J. Appl. Math. & Informatics Vol. 42(2024), No. 4, pp. 761 - 775
https://doi.org/10.14317/jami.2024.761

HAMILTONIAN PROPERTIES OF ENHANCED HONEYCOMB NETWORKS

M. SOMASUNDARI*, A. RAJKUMAR, F. SIMON RAJ AND A. GEORGE

Abstract. A cycle in a graph G that contains all of its vertices is said to be the Hamiltonian cycle of that graph. A Hamiltonian graph is one that has a Hamiltonian cycle. This article discusses how to create a new network from an existing one, such as the Enhanced Honeycomb Network $EHC(n)$, which is created by adding six new edges to each layer of the Honeycomb Network $HC(n)$. Enhanced honeycomb networks have $9n^2 + 3n - 6$ edges and $6n^2$ vertices. For every perfect sub-Honeycombe topology, this new network features six edge disjoint Hamiltonian cycles, which is an advantage over Honeycomb. Its diameter is $(2n + 1)$, which is nearly 50% lesser than that of the Honeycomb network. Using 3-bit grey code, we demonstrated that the Enhanced Honeycomb Network $EHC(n)$ is Hamiltonian.

AMS Mathematics Subject Classification : 05C12.
Key words and phrases : Enhanced honeycomb networks, interconnection networks, Hamiltonian graph.

1. Introduction

Interconnection networks are made up of many device types and the connections between them. A network topology is a way to connect one element to another and can take several forms depending on how it was built, including a tree, bus, mesh, star, ring, hypercube, and tori.

Honeycomb is an arrangement that is inspired by nature has been known before by scientists with its structural strengths and studied with them. Long before, people only know the honeycomb pattern from bee honeycombs. After technological inventions like microscopes, scientists found honeycomb structures in different natural formations. For example, Robert Hook found a cork has a

Received August 19, 2023. Revised January 10, 2024. Accepted March 7, 2024. *Corresponding author.

honeycomb-like cellular structure (Zhang et al, 2015) and there are many examples that have the honeycomb-like pattern in nature, such as insect eyes, marine skeletons, snowflakes, turtle and tortoise shells as in corks. Honeycomb has a lightweight, strong and rigid structure that scientists work on since the 20^{th} century (Hales, 2001). The first application of honeycomb is in 1914, Höfler and Renyi patented the first use of honeycombs as a structural element. The use of honeycomb has since grown year after year. Lester and Sandor J. (1985), satellite component research (Boudjemai et al. (2012)), bioengineering (Engelmary et al. 2008), multiprocessor interconnection networks design (Carle et al. (1999); Manuel et al. (2008)), station positioning for cellular phones (Nocetti et al. (2002), and chemical engineering (Rajan et al. (2012)) are just a few fields that use honeycomb applications.

Interconnection networks are generally represented with a graph $G = (V, E)$ where E is a set of edges and V is a set of nodes. A honeycomb pattern can be used as a graph for constructing interconnection networks. In Enhanced Honeycomb networks, each node can be labelled by using n-bit grey codes and consecutive labels differ from each other just in one bit.

In their investigation of the Hamiltonian characteristics of honeycomb meshes made in two different ways, Ayşe Nur Altinas Tankul et al. Using n-bit grey code, he discovered many Hamilton pathways for Honeycomb Meshes in any dimension. In this study, the labelling for the Enhanced Honeycomb networks is done using $3-$bit grey coding.

Hamiltonian properties of Enhanced Honeycomb meshes are investigated here. The Hamiltonian cycle is a path in a graph that passing once through every vertex and return to the starting vertex and the Hamiltonian graph is a graph in which a Hamiltonian cycle exists (Wilson, 1996). Hamiltonian cycles are important for the design of graphs, especially graphs with interconnection network topologies. Hamiltonian properties of graphs have been studied before such as on random graphs (Janson, 1994), interconnection graphs like star graph (Derakhshan and Hussak, 2013), hypercube graph (Karci and Selçuk, 2014 and Selçuk and Karci, 2017) and honeycomb+e (Simon Raj and George, 2015, Stojmenovic, 1997 and Dong et al., 2015).

The paper is organized as follows: Section 2 presented a Literature survey. Algorithms for Enhanced Honeycomb Networks and variants of Honeycomb Networks and Enhanced Honeycomb Networks are given in section 3. Hamiltonian properties of Enhanced Honeycomb Networks are explained in Section 4. In section 5, we have given an algorithm to find the Hamiltonian cycle.

2. Literature survey

Communication techniques and Honeycomb network topological features are presented in [6]. There are many parallel architectures built from meshes, including honeycomb and hexagonal networks. [10] discovered the fewest metric bases for hexagonal and honeycomb networks using the duality of these networks. For honeycomb and hexagonal networks, resolving sets and a few conditional resolving parameters are examined in [9]. Using n-bit grey code, we were able to determine the Hamiltonian properties of honeycomb meshes that are created in two distinct methods, as well as the various Hamiltonian routes for honeycomb meshes in any dimension. We also provided an approach for labelling the nodes of honeycomb meshes[18]. In Honeycomb networks $HC(n)$, [7] shows that any two vertices can be connected via a Hamiltonian path. $HC(n)$ is a maximal non-Hamiltonian graph as a result. We obtain a Hamiltonian cycle by joining the beginning vertex (u) and ending vertex (v) of a Hamiltonian path in $HC(n)$ through an edge. However, for $n \geq 2$, this new edge must cross $n-2$ edges.

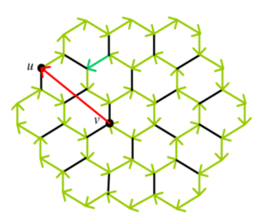

FIGURE 1. A Hamiltonian cycle in $HC(3) + uv$

Hamiltonian cycle in Honeycombe $HC(n) + e$, $n = 2,\ 3,\ 4$ without any edge crossings [15].

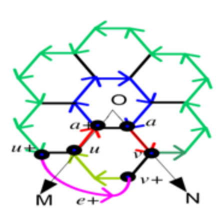

FIGURE 2. A Hamiltonian cycle in $HC(2) + e$

This idea can be extended to $n-$dimensional honeycomb networks, i.e a Hamiltonian cycle can be drawn for n-dimensional Honeycombe $(HC(n) + e)$. Using symmetrical nature, we can draw 5 more similar edges in $HC(n)$, the resulting graph is called the Enhanced Honeycomb network.

Drawing Algorithm for Enhanced Honeycomb Networks:

Input:

Step 1: Draw the Honeycomb Network (HC(n)) of dimension n.

*Step*2: Select a node in the first layer ($u^1_{1,1}$). Construct a path from $u^1_{1,1}$(initial node) to the boundary of the length $2n-2$ in the Zigzag manner (see the figure-5-red colour path). The terminal node is in the boundary with degree 4. Choose the nodes $u^1_{1,1}, u^2_{1,1}, u^3_{1,1} \cdots u^n_{1,1}$ in the path which is at an even distance to the initial node $u^1_{1,1}$. Clearly, $u^{(n)}_{1,1}$ is in the n^{th} layer which is at a distance of $2n-2$ from the initial node.

Step 3: Choose an adjacent node of $u^1_{1,1}$ in layer 1 call it as $u^1_{1,2}$. By repeating the same procedure in step 2, we get $u^1_{1,2}, u^2_{1,2}, u^3_{1,2}, \ldots.., u^n_{1,2}$ which are at distance $2n-2$ from $u^1_{1,2}$. Taking the next adjacent node in layer 1 call it as $u^1_{1,3}$. By repeating the same procedure in step 2, we get $u^1_{1,3}, u^2_{1,3}, u^3_{1,3}, \ldots.., u^n_{1,3}$ which are at distance $2n-2$ from $u^1_{1,3}$. Taking the next adjacent node in the layer 1 call it as $u^1_{1,4}$. By repeating the same procedure in step 2, we get $u^1_{1,4}, u^2_{1,4}, u^3_{1,4}, \ldots.., u^n_{1,4}$ which are at distance $2n-2$ from $u^1_{1,4}$. Taking the next adjacent node in the layer 1 call it as $u^1_{1,5}$. By repeating the same procedure in step 2, we get $u^1_{1,5}, u^2_{1,5}, u^3_{1,5}, \ldots.., u^n_{1,5}$ which are at distance $2n-2$ from $u^1_{1,5}$. Taking the next adjacent node in the layer 1 call it as $u^1_{1,6}$. By repeating the same procedure in step 2, we get $u^1_{1,6}, u^2_{1,6}, u^3_{1,6}, \ldots.., u^n_{1,6}$ which are at distance $2n-2$ from $u^1_{1,6}$.

Step 4: Connect any two nodes $u^n_{(1,i)}$ and $u^n_{(1,j)}$ by an edge if and only if $|i-j| = 1$ *or* 5, $n = 2, 3, 4, \cdots$

Output:

Note 2.1. *In figure* 4, *Green color edges* $\rightarrow$ *edges in the first layer*
Orange color edges $\rightarrow$ *edges in the second layer*
Pink color $\rightarrow$ *edges in the third layer*
Red color $\rightarrow$ *edges in the fourth layer*

3. Variants of Honeycomb and Enhanced Honeycomb networks

In this section, Enhanced Honeycomb Networks are derived from Honeycomb Networks.

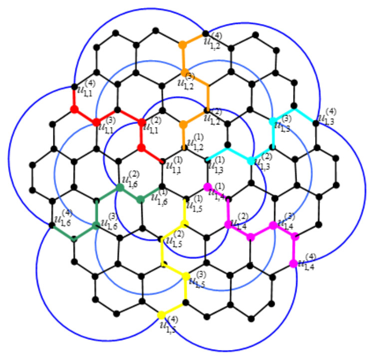

FIGURE 3. Enhanced Honeycomb Network of Dimension 4

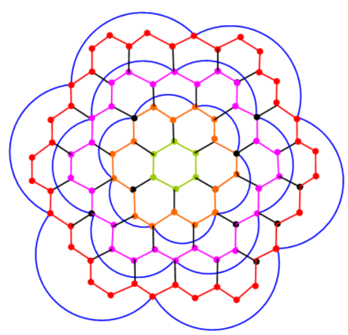

FIGURE 4. Enhanced Honeycomb Network of Dimension 4

FIGURE 5. Honeycomb Network of size two $HC(2)$

The number of vertices and edges of the $HC(n)$ are $6n^2$ and $9n^2 - 3n$, respectively, for $n \geq 1$ [8]. The parameter n of $HC(n)$ is called the dimension of $HC(n)$.

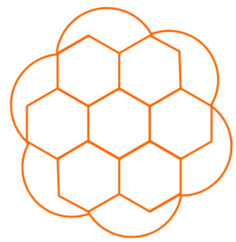

Figure 6. Enhanced Honeycomb Network of size two $EHC(2)$

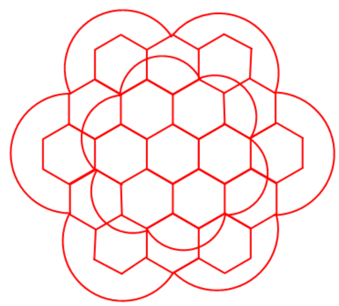

Figure 7. Enhanced Honeycomb Network of size four $EHC(3)$

Enhanced Honeycomb network $EHC(n)$ is derived from Honeycomb network $HC(n)$ by introducing six new edges on each layer. Enhanced Honeycomb Networks has $6n^2$ vertices and $9n^2 + 3n - 6$ edges.

The advantage of this new network over Honeycomb is that it has 6 edge-disjoint Hamiltonian cycles for every perfect sub-Honeycombe topology and diameter is $(2n + 1)$, which is almost 50% of the diameter of the Honeycomb network $4n - 1$. The crossing number of $EHC(n)$ is $3(n-1)(n-2)$ which is less than the crossing number of honeycomb torus. Here perfect Honeycombe topology is a graph derived from the Honeycomb network by introducing 6 new edges on the boundary layer only. This new network also called Mostar Honeycomb or Heightened Honeycomb networks.

4. Hamiltonian Properties of Enhanced Honeycomb Networks

Lemma 4.1. *Enhanced Honeycomb network of dimension two $EHC(2)$ is bipartite and Hamiltonian.*

Proof. From the drawing algorithm of Enhanced Honeycomb Networks, it is easy to see that it has no odd cycle, therefore $EHC(n)$ is bipartite. Here, $EHC(1)$ is a simple cycle which is Hamiltonian. We select all the nodes in layer 1 of $EHC(2)$ and start the cycle from the node "$a_1(110) \rightarrow b_1(100) \rightarrow c_1(010) \rightarrow d_1(001) \rightarrow e_1(101) \rightarrow f_1(111) \rightarrow a_2(110) \rightarrow b_2(100) \rightarrow c_2(010) \rightarrow d_2(001) \rightarrow e_2(101) \rightarrow f_2(111) \rightarrow a_3(110) \rightarrow b_3(100) \rightarrow c_3(010) \rightarrow d_3(001) \rightarrow e_3(101) \rightarrow f_3(111) \rightarrow a_4(110) \rightarrow b_4(100) \rightarrow c_4(010) \rightarrow d_4(001) \rightarrow e_4(101) \rightarrow f_4(111) \rightarrow a_1(110)$" (Figure-7 $- a$ & Figure-7 $- b$). This graph has a Hamiltonian path. Also, EHC is a Hamiltonian graph for $n = 2$. It is seen that $EHC(2)$ is a bipartite graph and Hamiltonian graphs as intuitively.

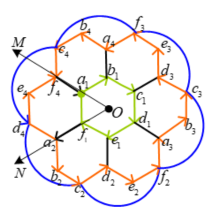

FIGURE 8. $EHC(2)$

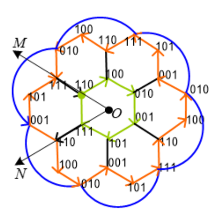

FIGURE 9. $EHC(2)$

Thus, we have found a Hamiltonian cycle in $EHC(2)$ and hence it is Hamiltonian. Using the symmetrical nature of the graph, we can find another five Hamiltonian cycles in $EHC(2)$. For example, the second Hamiltonian Cycle is given by the following closed sequence of nodes.

"$b_1(100) \rightarrow c_1(010) \rightarrow d_1(001) \rightarrow e_1(101) \rightarrow f_1(111) \rightarrow a_1(110) \rightarrow f_4(111) \rightarrow e_4(101) \rightarrow d_4(001) \rightarrow a_2(110) \rightarrow b_2(100) \rightarrow c_2(010) \rightarrow d_2(001) \rightarrow e_2(101) \rightarrow f_2(111) \rightarrow a_3(110) \rightarrow b_3(100) \rightarrow c_3(010) \rightarrow d_3(001) \rightarrow e_3(101) \rightarrow f_3(111) \rightarrow c_4(010) \rightarrow b_4(100) \rightarrow a_4(110) \rightarrow b_1(100)$".

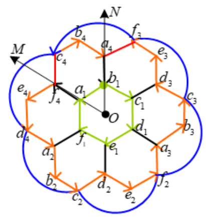

FIGURE 10. $EHC(2)$

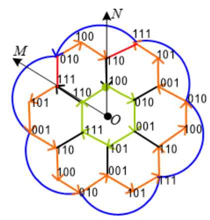

FIGURE 11. $EHC(2)$

We extend this idea to find an Hamiltonian cycle is in n dimensional Enhanced Honeycomb Networks-$EHC(n)$. □

Lemma 4.2. *Enhanced Honeycomb network $EHC(n)$ of dimension n is bipartite.*

Proof. By the drawing algorithm $EHC(n)$, it is clear that it has no odd cycle, therefore, $EHC(n)$ is Bipartite graph. □

Theorem 4.3. *Enhanced Honeycomb networks $EHC(n)$ are Hamiltonian.*

Proof. To prove the theorem, we select the starting node from any node with degree 3 in layer 1 of $EHC(3)$ and $EHC(4)$. We have two different starting nodes:

The starting node has three adjacent nodes with degree 3 which is denoted by a_1,
The starting node has three adjacent nodes with degree 3 which is denoted by b_1.

First strategy: We assume three adjacent of the starting nodes has 3−degree nodes in layer 1 and start the cycle from the node "$a_1(110) \rightarrow b_1(100) \rightarrow c_1(010) \rightarrow d_1(001) \rightarrow e_1(101) \rightarrow f_1(111) \rightarrow a_2(110) \rightarrow b_2(100) \rightarrow c_2(010) \rightarrow d_2(001) \rightarrow e_2(101) \rightarrow f_2(111) \rightarrow a_3(110) \rightarrow b_3(100) \rightarrow c_3(010) \rightarrow d_3(001) \rightarrow e_3(101) \rightarrow f_3(111) \rightarrow a_4(110) \rightarrow b_4(100) \rightarrow c_4(010) \rightarrow d_4(001) \rightarrow e_4(101) \rightarrow$

$f_4(111) \rightarrow a_1(110)$". We walk clockwise and counterclockwise in EHC when the dimension is odd or even. (Figure $8-a$ and Figure $9-a$)

Second strategy: We assume three adjacent of the starting nodes has $3-$degree nodes "$b_1 \rightarrow c_1 \rightarrow d_1 \rightarrow e_1 \rightarrow f_1 \rightarrow a_1$". We walk clockwise and counterclockwise in EHC when the dimension is odd or even. (Figure $8-b$ and Figure $9-b$). By symmetrical nature, we can start c_1, d_1, e_1 *and* f_1 from layer 1.

Now, we proved that Enhanced Honeycomb Network is a Hamiltonian graph. In Enhanced Honeycomb Network $EHC(n)$, the total number of layer edges are $(6n)^2$, the total number of curve edges are $6n-6$ and the total number of wheel edges are $(3n)^2 - 3n$. Therefore, the total number of edges of the $EHC(n)$ are $9n^2 + 3n - 6$. Here, $2(n-1)$ wheel edges will lie in a Hamiltonian cycle HC_1. A Hamiltonian number of a wheel edge is the number of Hamiltonian cycles passing through the wheel edge. Then the number of wheel edges passing through Hamiltonian cycle is $62(n-1)$ in HC_i where $1 \leq i \leq 6$.But in Hamiltonian cycle HC_1, the total numbers of uncovered wheel edges are $3n^2 - 13n + 10$ and layer edges are $2n-1$. In $Figure-8(a)$ and $Figure-8(b)$, we have shown 2 types of Hamiltonian cycle. Generally, we can get 6 Hamiltonian cycles from layer 1.

The total number of layer edges in HC_i are $6n^2 - 6n + 6 + 6n - 6 = 6n^2$.
The total number of wheel edges in HC_i is $6n-6$.
Curve edges in each layer of HC_i is 6.
The total number of uncovered edges in HC_i is $3n^2 - 3n - 6$.

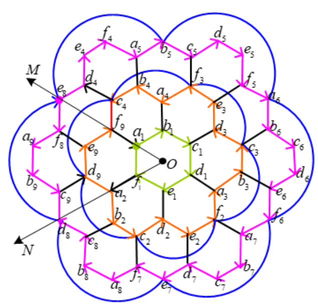

FIGURE 12. Enhanced Honeycomb Network of Dimension 3

$\square$

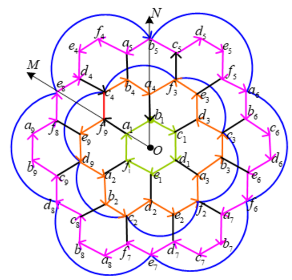

FIGURE 13. Enhanced Honeycomb Network of Dimension 3

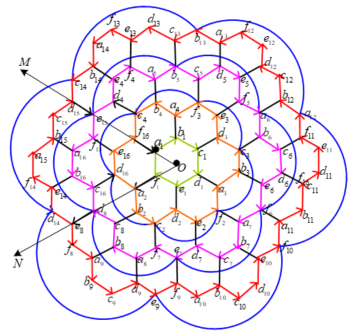

FIGURE 14. Enhanced Honeycomb Network of Dimension 4

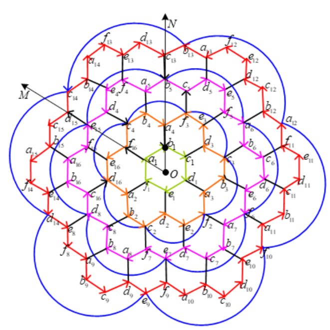

FIGURE 15. Enhanced Honeycomb Network of Dimension 4

Remark 4.4. (1) *Edges in the first layer the Hamiltonian count is* 5.
(2) *Wheel edges connected to the first layer the Hamiltonian cycle is* 2.
(3) *Hamiltonian count for curve edge is* 1.

Theorem 4.5. *The Enhanced Honeycomb network is a Hamiltonian graph for n dimension using* a_{n^2},b_{n^2},c_{n^2},d_{n^2},e_{n^2} *and* f_{n^2} *bit gray code, respectively.*

Proof. Every edge in layer 1 lies in atmost 5 Hamiltonian cycle. (f_1, a_2) lies in atmost 2 Hamiltonian cycle. By symmetrical nature, the other edges are connecting in layer 1 and layer 2 will present in atmost 2 Hamiltonian cycle. In Enhanced Honeycomb Network, curve edges are (c_4, d_4), (d_8, e_8) for dimension 2 and 3. In dimension 1, 2, 3 *and* 4, we have a_{1^2}, b_{1^2}, c_{1^2}, d_{1^2}, e_{1^2} and f_{1^2} bit gray codes, a_{2^2},b_{2^2},c_{2^2},d_{2^2},e_{2^2} and f_{2^2} bit grade codes, a_{3^2},b_{3^2},c_{3^2},d_{3^2},e_{3^2} and f_{3^2} bit gray codes and a_{4^2},b_{4^2},c_{4^2},d_{4^2},e_{4^2} and f_{4^2} bit gray code. For dimension n, the Hamiltonian graph for n dimension using a_{n^2},b_{n^2},c_{n^2},d_{n^2},e_{n^2} and f_{n^2}, bit gray code, respectively. □

An Algorithm to find a Hamiltonian cycle in EHC(n)

In order to build a Hamiltonian cycle in EHC(n) without any edge crossings, we have provided an additional algorithm.

Input:

Step 1: Let c_4 and d_4 denote the neighbor of any two vertices f_4 and a_2 in the clockwise respectively. Let us choose two vertices f_1 and a_1 in the first layer such that the f_1a_1 is in the edge set of $EHC(n)$.

Step 2: Let the center of $EHC(n)$ be O. Draw the axes M and N from O, such that M and N pass through f_1 and a_1 respectively. Clearly $\angle MON = 60°$.

Step 3: Let f_4 and a_2 be the two points of intersection of axes M and N with n^{th} layer, then the distance between f_4 and a_2 is $d(f_4, a_2) = 2n - 1$. If n is odd, then clearly a degree of d_8 and degree of e_8 is equal to four. i.e $d(d_8) = d(de_8) = 4$. If n is even, then $d(f_4) = d(a_2) = 3$.

Step 4: Starting from vertex a_1 (of the odd layer), let's move the cycle in a clockwise direction without crossing the axes M and N.

Step 5: Travel anticlockwise without crossing the axes M and N after reaching the second (even) stratum. Similar to this, move without crossing the axes M and N in the even layer in a clockwise manner and the odd layer in an anticlockwise way. The path must be maintained through edge c_4d_4 when n is even, and edge d_8e_8 when n is odd, after it has passed through the n^{th} layer without crossing the axes M and N. So, in $EHC(n)$, we can always discover a

Hamiltonian cycle.

Output:

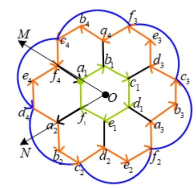

FIGURE 16. Enhanced Honeycomb Network of Dimension 2 $EHC(2)$

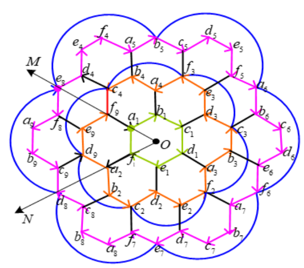

FIGURE 17. Enhanced Honeycomb Network of Dimension 3 $EHC(3)$

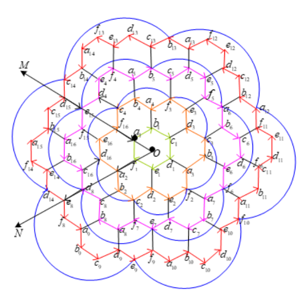

FIGURE 18. Enhanced Honeycomb Network of Dimension 4 $EHC(4)$

5. Result discussion

We have proved that the Enhanced Honeycomb network of dimension two $EHC(2)$ & $EHC(n)$ are bipartite and Hamiltonian. Enhanced Honeycomb networks $EHC(n)$ are Hamiltonian. The Enhanced Honeycomb network is a Hamiltonian graph for n dimension using a_{n^2},b_{n^2},c_{n^2},d_{n^2},e_{n^2} and f_{n^2} bit gray code, respectively.

6. Conclusion

This article deals with deriving a new network such as Enhanced honeycomb network $EHC(n)$ is derived from Honeycomb network $HC(n)$ by introducing six new edges on each layer. Enhanced honeycomb networks has $6n^2$ vertices and $9n^2 + 3n - 6$ edges. The advantage of this new network over Honeycomb is that it has 6 edge disjoint Hamiltonian cycles for every perfect sub-Honeycombe topology, and diameter is $(2n+1)$, which is almost 50% of the diameter of Honeycomb network. We get labeling of nodes in Enhanced Honeycomb networks EHC for any dimensions, using gray codes. Also, we find the upper bound for gray code bit number labeling of EHC.

Conflicts of interest : In this paper, there is no conflict of interest.

Data availability : In this paper, data availability is not applicable.

Acknowledgments : I would like to thank the reviewers.

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M. Somasundari received M. Phil. in Mathematics from Kodaikanal University in 2014. She worked as research assistant at Hindustan Institute of Technology and Science from 2019 to 2024. Her research interests include graph theory and fuzzy graph theory.

Department of Mathematics, Hindustan Institute of Technology and Science, Padur, Chennai, India.
e-mail: `soma.gvt@gmail.com`

A. Rajkumar received M.Sc. from University of Madras and Ph.D. at Hindustan Institute of Technology and Science. His research interests include fuzzy set theory and fuzzy graph theory.

Department of Mathematics, Hindustan Institute of Technology and Science, Padur, Chennai, India.
e-mail: `arajkumar@hindustanuniv.ac.in`

F. Simon Raj received M.Sc. from University of Madras and Ph.D. at Hindustan Institute of Technology and Science. His research interests include graph theory and fuzzy graph theory.

Department of Mathematics for Excellence, Saveetha School of Engineering, Saveetha institute of Medical and Technical Sciences, 602105.
e-mail: `simonrajf.sse@saveetha.com`

A. George received M. Tech. from Anna University and Ph.D. at Indian Institute of Technology Madras. He has been worked as Former Dean Academics, Department of Mathematics and Computer Applications, Periyar Maniammai University. His research interests include fuzzy metric spaces and graph theory.

Department of Mathematics, Periyar Maniammai University, Thanjavur, India.
e-mail: amalanathangeorge@gmail.com