J. Appl. Math. & Informatics Vol. **42**(2024), No. 4, pp. 777 - 783
https://doi.org/10.14317/jami.2024.777

INVESTIGATION OF BOUNDS FOR $\Re$ GRAPH VIA TOPOLOGICAL INDICES

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Abstract. Topoloical index is a numerical quantity which is correlates to properties of chemical compound . In this paper, we define operator graph namely, Edge $ss-$corona graph and we study structured properties of that graph. Also, establish the upper and lower bounds for First Zagreb index, Second Zagreb index, First Gourava index, SK_1 index, Forgotten topological index and EM_1 index of edge SS-corona graph.

2010 Mathematics Subject classification : 05C90, 05C12.
Key words and phrases : Edge SS-corona graph and topological index.

1. Introduction

The Harold Wiener was introduced path index in 1947, now it is renamed as Wiener index. [9, 4] The Wiener index was came out for finding boiling point of paraffin theoretically. That is Topological indices are working link function between molecular graph to numerical quantity. In that, few are listed below

The Gutman and Trinajestic[2] are define the Zagreb indices as

$$M_1(G) = \sum_{uv \in E(G)} d_u + d_v$$

$$M_2(G) = \sum_{uv \in E(G)} d_u d_v$$

The Kulli et al. [3] defined the first Gourava index as follows

$$GO_1(G) = \sum_{uv \in E(G)} [d_u + d_v + d_u d_v]$$

Received August 28, 2023. Revised December 28, 2023. Accepted May 22, 2024.
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The Furtula et.al.[1] defined the Forgotten topological index as

$$F(G) = \sum_{uv\in E(G)} [d_u + d_v]^2$$

The V. S. Shegehalli et al.[10] defined the SK indices as follows

$$SK_1(G) = \sum_{uv\in E(G)} \left[\frac{d_u + d_v}{2}\right]$$

The Milicevic et al.[8] defined the first reformulated zagreb index as follows

$$EM_1(G) = \sum_{uv\in E(G)} [d_u + d_v - 2]^2$$

The degree of vertex $v(d_G(v))$ is the number of vertices adjacent to v in G. The minimum and maximum degrees of a graph G are denoted by δ_G and Δ_G. Recently M. Manjunath et al. [7, 5, 6] had introduced new graph operator, motivated from that we will define a new class of operator graph namely Edge SS-corona graph.

Definition 1.1. Let G and H be two simple connected graphs with n_1, n_2 and m_1, m_2 vertices and edges respectively. Then the edge SS-Corona graph is a graph and is denoted by $G \ominus_{ss} H = \Re$, which is obtained by taking one copy of $S(G)$ with m_1-copies of $S(H)$ and each i^{th} vertex $[1 \leq i \leq m_1]$ of $I(G)$. [I(G) is a vertex set which are inserted vertices to each edge of G] is adjacent to each vertex of $S(H)$.

The number of vertices and edges in $\Re$ graph is $n_1 + m_1[n_2 + m_2 + 1]$ and $m_1[n_2 + 3m_2 + 2]$ respectively.

Edge partition of $\Re$ is as follows

(d_u, d_v)	$(d_G, n_2 + m_2 + 2)$	$(d_H + 1, 3)$	$(d_H + 1, n_2 + m_2 + 2)$	$(n_2 + m_2 + 2, 3)$
Number of edges	$2m_1$	$2m_1m_2$	m_1n_2	m_1m_2

2. Main results

Bounds on various topological indices of Edge $SS-$ Corona graph

In this section we formulate the bounds on the M_1, M_2, SK_1, EM_1, SK_1, GO_1 and Forgotten index of $\Re$.

Theorem 2.1. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for the M_1-index of $\Re$ is*

$$M_1[\Re] \leq 2m_1\Delta_G + \Delta_H m_1[2m_2 + n_2] + 2m_1[n_2 + m_2 + 2] + m_1n_2[n_2 + m_2 + 4] + m_1m_2[n_2 + m_2 + 5] + 8m_1m_2.$$

and

$$\begin{aligned}M_1[\Re] \geq{}& 2m_1\delta_G + \delta_H m_1[2m_2 + n_2] + 2m_1[n_2 + m_2 + 2] + m_1n_2[n_2 + m_2 + 4]\\&+ m_1m_2[n_2 + m_2 + 5] + 8m_1m_2.\end{aligned}$$

Proof. By using table values in definition of M_1 index, we have

$$\begin{aligned}M_1[\Re] ={}& 2m_1[d_G + n_2 + m_2 + 2] + 2m_1m_2[d_H + 1 + 3]\\&+ m_1n_2[d_H + 1 + n_2 + m_2 + 3] + m_1m_2[n_2 + m_2 + 2 + 3]\\&= 2m_1d_G + d_H[2m_1m_2 + m_1n_2] + 2m_1(n_2 + m_2 + 2) + 8m_1m_2\\&+ m_1n_2(n_2 + m_2 + 4) + m_1m_2(n_2 + m_2 + 5)\\&= 2m_1d_G + d_H m_1[2m_2 + n_2] + 2m_1(n_2 + m_2 + 2)\\&+ m_1n_2(n_2 + m_2 + 4) + m_1m_2(n_2 + m_2 + 5) + 8m_1m_2.\\M_1[\Re] \leq{}& 2m_1\Delta_G + \Delta_H m_1[2m_2 + n_2] + 2m_1[n_2 + m_2 + 2]\\&+ m_1n_2[n_2 + m_2 + 4] + m_1m_2[n_2 + m_2 + 5] + 8m_1m_2.\end{aligned}$$

Similarly

$$\begin{aligned}M_1[\Re] \geq{}& 2m_1\delta_G + \delta_H m_1[2m_2 + n_2] + 2m_1[n_2 + m_2 + 2] + m_1n_2[n_2 + m_2 + 4]\\&+ m_1m_2[n_2 + m_2 + 5] + 8m_1m_2.\end{aligned}$$

□

Theorem 2.2. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for GO_1-index of $\Re$ is*

$$\begin{aligned}GO_1[\Re] \leq{}& m_1(n_2 + m_2 + 2)[2(\Delta_G + 1) + m_2(\Delta_H + 6)] + 2m_1m_2[4(\Delta_H + 1) + 3]\\&+ m_1[m_2(\delta_H + 1) + 2\delta_G + 3m_2]\end{aligned}$$

and

$$\begin{aligned}GO_1[\Re] \geq{}& m_1(n_2 + m_2 + 2)[2(\delta_G + 1) + m_2(\delta_H + 6)] + 2m_1m_2[4(\Delta_H + 1) + 3]\\&+ m_1[m_2(\delta_H + 1) + 2\delta_G + 3m_2]\end{aligned}$$

Proof. By using table values in definition of GO_1 index, we have

$$\begin{aligned}GO_1[\Re] ={}& 2m_1[d_G + (n_2 + m_2 + 2) + d_G(n_2 + m_2 + 2)] + 2m_1m_2[(d_H + 1) + 3 + 3(d_H + 1)]\\&+ m_1n_2[(d_H + 1) + (n_2 + m_2 + 2) + (d_H + 1)(n_2 + m_2 + 2)] + m_1m_2[(n_2m_2 + 2)\\&+ 3(n_2 + m_2 + 2)]2m_1[(n_2 + m_2 + 2)[d_G + 1] + d_G] + 2m_1m_2[4(d_H + 1) + 3]\\&+ m_1m_2[(n_2 + m_2 + 2)(d_H + 2) + (d_H + 1)] + m_1m_2[4(n_2 + m_2 + 2) + 3]\\&= (n_2 + m_2 + 2)[2m_1(d_G + 1) + m_1m_2(d_H + 2) + 4m_1m_2] + 2m_1m_2[4(d_H + 1) + 3]\\&+ 2m_1d_G + m_1m_2(d_H + 1) + 3m_1m_2\\&= m_1(n_2m_2 + 2)[2(d_G + 1) + m_2(d_H + 6)] + 2m_1m_2[4(d_H + 1) + 3]\\&+ m_1[m_2(d_H + 1) + 2d_G + 3m_2]\\GO_1[\Re] \leq{}& m_1(n_2 + m_2 + 2)[2(\Delta_G + 1) + m_2(\Delta_H + 6)] + 2m_1m_2[4(\Delta_H + 1) + 3]\\&+ m_1[m_2(\delta_H + 1) + 2\delta_G + 3m_2].\end{aligned}$$

Similarly

$$GO_1[\Re] \geq m_1(n_2 + m_2 + 2)[2(\delta_G + 1) + m_2(\delta_H + 6)] + 2m_1m_2[4(\Delta_H + 1) + 3]$$

$$+ m_1[m_2(\delta_H + 1) + 2\delta_G + 3m_2].$$

□

Theorem 2.3. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for SK_1-index of $\Re$ is*

$$\begin{aligned} SK_1[\Re] &\leq m_1\Delta_G + m_1\Delta_H[m_2 + n_2/2] + m_1(n_2 + 5m_2 + 2) \\ &\quad + \frac{m_1}{2}[n_2(n_2 + m_2 + 3) + m_2(n_2 + m_{2+5})]. \end{aligned}$$

and

$$\begin{aligned} SK_1[\Re] &\geq m_1\delta_G + m_1\delta_H[m_2 + n_2/2] + m_1(n_2 + 5m_2 + 2) \\ &\quad + \frac{m_1}{2}[n_2(n_2 + m_2 + 3) + m_2(n_2 + m_{2+5})]. \end{aligned}$$

Proof. By using table values in definition of SK_1 index, we have

$$\begin{aligned} SK_1[\Re] &= 2m_1\left[\frac{d_G + n_2 + m_2 + 2}{2}\right] + 2m_1m_2\left[\frac{d_H + 1 + 3}{2}\right] \\ &\quad + m_1n_2\left[\frac{d_H + 1 + n_2 + m_2 + 2}{2}\right] + m_1m_2\left[\frac{n_2 + m_2 + 2 + 3}{2}\right] \\ &= m_1[d_G + n_2 + m_2 + 2] + m_1m_2[d_H + 4] + \frac{m_1n_2}{2}[d_H + n_2 + m_2 + 3] \\ &\quad + \frac{m_1m_2}{2}[n_2 + m_2 + 5] \\ &= m_1d_G + m_1d_H[m_2 + n_2/2] + m_1(n_2 + m_2 + 2) + 4m_1m_2 \\ &\quad + \frac{m_1}{2}[n_2(n_2 + m_2 + 3) + m_2(n_2 + m_2 + 5)]. \\ SK_1[\Re] &\leq m_1\Delta_G + m_1\Delta_H[m_2 + n_2/2] + m_1(n_2 + 5m_2 + 2) \\ &\quad + \frac{m_1}{2}[n_2(n_2 + m_2 + 3) + m_2(n_2 + m_{2+5})]. \end{aligned}$$

Similarly

$$\begin{aligned} SK_1[\Re] &\geq m_1\delta_G + m_1\delta_H[m_2 + n_2/2] + m_1(n_2 + 5m_2 + 2) \\ &\quad + \frac{m_1}{2}[n_2(n_2 + m_2 + 3) + m_2(n_2 + m_{2+5})]. \end{aligned}$$

□

Theorem 2.4. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for F-index of $\Re$ is*

$$\begin{aligned} F[\Re] &\leq 2m_1[\Delta_G + n_2 + m_2 + 1]^2 + 2m_1m_2[\Delta_H + 4]^2 + m_1n_2[\Delta_H + n_2 + m_2 + 3]^2 \\ &\quad + m_1m_2[n_2 + m_2 + 5]^3. \end{aligned}$$

and

$$\begin{aligned} F[\Re] &\geq 2m_1[\delta_G + n_2 + m_2 + 1]^2 + 2m_1m_2[\delta_H + 4]^2 + m_1n_2[\delta_H + n_2 + m_2 + 3]^2 \\ &\quad + m_1m_2[n_2 + m_2 + 5]^3. \end{aligned}$$

Proof. By using table values in definition of F index, we have

$$\begin{aligned}
F[\Re] &= 2m_1[d_G + n_2 + m_2 + 1]^2 + 2m_1m_2[d_H + 1 + 3]^2 \\
&+ m_1n_2[d_H + n_2 + m_2 + 2 + 1]^2 m_1m_2[n_2 + m_2 + 2 + 3]^2 \\
&= 2m_1[d_G + n_2 + m_2 + 1]^2 + 2m_1m_2[d_H + 4]^2 \\
&+ m_1n_2[d_H + n_2 + m_2 + 3]^2 + m_1m_2[n_2 + m_2 + 5]^3 \\
F[\Re] &\leq 2m_1[\Delta_G + n_2 + m_2 + 1]^2 + 2m_1m_2[\Delta_H + 4]^2 \\
&+ m_1n_2[\Delta_H + n_2 + m_2 + 3]^2 + m_1m_2[n_2 + m_2 + 5]^3.
\end{aligned}$$

Similarly

$$\begin{aligned}
F[\Re] &\geq 2m_1[\delta_G + n_2 + m_2 + 1]^2 + 2m_1m_2[\delta_H + 4]^2 \\
&+ m_1n_2[\delta_H + n_2 + m_2 + 3]^2 + m_1m_2[n_2 + m_2 + 5]^3.
\end{aligned}$$

□

Theorem 2.5. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for M_2-index of $\Re$ is*

$$M_2[\Re] \leq m_1(n_2 + m_2 + 2)[2\Delta_G + n_2(\Delta_H + 1) + 3m_2] + 6m_1m_2(\Delta_H + 1)$$

and

$$M_2[\Re] \geq m_1(n_2 + m_2 + 2)[2\delta_G + n_2(\delta_H + 1) + 3m_2] + 6m_1m_2(\delta_H + 1)$$

Proof. By using table values in definition of M_2 index, we have

$$\begin{aligned}
M_2[\Re] &= 2m_1[d_G(n_2 + m_2 + 2)] + 2m_1m_2[(d_H + 1).3] \\
&+ m_1n_2[(d_H + 1).(n_2 + m_2 + 2)] + m_1m_2[(n_2 + m_2 + 2).3] \\
&= 2m_1d_G(n_2 + m_2 + 2) + 6m_1m_2(d_H + 1) \\
&+ m_1n_2(d_H + 1)(n_2 + m_2 + 2) + 3m_1m_2(n_2 + m_2 + 2) \\
&= (n_2 + m_2 + 2)[2d_Gm_1 + m_1n_2(d_H + 1) + 3m_1m_2] + 6m_1m_2(d_H + 1) \\
&= m_1(n_2 + m_2 + 2)[2d_G + n_2(d_H + 1) + 3m_2] + 6m_1m_2(d_H + 1) \\
M_2[\Re] &\leq m_1(n_2 + m_2 + 2)[2\Delta_G + n_2(\Delta_H + 1) + 3m_2] + 6m_1m_2(\Delta_H + 1).
\end{aligned}$$

Similarly

$$M_2[\Re] \geq m_1(n_2 + m_2 + 2)[2\delta_G + n_2(\delta_H + 1) + 3m_2] + 6m_1m_2(\delta_H + 1).$$

□

Theorem 2.6. *Let $\Re$ be egde $SS-$ Corona graph of G and H, then bounds for EM_1-index of $\Re$ is*

$$\begin{aligned}
EM_1[\Re] &\leq 2m_1[\Delta_G + n_2 + m_2]^2 + 2m_1m_2[\Delta_H + 2]^2 + m_1n_2[\Delta_H + n_2 + m_2 + 2]^2 \\
&+ m_1m_2[n_2 + m_2 + 3]^2
\end{aligned}$$

and

$$EM_1[\Re] \geq 2m_1[\delta_G + n_2 + m_2]^2 + 2m_1m_2[\delta_H + 2]^2 + m_1n_2[\delta_H + n_2 + m_2 + 2]^2$$

$$+ m_1m_2[n_2 + m_2 + 3]^2$$

Proof. By using table values in definition of EM_1 index, we have

$$\begin{aligned} EM_1[\Re] &= 2m_1[d_G + n_2m_2 + 2 - 2]^2 + 2m_1m_2[d_H + 1 + 3 - 2]^2 \\ &+ m_1n_2[d_H + 1 + n_2 + m_2 + 3 - 2]^2 + m_1m_2[n_2 + m_2 + 2 + 3 - 2]^2. \\ &= 2m_1[d_G + n_2 + m_2]^2 + 2m_1m_2[d_H + 2]^2 + m_1n_2[d_H + n_2 + m_2 + 2]^2 \\ &+ m_1m_2[n_2 + m_2 + 3]^2. \\ EM_1[\Re] &\leq 2m_1[\Delta_G + n_2 + m_2]^2 + 2m_1m_2[\Delta_H + 2]^2 \\ &+ m_1n_2[\Delta_H + n_2 + m_2 + 2]^2 + m_1m_2[n_2 + m_2 + 3]^2. \end{aligned}$$

Similarly

$$\begin{aligned} EM_1[\Re] &\geq 2m_1[\delta_G + n_2 + m_2]^2 + 2m_1m_2[\delta_H + 2]^2 + m_1n_2[\delta_H + n_2 + m_2 + 2]^2 \\ &+ m_1m_2[n_2 + m_2 + 3]^2. \end{aligned}$$

□

3. Conclusion

In this work, we establish lower and upper bounds of M_1, M_2, GO_1, F, SK_1 and EM_1 of $\Re$ graph.

Conflicts of interest : We declare that we have no conflict of interest.

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