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A NEW CONTRACTION BY UTILIZING $\mathcal{H}$-SIMULATION FUNCTIONS AND Ω-DISTANCE MAPPINGS IN THE FRAME OF COMPLETE G-METRIC SPACES

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Abstract. In this manuscript, we formulate the notion of $\Omega(H,\theta)$-contraction on a self mapping $f : W \to W$, this contraction based on the concept of Ω-distance mappings equipped on G-metric spaces together with the concept of $\mathcal{H}$-simulation functions and the class of Θ-functions, we employ our new contraction to unify the existence and uniqueness of some new fixed point results. Moreover, we formulate a numerical example and a significant application to show the novelty of our results; our application is based on the significant idea that the solution of an equation in a certain condition is similar to the solution of a fixed point equation. We are utilizing this idea to prove that the equation, under certain conditions, not only has a solution as the Intermediate Value Theorem says but also that this solution is unique.

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1. Introduction

Since then, this theorem has been generalized by many mathematicians in two different ways for instance, some of them formulate spaces that are more generalized, such as G-metric spaces, b-metric spaces, and Ω-distance mappings for example see [2], [3], [5], [6], [8], [9] [10], [11], [13], [16], [17], [20], [21], [29]. The others generalized Banach Contraction, for examples [1], [4], [7], [8], [14], [15], [16], [18], [19], [23], [24], [25], [26], [27], [28], [30]. $w \in W$ is a fixed point of a self function f on W if $fw = w$. Also, we call $w \in W$ is a coincidence point of the two self functions $f_1, f_2 : W \to W$ if $f_1 w = f_2 w = \xi$ for some $\xi \in W$ where

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the point ξ is called the point of coincidence. It is obviously that, if $w = \xi$, then w is common fixed point of f_1 and f_2. or Introduction

2. Main results

Before we introduce our main result, it is necessary to introduce the following definitions

Definition 2.1. Suppose (W, d_G), equipped with Ω-distance mappings Ω. We say that W is bounded with respect to Ω if there exists $M > 0$ such that $\Omega(w, w^{'}, w^{''}) \leq M$ for $w, w^{'}, w^{''} \in W$.

Definition 2.2. Suppose (W, d_G), equipped with Ω-distance mappings Ω. We called the self mapping $f : W \to W$ is a $\Omega(H, \theta)$-contraction if there exist $H \in \mathcal{H}$, $\theta \in \Theta$ and $\lambda \in [0, 1)$ such that for all $w, w^{'}, w^{''} \in W$, we have:

$$1 \leq H\Bigg(\theta\Omega(fw, fw^{'}, fw^{''}), \theta\lambda T(w, w^{'}, w^{''})\Bigg), \tag{1}$$

where

$$T(w, w^{'}, w^{''}) = \max\Bigg\{\Omega(w, w^{'}, w^{''}), \Omega(w, fw, fw), \Omega(w^{'}, fw^{'}, fw^{'})\Bigg\}.$$

Lemma 2.3. *If f is $\Omega(H, \theta)$- contraction, and $w, w^{'}, w^{''} \in W$, then:*
1. *If $0 < T(w, w^{'}, w^{''})$, then $\Omega(fw, fw^{'}, fw^{''}) < T(w, w^{'}, w^{''})$,*
2. *If $T(w, w^{'}, w^{''}) = 0$, then $\Omega(fw, fw^{'}, fw^{''})$=0.*

Proof. Assume $T(w, w^{'}, w^{''}) > 0$, then

$$\begin{aligned} 1 &\leq H\Bigg(\theta\Omega(fw, fw^{'}, fw^{''}), \theta\lambda T(w, w^{'}, w^{''})\Bigg) \\ &\leq \frac{\theta\lambda T(w, w^{'}, w^{''})}{\theta\Omega(fw, fw^{'}, fw^{''})}. \end{aligned} \tag{2}$$

Since θ is non-decreasing. We get $\Omega(fw, fw^{'}, fw^{''}) \leq \lambda T(w, w^{'}, w^{''})$. □

Proof. Let $T(w, w^{'}, w^{''}) = 0$, then

$$\begin{aligned} 1 &\leq \theta\Omega(fw, fw^{'}, fw^{''}) \\ &\leq \theta\lambda T(w, w^{'}, w^{''}) \\ &= 1. \end{aligned} \tag{3}$$

This implies that $\Omega(fw, fw^{'}, fw^{''}) = 0$. □

Lemma 2.4. *Suppose (W, d_G), $H \in \mathcal{H}$ and Ω_f be a Ω- distance on W. $f : W \to W$ is called an $\Omega(H, \theta)$- contraction w.r.t $\mathcal{H}$.*
Then Ω_f contains only one element.

Proof. First, we claim that $\forall u \in \Omega_f$. Then $\Omega(u,u,u) = 0$,

$$\begin{aligned} 1 \quad &\leq H\bigg(\theta\Omega(fu,fu,fu), \theta\lambda T(u,u,u)\bigg) \\ &\leq \frac{\theta\lambda T(u,u,u)}{\theta\Omega(fu,fu,fu)} \\ &= \frac{\theta\lambda T(u,u,u)}{\theta\Omega(u,u,u)}. \end{aligned} \tag{4}$$

This means that

$$\begin{aligned} \theta\Omega(u,u,u) \quad &\leq \theta\lambda \max\{\Omega(u,u,u), \Omega(u,fu,fu), \Omega(u,fu,fu)\} \\ &= \theta\lambda\{\Omega(u,u,u)\} \ where \ \lambda \in [0,1). \end{aligned} \tag{5}$$

Since the class Θ is non-decreasing functions so we have:

$$\Omega(u,u,u) < \Omega(u,u,u).$$

A contradiction. Thus, $\Omega(u,u,u) = 0$.
Now, assume $\exists\ u, v \in \Omega_f$ s.t $fu = u,\ fv = v$, since f is $\Omega(H,\theta)$- contraction, we have:

$$\begin{aligned} \theta\Omega(u,u,v) \quad &= \theta\Omega(fu,fu,fv) \\ &\leq \theta\lambda T(u,u,v) \\ &= \theta\lambda \max\{\Omega(u,u,v), \Omega(u,fu,fu), \Omega(u,fu,fu)\} \\ &= \theta\lambda\Omega(u,u,v). \end{aligned} \tag{6}$$

Therefore $\Omega(u,u,v) < \Omega(u,u,v)$,which is contradiction, hence $\Omega(u,u,v) = 0$. Since $\Omega(v,v,v) = 0$ by taking into consideration the definition of Ω- distance mapping condition 3, we have u=v. □

Theorem 2.5. *Assume that (W, d_G) is a complete and equipped with Ω-distance mappings Ω and W is bounded w.r.t Ω. Assume $\exists\ H \in \mathcal{H}$, $\theta \in \Theta$ and $\lambda \in [0,1)$ such that the self mapping $f : W \to W$ is $\Omega(H,\theta)$- contraction.*
If one of the following condition hold:

1- *IF f is continuous,*
2- *$\forall\ u \in W$ if $fu \neq u$, then $\inf\{\Omega(w, fw, u) : w \in W\} > 0$.*

Then Ω_f has only one element.

Proof. construct the sequence (w_n), starting arbitrary by w_0 and $w_n = f^n(w_0)$ for $n \in \mathbb{N}$
By using condition (1), we have

$$1 \quad \leq H\bigg(\theta\Omega(fw_{n-1}, fw_{m-1}, fw_{m-1}), \theta\lambda T(w_{n-1}, w_{m-1}, w_{m-1})\bigg) \tag{7}$$

Now,

$$\begin{aligned}
&\theta\Omega(w_n, w_m, w_m)\\
&= \theta\Omega(fw_{n-1}, fw_{m-1}, fw_{m-1})\\
&\leq \theta\lambda T(w_{n-1}, w_{m-1}, w_{m-1})\\
&= \theta\lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{m-1}), \Omega(w_{n-1}, w_n, w_n), \Omega(w_{m-1}, w_m, w_m)\}.
\end{aligned} \tag{8}$$

Since the class Θ is non-decreasing functions so we have:

$$\begin{aligned}
&\Omega(w_n, w_m, w_m)\\
&\leq \lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{m-1}), \Omega(w_{n-1}, w_n, w_n), \Omega(w_{m-1}, w_m, w_m)\}.
\end{aligned} \tag{9}$$

To prove that $\Omega(w_{m-1}, w_m, w_m) \leq \Omega(w_{n-1}, w_n, w_n)$ for $m > n$, choose m = n+k for some $k \in \mathbb{N}$, then

$$\begin{aligned}
&\theta\Omega(w_{m-1}, w_m, w_m)\\
&= \theta\Omega(w_{n+k-1}, w_{n+k}, w_{n+k})\\
&\leq \theta\lambda \max\{\Omega(w_{n+k-2}, w_{n+k-1}, w_{n+k-1}), \Omega(w_{n+k-1}, w_{n+k}, w_{n+k})\}.
\end{aligned} \tag{10}$$

Therefore,

$$\begin{aligned}
&\Omega(w_{n+k-1}, w_{n+k}, w_{n+k})\\
&\leq \lambda \max\{\Omega(w_{n+k-2}, w_{n+k-1}, w_{n+k-1}), \Omega(w_{n+k-1}, w_{n+k}, w_{n+k})\}\\
&= \lambda\Omega(w_{n+k-2}, w_{n+k-1}, w_{n+k-1})\\
&\leq \lambda^2\Omega(w_{n+k-3}, w_{n+k-2}, w_{n+k-2})\\
&\vdots\\
&\leq \lambda^k\Omega(w_{n-1}, w_n, w_n).
\end{aligned} \tag{11}$$

Thus,

$$\Omega(w_n, w_m, w_m) \leq \lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{m-1}), \Omega(w_{n-1}, w_n, w_n)\}, \tag{12}$$

$$\Omega(w_{n-1}, w_{m-1}, w_{m-1}) \leq \lambda \max\{\Omega(w_{n-2}, w_{m-2}, w_{m-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}.$$

So,

$$\begin{aligned}
&\Omega(w_n, w_m, w_m)\\
&\leq \lambda \max\{\lambda max\{\Omega(w_{n-2}, w_{m-2}, w_{m-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}, \Omega(w_{n-1}, w_n, w_n)\}\\
&= \lambda^2 \max\{\Omega(w_{n-2}, w_{m-2}, w_{m-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1}), \Omega(w_{n-1}, w_n, w_n)\}\\
&= \lambda^2 \max\{\Omega(w_{n-2}, w_{m-2}, w_{m-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}.
\end{aligned} \tag{13}$$

By induction, we get:

$$\Omega(w_n, w_m, w_m) \leq \lambda^n \max\{\Omega(w_0, w_{m-n}, w_{m-n}), \Omega(w_0, w_1, w_1)\}.$$

Since W is bounded w.r.t. Ω. Then $\exists\ M > 0$, such that

$$\Omega(w_n, w_m, w_m) \leq \lambda^n M.$$

Taking the limit as $n \to +\infty$, we get:

$$\lim_{n\to\infty} \Omega(w_n, w_m, w_m) = 0. \tag{14}$$

If $n \leq m \leq l$, then

$$\begin{aligned} &\theta\Omega(w_n, w_m, w_l) \\ &\leq \theta\lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{l-1}), \Omega(w_{n-1}, w_n, w_n), \Omega(w_{m-1}, w_m, w_m)\} \\ &= \theta\lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{l-1}), \Omega(w_{n-1}, w_n, w_n). \end{aligned} \quad (15)$$

Thus,

$$\Omega(w_n, w_m, w_l) \leq \lambda \max\{\Omega(w_{n-1}, w_{m-1}, w_{l-1}), \Omega(w_{n-1}, w_n, w_n).$$

$$\begin{aligned} &\Omega(w_{n-1}, w_{m-1}, w_{l-1}) \\ &\leq \lambda \max\{\Omega(w_{n-2}, w_{m-2}, w_{l-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1}), \Omega(w_{m-2}, w_{m-1}, w_{m-1})\} \\ &= \lambda \max\{\Omega(w_{n-2}, w_{m-2}, w_{l-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}. \end{aligned} \quad (16)$$

So,

$$\begin{aligned} &\Omega(w_n, w_m, w_l) \\ &\leq \lambda \max\{\lambda \max\{\Omega(w_{n-2}, w_{m-2}, w_{l-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}, \Omega(w_{n-1}, w_n, w_n)\} \\ &\leq \lambda^2 \max\{\Omega(w_{n-2}, w_{m-2}, w_{l-2}), \Omega(w_{n-2}, w_{n-1}, w_{n-1})\}. \end{aligned} \quad (17)$$

By induction , we get:

$$\Omega(w_n, w_m, w_l) \leq \lambda^n \max\{\Omega(w_0, w_{m-n}, w_{l-n}), \Omega(w_0, w_1, w_1)\}.$$

Since W is bounded w.r.t. Ω. Then $\exists\ M > 0$, such that

$$\Omega(w_n, w_m, w_l) \leq \lambda^n M.$$

Taking the limit as n approach to infinity

$$\lim_{n\to\infty} \Omega(w_n, w_m, w_l) = 0.$$

Then (w_n) is a G- Cauchy sequence since (W, d_G) is complete. Then $\exists\ \beta \in W$ such that $w_n \to \beta$.

- If f is a continuous function, then $w_{n+1} = f_{w_n} \to f_\beta = \beta$.
- If f is any mapping, by utilizing the lower semi continuity of Ω, then

$$\begin{aligned} \Omega(w_n, w_m, u) \quad &\leq \liminf_{p\to\infty} \Omega(w_n, w_m, w_p) \\ &< \epsilon. \end{aligned} \quad (18)$$

Since $fu \neq u$. We have for every $\epsilon > 0$, we get

$$\begin{aligned} 0 \quad &< \inf\{\Omega(w, fw, u) : w \in W\} \\ &\leq \inf\{\Omega(w_n, w_{n+1}, u) : n \in \mathbb{N}\} \\ &< \epsilon. \end{aligned} \quad (19)$$

Which is a contraction.
Thus $fu = u$ the uniqueness of u follows from lemma 2.1.
Which complete the prove. □

Corollary 2.6. *Assume* (W, d_G) *is a complete and equipped with* Ω*-distance mappings* Ω *and* W *is bounded with respect to* Ω. *Assume* $\exists\ \gamma \in [0,1)$ *and self mapping* $f : W \to W$ *satisfy the condition* $\forall\ w, w^{'}, w^{''} \in W$, *we have:*

$$1 \le e^{\gamma T(w,w^{'},w^{''}) - \Omega(fw,fw^{'},fw^{''})}.$$

If one of the following condition satisfy:

1- f *is continuous mapping,*
2- $\forall\ u \in W$ *if* $fu \neq u$, *then* $\inf\{\Omega(w, fw, u) : w \in W\} > 0$.

Then Ω_f *has only one element.*

Proof. Define $H : [0, +\infty) \times [0, +\infty) \to [1, +\infty)$ via $H(\frac{w^{'}}{w}) = \frac{(w^{'})^{\lambda}}{w}$,
and $\theta : [0, +\infty) \to [1, +\infty)$ via $\theta(w) = e^{w}$.
Now

$$\begin{aligned} 1 &\le e^{\gamma T(w,w^{'},w^{''}) - \Omega(fw,fw^{'},fw^{''})} \\ &= \frac{e^{\gamma T(w,w^{'},w^{''})}}{e^{\Omega(fw,fw^{'},fw^{''})}}. \end{aligned} \tag{20}$$

If $\gamma = \lambda^2$, then $\lambda \in [0, 1)$, so

$$\begin{aligned} 1 &\le \frac{e^{\lambda^2 T(w,w^{'},w^{''})}}{e^{\Omega(fw,fw^{'},fw^{''})}} \\ &= \frac{e^{\lambda T(w,w^{'},w^{''})^{\lambda}}}{e^{\Omega(fw,fw^{'},fw^{''})}} \\ &= H\bigg(\theta\Omega(fw, fw^{'}, fw^{''}), \theta\lambda T(w, w^{'}, w^{''})\bigg). \end{aligned} \tag{21}$$

□

Corollary 2.7. *Assume* (W, d_G) *is a complete and equipped with* Ω*-distance mappings* Ω *and* W *is bounded with respect to* Ω. *Assume* $\exists\ \gamma \in [0,1)$ *and self mapping* $f : W \to W$ *satisfy the condition*
$\forall\ w, w^{'}, w^{''} \in W$, *we have*

$$\Omega(fw, fw^{'}, fw^{''}) \le \gamma T(w, w^{'}, w^{''}).$$

If one of the following condition satisfy:

1- f *is continuous mapping,*
2- $\forall\ u \in W$ *if* $fu \neq u$, *then* $\inf\{\Omega(w, fw, u) : w \in W\} > 0$.

Then Ω_f *has only one element.*

Proof. By using corollary 3.1, we have:

$$1 \le e^{\gamma T(w,w^{'},w^{''}) - \Omega(f_w,fw^{'},fw^{''})} = \frac{e^{\lambda T(w,w^{'},w^{''})^{\lambda}}}{e^{\Omega(fw,fw^{'},fw^{''})}}. \tag{22}$$

So, we get:

$$e^{\Omega(fw,fw',fw'')} \leq e^{\lambda T(w,w',w'')^{\lambda}}.$$

Applying the logarithm on both side, we get:
$\Omega(fw,fw',fw'') \leq \lambda^2 T(w,w',w'') = \gamma T(w,w',w'')$. □

Example 2.8. Consider the following mapping:

$$fw = 1 - \frac{w^n}{D + w^n} \text{ , } where\ n \in \mathbb{N}\ and\ M > n.$$

Then Ω_f consists of only one element on $[0,1]$.
To show this, let $W = [0,1]$ and define the following mappings:
$H : [1,+\infty) \times [1,+\infty) \to \mathbb{R}$, $\theta : [0,+\infty) \to [1,+\infty)$ by $H(v_1, v_2) = 1 + \ln\left(\frac{v_2}{v_1}\right)$, $\theta(v) = e^v, \forall v \in R$ respectively, then $H \in \mathcal{H}\ and\ \theta \in \Theta$.
Also, define: $d_G : W \times W \times W \to [0,+\infty)$ via $d_G(w,w',w') = |w - w'| + |w' - w''| + |w - w''|$, then (W, d_G) is a complete G-metric space.
Furthermore, define $\Omega : W \times W \times W \to [0,+\infty)$ via $\Omega(w,w',w'') = |w - w'| + |w - w''|$, Ω is a Ω-distance mapping.
Now, equip (W, d_G) with Ω, for all $w, w', w' \in W$, we have:

$$\begin{aligned}
&\Omega(fw, fw', fw'') \\
&= |fw - fw'| + |f - fw''| \\
&= |1 - \frac{(w)^n}{D+(w)^n} - (1 - \frac{(w')^n}{D+(w')^n})| + |1 - \frac{(w)^n}{D+(w)^n} - (1 - \frac{(w'')^n}{D+(w'')^n})| \\
&\leq \frac{1}{D^2}\Big[|(w^n)(D+(w')^n) - (w')^n(D+w^n)| \\
&+|(w^n)(D+(w'')^n) - (w']^n(D+(w')^n)|\Big] \\
&= \frac{1}{D}\Big[|(w)^n - (w')^n| + |w^n - (w'')^n|\Big] \\
&= \frac{1}{D}\Big[|(w)-(w')|((w)^{n-1} + (w')(w)^{n-2} + \cdots + (w)(w')^{n-2} + (w')^{n-1}) \\
&\quad +|(w)-(w'')|(w)^{n-1} + (w'')(w)^{n-2} + \cdots + (w)(w'')^{n-2} + (w'')^{n-1})\Big] \\
&\leq \frac{n}{D}[|(w)-(w')| + |(w)-(w')|]
\end{aligned}$$

$$= \lambda\Omega(w, w^{'}, w^{''}) \ with \ \lambda = \frac{n}{D}$$

$$\leq \lambda T(w, w^{'}, w^{''}).$$

Now,

$$\Omega(fw, fw^{'}, fw^{''}) \leq \lambda T(w, w^{'}, w^{''}),$$

iff

$$e^{\Omega(fw, fw^{'}, fw^{''})} \leq e^{(\lambda Tw, w^{'}, w^{''})},$$

iff

$$1 \leq \frac{e^{\lambda T(w, w^{'}, w^{''})}}{e^{\Omega(fw, fw^{'}, fw^{''})}},$$

iff

$$1 \leq 1 + \ln\left(\frac{e^{\lambda T(w, w^{'}, w^{''})}}{e^{\Omega(fw, fw^{'}, fw^{''})}}\right),$$

iff

$$1 \leq H\bigg(\theta\Omega(fw, fw^{'}, fw^{''}, \theta\lambda T(w, w^{'}, w^{''})\bigg).$$

Hence, f satisfy the conditions of $\Omega(\mathcal{H}, \theta)$-contraction.
Theorem 2.5 ensures that Ω_f has only one element.

3. Application

In this application, we will prove that the following equation:

$$w^{n+1} + Dw + D = 0, where\ D > n\ and\ n \in \mathbb{N}, \tag{23}$$

has not only a solution in the unit interval $[0, 1]$ as intermediate value theorem, but also, the solution is unique.
To prove this, it is equivalent to prove that the following mapping has a unique fixed point in the unit interval $[0, 1]$.

$$f(w) = 1 - \frac{w^n}{D + w^n}, where\ D > n\ and\ n \in \mathbb{N}.$$

Example 2.8 Ensures that f has a unique fixed point and so the equation 23 has a unique solution.

Example 3.1. If $n = 99$ and $D = 100$ in Equation 3 by utilizing MATLAB simulator we get that the fixed point of f is $w = 0.97598925061853$ and so, it is the unique solution of the Equation 23.

4. Conclusion

In this study, we unify some significant new fixed point results based on our contraction namely, $\Omega(H,\theta)$-contraction. We formulated some numerical examples and a significant application to show the novelty of our results. This application based on the significant idea that the solution of a equation in a certain conditions is typical to solution of fixed point equation. we utilized this idea to prove that this equation not only has solution as the Intermediate value Theorem says but also, this solution is unique.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : The data used to support the findings of this study are included within the article.

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