J. Appl. Math. & Informatics Vol. **42**(2024), No. 4, pp. 831 - 854
https://doi.org/10.14317/jami.2024.831

SOLVING BI-OBJECTIVE TRANSPORTATION PROBLEM UNDER NEUTROSOPHIC ENVIRONMENT

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Abstract. The transportation problem (TP) is one of the earliest and the most significant implementations of linear programming problem (LPP). It is a specific type of LPP that mostly works with logistics and it is connected to day-to-day activities in our everyday lives. Nowadays decision makers (DM's) aim to reduce the transporting expenses and simultaneously aim to reduce the transporting time of the distribution system so the bi-objective transportation problem (BOTP) is established in the research. In real life, the transportation parameters are naturally uncertain due to insufficient data, poor judgement and circumstances in the environment, etc. In view of this, neutrosophic bi-objective transportation problem (NBOTP) is introduced in this paper. By introducing single-valued trapezoidal neutrosophic numbers (SVTrNNs) to the co-efficient of the objective function, supply and demand constraints, the problem is formulated. The DM's aim is to determine the optimal compromise solution for NBOTP. The extended weighted possibility mean for single-valued trapezoidal neutrosophic numbers based on [40] is proposed to transform the single-valued trapezoidal neutrosophic BOTP (SVTrNBOTP) into its deterministic BOTP. The transformed deterministic BOTP is then solved using the dripping method [10]. Numerical examples are provided to illustrate the applicability, effectiveness and usefulness of the solution approach. A sensitivity analysis (SA) determines the sensitivity ranges for the objective functions of deterministic BOTP. Finally, the obtained optimal compromise solution from the proposed approach provides a better result as compared to the existing approaches and conclusions are discussed for future research.

AMS Mathematics Subject Classification : 90B06, 90C29, 90C70.
Key words and phrases : Single-valued trapezoidal neutrosophic numbers, possibilistic mean, weighted possibility mean, dripping method, ideal solution, optimal compromise solution.

Received September 22, 2023. Revised March 27, 2024. Accepted March 29, 2024.
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1. Introduction

The transportation problem (TP) is an intriguing method of management sciences, which can be solved as a problem of linear programming (LP). The basic goal of a TP is viewed as a logistics challenge to decide how and when to transport commodities from different sources to different destinations with a minimal cost or maximum profit. TP was introduced by Hitchcock [1] in 1941 and then the solution procedure for finding the optimal solution was developed by Koopmans [2] in 1947. The zero point method is introduced by Pandian and Natarajan [3] for finding an optimal solution to the TP. The TP has applications in a variety of fields including industry, allocation, planning, communication networks and scheduling. These transportation problems have a single objective. In actual circumstances, every business seeks to accomplish a number of goals while arranging for the delivery of commodities. In order to make decisions about reaching several goals at once, multiple objectives (MO) is established. An algorithm was proposed by Aneja et al. [4] to determine non dominated extreme points for bi-criteria TP. Isermann [5] presented the algorithm to yield a set of efficient solutions to solve multi-objective TP (MOTP). Numerous researchers such as Gupta et al. [6], Ringuest et al. [7], Kasana et al. [8], Bai et al. [9], Pandian and Anuradha [10], Nomani et al. [11] and Kaur et al. [12] have employed various approaches for solving MOTP to determine the optimal compromise solution.

Real world circumstances might use transportation in a variety of ways, but it is more difficult to calculate the exact expense of transportation due to certain unpredictable elements like price of fuel, traffic delays, road conditions and so on. In order to deal with the challenges constructively, the characteristics of the problems can be modelled as uncertain and ambiguous. Zadeh [13] was the first to introduce the fuzzy sets (FS) which provides the degree of membership function (MF) in 1965. In 1978, the possibility theory of a fuzzy set was proposed by Zadeh [14] and the possibilistic mean and variance of fuzzy numbers was introduced by Carlsson and Fuller [15]. Various researchers such as Gupta et al. [16], Dhanasekar et al. [17], Singh et al. [18], Bagheri et al. [19], Malihe Niksirat [20] and Kacher et al. [21] have utilized the various methods for solving fuzzy MOTP to determine the optimal compromise solution. Revathi et al. [22] constructed a chance constraint model for uncertain MOTP and then solved using the neutrosophic compromise approach to obtain the pareto optimal solution. Bodkhe [23] developed fuzzy programming technique based on exponential MF for solving MOTP under fuzzy environment. Miah et al. [24] developed fuzzy programming technique based on exponential and hyperbolic MF for solving MOTP under fuzzy environment to obtain the optimal compromise solution.

Fuzzy numbers are commonly used to describe imprecise data, they may not be suitable for some situations where uncertainty and hesitation both exist. In this case, the IFS which is an extension of FS was introduced by Atanassov [25] to handle both the degree of MF and non-membership function (NMF). An accuracy function was applied by Ebrahimnejad and Verdegay [26] and Mahmoodirad et al. [27] to solve the IFTP. Wan et al. [28] defined the possibility mean, variance and covariance of triangular IF numbers. In recent research, Garai et al. [29] proposed the concept of mean, variance and covariance for solving multi-item generalized IF inventory model. Roy et al. [30] proposed the intuitionistic fuzzy programming approach (IFPA) and goal programming approach (GPA) to solve IFMOTP. Ghosh et al. [31] utilized the FPA, IFPA and GPA to solve the multi-objective transportation problem (MOTP) with fixed charge three dimensional problem under intuitionistic environment to obtain a pareto optimal solution. Mahajan and Gupta [32] utilized a variety of MFs to solve fully IFMOTP. Ahmadini and Ahmad [33] proposed the different MFs under neutrosophic environment for solving IFMOLPP. Bera and Mondal [34] developed ordered weighted average operator and average value approach for solving MOTP under triangular intuitionistic fuzzy environment.

IFS considers both the degree of MF and NMF but it is unable to address the indeterminacy nature of reality. To address these issues, Smarandache [35] introduced the concept of the neutrosophic set (NS). NS takes into account both the degree of truth MF and falsity MF together with the indeterminacy degree when making decisions. The existence of an element of degree of indeterminacy within the set is the primary distinction between these sets. After discussing the special forms of single-valued neutrosophic numbers, Deli and Subas [36] applied the weighted aggregation operator of SVTrNNs to solve multi-criteria decision-making problems. The single-valued neutrosophic sets (SVNS) was introduced by Wang et al. [37] in order to analyse the relations and operations over SVNS. Risk Allah et al. [38] proposed the neutrosophic compromise programming approach (NCPA) to solve the MO transportation problem under neutrosophic environment. Khalifa et al.[39] proposed the KKM approach and then applied the dual and optimality conditions to the inverse capacitated TP. Kiran Khatter [40] introduced the $\langle \alpha, \beta, \gamma \rangle$ cut set of single-valued triangular neutrosophic numbers (SVTNN) and possibilistic mean of truth, indeterminacy and falsity MF for SVTNN to convert the neutrosophic LPP into its equivalent deterministic LPP. The possibility mean, variance and standard deviation of single-valued neutrosophic numbers was introduced by Garai et al.[41] to solve multi-attribute decision-making problems. Sandhiya and Anuradha [42] discussed the fixing point approach to determine the neutrosophic efficient solution and neutrosophic optimal compromise solution for solving neutrosophic MO assignment problem.

Sensitivity analysis (SA) is the technique that examines the impact of changing the coefficients of the objective function, as well as the supply and demand constraints and validates the sensitivity ranges. SA identifies an increase or decrease in the value of objective function and reveals the change in the optimal solution due to parameter variation. The possible change in the sensitivity ranges can range from zero to a substantial change. The main purpose of SA is to identify the sensitivity ranges without affecting the current optimal solution, which is a special estimation to minimize the risk of erroneous solutions. Intrator and Paroush [43] and Arsham [44] examined the SA to solve the TP and some intriguing findings were obtained. Doustdargholi et al. [45] investigated the SA of right-hand-side parameter to solve the TP. Badra [46] introduced SA to solve MOTP. Bhatia and Kumar [47] developed an approach based on tabular representation to address the SA of fuzzy TP. Ravinder Reddy et al. [48] carried out the concept of SA on fuzzy TP. Table 1 lists the related works by different authors in single-valued trapezoidal neutrosophic TP.

Table 1. Literature survey of single-valued trapezoidal NTP

Authors	**Objective**		**Transformation techniques**			
	Single	Multi	SF	RF	COG	WPM
Thamaraiselvi and Santhi [49]	✓		✓			
Singh et al. [50]	✓		✓			
Sikannan et al.[51]	✓				✓	
Saini et al. [52]	✓		✓			
Umamageswari and Uthara [53]	✓			✓		
Kumar et al. [54]	✓		✓			
Dhouib [55]	✓		✓			

Note: SF – Score function, RF – Ranking function, COG – Centre of gravity, WPM – Weighted possibility mean

Based on the above literature survey, there are some gaps in the evaluation which are presented below:

- It is evident that the work performed so far focuses mostly on SOTP under neutrosophic environment using different approaches. The performance of SA to solve SOTP is very rare.
- To the best of our knowledge, MOTP under neutrosophic environment and also the performance of SA to solve MOTP have received less attention.
- The multi-objective environment on TP is necessary when DM's are involved in an economic competition that results in various conflicting and non-commensurable objectives. In some situations, the parameters cannot be precisely predicted due to constant changes in the market such as in the field of building construction, medical treatment, and so on. In these situations, neutrosophic theory plays an important role. This motivates us to study the multi-objective TP under a neutrosophic environment.
- In this study, considering all these gaps, BOTP under neutrosophic environment is formulated.

Hence, the novelties of the presented work are summarised as follows:

In this paper, we have formulated the bi-objective TP under neutrosophic environment in which the first and second objective represent the expense of transportation and the time of transportation. We have extended the weighted possibility mean for single-valued trapezoidal neutrosophic numbers based on

[40]. The extended weighted possibility mean is used to transform the single-valued trapezoidal neutrosophic BOTP into its equivalent deterministic BOTP. The reduced problem is solved using the dripping method [10] to obtain the optimal compromise solution. Finally, to determine the sensitivity ranges of the objective function for the deterministic BOTP, the sensitivity analysis is performed.

The paper is classified into following categories: Section 2 follows with basic concepts and preliminaries. The possibilistic mean and weighted possibility mean of SVTrNNs are extended in Sections 3 and 4. The mathematical formulation of BOTP under neutrosophic nature are represented in Section 5. Section 6 presents the solution approach to obtain optimal compromise solution while Section 7 depicts a numerical illustration with results and discussions. In Section 8, a comparison of the solution approach is illustrated with the other existing approaches while Section 9 incorporates the final conclusions.

2. Preliminaries and Essential Definitions of Neutrosophic sets

Some fundamental definitions related to single-valued neutrosophic sets, single-valued trapezoidal neutrosophic numbers and its arithmetic operations have been discussed in [35] and [36]. The definition of $\langle \alpha, \beta, \gamma \rangle$ -cut set of single-valued trapezoidal neutrosophic number is discussed in [41].

Definition 2.1. Neutrosophic set [35]: Let X be a universe discourse. A neutrosophic set L in X is characterized by a truth MF $A_{\bar{L}^N}(x)$, indeterminacy MF $B_{\bar{L}^N}(x)$ and a falsity MF $C_{\bar{L}^N}(x)$. $A_{\bar{L}^N}(x)$, $B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$ are real standard elements of [0,1].It can be written as

$$\bar{L}^N = \{\langle x, A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x)\rangle : x \in X, A_{\bar{L}^N}(x), B_{\bar{L}^N}(x), C_{\bar{L}^N}(x) \in]\, 0^-, 1^+ \,[\}$$

There is no restriction on the sum of $A_{\bar{L}^N}(x)$, $B_{\bar{L}^N}(x)$ and $C_{\bar{L}^N}(x)$, so $0^- \leq A_{\bar{L}^N}(x) + B_{\bar{L}^N}(x) + C_{\bar{L}^N}(x) \leq 3^+$

Definition 2.2. Neutrosophic efficient solution [42]: A feasible solution U° is said to be efficient solution to the problem if there exists no other feasible X° such that $\tilde{Z}^{1(N)}(X^\circ) \leq \tilde{Z}^{1(N)}(U^\circ)$ and $\tilde{Z}^{2(N)}(X^\circ) \leq \tilde{Z}^{2(N)}(U^\circ)$ (or) $\tilde{Z}^{1(N)}(X^\circ) < \tilde{Z}^{1(N)}(U^\circ)$ and $\tilde{Z}^{2(N)}(X^\circ) > \tilde{Z}^{2(N)}(U^\circ)$. Otherwise, it is called non-efficient solution to the problem.

Definition 2.3. Neutrosophic optimal compromise solution [42]: An optimal compromise solution $(\tilde{Z}^{1(N)}(U^\circ), \tilde{Z}^{2(N)}(V^\circ))$ is an efficient solution which is closest to the ideal solution $(\tilde{Z}^{1(N)}(X^\circ), \tilde{Z}^{2(N)}(Y^\circ))$ where $\tilde{Z}^{1(N)}(X^\circ)$ is an optimal compromise solution to the first objective problem with all constraints and $\tilde{Z}^{2(N)}(Y^\circ)$ is an optimal solution of the second objective problem with all constraints.

Graphical representation for $\langle \alpha, \beta, \gamma \rangle$ -cut set of single-valued trapezoidal neutrosophic number: Figure 1 represents the α-cut of truth, β-cut of indeterminacy and γ-cut of falsity membership functions for single-valued trapezoidal neutrosophic number. Let $\tilde{r} = (1, 3, 5, 8; 0.8, 0.5, 0.2)$ be a single-valued trapezoidal neutrosophic number. If the element $5 \in \tilde{r}$ then the α-cut of truth MF is $\eta_{\tilde{r}}(5) = 0.8$, β-cut of indeterminacy MF is $\rho_{\tilde{r}}(5) = 0.5$ and γ-cut of falsity MF is $\delta_{\tilde{r}}(5) = 0.2$

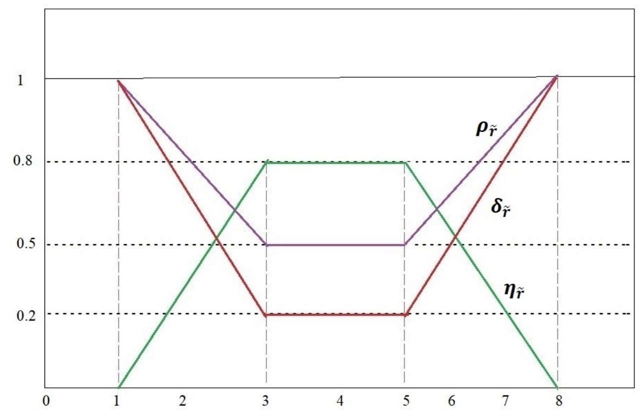

FIGURE 1. α-cut of truth, β-cut of indeterminacy and γ-cut of falsity MF for single-valued trapezoidal neutrosophic number

3. Possibilistic mean of single-valued trapezoidal neutrosophic number

Experts confront a variety of difficulties when making decisions in real-world scenarios including short deadlines, sparse data and lack of in-depth knowledge of the problem. So in this paper, we consider the investment period that is uncertain from the point of view of possibilistic analysis. Possibilistic programming is one of the most promising tools for handling mathematical programming problems with ambiguous parameters. Most of the techniques available in the literature consider arithmetic mean and weighted distance concept to transform the single-valued trapezoidal neutrosophic numbers into its deterministic numbers. Here in this paper, we have proposed the extended weighted possibility mean for single-valued trapezoidal neutrosophic numbers based on [40] to transform the single-valued trapezoidal neutrosophic numbers into its deterministic numbers. The possibilistic mean in neutrosophic environment helps to determine the risk attitude of the DM's whether he/she is a risk-taker or risk-averse.

3.1. Possibilistic mean of truth MF for single-valued trapezoidal neutrosophic number. Let $\tilde{r} = (s^1_{\tilde{r}}, s^2_{\tilde{r}}, s^3_{\tilde{r}}, s^4_{\tilde{r}}; k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}})$ be single-valued trapezoidal neutrosophic number where $s^1_{\tilde{r}}, s^2_{\tilde{r}}, s^3_{\tilde{r}}, s^4_{\tilde{r}} \in X$ and $k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}} \in [0, 1]$ are real numbers. Let $\tilde{r}(\alpha) = \{y|T_{\tilde{r}}(y) \geq \alpha : y \in Y, 0 \leq \alpha \leq k_{\tilde{r}}\}$ is defined as $\langle\alpha\rangle$-cut set of single-valued trapezoidal neutrosophic number. Using [40], the lower possibilistic mean of truth MF for single valued trapezoidal neutrosophic number is as follows:

$$g_*(\tilde{r}(\alpha)) = \left[\frac{s^1_{\tilde{r}} + 2s^2_{\tilde{r}}}{3}\right] k^2_{\tilde{r}} \tag{1}$$

The upper possibilistic mean of truth MF for single-valued trapezoidal neutrosophic number is developed using [40] which is represented as follows:

$$g^*(\tilde{r}(\alpha)) = 2\int_0^{k_{\tilde{r}}} \alpha\tilde{r}^*(\alpha)d\alpha$$

$$g^*(\tilde{r}(\alpha)) = 2\int_0^{k_{\tilde{r}}} \alpha\left[s^4_{\tilde{r}} - \frac{\alpha}{k_{\tilde{r}}}(s^4_{\tilde{r}} - s^3_{\tilde{r}})\right] d\alpha$$

$$g^*(\tilde{r}(\alpha)) = 2\left[\frac{s^4_{\tilde{r}}}{2} - \frac{s^4_{\tilde{r}}}{3} + \frac{s^3_{\tilde{r}}}{3}\right] k^2_{\tilde{r}}$$

$$g^*(\tilde{r}(\alpha)) = \left[\frac{s^4_{\tilde{r}} + 2s^3_{\tilde{r}}}{3}\right] k^2_{\tilde{r}} \tag{2}$$

Using (1) and (2), the possibilistic mean of truth MF for single-valued trapezoidal neutrosophic number $g(\tilde{r}(\alpha))$ is explained as follows:

$$g(\tilde{r}(\alpha)) = \frac{g_*(\tilde{r}(\alpha)) + g^*(\tilde{r}(\alpha))}{2}$$

$$g(\tilde{r}(\alpha)) = \left[\frac{s^1_{\tilde{r}} + 2s^2_{\tilde{r}} + 2s^3_{\tilde{r}} + s^4_{\tilde{r}}}{6}\right] k^2_{\tilde{r}} \tag{3}$$

3.2. Possibilistic mean of indeterminacy MF for single-valued trapezoidal neutrosophic number. Let $\tilde{r}(\beta) = \{y|I_{\tilde{r}}(y) \geq \beta : y \in Y, l_{\tilde{r}} \leq \beta \leq 1\}$ is defined as $\langle\beta\rangle$-cut set of single-valued trapezoidal neutrosophic number. Using [40], the lower possibilistic mean of indeterminacy MF for single-valued trapezoidal neutrosophic number is as follows:

$$g_*(\tilde{r}(\beta)) = \left[(s^2_{\tilde{r}} - l_{\tilde{r}}s^1_{\tilde{r}})(1 + l_{\tilde{r}}) - \frac{2(s^2_{\tilde{r}} - s^1_{\tilde{r}})}{3}(1 + l_{\tilde{r}} + l^2_{\tilde{r}})\right] \tag{4}$$

The upper possibilistic mean of indeterminacy MF for single-valued trapezoidal neutrosophic number is developed using [40] which is represented as follows:

$$g^*(\tilde{r}(\beta)) = 2\int_{l_{\tilde{r}}}^1 \beta\tilde{r}^*(\beta)d\beta$$

$$g^*(\tilde{r}(\beta)) = 2\int_{l_{\tilde{r}}}^1 \beta\left[\frac{s^3_{\tilde{r}} - l_{\tilde{r}}s^4_{\tilde{r}} + \beta(s^4_{\tilde{r}} - s^3_{\tilde{r}})}{1 - l_{\tilde{r}}}\right] d\beta$$

$$g^*(\tilde{r}(\beta)) = 2\left[\frac{(s^3_{\tilde{r}} - l_{\tilde{r}}s^4_{\tilde{r}})}{2}(1 + l_{\tilde{r}}) + \frac{(s^4_{\tilde{r}} - s^3_{\tilde{r}})}{3}(1 + l_{\tilde{r}} + l^2_{\tilde{r}})\right]$$

$$g^*(\tilde{r}(\beta)) = \left[(s^3_{\tilde{r}} - l_{\tilde{r}}s^4_{\tilde{r}})(1 + l_{\tilde{r}}) + \frac{2(s^4_{\tilde{r}} - s^3_{\tilde{r}})}{3}(1 + l_{\tilde{r}} + l^2_{\tilde{r}})\right] \tag{5}$$

Using (4) and (5), the possibilistic mean of indeterminacy MF for single-valued trapezoidal neutrosophic number $g(\tilde{r}(\beta))$ is explained as follows:

$$g(\tilde{r}(\beta)) = \frac{g_*(\tilde{r}(\beta)) + g^*(\tilde{r}(\beta))}{2}$$

$$g(\tilde{r}(\beta)) = \left[\frac{\begin{array}{r}(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4)l_{\tilde{r}} \\ - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4)l_{\tilde{r}}^2\end{array}}{6} \right] \tag{6}$$

3.3. Possibilistic mean of falsity MF for single-valued trapezoidal neutrosophic number. Let $\tilde{r}(\gamma) = \{y | F_{\tilde{r}}(y) \geq \gamma : y \in Y, m_{\tilde{r}} \leq \gamma \leq 1\}$ is defined as $\langle\gamma\rangle$- cut set of single-valued trapezoidal neutrosophic number. Using [40], the lower possibilistic mean of falsity MF for single-valued trapezoidal neutrosophic number is as follows:

$$g_*(\tilde{r}(\gamma)) = \left[(s_{\tilde{r}}^2 - m_{\tilde{r}} s_{\tilde{r}}^1)(1 + m_{\tilde{r}}) - \frac{2(s_{\tilde{r}}^2 - s_{\tilde{r}}^1)}{3}(1 + m_{\tilde{r}} + m_{\tilde{r}}^2) \right] \tag{7}$$

The upper possibilistic mean of falsity MF for single-valued trapezoidal neutrosophic number is developed using [40] which is represented as follows:

$$g^*(\tilde{r}(\gamma)) = 2\int_{m_{\tilde{r}}}^{1} \gamma \tilde{r}^*(\gamma) d\gamma$$

$$g^*(\tilde{r}(\gamma)) = 2\int_{m_{\tilde{r}}}^{1} \gamma \left[\frac{s_{\tilde{r}}^3 - m_{\tilde{r}} s_{\tilde{r}}^4 + \gamma(s_{\tilde{r}}^4 - s_{\tilde{r}}^3)}{1 - m_{\tilde{r}}} \right] d\gamma$$

$$g^*(\tilde{r}(\gamma)) = 2\left[\frac{(s_{\tilde{r}}^3 - m_{\tilde{r}} s_{\tilde{r}}^4)}{2}(1 + m_{\tilde{r}}) + \frac{(s_{\tilde{r}}^4 - s_{\tilde{r}}^3)}{3}(1 + m_{\tilde{r}} + m_{\tilde{r}}^2) \right]$$

$$g^*(\tilde{r}(\gamma)) = \left[(s_{\tilde{r}}^3 - m_{\tilde{r}} s_{\tilde{r}}^4)(1 + m_{\tilde{r}}) + \frac{2(s_{\tilde{r}}^4 - s_{\tilde{r}}^3)}{3}(1 + m_{\tilde{r}} + m_{\tilde{r}}^2) \right] \tag{8}$$

Using (7) and (8), the possibilistic mean of falsity MF for single-valued trapezoidal neutrosophic number $g(\tilde{r}(\gamma))$ is explained as follows:

$$g(\tilde{r}(\gamma)) = \frac{g_*(\tilde{r}(\gamma)) + g^*(\tilde{r}(\gamma))}{2}$$

$$g(\tilde{r}(\gamma)) = \left[\frac{\begin{array}{r}(2s_{\tilde{r}}^1 + s_{\tilde{r}}^2 + s_{\tilde{r}}^3 + 2s_{\tilde{r}}^4) - (s_{\tilde{r}}^1 - s_{\tilde{r}}^2 - s_{\tilde{r}}^3 + s_{\tilde{r}}^4)m_{\tilde{r}} \\ - (s_{\tilde{r}}^1 + 2s_{\tilde{r}}^2 + 2s_{\tilde{r}}^3 + s_{\tilde{r}}^4)m_{\tilde{r}}^2\end{array}}{6} \right] \tag{9}$$

4. Weighted possibility mean for single-valued trapezoidal neutrosophic number [40]

Let $\tilde{r} = (s^1_{\tilde{r}}, s^2_{\tilde{r}}, s^3_{\tilde{r}}, s^4_{\tilde{r}}; k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}})$ be single-valued trapezoidal neutrosophic number and the possibilistic mean of truth, indeterminacy and falsity MF denoted by $g(\tilde{r}(\alpha))$, $g(\tilde{r}(\beta))$ and $g(\tilde{r}(\gamma))$ respectively. The weighted possibility mean is defined as $B(\tilde{r}, \mu)$ in which the DM's always aim to maximize the degree of truth membership, minimize the degree of indeterminacy and falsity membership, as follows:

$$B(\tilde{r}, \mu) = \mu g(\tilde{r}(\alpha)) + (1-\mu) g(\tilde{r}(\beta)) + (1-\mu) g(\tilde{r}(\gamma)) \tag{10}$$

Remark 4.1. The single-valued trapezoidal neutrosophic number can be represented by $\tilde{r} = (s^1_{\tilde{r}}, s^2_{\tilde{r}}, s^3_{\tilde{r}}, s^4_{\tilde{r}}; k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}})$.If $s^2_{\tilde{r}} = s^3_{\tilde{r}}$ then the single-valued trapezoidal neutrosophic number reduces to the single-valued triangular neutrosophic number given by $\tilde{r} = (s^1_{\tilde{r}}, s^2_{\tilde{r}}, s^4_{\tilde{r}}; k_{\tilde{r}}, l_{\tilde{r}}, m_{\tilde{r}})$.

5. Problem description and formulation

In reality, the transportation parameters are uncertain due to uncontrollable factors such as distance travelled, weather, traffic conditions, weight of the load, etc. The supply remains uncertain due to unpredictable circumstances such as condition of the climate, shortage of labour, etc. The demand also remains uncertain due to inaccurate forecasting, fluctuating demand, unpredictable delivery delays, etc. In real life, every business enterprise aims to achieve multiple goals at the same time in order to become more profitable. So, the DM's need to handle multiple objectives, which may be considered as transportation cost, delivery time, degradation of breakable/perishable items, profit, etc. This study considers the bi-objective transportation problem under neutrosophic environment with the transportation cost as the first objective and the time of transportation as the second objective. Our main goal is to obtain the optimal compromise solution for the BOTP under neutrosophic environment. The mathematical formulation of the problem is represented as follows:

$$(G)\ Minimize\ \tilde{Z}^{(1)N}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^N_{ij}\tilde{x}^N_{ij};$$

$$Minimize\ \tilde{Z}^{(2)N}(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{t}^N_{ij}\tilde{x}^N_{ij};$$

$$Subject\ to: \sum_{j=1}^{n} \tilde{x}^N_{ij} = \tilde{a}^N_i, i = 1, 2, ..., m \tag{11}$$

$$\sum_{i=1}^{m} \tilde{x}^N_{ij} = \tilde{b}^N_j, j = 1, 2, ..., n \tag{12}$$

$$\tilde{x}^N_{ij} \geq 0, for\ all\ i\ and\ j \tag{13}$$

Here $\tilde{a}_i^N = (a_i^1, a_i^2, a_i^3, a_i^4; a_i^{'1}, a_i^{'2}, a_i^{'3})$ for $i = 1, 2, ..., m$ refers to the single-valued trapezoidal neutrosophic supply at i^{th} origin and $\tilde{b}_j^N = (b_j^1, b_j^2, b_j^3, b_j^4; b_j^{'1}, b_j^{'2}, b_j^{'3})$ for $j = 1, 2, ..., n$ refers to the single-valued trapezoidal neutrosophic demand at j^{th} destination; $\tilde{c}_{ij}^N = (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4; c_{ij}^{'1}, c_{ij}^{'2}, c_{ij}^{'3})$ and $\tilde{t}_{ij}^N = (t_{ij}^1, t_{ij}^2, t_{ij}^3, t_{ij}^4; t_{ij}^{'1}, t_{ij}^{'2}, t_{ij}^{'3})$ denote the first and second objective of single valued trapezoidal neutrosophic transportation cost and transportation time transported from i^{th} source to j^{th} destination respectively. The single-valued trapezoidal neutrosophic variable $\tilde{x}_{ij}^N = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4; x_{ij}^{'1}, x_{ij}^{'2}, x_{ij}^{'3})$ denotes the products transported from i^{th} source to j^{th} destination.

BOTP under neutrosophic environment problem is preferable to handle an industrial transportation system. The solution approach is discussed in the following section to determine the optimal compromise solution for the given problem.

6. Solution approach

The dripping method [10] is utilized to find efficient solutions which in turn lead to optimal compromise solution for solving the NBOTP. The proposed approach proceeds as follows:

Step 1 Consider the single-valued trapezoidal neutrosophic bi-objective transportation problem (G).

Step 2 Transform the problem (G) into its equivalent weighted possibility mean representation problem using equation (10). Then compute the possibilistic mean values for truth $g(\tilde{r}(\alpha))$, indeterminacy $g(\tilde{r}(\beta))$ and falsity $g(\tilde{r}(\gamma))$ MF using the possibilistic mean of truth MF (3), possibilistic mean of indeterminacy MF (6) and possibilistic mean of falsity MF (9).

Step 3 Let μ be any value between 0 and 1. Here $\mu = 0$ is chosen to reduce the weighted possibility mean BOTP to its deterministic BOTP.

Step 4 Check whether the reduced deterministic BOTP is balanced. If not, balance it and then solve $Z^{(1)}(x)$ and $Z^{(2)}(x)$ of deterministic BOTP to acquire the optimal solution using the zero point method [3].

Step 5 Consider the optimal solution of $Z^{(1)}(x)$ as a feasible solution in $Z^{(2)}(x)$. Then solve $Z^{(1)}(x)$ to $Z^{(2)}(x)$ using the dripping method [10] until the optimal solution of $Z^{(2)}(x)$ is reached to obtain the set of all efficient solutions.

Step 6 Consider the optimal solution of $Z^{(2)}(x)$ as a feasible solution in $Z^{(1)}(x)$. Then solve $Z^{(2)}(x)$ to $Z^{(1)}(x)$ using the dripping method [10] until the optimal solution of $Z^{(1)}(x)$ is reached to obtain the set of all efficient solutions.

Step 7 Evaluate the optimal compromise solution for deterministic BOTP from the set of all efficient solutions obtained using Step 5 and Step 6. Finally, to compute the neutrosophic optimal compromise solution substitute the optimal allocations of deterministic BOTP in the problem (G).

Step 8 To analyse the sensitivity range for objective function coefficients (cost and time) of basic variables of deterministic BOTP which are obtained in Step 7.

Step 9 To find the sensitivity range for the first objective $Z^{(1)}(x)$
(i) Vary the cost parameter of one basic variable at a time while keeping the other cells at their initial values.
(ii) Let c_{ij} be the cost coefficient of the basic variable then change the cost coefficient c_{ij} to $c_{ij} + \eta$ where η is the sensitivity range.
(iii) Calculate the new $u_i's$ and $v_j's$ to determine whether the current solution remains optimal.
(iv) Using $\bar{c}_{ij} = u_i + v_j - c_{ij} \leq 0$ determine the ranges of all the non-basic variables.
(v) Sensitivity range over c_{ij} can vary by maintaining the optimality of the solution given by $c_{ij} - \eta \leq c_{ij} \leq c_{ij} + \eta$ from the obtained ranges of non-basic variables.
Step 10 Repeat Step 9 to determine the sensitivity range for cost coefficient of the other basic variables.
Step 11 To determine the sensitivity range for the second objective $Z^{(2)}(x)$ repeat Step 9 and Step 10.
Step 12 To acquire the neutrosophic optimal compromise solution for other values of μ and the corresponding sensitivity ranges repeat the above steps.

The solution approach for solving the problem (G) is demonstrated using numerical examples which are given in the following section.The primary objective for solving the problem (G) is to determine the optimal compromise solution to deliver the products from 'm' sources to 'n' destinations at minimal cost and time.

7. Numerical Example 1

Every year, Neyveli thermal power station in Tamil Nadu, India imports ten million tons of coal through shipping containers. The famous cities of Tamil Nadu such as Cuddalore, Thanjavur and Ramanathapuram are the three main regions for coal mining. There are four ports in Tamil Nadu where this coal can be imported namely Nagapattinam port, Ennore port, V.O. Chidambaranar port and Tuticorin port. In order to generate electricity, coal must be transported to every power plant in bulk. The weekly capacities of the coal are $\tilde{a}_1^N$, $\tilde{a}_2^N$ and $\tilde{a}_3^N$ units and the weekly requirements of the coal are $\tilde{b}_1^N$, $\tilde{b}_2^N$ and $\tilde{b}_3^N$ units respectively. Let us assume that there are two objectives $\tilde{Z}^{(1)N}(x)$ and $\tilde{Z}^{(2)N}(x)$ to be considered: (i) the minimization of total transportation cost $\tilde{c}_{ij}^N$ of shipment for importing coal; and (ii) the minimization of total transportation time $\tilde{t}_{ij}^N$ of shipment for importing coal. The problem (G) is modelled as fully single-valued trapezoidal neutrosophic BOTP (SVTrNBOTP) where all the coefficients of objective function, coefficients of supply constraints and demand constraints are given in Table 2.

TABLE 2. SVTrNBOTP

		P_1	P_2	P_3	Supply
M_1	$\tilde{Z}^{(1)N}$	(4,8,11,15;0.6,0.3,0.2)$\tilde{c}_{11}^N$	(5,7,9,11;0.7,0.5,0.5)	(6,10,13,15;0.7,0.3,0.4)	$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$	(6,7,9,12;0.8,0.1,0.2)$\tilde{t}_{11}^N$	(3,5,6,8;0.6,0.5,0.4)	(2,3,5,7;0.5,0.1,0.3)	
M_2	$\tilde{Z}^{(1)N}$	(3,9,10,12;0.7,0.3,0.2)	(10,12,15,17;0.6,0.7,0.6)	(1,3,7,10;0.6,0.3,0.2)	$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$	(4,8,11,15;0.6,0.3,0.2)	(7,9,11,13;0.6,0.4,0.3)	(9,11,14,16;0.5,0.4,0.7)	
M_3	$\tilde{Z}^{(1)N}$	(6,7,9,12;0.8,0.1,0.2)	(2,3,5,7;0.5,0.1,0.3)	(3,5,6,8;0.6,0.5,0.4)	$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$	(4,7,10,12;0.5,0.6,0.6)	(4,8,11,15;0.6,0.3,0.2)	(5,8,10,14;0.3,0.6,0.6)	
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	

where $\tilde{a}_1^N = (9, 11, 14, 16; 0.8, 0.4, 0.7)$, $\tilde{a}_2^N = (5, 7, 9, 12; 0.8, 0.3, 0.2)$, $\tilde{a}_3^N = (3, 5, 6, 8; 0.6, 0.5, 0.4)$, $\tilde{b}_1^N = (9, 10, 12, 15; 0.8, 0.7, 0.4)$, $\tilde{b}_2^N = (3, 9, 10, 12; 0.7, 0.3, 0.3)$, $\tilde{b}_3^N = (7, 9, 11, 13; 0.6, 0.4, 0.3)$ Using Step 2, the weighted possibility mean representation for SVTrNBOTP is shown in Table 3.

TABLE 3. WPM representation of SVTrNBOTP

		P_1	P_2	P_3	Supply
M_1	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{11}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{11}^N(\beta)) + g(\tilde{c}_{11}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{12}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{12}^N(\beta)) + g(\tilde{c}_{12}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{13}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{13}^N(\beta)) + g(\tilde{c}_{13}^N(\gamma))\right]$	$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{t}_{11}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{11}^N(\beta)) + g(\tilde{t}_{11}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{12}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{12}^N(\beta)) + g(\tilde{t}_{12}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{13}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{13}^N(\beta)) + g(\tilde{t}_{13}^N(\gamma))\right]$	
M_2	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{21}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{21}^N(\beta)) + g(\tilde{c}_{21}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{22}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{22}^N(\beta)) + g(\tilde{c}_{22}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{23}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{23}^N(\beta)) + g(\tilde{c}_{23}^N(\gamma))\right]$	$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{t}_{21}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{21}^N(\beta)) + g(\tilde{t}_{21}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{22}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{22}^N(\beta)) + g(\tilde{t}_{22}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{23}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{23}^N(\beta)) + g(\tilde{t}_{23}^N(\gamma))\right]$	
M_3	$\tilde{Z}^{(1)N}$	$\mu\left[g(\tilde{c}_{31}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{31}^N(\beta)) + g(\tilde{c}_{31}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{32}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{32}^N(\beta)) + g(\tilde{c}_{32}^N(\gamma))\right]$	$\mu\left[g(\tilde{c}_{33}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{c}_{33}^N(\beta)) + g(\tilde{c}_{33}^N(\gamma))\right]$	$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$	$\mu\left[g(\tilde{t}_{31}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{31}^N(\beta)) + g(\tilde{t}_{31}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{32}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{32}^N(\beta)) + g(\tilde{t}_{32}^N(\gamma))\right]$	$\mu\left[g(\tilde{t}_{33}^N(\alpha))\right] + (1-\mu)\left[g(\tilde{t}_{33}^N(\beta)) + g(\tilde{t}_{33}^N(\gamma))\right]$	
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	

where $\tilde{a}_1^N = \mu\left[g(\tilde{a}_1^N(\alpha))\right] + (1-\mu)\left[g(\tilde{a}_1^N(\beta)) + g(\tilde{a}_1^N(\gamma))\right]$, $\tilde{a}_2^N = \mu\left[g(\tilde{a}_2^N(\alpha))\right] + (1-\mu)\left[g(\tilde{a}_2^N(\beta)) + g(\tilde{a}_2^N(\gamma))\right]$, $\tilde{a}_3^N = \mu\left[g(\tilde{a}_3^N(\alpha))\right] + (1-\mu)\left[g(\tilde{a}_3^N(\beta)) + g(\tilde{a}_3^N(\gamma))\right]$, $\tilde{b}_1^N = \mu\left[g(\tilde{b}_1^N(\alpha))\right] + (1-\mu)\left[g(\tilde{b}_1^N(\beta)) + g(\tilde{b}_1^N(\gamma))\right]$, $\tilde{b}_2^N = \mu\left[g(\tilde{b}_2^N(\alpha))\right] + (1-\mu)\left[g(\tilde{b}_2^N(\beta)) + g(\tilde{b}_2^N(\gamma))\right]$, $\tilde{b}_3^N = \mu\left[g(\tilde{b}_3^N(\alpha))\right] + (1-\mu)\left[g(\tilde{b}_3^N(\beta)) + g(\tilde{b}_3^N(\gamma))\right]$
Now to compute the possibilistic mean values for truth $g(\tilde{r}(\alpha))$, indeterminacy $g(\tilde{r}(\beta))$ and falsity $g(\tilde{r}(\gamma))$ MF using the possibilistic mean of truth MF (3), possibilistic mean of indeterminacy MF (6) and possibilistic mean of falsity MF (9).

Choose the first cell value

$$\mu\left[g(\tilde{c}_{11}^N(\alpha))\right]+(1-\mu)\left[g(\tilde{c}_{11}^N(\beta))+g(\tilde{c}_{11}^N(\gamma))\right] \tag{14}$$

The possibilistic mean of truth MF $\left[g(\tilde{c}_{11}^N(\alpha))\right]$ using (3) is

$$g(\tilde{c}_{11}^N(\alpha))=\left[\frac{s_{\tilde{r}}^1+2s_{\tilde{r}}^2+2s_{\tilde{r}}^3+s_{\tilde{r}}^4}{6}\right]k_{\tilde{r}}^2=\left[\frac{(4)+2(8)+2(11)+15}{6}\right](0.6)^2=3.42$$

The possibilistic mean of indeterminacy MF $\left[g(\tilde{c}_{11}^N(\beta))\right]$ using (6) is

$$\begin{aligned}&g(\tilde{c}_{11}^N(\beta))\\&=\left[\frac{(2s_{\tilde{r}}^1+s_{\tilde{r}}^2+s_{\tilde{r}}^3+2s_{\tilde{r}}^4)-(s_{\tilde{r}}^1-s_{\tilde{r}}^2-s_{\tilde{r}}^3+s_{\tilde{r}}^4)l_{\tilde{r}}-(s_{\tilde{r}}^1+2s_{\tilde{r}}^2+2s_{\tilde{r}}^3+s_{\tilde{r}}^4)l_{\tilde{r}}^2}{6}\right]\\&=\left[\frac{(2(4)+8+11+2(15))-(4-8-11+15)(0.3)-(4+2(8)+2(11)+(15))(0.3)^2}{6}\right]\\&=8.64\end{aligned}$$

The possibilistic mean of falsity MF $\left[g(\tilde{c}_{11}^N(\gamma))\right]$ using (9) is

$$\begin{aligned}&g(\tilde{c}_{11}^N(\gamma))\\&=\left[\frac{(2s_{\tilde{r}}^1+s_{\tilde{r}}^2+s_{\tilde{r}}^3+2s_{\tilde{r}}^4)-(s_{\tilde{r}}^1-s_{\tilde{r}}^2-s_{\tilde{r}}^3+s_{\tilde{r}}^4)m_{\tilde{r}}-(s_{\tilde{r}}^1+2s_{\tilde{r}}^2+2s_{\tilde{r}}^3+s_{\tilde{r}}^4)m_{\tilde{r}}^2}{6}\right]\\&=\left[\frac{(2(4)+8+11+2(15))-(4-8-11+15)(0.2)-(4+2(8)+2(11)+(15))(0.2)^2}{6}\right]\\&=9.12\end{aligned}$$

Now replacing the above values in (14) we obtain $\mu(3.42)+(1-\mu)(8.64+9.12)$. In the same manner, we compute the values of possibilistic mean of truth, indeterminacy and falsity MF of the other cells in Table 3. Now the reduced WPM of SVTrNBOTP is shown in Table 4.

Table 4. WPM of SVTrNBOTP

		P_1	P_2	P_3	Supply
M_1	$\tilde{Z}^{(1)N}$	$\mu(3.42)+(1-\mu)$ $(8.64+9.12)$	$\mu(3.42)+(1-\mu)$ $(8.64+9.12)$	$\mu(3.42)+(1-\mu)$ $(8.64+9.12)$	$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$	$\mu(5.33)+(1-\mu)$ $(8.55+8.26)$	$\mu(1.98)+(1-\mu)$ $(4.12+4.62)$	$\mu(1.04)+(1-\mu)$ $(4.5+4.6)$	
M_2	$\tilde{Z}^{(1)N}$	$\mu(4.32)+(1-\mu)$ $(7.57+7.94)$	$\mu(4.86)+(1-\mu)$ $(6.88+8.64)$	$\mu(1.86)+(1-\mu)$ $(4.74+5.06)$	$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$	$\mu(3.42)+(1-\mu)$ $(8.64+9.12)$	$\mu(3.6)+(1-\mu)$ $(8.4+9)$	$\mu(3.12)+(1-\mu)$ $(10.5+6.37)$	
M_3	$\tilde{Z}^{(1)N}$	$\mu(5.33)+(1-\mu)$ $(8.55+8.26)$	$\mu(1.04)+(1-\mu)$ $(4.5+4.6)$	$\mu(1.98)+(1-\mu)$ $(4.12+4.62)$	$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$	$\mu(2.08)+(1-\mu)$ $(5.26+5.26)$	$\mu(3.42)+(1-\mu)$ $(8.64+9.12)$	$\mu(0.82)+(1-\mu)$ $(5.93+5.93)$	
Demand		$\tilde{b}_1^N$	$\tilde{b}_2^N$	$\tilde{b}_3^N$	

where $\tilde{a}_1^N = \mu(8) + (1-\mu)(10.5 + 6.37)$, $\tilde{a}_2^N = \mu(5.22) + (1-\mu)(8.74 + 8.77)$, $\tilde{a}_3^N = \mu(1.98) + (1-\mu)(4.12 + 4.62)$, $\tilde{b}_1^N = \mu(7.25) + (1-\mu)(5.88 + 9.72)$, $\tilde{b}_2^N = \mu(4.32) + (1-\mu)(7.57 + 7.57)$, $\tilde{b}_3^N = \mu(3.6) + (1-\mu)(8.4 + 9)$

As in Step 3, transform the WPM of SVTrNBOTP into its equivalent deterministic BOTP by choosing the value of $\mu = 0$ where $0 \leq \mu \leq 1$ as shown in Table 5.

TABLE 5. BOTP

	P_1	P_2	P_3	Supply
M_1	$18 Z^{(1)}$ $Z^{(2)} 17$	12 9	19 9	17
M_2	16 18	16 18	10 17	18
M_3	17 11	9 18	9 12	9
Demand	16	15	18	

Using Step 4, check BOTP is balanced, if not balance it. By zero point method [3], the optimal solution of $Z^{(1)}(x)$ is 531 at the allocations $x_{11} = 2, x_{12} = 15, x_{21} = 9, x_{23} = 9, x_{33} = 9, x_{41} = 5$ and the rest are all zero. Similarly, the optimal solution of $Z^{(2)}(x)$ is 560 at the allocations $x_{12} = 15, x_{13} = 2, x_{21} = 2, x_{23} = 16, x_{31} = 9, x_{41} = 5$ and the rest are all zero.
As in Step 5, consider the optimal solution of $Z^{(1)}(x)$ as a feasible solution of $Z^{(2)}(x)$.

TABLE 6

	P_1	P_2	P_3	Supply
M_1	17 **(2)**	9 **(15)**	9	17
M_2	18 **(9)**	18	17 **(9)**	18
M_3	11	18	12 **(9)**	9
M_4	0 **(5)**	0	0	5
Demand	16	15	18	

Thus, (531,592) is the bi-objective value of BOTP for the feasible allocation $x_{11} = 2, x_{12} = 15, x_{21} = 9, x_{23} = 9, x_{33} = 9, x_{41} = 5$ and the rest are all zero.

Using dripping method [10], we construct a rectangular loop which is shown in Table 7.

TABLE 7

	P_1	P_2	P_3	Supply
M_1	17 $\mathbf{(2-\delta)}$	9 **(15)**	9 $\mathbf{(\delta)}$	17
M_2	18 $\mathbf{(9+\delta)}$	18	17 $\mathbf{(9-\delta)}$	18
M_3	11	18	12 **(9)**	9
M_4	0 **(5)**	0	0	5
Demand	16	15	18	

As a result, the transportation time of $Z^{(2)}(x)$ is $592-7\delta$ and the transportation cost of $Z^{(1)}(x)$ is $531 + 7\delta$ for $\delta \in \{1, 2\}$. Thus, the BOTP has the bi-objective value $(531 + 7\delta, 592 - 7\delta)$ for the feasible allocation $x_{11} = 2 - \delta, x_{12} = 15, x_{13} = \delta, x_{21} = 9 + \delta, x_{23} = 9 - \delta, x_{33} = 9, x_{41} = 5$ and the rest are all zero. For the highest value of $\delta = 2$, the transportation time of $Z^{(2)}(x)$ is 578 and the transportation cost of $Z^{(1)}(x)$ is 545. Thus, the BOTP has the bi-objective value (545,578) for the feasible allocation $x_{12} = 15, x_{13} = 2, x_{21} = 11, x_{23} = 7, x_{33} = 9, x_{41} = 5$ and the rest are all zero. At this stage, the optimal solution of $Z^{(2)}(x)$ is not attained. So, again we construct a rectangular loop.

TABLE 8

	P_1	P_2	P_3	Supply
M_1	17	9 **(15)**	9 **(2)**	17
M_2	18 $\mathbf{(11-\delta)}$	18	17 $\mathbf{(7+\delta)}$	18
M_3	11 $\mathbf{(\delta)}$	18	12 $\mathbf{(9-\delta)}$	9
M_4	0 **(5)**	0	0	5
Demand	16	15	18	

As a result, the transportation time of $Z^{(2)}(x)$ is $578-2\delta$ and the transportation cost of $Z^{(1)}(x)$ is $545+2\delta$ for $\delta \in \{1,2,...,9\}$. Thus, the BOTP has the bi-objective value for the feasible allocation $x_{12}=15, x_{13}=2, x_{21}=11-\delta, x_{23}=7+\delta, x_{33}=9-\delta, x_{41}=5$ and the rest are all zero. For the highest value of $\delta=9$, the transportation time of $Z^{(2)}(x)$ is 560 and the transportation cost of $Z^{(1)}(x)$ is 563. Thus, the BOTP has the bi-objective value (563,560) for the feasible allocation $x_{12}=15, x_{13}=2, x_{21}=2, x_{23}=16, x_{33}=9, x_{41}=5$ and the rest are all zero. The computation terminates at this stage as the optimal solution of $Z^{(2)}(x)$ is attained. Therefore, the set of all efficient solutions obtained from $Z^{(1)}(x)$ to $Z^{(2)}(x)$ are $\{(531,592),(545,578),(563,560)\}$

In the same manner, the set of all efficient solutions obtained from $Z^{(2)}(x)$ to $Z^{(1)}(x)$ are $\{(563,560),(545,578),(531,592)\}$

The following solutions obtained by combining the set of all efficient solutions of $Z^{(1)}(x)$ to $Z^{(2)}(x)$ and $Z^{(2)}(x)$ to $Z^{(1)}(x)$ for the given problem are as follows:

- Ideal solution : (531,592)
- Efficient solutions : $\{(531,592),(545,578),(563,560)\}$
- Optimal compromise solution for BOTP : (545,578)
- Neutrosophic optimal compromise solution for SVTrNBOTP: $\{(154,290,374,469;0.6,0.5,0.5),(201,318,409,537;0.3,0.6,0.7)\}$

Finally, the obtained neutrosophic optimal compromise solution is $(154,290,374,469;0.6,0.5,0.5),(201,318,409,537;0.3,0.6,0.7)$ at the allocations $x_{12}=15, x_{13}=2, x_{21}=11, x_{23}=7, x_{33}=9, x_{41}=5$ and the rest are all zero. For better understanding, the obtained solutions are plotted using the MATLAB which are shown in Figure 2.

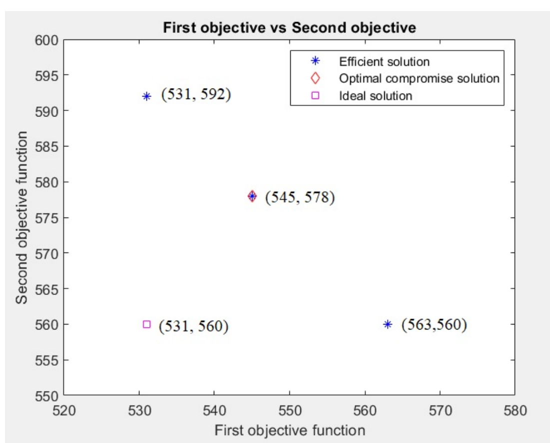

FIGURE 2. Graphical representation of all solutions obtained by solution approach

We now explore the sensitivity analysis for the deterministic BOTP to describe the behaviour of sensitivity ranges.
By Step 8, analyse the sensitivity range for objective function coefficients (cost and time) of basic variables of deterministic BOTP which are obtained in Step 7.
To find the sensitivity range for the first objective $Z^{(1)}(x)$:
As in Step 9(i), vary the cost parameter of one basic variable at a time while keeping the other cells at their initial values.
Using Step 9(ii), we consider c_{12} be the cost coefficient of the basic variable. Then change the parameter of basic variable c_{12} to $c_{12}+\eta$ which is shown in Table 9.

TABLE 9. Changing the parameter of basic cell c_{12}

	P_1	P_2	P_3	u_i
M_1	18	$12+\eta$ **(15)**	19 **(2)**	9
M_2	16 **(11)**	16	10 **(7)**	0
M_3	17	9	9 **(9)**	-1
M_4	0 **(5)**	0	0	-16
v_j	16	$3+\eta$	10	

By Step 9(iii), calculate the new $u_i's$ and $v_j's$ to determine whether the current solution remains optimal.
As in Step 9(iv), using $\bar{c}_{ij}=u_i+v_j-c_{ij}\leq 0$ determine the ranges of non-basic variables.
First, consider the non-basic cell (3,2) and calculate the range which is given by $\bar{c}_{32}=u_3+v_2-c_{32}=-1+3+\eta-9=-7+\eta\leq 0$
In the same manner, we can calculate the ranges of the other non-basic cells.
Using Step 9(v), the sensitivity range over c_{12} can vary as $-\infty<\eta\leq 7$ and its corresponding sensitivity ranges for the basic cell c_{12} is $-\infty<c_{12}\leq 19$
Repeat the above steps to determine the sensitivity range of cost coefficient of the other basic variables.
To determine the sensitivity range for the second objective $Z^{(2)}(x)$ repeat Step 9 and Step 10.
The sensitivity ranges for objective function coefficients of basic variables of $Z^{(1)}(x)$ and $Z^{(2)}(x)$ are shown in Table 10.

TABLE 10. Sensitivity ranges for $Z^{(1)}(x)$ and $Z^{(2)}(x)$

Sensitivity ranges for c_{ij} in $Z^{(1)}(x)$	Sensitivity ranges for t_{ij} in $Z^{(2)}(x)$
$-\infty < c_{12} \leq 19$	$-\infty < t_{12} \leq 10$
$12 \leq c_{13} < \infty$	$8 \leq t_{13} \leq 16$
$10 \leq c_{21} \leq 18$	$17 \leq t_{21} \leq 25$
$8 \leq c_{23} \leq 17$	$16 \leq t_{23} \leq 18$
$-\infty < c_{33} \leq 11$	$10 \leq t_{33} \leq 18$
$-\infty < c_{41} \leq 6$	$-\infty < t_{41} \leq 1$

7.1. Numerical Example 2. Petrochemicals have a massive role in society. Petrochemicals play a fundamental part in many aspects of our daily lives, including the carpets we use to adorn our houses, the clothes we wear, plastic bottles, fertilisers we use to produce crops, tyres, paints, medications, cosmetics and more. Tamil Nadu Petroproducts Ltd (TPL) is one of the Chennai-based petrochemicals manufacturers. It is the first company in the world to produce linear alkylbenzene (LAB). LAB is produced by three petrochemical plants in TPL and is exported to three destinations namely Taiwan, Japan and Netherlands. The weekly capacities of the petrochemicals are $\tilde{a}_1^N$, $\tilde{a}_2^N$ and $\tilde{a}_3^N$ units and the weekly requirements of the petrochemicals are $\tilde{b}_1^N$, $\tilde{b}_2^N$, $\tilde{b}_3^N$ and $\tilde{b}_4^N$ units respectively. Let us assume that there are two objectives $\tilde{Z}^{(1)N}(x)$ and $\tilde{Z}^{(2)N}(x)$ to be considered: (i) the minimization of total transportation cost $\tilde{c}_{ij}^N$ of shipment for exporting petrochemicals; and (ii) the minimization of total transportation time $\tilde{t}_{ij}^N$ of shipment for exporting petrochemicals. The problem (G) is modelled as SVTrNBOTP where all the coefficients of objective function, coefficients of supply constraints and demand constraints are given in Table 11.

TABLE 11. SVTrNBOTP

		P_1		P_2		P_3		P_3		Supply
M_1	$\tilde{Z}^{(1)N}$	$\tilde{c}_{11}^N$		$\tilde{c}_{12}^N$		$\tilde{c}_{13}^N$		$\tilde{c}_{14}^N$		$\tilde{a}_1^N$
	$\tilde{Z}^{(2)N}$		$\tilde{t}_{11}^N$		$\tilde{t}_{12}^N$		$\tilde{t}_{13}^N$		$\tilde{t}_{14}^N$	
M_2	$\tilde{Z}^{(1)N}$	$\tilde{c}_{21}^N$		$\tilde{c}_{22}^N$		$\tilde{c}_{23}^N$		$\tilde{c}_{24}^N$		$\tilde{a}_2^N$
	$\tilde{Z}^{(2)N}$		$\tilde{t}_{21}^N$		$\tilde{t}_{22}^N$		$\tilde{t}_{23}^N$		$\tilde{t}_{24}^N$	
M_3	$\tilde{Z}^{(1)N}$	$\tilde{c}_{31}^N$		$\tilde{c}_{32}^N$		$\tilde{c}_{33}^N$		$\tilde{c}_{34}^N$		$\tilde{a}_3^N$
	$\tilde{Z}^{(2)N}$		$\tilde{t}_{31}^N$		$\tilde{t}_{32}^N$		$\tilde{t}_{33}^N$		$\tilde{t}_{34}^N$	
Demand		$\tilde{b}_1^N$		$\tilde{b}_2^N$		$\tilde{b}_3^N$		$\tilde{b}_4^N$		

where $\tilde{c}_{11}^N = (6, 10, 13, 15; 0.7, 0.3, 0.4)$; $\tilde{c}_{12}^N = (18, 22, 30, 34; 0.5, 0.1, 0.4)$; $\tilde{c}_{13}^N = (4, 8, 11, 15; 0.6, 0.3, 0.2)$; $\tilde{c}_{14}^N = (26, 28, 30, 35; 0.9, 0.1, 0.1)$; $\tilde{c}_{21}^N = (18, 24, 30, 35; 0.5, 0.1, 0.3)$; $\tilde{c}_{22}^N = (26, 28, 30, 35; 0.9, 0.1, 0.1)$; $\tilde{c}_{23}^N = (14, 7, 21, 28; 0.8, 0.2, 0.3)$;

$\tilde{c}_{24}^N = (18, 20, 22, 25; 0.7, 0.5, 0.5)$; $\tilde{c}_{31}^N = (20, 26, 30, 31; 0.9, 0.1, 0.1)$; $\tilde{c}_{32}^N = (9, 11, 14, 16; 0.5, 0.4, 0.7)$; $\tilde{c}_{33}^N = (25, 35, 44, 50; 0.9, 0.2, 0.1)$; $\tilde{c}_{34}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$; $\tilde{t}_{11}^N = (14, 7, 21, 28; 0.8, 0.2, 0.6)$; $\tilde{t}_{12}^N = (4, 8, 11, 15; 0.6, 0.3, 0.2)$; $\tilde{t}_{13}^N = (15, 17, 19, 22; 0.6, 0.4, 0.5)$; $\tilde{t}_{14}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$; $\tilde{t}_{21}^N = (15, 18, 20, 22; 0.7, 0.5, 0.5)$; $\tilde{t}_{22}^N = (3, 5, 6, 8; 0.6, 0.5, 0.4)$; $\tilde{t}_{23}^N = (12, 15, 19, 22; 0.6, 0.4, 0.5)$; $\tilde{t}_{24}^N = (15, 17, 19, 22; 0.4, 0.8, 0.4)$; $\tilde{t}_{31}^N = (18, 20, 22, 25; 0.7, 0.5, 0.5)$; $\tilde{t}_{32}^N = (5, 8, 10, 14; 0.3, 0.6, 0.6)$; $\tilde{t}_{33}^N = (9, 11, 14, 16; 0.5, 0.4, 0.7)$; $\tilde{t}_{34}^N = (15, 18, 20, 22; 0.7, 0.5, 0.5)$; $\tilde{a}_1^N = \tilde{a}_2^N = (25, 35, 44, 50; 0.9, 0.2, 0.1)$; $\tilde{a}_3^N = (68, 75, 80, 85; 0.9, 0.1, 0.2)$; $\tilde{b}_1^N = (19, 25, 30, 35; 0.5, 0.1, 0.2)$; $\tilde{b}_2^N = (26, 28, 30, 35; 0.9, 0.1, 0.2)$; $\tilde{b}_3^N = (35, 45, 50, 55; 0.9, 0.1, 0.2)$; $\tilde{b}_4^N = (45, 55, 60, 64; 0.9, 0.1, 0.3)$

Using Steps 1 to 7, we obtain the optimal compromise solution for deterministic BOTP as (7395,7177). Finally, the obtained neutrosophic optimal compromise solution is $\{(3139, 3576, 5180, 6262; 0.5, 0.5, 0.7), (3717, 4202, 5350, 6350; 0.3, 0.8, 0.6)\}$ at the allocations $x_{11} = 37, x_{13} = 38, x_{21} = 16, x_{23} = 52, x_{32} = 59, x_{34} = 91, x_{44} = 8$ and the rest are all zero. Similarly, we can calculate the sensitivity range for objective function coefficient of basic variables of $Z^{(1)}(x)$ and $Z^{(2)}(x)$ which are shown in Table 12.

TABLE 12. Sensitivity ranges for $Z^{(1)}(x)$ and $Z^{(2)}(x)$

Sensitivity ranges for c_{ij} in $Z^{(1)}(x)$	Sensitivity ranges for t_{ij} in $Z^{(2)}(x)$
$-\infty < c_{11} \le 77$	$-\infty < t_{11} \le 33$
$-\infty < c_{13} \le 55$	$-\infty < t_{13} \le 32$
$32 \le c_{21} \le 57$	$26 \le t_{21} < \infty$
$-8 \le c_{23} \le 32$	$22 \le t_{23} \le 44$
$29 \le c_{24} \le 51$	$-\infty < t_{24} \le 24$
$-\infty < c_{32} \le 27$	$-\infty < t_{32} \le 15$
$17 \le c_{34} \le 77$	$-2 \le t_{34} < \infty$
$-19 \le c_{44} \le 3$	$-5 \le t_{44} \le 16$

8. Results and Discussions

To assess the performance of the proposed method, the neutrosophic optimal compromise solution is compared with the existing approaches such as linear membership approach (LMA) [33], hyperbolic membership approach (HMA) [33], fuzzy programming approach (FPA) [31] and neutrosophic compromise programming approach (NCPA) [38]. The neutrosophic optimal compromise solution of these methods are shown in Table 13.

TABLE 13. Comparison between proposed method with existing approaches

Methods	Example 1: Neutrosophic optimal compromise solution	Example 1: Values of Decision variable
Proposed method	{(154,290,374,469;0.6,0.5,0.5), (201,318,409,537;0.3,0.6,0.7)}	$x_{12}=15, x_{13}=2, x_{21}=11,$ $x_{23}=7, x_{33}=9, x_{41}=5$
LMA [33]	{(154,242,364,481;0.6,0.5,0.5), (251,348,447,541;0.3,0.6,0.7)}	$x_{11}=2, x_{12}=15, x_{21}=1,$ $x_{23}=17, x_{31}=8, x_{33}=1, x_{41}=5$
HMA [33]	{(154,242,364,481;0.6,0.5,0.5), (251,348,447,541;0.3,0.6,0.7)}	$x_{11}=2, x_{12}=15, x_{21}=1,$ $x_{23}=17, x_{31}=8, x_{33}=1, x_{41}=5$
FPA [31]	{(155,286,374,471;0.6,0.5,0.5), (205,320,412,536;0.3,0.6,0.7)}	$x_{12}=15, x_{13}=2, x_{21}=10,$ $x_{23}=8, x_{31}=1, x_{33}=8, x_{41}=5$
NCPA [38]	{(154,242,364,481;0.6,0.5,0.5), (251,348,447,541;0.3,0.6,0.7)}	$x_{11}=2, x_{12}=15, x_{21}=1,$ $x_{23}=17, x_{31}=8, x_{33}=1, x_{41}=5$
	Example 2: Neutrosophic Optimal compromise solution	**Example 2: Values of Decision variable**
Proposed method	{(3139,3576,5180,6262;0.5,0.5,0.7), (3717,4202,5350,6350;0.3,0.8,0.6)}	$x_{11}=37, x_{13}=38, x_{21}=16, x_{23}=52,$ $x_{24}=7, x_{32}=59, x_{34}=91, x_{44}=8$
LMA [33]	{(3826,4982,6147,7172;0.5,0.5,0.7), (3574,3924,5101,6134;0.3,0.8,0.7)}	$x_{11}=42, x_{13}=33, x_{21}=3, x_{23}=16, x_{24}=56,$ $x_{32}=59, x_{33}=41, x_{34}=50, x_{41}=8$
HMA [33]	{(3837,4810,6152,7176;0.5,0.5,0.7), (3559,3880,5098,6146;0.3,0.8,0.7)}	$x_{11}=45, x_{13}=30, x_{23}=18, x_{24}=57,$ $x_{32}=59, x_{33}=42, x_{34}=49, x_{41}=8$
FPA [31]	{(3826,4982,6147,7172;0.5,0.5,0.7), (3574,3924,5101,6134;0.3,0.8,0.7)}	$x_{11}=42, x_{13}=33, x_{21}=3, x_{23}=16, x_{24}=56,$ $x_{32}=59, x_{33}=41, x_{34}=50, x_{41}=8$
NCPA [38]	{(3826,4982,6147,7172;0.5,0.5,0.7), (3574,3924,5101,6134;0.3,0.8,0.7)}	$x_{11}=42, x_{13}=33, x_{21}=3, x_{23}=16, x_{24}=56,$ $x_{32}=59, x_{33}=41, x_{34}=50, x_{41}=8$

From Table 13, it is clear that the obtained optimal compromise solution for Example 1 using the proposed method provides the same result as compared to the existing approaches while Example 2 provides the better result as compared to the existing approaches. For better understanding, the obtained optimal compromise solution compared with the existing approaches which is shown graphically in Figure 3 and Figure 4.

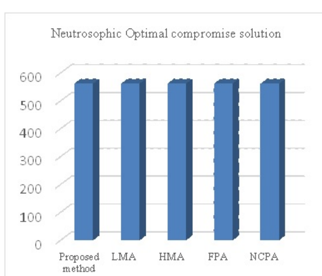

Figure 3. Comparison of Example 1 between existing approaches

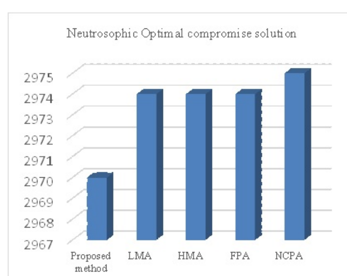

Figure 4. Comparison of Example 2 between existing approaches

9. Conclusions and future scopes

In the literature survey, the study of bi-objective transportation problem under neutrosophic environment which receives less attention. In this study, we have considered the bi-objective transportation problem under neutrosophic environment. The extended weighted possibility mean for single-valued trapezoidal neutrosophic numbers based on [40] is utilized to transform the SVTrNBOTP into its equivalent deterministic BOTP. The transformed deterministic BOTP is then solved using the dripping method [10] to determine the optimal compromise solution. This method can assist DM's in decision making process to provide an optimal compromise solution for real life problems. To illustrate the utility and applicability of the solution approach, numerical examples are validated and a comparison is made between the proposed method and the existing approaches. Finally, a sensitivity analysis is performed to determine the sensitivity ranges for the problem. The neutrosophic data facilitates a reasonable and practicable way for DMs to tackle decision-making problems by managing indeterminacy and providing an effective framework for analysis and synthesis of complex decision scenarios. The limitations in predicting the solutions of qualitative and complex data due to the computational complexity in handling higher dimensional problems can be resolved using evolutionary algorithms. For the future perspective, the content of this study can open a new dimension towards the neutrosophic bi-objective fractional TP. Additionally, this problem can be viewed as a multi-item type-2 fuzzy problem or type-2 neutrosophic fuzzy problem.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable

References

1. F.L. Hitchcock, *The distribution of a product from several sources to numerous localities*, Journal of Mathematics and Physics **20** (1941), 224-230.
2. C. Koopmans, *Optimum utilization of the transportation system*, Econometrica: Journal of the Econometric Society (1949), 136-146.
3. P. Pandian and G. Natarajan, *A new method for finding an optimal solution for transportation problems*, International Journal of Mathematical Sciences and Engineering Applications **4** (2010), 59-65.
4. Y.P. Aneja and K.P. Nair, *Bicriteria transportation problem*, Management Science **25** (1979), 73-78.
5. H. Isermann, *The enumeration of all efficient solutions for a linear multiple-objective transportation problem*, Naval Research Logistics Quarterly **26** (1979), 123-139.
6. B.I.N.A. Gupta and R.E.E.T.A. Gupta, *Multi-criteria simplex method for a linear multiple objective transportation problem*, Indian Journal of Pure and Applied Mathematics **14** (1983), 222-232.
7. J.L. Ringuest and D.B. Rinks, *Interactive solutions for the linear multi-objective transportation problem*, European Journal of Operational Research **32** (1987), 96-106.

8. H.S. Kasana and K.D. Kumar, *An efficient algorithm for multi-objective transportation problems*, Asia-Pacific Journal of Operational Research **17** (2000), 27.
9. G. Bai and L. Yao, *A simple algorithm for the multi-objective transportation model*, International Conference on Business Management and Electronic Information **2** (2011), 479-482.
10. P. Pandian and D. Anuradha, *A new method for solving bi-objective transportation problems*, Australian Journal of Basic and Applied Sciences **5** (2011), 67-74.
11. M.A. Nomani, I. Ali and A. Ahmed, *A new approach for solving multi-objective transportation problems*, International Journal of Management Science and Engineering Management **12** (2017), 165-173.
12. L. Kaur, M. Rakshit and S. Singh, *A new approach to solve multi-objective transportation problem*, Applications and Applied Mathematics: An International Journal **13** (2018), 10.
13. L.A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338-353.
14. L.A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems **32** (1978), 3-28.
15. C. Carlsson and R. Fuller, *On possibilistic mean value and variance of fuzzy numbers*, European Journal of Operational Research **122** (2001), 315-326.
16. A. Gupta, A. Kumar and A. Kaur, *Mehar's method to find exact fuzzy optimal solution of unbalanced fully fuzzy multi-objective transportation problems*, Optimization Letters **6** (2012), 1737-1751
17. S. Dhanasekar, S. Hariharan and P. Sekar, *Fuzzy Hungarian MODI algorithm to solve fully fuzzy transportation problems*, International Journal of Fuzzy Systems **19** (2017), 1479-1491.
18. P. Singh, S. Kumari and P. Singh, *Fuzzy efficient interactive goal programming approach for multi-objective transportation problems*, International Journal of Applied and Computational Mathematics **3** (2017), 505-525.
19. M. Bagheri, A. Ebrahimnejad, S. Razavyan, F. Hosseinzadeh Lotfi and N. Malekmohammadi, *Solving the fully fuzzy multi-objective transportation problem based on the common set of weights in DEA*, Journal of Intelligent and Fuzzy Systems **39** (2020), 3099-3124.
20. M. Niksirat, *A New Approach to Solve Fully Fuzzy Multi-Objective Transportation Problem*, Fuzzy Information and Engineering **39** (2022), 1-12.
21. Y. Kacher and P. Singh, *Fuzzy harmonic mean technique for solving fully fuzzy multi-objective transportation problem*, Journal of Computational Science **63** (2022), 101782.
22. A.N. Revathi, S. Mohanaselvi and B. Said, *An efficient neutrosophic technique for uncertain multi objective transportation problem*, Neutrosophic Sets and Systems **53** (2023), 27.
23. S.G. Bodkhe, *Multi-objective transportation problem using fuzzy programming techniques based on exponential membership functions*, International Journal of Statistics and Applied Mathematics **8** (2023), 20-24.
24. M.M. Miah, A. AlArjani, A. Rashid, A.R. Khan, M.S. Uddin and E.A. Attia, *Multi-objective optimization to the transportation problem considering non-linear fuzzy membership functions* AIMS Mathematics **8** (2023), 10397-10419.
25. K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87-96.
26. A. Ebrahimnejad and J.L. Verdegay, *A new approach for solving fully intuitionistic fuzzy transportation problems*, Fuzzy Optimization and Decision Making **17** (2018), 447–474.
27. A. Mahmoodirad, T. Allahviranloo and S. Niroomand, *A new effective solution method for fully intuitionistic fuzzy transportation problem*, Soft Computing (2018), 1-10.
28. S.P. Wan, D.F. Li and Z.F. Rui, *Possibility mean, variance and covariance of triangular intuitionistic fuzzy numbers*, Journal of Intelligent and Fuzzy Systems **24** (2013), 847-858.
29. T. Garai, D. Chakraborty and T.K. Roy, *A multi-item generalized intuitionistic fuzzy inventory model with inventory level dependent demand using possibility mean, variance and covariance*, Journal of Intelligent and Fuzzy Systems **35** (2018), 1021-1036.
30. S.K. Roy, A. Ebrahimnejad, J.L. Verdegay and S. Das, *New approach for solving intuitionistic fuzzy multi-objective transportation problem*, Sadhana **43** (2018), 1-12.

31. S. Ghosh, S.K. Roy, A. Ebrahimnejad and J.L. Verdegay, *Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem*, Complex and Intelligent Systems **7** (2021), 1009-1023.
32. S. Mahajan and S.K. Gupta, *On fully intuitionistic fuzzy multi-objective transportation problems using different membership functions*, Annals of Operations Research **296** (2021), 211-241.
33. A.A.H. Ahmadini and F. Ahmad, *Solving intuitionistic fuzzy multi-objective linear programming problem under neutrosophic environment*, AIMS Mathematics **6** (2021), 4556-4580.
34. R.K. Bera and S.K. Mondal, *A multi-objective transportation problem under quantity dependent credit period and cost structure policies in triangular intuitionistic fuzzy environment*, Engineering Applications of Artificial Intelligence **123** (2023), 106396.
35. F. Smarandache, *A unifying field in logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, American Research Press, Rehoboth, New York, 1999.
36. I. Deli and Y. Subas, *Single valued neutrosophic numbers and their applications to multicriteria decision making problem*, Neutrosophic Sets and Systems **2** (2014), 1-13.
37. H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, *Single valued neutrosophic sets*, Multispace and Multistructure **4** (2010), 410–413.
38. R.M. Rizk-Allah, A.E. Hassanien and M. Elhoseny, *A multi-objective transportation model under neutrosophic environment*, Computers and Electrical Engineering **69** (2018), 705-719.
39. H.A.E.W. Khalifa, P. Kumar and S. Mirjalili, *A KKM approach for inverse capacitated transportation problem in neutrosophic environment*, Sadhana **46** (2021), 1-8.
40. K. Khatter, *Neutrosophic linear programming using possibilistic mean*, Soft Computing **24** (2020), 16847-16867.
41. T. Garai, S. Dalapati, H. Garg and T.K. Roy, *Possibility mean, variance and standard deviation of single-valued neutrosophic numbers and its applications to multi-attribute decision-making problems*, Sadhana **24** (2020), 18795-18809.
42. S. Sandhiya and D. Anuradha *Solving bi-objective assignment problem under neutrosophic environment*, Reliability: Theory and Applications **17** (2022), 164-175.
43. J. Intrator and J. Paroush, *Sensitivity analysis of the classical transportation problem:A combinatorial approach*, Computers and Operations Research **4** (1977), 213-226.
44. H. Arsham, *Postoptimality analyses of the transportation problem*, Journal of the Operational Research Society **43** (1992), 121-139.
45. S. Doustdargholi, D.D. Asl and V. Abasgholipour, *Sensitivity analysis of righthand-side parameter in transportation problem*, Applied Mathematical Sciences **3** (2009), 1501-1511.
46. N.M. Badra, *Sensitivity analysis of transportation problems*, Journal of Applied Sciences Research **3** (2007), 668-675.
47. N. Bhatia and A. Kumar, *A new method for sensitivity analysis of fuzzy transportation problems*, Journal of Intelligent and Fuzzy Systems **25** (2013), 167-175.
48. K. Ravinder Reddy, Ch. Rajitha and L.P. Raj Kumar, *Sensitivity analysis in fuzzy transportation problems with trapezoidal fuzzy numbers*, IOSR Journal of Mathematics **18** (2022), 16-22.
49. A. Thamaraiselvi and R. Santhi, *A new approach for optimization of real life transportation problem in neutrosophic environment*, Mathematical Problems in Engineering **2016** (2016), 1-9.
50. A. Singh, A. Kumar and S.S. Appadoo, *Modified approach for optimization of real life transportation problem in neutrosophic environment*, Mathematical Problems in Engineering **2017** (2017).
51. K.P. Sikkannan and V. Shanmugavel, *Unraveling neutrosophic transportation problem using costs mean and complete contingency cost table*, Neutrosophic Sets and Systems **29** (2019), 165–173.

52. R.K. Saini, A. Sangal and M. Manisha, *Application of single valued trapezoidal neutrosophic numbers in transportation problem*, Neutrosophic Sets and Systems **35** (2020), 33.
53. R.M. Umamageswari and G. Uthra, *Generalized single valued neutrosophic trapezoidal numbers and their application to solve transportation problem*, Studia Rosenthaliana **11** (2020), 164-170.
54. A. Kumar, R. Chopra and R.R. Saxena, *An efficient enumeration technique for a transportation problem in neutrosophic environment*, Neutrosophic Sets and System **47** (2021), 354-365.
55. S. Dhouib, *Solving the single-valued trapezoidal neutrosophic transportation problems through the novel dhouib-matrix-TP1 heuristic*, Mathematical Problems in Engineering **2021** (2021), 1-11.

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