, D_2]$ be two generalized symmetric reverse biderivations of R with associated reverse biderivations D_1 and D_2. The main aim of the present paper is to establish conditions of orthogonality for symmetric reverse biderivations and symmetric generalized reverse biderivations in R.

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1. Introduction

The study of symmetric bi-derivations was initiated by G. Maksa [10,11]. Later J.Vukman[12,13] studied some results concerning symmetric bi-derivations on prime and semi primerings. Bresar and Vukman [14] introduced the notation of orthogonality for two derivations in semiprime rings and proved some conditions of orthogonality between them. Daif, Elsaviad and Haetinger [15] obtained orthogonality conditions between derivations and biderivations in semi prime rings. Abdul Rhman [1] proved orthogonality conditions for a pair of reverse derivations in semiprime ring. C. Jaya Subba Reddy et al. [2-7] proved some results on orthogonality of symmetric biderivations and generalized biderivations in semi prime, prime rings. M.S.Venigul and N.Argac [16] studied orthogonal derivations. E.K. Sogutcu [9] studied results in Lie ideals with symmetric reverse biderivations in semiprime rings. Recently, C.Jaya Subba Reddy et al. [8,18,17,19] have investigated orthogonal symmetric reverse bi-(σ, τ)-derivations, generalized reverse

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bi-(σ,τ)-derivations in semiprime rings, orthogonal generalized reverse (σ,τ)-derivations in semiprime Γ-rings and orthogonal generalized (σ,τ)-derivations in semiprime Γ- near rings . In the present paper, we establish equivalent conditions for the orthogonality of symmetric reverse biderivations and generalized symmetric reverse biderivations of a semiprime ring.

2. Preliminaries

Let R be a semi prime ring. Recall that R is semi-prime if $uRu = \{0\}$ implies $u = 0$ for any $u \in R$. A map D from $R \times R$ into R is termed as symmetric if $D(u,v) = D(v,u)$ for all $u, v \in R$. A symmetric bi-additive map D from $R \times R$ into R is termed a biderivation if $D(uv,w) = D(u,w)v + uD(v,w)$ for all $u, v, w \in R$ and $D(u, vw) = D(u,v)w + vD(u,w)$ for all $u, v, w \in R$. A bi-additive map D from $R \times R$ into R is termed a symmetric reverse biderivation if $D(uv,w) = D(v,w)u + vD(u,w)$ for all $u, v, w \in R$ and $D(u, vw) = D(u,w)v + wD(u,v)$ for all $u, v, w \in R$. A bi-additive map $\delta \colon R \times R$ into R is termed a symmetric generalized biderivation if there is a biderivation $D \colon R \times R$ into R such that $\delta(uv,w) = \delta(u,w)v + uD(v,w)$ for all $u, v, w \in R$ and $\delta(u, vw) = \delta(u,v)w + vD(u,w)$ for all $u, v, w \in R$. A bi-additive map $\delta \colon R \times R$ into R is termed a symmetric generalized reverse biderivation if there is a biderivation $D \colon R \times R$ into R such that $\delta(uv,w) = \delta(v,w)u + vD(u,w)$ for all $u, v, w \in R$ and $\delta(u, vw) = \delta(u,w)v + wD(u,v)$ for all $u, v, w \in R$. Two reverse derivations d_1 and d_2 are called orthogonal if $d_1(u)Rd_2(v) = 0 = d_2(v)Rd_1(u)$ for all $u, v \in R$. Let R be a semi-prime ring, then two symmetric reverse biderivations are said to be orthogonal if $D_1(u,v)RD_2(v,w) = \{0\} = D_2(v,w)RD_1(u,v)$ for all $u, v, w \in R$. Two generalized reverse biderivations δ_1 and δ_2 associated with the reverse biderivations D_1, D_2 are said to be orthogonal if $\delta_1(u,v)R\delta_2(v,w) = \{0\} = \delta_2(v,w)R\delta_1(u,v)$ for all $u, v, w \in R$.

Lemma 1:[11, Lemma 1] Let R be a 2-torsion free semi-prime ring and a and b be the elements of R. Then the following conditions are equivalent:

(1) $aub = 0$, $\forall u \in R$.
(2) $bua = 0$, $\forall u \in R$.
(3) $aub + bua = 0$, $\forall u \in R$.

If one of the above conditions is fulfilled, then $ab = ba = 0$.

Lemma 2: [4, Lemma 2] Let R be a semi-prime ring. Suppose that two bi-additive mappings $D_1, D_2 : R \times R \to R$ satisfy $D_1(u,v)RD_2(u,v) = 0$, $\forall u, v \in R$, then $D_1(u,v)RD_2(v,w) = \{0\}$, $\forall u, v, w \in R$.

3. Main results

Symmetric Reverse Biderivations

Theorem 1: Let R be a 2-torsion free semi-prime ring. Then the following conditions are equivalent:

(1) Two reverse biderivations D_1 and D_2 are orthogonal.
(2) $D_1(u,v)D_2(v,w) + D_2(u,v)D_1(v,w) = 0$, $\forall\, u,v,w \in R$.

Proof:
(1) $\Longrightarrow$ (2)
Suppose that D_1 and D_2 are orthogonal reverse biderivations. Then,

$$D_1(u,v)RD_2(v,w) = \{0\} \quad \text{and} \quad D_2(u,v)RD_1(v,w) = \{0\},\ \forall\, u,v,w \in R.$$

which means

$$D_1(u,v)D_2(v,w) = 0 \quad \text{and} \quad D_2(u,v)D_1(v,w) = 0 \quad (\text{Since } 1 \in R)$$

and so

$$D_1(u,v)D_2(v,w) + D_2(u,v)D_1(v,w) = 0,\ \forall\, u,v,w \in R.$$

(2) $\Longrightarrow$ (1)
Suppose that

$$D_1(u,v)D_2(v,w) + D_2(u,v)D_1(v,w) = 0,\ \forall\, u,v,w \in R. \tag{3.1}$$

We prove that D_1 and D_2 are orthogonal.
Replacing w by uw, $u \in R$ in equation (3.1) and using (3.1), we get

$$D_1(u,v)D_2(v,uw) + D_2(u,v)D_1(v,uw) = 0,$$

$$D_1(u,v)wD_2(v,u) + D_2(u,v)wD_1(v,u) = 0.$$

By Lemma 1, we get

$$D_1(u,v)RD_2(v,u) = \{0\}.$$

By Lemma 2, we get

$$D_1(u,v)RD_2(v,w) = \{0\}.$$

Therefore, D_1 and D_2 are orthogonal reverse biderivations.

Theorem 2: Let R be a 2-torsion free semi-prime ring. Then the following conditions are equivalent:

(1) Two reverse biderivations D_1 and D_2 are orthogonal.
(2) $D_1D_2 = 0$.

Proof: (1) $\Longrightarrow$ (2)
Suppose that D_1 and D_2 are orthogonal. We prove that $D_1D_2 = 0$.
Since D_1 and D_2 are orthogonal, we have

$$D_1(u,v)RD_2(v,w) = 0,\ \forall\, u,v,w \in R,$$

$$D_1(D_1(u,v)rD_2(v,w),m) = 0,\ \forall\, u,v,w,r,m \in R,$$

$$D_1(D_2(v,w),m)rD_1(u,v) + D_2(v,w)D_1(r,m)D_1(u,v)$$
$$+rD_2(v,w)D_1(D_1(u,v),m) = 0,$$

$$D_1D_2(v,w)rD_1(u,v) = 0. \quad (\text{By the orthogonality of } D_1 \text{ and } D_2)$$

In particular, if we put $u = D_2(v,w)$ in the above equation, we get

$$D_1D_2(v,w)rD_1(D_2(v,w),v) = 0,$$

$$D_1D_2(v,w)rD_1D_2(v,w) = 0,$$

$$D_1D_2(v,w) = 0, \quad \text{(By semi-primeness of } R)$$
$$D_1D_2 = 0.$$

(2) $\implies$ (1)
Let D_1 and D_2 be two reverse biderivations such that $D_1D_2 = 0$. We prove that D_1 and D_2 are orthogonal.

$$\begin{aligned}
&D_1D_2(uv,w)\\
&= D_1(D_2(uv,w),m),\\
&= D_1(D_2(v,w)u + vD_2(u,w),m),\\
&= D_1(D_2(v,w)u,m) + D_1(vD_2(u,w),m),\\
&= D_1(u,m)D_2(v,w) + uD_1(D_2(v,w),m) + D_1(D_2(u,w),m)v\\
&+ D_2(u,w)D_1(v,m),\\
&= D_1(u,m)D_2(v,w) + uD_1D_2(v,w) + D_1D_2(u,w)v + D_2(u,w)D_1(v,m),\\
&= uD_1D_2(v,w) + D_1(u,m)D_2(v,w) + D_2(u,w)D_1(v,m) + D_1D_2(u,w)v.
\end{aligned}$$

Therefore,

$$D_1D_2(uv,w) = uD_1D_2(v,w)+D_1(u,m)D_2(v,w)+D_2(u,w)D_1(v,m)+D_1D_2(u,w)v,$$
$$0 = D_1(u,m)D_2(v,w) + D_2(u,w)D_1(v,m) \quad \text{(Since } D_1D_2 = 0).$$

In particular,

$$D_1(u,w)D_2(v,w) + D_2(u,w)D_1(v,w) = 0,$$
$$D_1(u,w)D_2(w,v) + D_2(u,w)D_1(w,v) = 0. \quad \text{(Since } D_1 \text{ and } D_2 \text{ are symmetric)}$$

By Theorem 1, we conclude that D_1 and D_2 are orthogonal.

Theorem 3: Let R be a 2-torsion free semi-prime ring. Then the following conditions are equivalent:

(1) Two reverse biderivations D_1 and D_2 are orthogonal.
(2) $D_1(u,v)D_2(v,w) = 0$ or $D_2(u,v)D_1(v,w) = 0$.

Proof: (1) $\implies$ (2)
Suppose that the reverse biderivations D_1 and D_2 are orthogonal.
We prove $D_1(u,v)D_2(v,w) = 0$ or $D_2(u,v)D_1(v,w) = 0$. By the definition of orthogonality of D_1 and D_2, we have

$$D_1(u,v)RD_2(v,w) = \{0\} = D_2(v,w)RD_1(u,v).$$

We can write

$$D_1(u,v)D_2(v,w) = 0 = D_2(v,w)D_1(u,v). \quad \text{(Since } 1 \in R)$$

Hence, $D_1(u,v)D_2(v,w) = 0$ which is one of the required conditions. Similarly, we can prove $D_2(u,v)D_1(v,w) = 0$.
(2) $\implies$ (1)
Suppose

$$D_1(u,v)D_2(v,w) = 0, \forall\, u,v,w \in R. \tag{3.2}$$

We prove that D_1 and D_2 are orthogonal.
Replace w by $uw, u \in R$ in the above equation (3.2) and using (3.2), we get

$$D_1(u,v)D_2(v,uw) = 0.$$
$$D_1(u,v)wD_2(v,u) = 0.$$
$$D_1(u,v)wD_2(u,v) = 0. \quad \text{(Since } D_2 \text{ is symmetric)}$$
$$D_1(u,v)RD_2(u,v) = \{0\}.$$

By Lemma 2, we have $D_1(u,v)RD_2(v,w) = 0, \forall\, u,v,w \in R$.
Therefore, D_1 and D_2 are orthogonal.
Similarly, by supposing $D_2(u,v)D_1(v,w) = 0$, we can prove that D_1 and D_2 are orthogonal.

Theorem 4: Let R be a 2-torsion free semi-prime ring. Two symmetric reverse biderivations D_1 and D_2 are orthogonal if and only if D_1D_2 is a bi-derivation.

Proof:
We have to show that D_1 and D_2 are orthogonal $\iff$ D_1D_2 is a bi-derivation.
Suppose that D_1 and D_2 are orthogonal reverse biderivations.
By the definition of orthogonality of D_1 and D_2, we can write

$$D_1(u,v)RD_2(v,w) = 0 = D_2(u,v)RD_1(v,w), \quad \forall\, u,v,w \in R. \tag{3.3}$$

We have to prove that D_1D_2 is a bi-derivation. Using the same process we followed in Theorem 2, we obtain

$$D_1D_2(uv,w) = uD_1D_2(v,w) + D_1(u,m)D_2(v,w) + D_2(u,w)D_1(v,m)$$
$$+D_1D_2(u,w)v, \forall\, u,v,w,m \in R.$$

Replacing $m = w$ in the above equation,

$$D_1D_2(uv,w) = uD_1D_2(v,w)+D_1(u,w)D_2(v,w)+D_2(u,w)D_1(v,w)+D_1D_2(u,w)v.$$

Using the orthogonality of D_1 and D_2, the above equation becomes

$$D_1D_2(uv,w) = uD_1D_2(v,w) + D_1D_2(u,w)v, \forall\, u,v,w \in R.$$

Hence, D_1D_2 is a bi-derivation.
Conversely, suppose that D_1D_2 is a bi-derivation.
We prove that D_1 and D_2 are orthogonal. Since D_1D_2 is a bi-derivation, we can have

$$D_1D_2(uv,w) = D_1D_2(u,w)v + uD_1D_2(v,w). \tag{3.4}$$

By the earlier discussion, we can write

$$D_1D_2(uv,w) = uD_1D_2(v,w)+D_1(u,w)D_2(v,w)+D_2(u,w)D_1(v,w)+D_1D_2(u,w)v$$
$$\forall\, u,v,w \in \mathbb{R}. \tag{3.5}$$

Combining equations (3.4) and (3.5), we obtain

$$D_1D_2(u,w)v + uD_1D_2(v,w)$$
$$= uD_1D_2(v,w) + D_1(u,w)D_2(v,w) + D_2(u,w)D_1(v,w) + D_1D_2(u,w)v$$
$$\forall\, u,v,w \in \mathbb{R},$$
$$0 = D_1(u,w)D_2(v,w) + D_2(u,w)D_1(v,w).$$

Hence, we can conclude that D_1 and D_2 are orthogonal.

Generalized Symmetric Reverse Biderivations

Theorem 5:
Let R be a 2-torsion free semi-prime ring. Let $[\delta_1, D_1]$ and $[\delta_2, D_2]$ be orthogonal generalized symmetric reverse biderivations of R, then the following conditions are equivalent:

(1) δ_1 and δ_2 are orthogonal generalized reverse biderivations.
(2) (a) $\delta_1(u,v)\delta_2(v,w) + \delta_2(u,v)\delta_1(v,w) = 0$.
(b) $D_1(u,v)\delta_2(v,w) + D_2(u,v)\delta_1(v,w) = 0, \quad \forall\, u,v,w \in R$.

Proof: (1) $\Rightarrow$ (2)
$[\delta_1, D_1]$ and $[\delta_2, D_2]$ are orthogonal generalized symmetric reverse biderivations of R. By the definition of orthogonality, we can write

$$\delta_1(u,v)r\delta_2(v,w) = 0, \quad \forall\, u,v,w,r \in R. \tag{3.6}$$

By Lemma 1, we obtain,

$$\delta_1(u,v)\delta_2(v,w) = \delta_2(v,w)\delta_1(u,v) = 0 = \delta_2(u,v)\delta_1(v,w), \tag{3.7}$$

which means

$$\delta_1(u,v)\delta_2(v,w) + \delta_2(v,w)\delta_1(u,v) = 0, \quad \forall\, u,v,w \in R,$$
$$\delta_1(u,v)\delta_2(v,w) + \delta_2(u,v)\delta_1(v,w) = 0.$$

Hence, Condition 2(a) is proved.

To Prove Condition 2(b): We have $\delta_1(u,v)\delta_2(v,w) = 0$. (By equation (3.7)) Replacing u by ur, $r \in R$ in the above equation and using (3.6), we get

$$rD_1(u,v)\delta_2(v,w) = 0, \quad \forall\, u,v,w,r \in R,$$
$$D_1(u,v)\delta_2(v,w)rD_1(u,v)\delta_2(v,w) = 0,$$
$$D_1(u,v)\delta_2(v,w) = 0. \quad \text{(Since } R \text{ is semi-prime)} \tag{3.8}$$

Replacing u by ru, $r \in R$, in (3.8) and using (3.8), we get

$$D_1(u,v)r\delta_2(v,w) = 0.$$

By Lemma 1, we have

$$D_1(u,v)\delta_2(v,w) = \delta_2(v,w)D_1(u,v) = 0.$$

Hence,

$$D_1(u,v)\delta_2(v,w) = 0. \tag{3.9}$$

Again, we have $\delta_2(u,v)\delta_1(v,w) = 0$. (By equation (3.7)) Replacing u by ur, $r \in R$, in the above equation and using the condition of orthogonality, we obtain

$$rD_2(u,v)\delta_1(v,w) = 0, \quad \forall\, u,v,w,r \in R,$$
$$D_2(u,v)\delta_1(v,w)rD_2(u,v)\delta_1(v,w) = 0,$$
$$D_2(u,v)\delta_1(v,w) = 0. \quad \text{(Since } R \text{ is semi-prime)} \tag{3.10}$$

Replacing u by ru, $r \in R$ in (3.10) and using (3.10), we get

$$D_2(u,v)r\delta_1(v,w) = 0.$$

By Lemma 1, we have

$$D_2(u,v)\delta_1(v,w) = \delta_1(v,w)D_2(u,v) = 0.$$

Hence,

$$D_2(u,v)\delta_1(v,w) = 0. \tag{3.11}$$

Therefore,

$$D_1(u,v)\delta_2(v,w) + D_2(u,v)\delta_1(v,w) = 0.$$

Hence condition (ii) is proved.

(2) $\Rightarrow$ (1)

Suppose that

$$\delta_1(u,v)\delta_2(v,w) + \delta_2(u,v)\delta_1(v,w) = 0 \quad \forall\ u,v,w \in \mathbb{R}. \tag{3.12}$$

and

$$D_1(u,v)\delta_2(v,w) + D_2(u,v)\delta_1(v,w) = 0 \quad \forall\, u,v,w \in \mathbb{R}. \tag{3.13}$$

We prove that δ_1 and δ_2 are orthogonal generalized reverse biderivations.
Replacing u by ru, $r \in R$, in (3.12) and using (3.13), we get

$$\delta_1(u,v)r\delta_2(v,w) + \delta_2(u,v)r\delta_1(v,w) = 0,$$

$$\delta_1(u,v)R\delta_2(v,w) + \delta_2(u,v)R\delta_1(v,w) = 0.$$

By Lemma 1, we can write $\delta_1(u,v)\delta_2(v,w) = 0$. Hence, δ_1 and δ_2 are orthogonal.

Theorem 6:

Let R be a 2-torsion free semi-prime ring and $[\delta_1, D_1]$ and $[\delta_2, D_2]$ be orthogonal generalized symmetric reverse biderivations of R, then the following conditions are equivalent:

(1) δ_1 and δ_2 are orthogonal generalized reverse biderivations.

(2) $\delta_1(u,v)\delta_2(v,w) = D_1(u,v)\delta_2(v,w) = 0, \quad \forall\ u,v,w \in R.$

Proof: (1) $\Rightarrow$ (2)

Suppose that $[\delta_1, D_1]$ and $[\delta_2, D_2]$ are orthogonal generalized symmetric reverse biderivations of R, we prove the condition (2).

By the definition of orthogonality, we can write

$$\delta_1(u,v)r\delta_2(v,w) = 0, \quad \forall\ u,v,w \in R.$$

By Lemma 1, we get,

$$\delta_1(u,v)\delta_2(v,w) = 0. \tag{3.14}$$

Replacing u by ur, $r \in R$, in the above equation (3.14) and using the condition of orthogonality, we get

$$rD_1(u,v)\delta_2(v,w) = 0,$$

$$D_1(u,v)\delta_2(v,w)rD_1(u,v)\delta_2(v,w) = 0,$$

$$D_1(u,v)\delta_2(v,w) = 0. \quad \text{(Since } R \text{ is semi-prime)} \tag{3.15}$$

Replacing u by ru, $r \in R$, in (3.15) and using (3.15), we get

$$D_1(u,v)r\delta_2(v,w) = 0.$$

By Lemma 1, we have

$$D_1(u,v)\delta_2(v,w) = 0. \tag{3.16}$$

From (3.14) and (3.16), the condition (2) is proved.

(2) $\Rightarrow$ (1)

Suppose that

$$\delta_1(u,v)\delta_2(v,w) = 0. \tag{3.17}$$

$$D_1(u,v)\delta_2(v,w) = 0, \forall\, u,v,w \in R. \tag{3.18}$$

We prove that δ_1 and δ_2 are orthogonal generalized reverse biderivations. Replacing u by ru, $r \in R$ in (3.17) and using (3.18), we get

$$\delta_1(u,v)r\delta_2(v,w) = 0, \quad \forall\, u,v,w,r \in R.$$

Hence, δ_1 and δ_2 are orthogonal generalized reverse biderivations.

Discussion : The present manuscript is primarily intended for algebraists exploring maps of various algebraic structures that exhibit new properties, illustrating derivations, generalized derivations, and related concepts. For many years,algebraists have found it intriguing to explore the conditions of orthogonality between two derivations and biderivations in various algebraic structures. In this paper, we have delved into the structure and properties of symmetric reverse biderivations and generalized symmetric reverse biderivations within the framework of semiprime rings. The main objective is to establish conditions under which these biderivations are orthogonal. The results presented offer a significant contribution to the theory of symmetric reverse biderivations and generalized symmetric reverse biderivations in ring theory, particularly within the context of semiprime rings. While proving these results, we derive necessary and sufficient conditions for the orthogonality of symmetric reverse biderivations and generalized symmetric reverse biderivations in semiprime rings. These conditions help in understanding the interplay between these mathematical objects and their underlying algebraic structures. By examining the relationships between the generalized symmetric reverse biderivations $[\delta_1, D_1]$ and $[\delta_2, D_2]$ and their associated reverse biderivations D_1 and D_2, we can delineate the constraints under which these biderivations can be considered orthogonal.

The conditions for orthogonality we have established enrich the current understanding of symmetric reverse biderivations and generalized symmetric reverse biderivations in ring theory, particularly in the specialized setting of semiprime rings. This can serve as a foundation for further theoretical advancements in the study of more complex algebraic systems. By focusing on semiprime rings, the results provide structural insights that can be extrapolated to other types of rings and algebraic structures. Understanding the behavior of reverse biderivations and generalized reverse biderivations in semiprime rings can potentially shed light on their behavior in more general or more specific ring classes.The theoretical results obtained can find applications in other mathematical disciplines such as differential algebra, non commutative geometry, and theoretical physics. For instance, in non commutative geometry, biderivations play a role in defining differential structures on non commutative spaces. Similarly,in theoretical physics, the symmetry and orthogonality of certain algebraic structures can have implications in the formulation of physical theories.

Future Research : While this paper focuses on semiprime rings, it would be interesting to investigate whether similar orthogonality conditions hold in other classes of rings, such as prime rings or Γ- rings with other specific properties. The concept of generalized reverse biderivations can be extended to higher-dimensional structures or modules, where the orthogonality conditions may exhibit more complex behaviors. Developing computational methods to verify the orthogonality conditions for given rings and generalized reverse biderivations can be a valuable tool for researchers working with specific algebraic structures.

Conclusion : The study of symmetric reverse biderivations and generalized symmetric reverse biderivations in semiprime rings, and the conditions for their orthogonality, presents a significant step forward in the field of ring theory. The results not only enhance the theoretical understanding of these algebraic constructs but also pave the way for future research and applications across various fields of mathematics and physics. The established conditions of orthogonality provide a new perspective on the interaction between reverse bi derivations and biderivations, contributing to the broader understanding of algebraic structures.

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