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Technical Note

A study on the uncertainty of setpoint for reactor trip system of NPPs considering rectangular distributions

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ABSTRACT

The setpoint of the reactor trip system shall be set to consider the measurement uncertainty of the instrument channel and provide a reasonable and sufficient margin between the analytical limit and the trip setpoint.

A comparative analysis was conducted to find out an appropriate uncertainty combination method through an example problem. The four methods were evaluated; 1) ISA-67.04.01 method, 2) the GUM95 method, 3) the modified GUM method developed by Fotowicz, and 4) the modified IEC61888 method proposed by authors for the pressure instrument channel presented in ISA-RP67.04.02 example. The appropriateness of each method was validated by comparing it with the result of Monte Carlo simulation.

As a result of the evaluation, all methods are appropriate when all measurement uncertainty elements are normally distributed as expected. But ISA-67.04 method and GUM95 method overestimated the channel uncertainty if there is a dominant input element with rectangular distribution among the uncertainty input elements.

Modified GUM95 methods developed by Fotowicz and modified IEC61888 method by authors are able to produce almost the same level of channel uncertainty as the Monte Carlo method, even when there is a dominant rectangular distribution among the uncertainty components, without computer-assisted simulations.

1. Introduction

The reactor trip system of a nuclear power plant has trip setpoints that trigger the automatic reactor trip when the plant is in an abnormal state. The setpoint of the reactor trip system shall be set to consider the measurement uncertainty of the instrument channel and provides a sufficient margin between the analytical limit and the trip setpoint. However, if the trip setpoint is set too conservatively it can reduce safety and operability by initiating an unnecessary reactor trip or limiting plant operation. Therefore, the trip setpoint is set considering the measurement uncertainty of the instrument channel appropriately.

The current uncertainty for the reactor trip system setpoints in nuclear power plants complies with the uncertainty combination methodology of IEC 61888 [1] and ISA67.04–01 [2]. The uncertainty evaluation for calibration of instruments important to the reactor trip system of nuclear power plants secures traceability through a calibration certificate, in which the calibration certification authorities use the method of GUM95 [3].

The National Metrology Institute ensures equivalence and traceability of international measurement standards under the International Metrology Commission Mutual Recognition Convention (CIPM MRA), mutually recognizes measurement results in international trade, and the calibration certification body applies the International Laboratory Accreditation Cooperation (ILAC) calibration uncertainty policy [4], ISO/IEC 17025 [5] and IEC 115 [6].

Generally, it is known that the uncertainty combination methods in IEC 61888, ISA67.04–01, and GUM95 result in accurate channel uncertainty if all input elements have a normal distribution, but do not result in accurate uncertainty for the non-normal distribution. GUM95 suggested that Monte Carlo Method should be used if the uncertainty of the non-normal distribution is large compared with that of the normal distribution [3]. Fotowicz suggested a modified k-factors to overcome the over-estimation of GUM95 when combining rectangular distributions with large standard deviation compared to that of the normal distributions [9]. Moszczynsky et al. used the convolution method to find the combined distribution of a rectangular distribution and a normal distribution and calculated the k factor using an excel sheet. This method was verified by the use of MCM [10]. Dietrich also showed the combined distribution of a rectangular distribution and a normal

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Nomenclature

CLT	Central Limit Theorem
DR	Drift
EE	Environmental Effect
MTE	Measurement and Test Equipment
NPP	Nuclear Power Plant
PS	Power Supply
RA	Reference Accuracy
SRSS	Square Root of the Sum of the Squares
TE	Temperature Effect

distribution using convolution technique which is an analytical method [16].

To calculate the uncertainty of the system, the information on the probability distribution functions (pdfs), means, and standard deviations of the input elements are required. This information is usually provided by the manufactures. But for some input elements only upper and lower limits are given instead of the pdfs. For this case pdf should be determined based on the information given. If the only information provided is the upper and lower limits, it is proved that the uniform distribution is the most appropriate distribution by the principle of maximum entropy [7,8]. GUM95 also states that a rectangular distribution can be used as a probability density function if useful information for the input term is given only upper and lower limits [3]. Drift can be a possible example. As the number of years of operation of nuclear power plants has increased, drift data from safety system instruments has been accumulated, and there has been a movement in the industry to use this 95/95 drift value to calculate setpoints, and revisions to regulatory documents to reflect this are also in progress [12]. Appendix A provides how rectangular distribution is developed by the principle of maximum entropy [7] where the only information given is the upper and lower limit.

The analytic convolution of normal distribution and rectangular distribution is not easy for hand calculation and some methods have been proposed to resolve this difficulty [9–11].

In this paper, the modified IEC61888 method is proposed and evaluated with the four methods presented in References 1, 2, 3, and 9. The measurement uncertainty of the instrument channel was calculated through example, and then the Monte Carlo method verified the selected methods which one among them is acceptable.

2. Uncertainty evaluation method

2.1. Method used in nuclear power plant

The method of calculating the uncertainty of the instrument channel used in the nuclear power plant is described in References [1,2]. The trip system setpoints for nuclear power plants were evaluated in accordance with the methodology of IEC 61888 [1]. In this methodology, uncertainty is combined by the SRSS (Square Root of Sum of Square) term for elements with normal uncertainty, and the elements with non-normal distribution are arithmetically combined to the SRSS term. In other words, the combining formula for uncertainty is represented by Eq. (1).

$$\begin{aligned} CU_{61888} &= \sqrt{(A^2+B^2)}+C \\ &= (A,B)_{SRSS}+C \\ \text{where } (A,B)_{SRSS} &\equiv \sqrt{(A^2+B^2)} \end{aligned} \qquad \text{(Eq. 1)}$$

CU_{61888} is the channel uncertainty, and the sign of CU is ± . A and B are the uncertainty of the input elements constituting the channel uncertainty, which is random, independent, and has the characteristics of a normal distribution. Bias or dependent uncertainty is expressed as C. However, in Eq. (1), it is unclear how to deal with the uncertainty of the input element that is random and independent but does not have the characteristics of a normal distribution (e.g., a rectangular distribution).

To clear this ambiguity, ISA 67.04.01 [2], which was endorsed by U.S.NRC through RG 1.105 [12], further subdivided term C of Eq. (1) to reflect non-normal distributions as terms F, L, and M in Eq. (2).

$$\begin{aligned} CU_{67.04} &= \sqrt{A^2+B^2+C^2+(D+E)^2}+|F|+L-M \\ &= (A,B,C,(D+E))_{SRSS}+|F|+L-M \\ \text{where } & (A,B,C,(D+E))_{SRSS} \equiv \sqrt{A^2+B^2+C^2+(D+E)^2} \end{aligned} \qquad \text{(Eq. 2)}$$

and where $CU_{67.04}$ is the channel uncertainty, where the sign of CU is ±.

A, B, C = random, independent, and approximately normally distributed terms
D and E = random dependent uncertainty terms that are independent of terms A, B, and C
F = non-normally distributed uncertainties and/or biases (unknown sign)
L & M = biases with the known sign

It is important to note that, unlike GUM95, the value of each term in Eq. (2) is not a standard uncertainty, but an uncertainty value with 95 % of confidence level. This means that a coverage factor in GUM95 is reflected on each element in SRSS term and F term separately.

2.2. GUM95 methodology

The methodology described in 'Guide to the expression of Uncertainty in Measurement (GUM95)' can be summarized in the following major steps.

2.2.1. Estimation of uncertainty of input elements

This step estimates the uncertainty for all input elements. According to GUM95, uncertainty can be basically classified into two types: Type A and Type B uncertainty. Type A uncertainty is an uncertainty that deals with the source of uncertainty from the standard deviation obtained by repeated measurements, and estimated as the standard deviation of a mean obtained from repeated measurements. Type B uncertainty is an uncertainty determined from other sources of information, based on careful analysis through observation or accurate scientific judgment using all available information about the measurement procedure.

2.2.2. Combined standard uncertainty

The GUM95 uncertainty framework is a widely used method for evaluating the uncertainty of a measurement. It is based on the law of propagation of uncertainty (LPU), which states that the uncertainty of a measurement can be determined by analyzing the uncertainties of the individual inputs that contribute to the measurement.

To calculate the combined standard uncertainty, the GUM95 framework extends the measurement model to the Taylor series and simplifies the expression by considering only the first-order term. This approximation is valid because the uncertainty is typically a small value compared to the measurement.

In this way, a model in which the measured value, y, is expressed as a function of N variables, $x_1,\ldots,x_N,$ can be represented by the following general expression for the propagation of uncertainty:

$$y = f(x_1,\ldots,x_N) \qquad \text{(Eq. 3)}$$

$$u_c^2 = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\frac{\partial f}{\partial x_i}\right)\left(\frac{\partial f}{\partial x_j}\right)\mathrm{cov}(x_i,x_j) \qquad \text{(Eq. 4)}$$

where, u_c is the combined standard uncertainty of measurand y, and x_i is the uncertainty of the i_{th} input element. If no correlation exists between input elements, the second term in Eq. (4) is deleted as shown Eq. (5).

$$u_c^2 = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u_{x_i}^2 \quad \text{(Eq. 5)}$$

If the measurand y is linear to the input element x_i, such as $y = x_1 + x_2 + \ldots + x_N$ and, the combined uncertainty u_c for the measurand y can be expressed as Eq. (6).

$$u_c = \sqrt{u_{x_1}^2 + u_{x_2}^2 + \ldots + u_{x_N}^2} \quad \text{(Eq. 6)}$$

2.2.3. Estimation of a coverage factor

The value calculated by Eq. (6) corresponds to one standard deviation (inclusive rate of about 68.2 %) interval value. To obtain the confidence level we want, we assume that the GUM95 method follows Student's t-distribution. The effective degree of freedom v_{eff} for t-distribution can be calculated using the Welch-Satterthwaite formula.

$$v_{eff} = \frac{u_y^4}{\sum_{i=1}^{N} \frac{u_{x_i}^4}{v_{x_i}}} \quad \text{(Eq. 7)}$$

where v_{x_i} is the degree of freedom for the i_{th} input element. The coverage factor (k_p) corresponding v_{eff} can be found in Table G.2 of GUM95 [3]. Usually, a coverage factor value corresponding to 95 % of the confidence level is used.

2.2.4. Expanded uncertainty

Expanded uncertainty U_p can be obtained by multiplying the coverage factor k_p to the combined standard uncertainty u_c. It has the sign ±.

$$U_p = k_p \cdot u_c \quad \text{(Eq. 8)}$$

The GUM95 method is based on the central limit theorem (CLT). When a large number of distributions are combined, the resultant distribution approximately follows a normal distribution. If the distribution of input element is asymmetric or one or more input elements which are not normally distributed have significantly larger standard deviations than other input elements, CLT may not be applicable. In addition, the GUM95 method is not appropriate for the nonlinear model. And the effectiveness of the Welch-Satterthwaite formula which is used to calculate the effective degree of freedom of mixture of Type A data and Type B data is still controversial [14]. In this case, it is recommended using the Monte Carlo method as an alternative.

2.3. Fotowicz methodology

The GUM95 method may not be valid as mentioned above if one or more input elements have a significantly large value than the other input elements, which are non-normally distributed. Fotowicz proposed a method to obtain the coverage factor, especially when the distribution of input elements with large values is rectangular distribution [9]. The method proposed by Fotowicz is as follows.

2.3.1. Combined standard uncertainty

The combined standard uncertainty is obtained as the same way of the GUM95 [3] method, SRSS by Eq. (6).

2.3.2. Parameter r_u

Fotowicz introduced the parameter r_u to reflect the influence of the input element having a rectangular distribution.

$$r_u = \frac{|u_k(y)|}{\sqrt{u_c^2(y) - u_k^2(y)}} \quad \text{(Eq. 9)}$$

where $u_k(y)$ is the largest contribution of the input quantity having a rectangular distribution.

2.3.3. Coverage factor k_{RN}

Coverage factor k_{RN} depends on r_u value. The detail information on r_u and k_{RN} can be found in Ref. [9]. The coverage factor k_{RN} is value corresponding to 95 % of the confidence level of uncertainty.

2.3.4. Expanded uncertainty

Expanded uncertainty can be obtained by multiplying the coverage factor k_{RN} to the combined standard uncertainty.

$$U_p = k_{RN} \cdot u_c \quad \text{(Eq. 10)}$$

2.4. Modified IEC61888 method

To overcome the deficiency of IEC61888 method which results in the conservative result when the non-normally distributed inputs exist, we proposed a modified form of the uncertainty combination method used in IEC61888 as follows:

$$\begin{aligned} \mathrm{CU}_{M-61888} &= \lambda \cdot \sqrt{A^2 + B^2 + C^2 + (D+E)^2 + \sum_{i=1}^{N} {F_i}^2} \\ &= \lambda \cdot \left(A, B, C, (D+E), \sum_{i=1}^{N} F_i\right)_{SRSS} \end{aligned} \quad \text{(Eq. 11)}$$

where λ = compensation factor which considers the contribution of the rectangular distribution and depends on r_{RN}

r_{RN} = ratio of uncertainties of rectangular distributions to those of normal distributions

$$r_{RN} = \sqrt{\frac{\sum_{i=1}^{N} {F_i}^2}{A^2 + B^2 + C^2 + (D+E)^2}} \quad \text{(Eq. 12)}$$

where F_i = rectangularly distributed uncertainty with the confidence level of 95 %

Other terms are same as defined in Eq. (2).

The compensation factor λ is given Table 1 and can be illustrated as Fig. 1. This λ value is obtained by comparing the uncertainty value calculated by Eq. (11) with the value obtained by Monte Carlo simulation.

3. Example problem and uncertainty data

The example problem used in this paper is taken from Annex L of ISA-RP67.04.02–2010 [15], which aims to be familiar to nuclear power plant (NPP) industry users through a sample uncertainty calculation of NPP trip setpoint. The example channel consists of two modules. Module 1 is a pressure transmitter, and Module 2 is a bistable. A power supply is common to Module 1 and Module 2.

Table 1
Compensation factor λ.

r_{RN} up to value	λ	r_{RN} up to value	λ	r_{RN} up to value	λ
0.06	1.00	0.6	1.05	3.4	1.04
0.2	1.01	0.8	1.06	4.0	1.03
0.3	1.02	1.7	1.07	5.4	1.02
0.4	1.03	2.2	1.06	10.0	1.01
0.5	1.04	2.6	1.05	∞	1.00

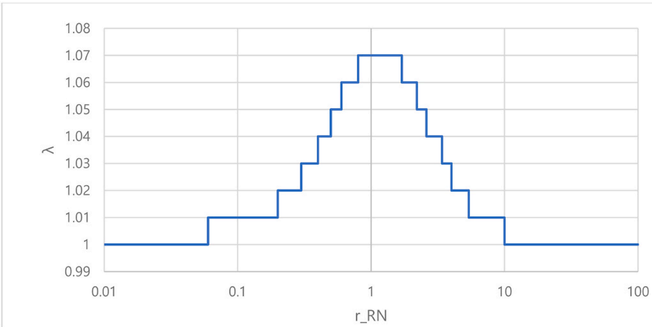


Fig. 1. Compensation factor λ.

3.1. *Example problem*

High pressure trip of the containment building triggers one of several inputs of the steam isolation logic in mitigation for a high energy pipe break accident.

3.2. *Uncertainty data*

The uncertainty for each input element used in the example problem is summarized in Table 2. The standard uncertainty was obtained by dividing the uncertainty value in Table 2 by the coverage factor of 1.96 which corresponds to the 95 % confidence level of normal distribution. If the distribution is a rectangular one, the standard uncertainty was obtained by dividing a semi-range by $\sqrt{3}$.

Channel uncertainties were calculated using different methods for the following three cases.

Case 1: All input elements have normal distributions,
Case 2: EE_1, DR_1, and DR_2 have rectangular distributions and the others have normal distributions,
Case 3: EE_1, DR_1, and DR_2 have rectangular distributions and the others have normal distributions. The magnitude of EE_1 is reduced to moderate value, i.e., 0.5 (1/10 of the original value).

4. Evaluation of uncertainty

4.1. *Case 1: all input elements have normal distributions*

4.1.1. *ISA67.04 (IEC 61888) method*

Uncertainty is obtained for module 1 and module 2, respectively, by the SRSS method, and the channel uncertainty is the combination of uncertainties of module 1 and module 2. The uncertainty of module 1 and module 2 can be calculated as follows. The unit of pressure uncertainty in ISA RP 67.04.02 [15] is psi.

$$
\begin{aligned}
e_1 &= \sqrt{RA_1^2 + \overleftrightarrow{DR_1^2} + \overleftrightarrow{EE_1^2} + PS_1^2 + MTE_1^2} \\
&= \left(0.750,\ \overleftrightarrow{0.200},\ \overleftrightarrow{5.0},\ 0.015,\ 0.375\right)_{SRSS} \\
&\cong 5.0738
\end{aligned}
$$

$$
\begin{aligned}
e_2 &= \sqrt{RA_2^2 + \overleftrightarrow{DR_2^2} + TE_2^2 + PS_2^2 + MTE_2^2} \\
&= \left(0.1875,\ \overleftrightarrow{0.3750},\ 0.0075,\ 0.0150,\ 0.3750\right)_{SRSS} \\
&\cong 0.5627
\end{aligned}
$$

The channel uncertainty CU is

$$
\begin{aligned}
CU_{67.04} &= \sqrt{e_1^2 + e_2^2} \\
&= (5.0738,\ 0.5627)_{SRSS} \cong 5.1049
\end{aligned}
$$

4.1.2. *GUM95 method*

The combined standard uncertainty is calculated.

$$
\begin{aligned}
u_c &= \left(RA_1^{SD},\ \overleftrightarrow{DR_1^{SD}},\ \overleftrightarrow{EE_1^{SD}},\ \dots,\ TE_2^{SD},\ PS_2^{SD},\ MTE_2^{SD}\right)_{SRSS} \\
&= \left(0.383,\ \overleftrightarrow{0.102},\ \overleftrightarrow{2.551},\ \dots,\ 0.0038,\ 0.0077,\ 0.1913\right)_{SRSS} \\
&\cong \sqrt{6.7836} \cong 2.6045
\end{aligned}
$$

where X^{SD} is the standard deviation of input element X.

Effective degree of freedom can be calculated using Eq. (7). Since u_{x_i} is a Type B uncertainty with known probability distribution, ν_{x_i} is ∞, and accordingly, ν_{eff} is infinity. In Table G.2 of GUM95 [3], where $\nu_{eff} = \infty$, the coverage factor k_p corresponding to the 95 % of confidence level is 1.96. i.e., $k_p = 1.96$. Expanded uncertainty U_p, can be obtained as $U_p = k_p u_c(y)$;

$$U_{p-GUM95} = (1.96)\cdot(2.6045) \cong 5.1049$$

This result is the same as the value calculated in 4.1.1.

4.1.3. *Fotowicz method*

Since the method of obtaining the combined standard uncertainty u_c is the same as the GUM95 method, $u_c = 2.6$. Since there is no rectangular distribution, $r_u = 0$ from Eq. (9), and $k_{RN} = 1.96$ for $r_u = 0$ in Table 1 of Fotowicz [9].

Expanded uncertainty $U_p = k_{RN}\, u_c(y)_p$ is

Table 2
Uncertainty and probability density function for each input element.

Input element	Case 1			Case 2			Case 3		
	Uncertainty	Distribution[a]	Standard Uncertainty	Uncertainty/ Semi-range[b]	Distribution[a]	Standard Uncertainty	Uncertainty/ Semi-range[b]	Distribution[a]	Standard Uncertainty
Module 1									
RA_1	0.75	N	0.3827	0.75	N	0.3827	0.75	N	0.3827
DR_1	0.2	N	0.1020	0.2	R	0.1155	0.2	R	0.1155
EE_1	5.0	N	2.5510	5.0	R	2.8868	0.5	R	0.2887
PS_1	0.015	N	0.0077	0.015	N	0.0077	0.015	N	0.0077
MTE_1	0.375	N	0.1913	0.375	N	0.1913	0.375	N	0.1913
Module 2									
RA_2	0.1875	N	0.0957	0.1875	N	0.0957	0.1875	N	0.0957
DR_2	0.375	N	0.1913	0.375	R	0.2165	0.375	R	0.2165
TE_2	0.0075	N	0.0038	0.0075	N	0.0038	0.0075	N	0.0038
PS_2	0.015	N	0.0077	0.015	N	0.0077	0.015	N	0.0077
MTE_2	0.375	N	0.1913	0.375	N	0.1913	0.375	N	0.1913

[a] N: Normal distribution, R: Rectangular Distribution.
[b] If the distribution is a rectangular one, then the value corresponds to the semi-range.

$U_{p-Fotowicz} = (1.96)\cdot(2.6045) \cong 5.1049$

4.1.4. Modified IEC61888 method

Uncertainty was obtained by the SRSS method, and the channel uncertainty is obtained by multiplying λ to the uncertainty obtained by the SRSS method. For this case, $r_{RN} = 0$ and corresponding λ is 1 from Table 1.

$$\begin{aligned} CU_{M-61888} &= \lambda\cdot\left(RA_1, \overleftrightarrow{DR_1}, \overleftrightarrow{EE_1}, \ldots, TE_2, PS_2, MTE_2\right)_{SRSS} \\ &= 1.0\cdot\left(0.75, \overleftrightarrow{0.2}, \overleftrightarrow{5.0}, \ldots, 0.0075, 0.015, 0.375\right)_{SRSS} \\ &\cong 5.1049 \end{aligned}$$

Results are summarized in Table 3. As shown in Table 3, when the distribution of input elements is normal distribution, all methods give the same results.

4.2. Case 2: rectangular and normal distribution

EE_1, DR_1, and DR_2 have rectangular distributions and the others have normal distributions.

4.2.1. ISA67.04 method

Since EE_1, DR_1, and DR_2 are not normal distributions (i.e., rectangular distribution), according to Eq. (2), the uncertainty of module 1 and module 2 can be calculated as follows:

$$\begin{aligned} e_1 &= (RA_1, PS_1, MTE_1)_{SRSS} + \left|\overleftrightarrow{DR_1} + \overleftrightarrow{EE_1}\right| \\ &= (0.750, 0.015, 0.375)_{SRSS} + \left|\overleftrightarrow{0.20} + \overleftrightarrow{5.0}\right| \cong 6.0387 \\ e_2 &= (RA_2, TE_2, PS_2, MTE_2)_{SRSS} + \left|\overleftrightarrow{DR_2}\right| \\ &= (0.1875, 0.0075, 0.015, 0.375)_{SRSS} + \left|\overleftrightarrow{0.375}\right| \cong 0.7946 \end{aligned}$$

Channel uncertainty CU is

$$\begin{aligned} CU_{67.04} &= \sqrt{e_1^2 + e_2^2} \\ &= (6.0387, 0.7946)_{SRSS} \cong 6.0907 \end{aligned}$$

4.2.2. GUM95 method

The combined standard uncertainty u_c is calculated.

$$\begin{aligned} u_c &= \left(RA_1^{SD}, \overleftrightarrow{DR_1^{SD}}, \overleftrightarrow{EE_1^{SD}}, \ldots, TE_2^{SD}, PS_2^{SD}, MTE_2^{SD}\right)_{SRSS} \\ &= \left(0.3827, \overleftrightarrow{0.1157}, \overleftrightarrow{2.8868}, \ldots, 0.0038, 0.0077, 0.1913\right)_{SRSS} \\ &\cong 2.9364 \end{aligned}$$

The coverage factor $k_p = t_p(\nu_{eff}) = 1.96$ as in section 4.1.2 and the expanded uncertainty U_p is

$$U_{p-GUM95} = k_p u_c(y) = (1.96)\cdot(2.9364) \cong 5.7553$$

4.2.3. Fotowicz method

Since the method to get the combined standard uncertainty u_c is same as GUM95 method, we can get $u_c = 2.9364$.

Using Eq. (9) we can get r_u.

$$\begin{aligned} r_u &= \frac{|u_i(y)|}{\sqrt{u_c^2(y) - u_i^2(y)}} \\ &= \frac{|2.8868|}{\sqrt{(2.9364)^2 - (2.8868)^2}} = 5.3715 \end{aligned}$$

And from Table 1 of Reference 9 we can get $k_{RN} = 1.68$ where $r_u = 5.3715$.

The expanded uncertainty U_p is

$$U_{p\_Fotowicz} = k_{RN} u_c(y) = (1.68)\cdot(2.9364) = 4.9332$$

4.2.4. Modified IEC61888 method

Uncertainty was obtained by the SRSS method, and the channel uncertainty is obtained by multiplying λ to the uncertainty obtained by the SRSS method.

For this case, $r_{RN} = 5.08$ and corresponding λ is 1.02 from Table 1.

$$\begin{aligned} CU_{M-61888} &= \lambda\cdot\left(RA_1, \overleftrightarrow{DR_1}, \overleftrightarrow{EE_1}, \ldots, TE_2, PS_2, MTE_2\right)_{SRSS} \\ &= 1.02\cdot\left(0.75, \left(\overleftrightarrow{0.95\cdot 0.2}\right), \left(\overleftrightarrow{0.95\cdot 5}\right), \ldots, 0.0075, 0.015, 0.375\right)_{SRSS} \\ &\cong 4.9557 \end{aligned}$$

Results are summarized in Table 3. The ISA67.04 and GUM95 method show the larger uncertainty than the other two methods in Case 2.

4.3. Case 3: rectangular (0.1 times of EE_1) and normal distribution

EE_1, DR_1, and DR_2 have rectangular distributions and the others have normal distributions. The magnitude of EE_1 is reduced to 0.5 (1/10 of Case 2).

4.3.1. ISA67.04 method

Since EE_1, DR_1, and DR_2 are not normal distributions, according to Eq. (2), the uncertainty of module 1 and module 2 can be calculated as follows.

$$\begin{aligned} e_1 &= (RA_1, PS_1, MTE_1)_{SRSS} + \left|\overleftrightarrow{DR_1} + \overleftrightarrow{EE_1}\right| \\ &= (0.750, 0.015, 0.375)_{SRSS} + \left|\overleftrightarrow{0.20} + \overleftrightarrow{0.5}\right| \cong 1.5387 \end{aligned}$$

$$\begin{aligned} e_2 &= (RA_2, TE_2, PS_2, MTE_2)_{SRSS} + \left|\overleftrightarrow{DR_2}\right| \\ &= (0.1875, 0.0075, 0.015, 0.375)_{SRSS} + \left|\overleftrightarrow{0.375}\right| \cong 0.7946 \end{aligned}$$

Channel uncertainty CU is

$$\begin{aligned} CU_{67.04} &= \sqrt{e_1^2 + e_2^2} \\ &= (1.5387, 0.7946)_{SRSS} = 1.7317 \end{aligned}$$

4.3.2. GUM95 method

The combined standard uncertainty u_c is calculated.

$$\begin{aligned} u_c &= \left(RA_1^{SD}, \overleftrightarrow{DR}_1^{SD}, \overleftrightarrow{EE}_1^{SD}, \ldots, TE_2^{SD}, PS_2^{SD}, MTE_2^{SD}\right)_{SRSS} \\ &= \left(0.3827, \overleftrightarrow{0.1155}, \overleftrightarrow{0.2887}, \ldots, 0.0038, 0.0077, 0.1913\right)_{SRSS} \\ &\cong 0.6103 \end{aligned}$$

The coverage factor $k_p = t_p(\nu_{eff}) = 1.96$ as in section 4.1.2 and the expanded uncertainty U_p is

$$U_{p\_GUM95} = k_p u_c(y) = (1.96)\cdot(0.6103) = 1.1962$$

4.3.3. Fotowicz method

Since the method to get the combined standard uncertainty u_c is same as GUM95 method, we can get $u_c = 0.6103$.

Using Eq. (9) we can get r_u.

$$r_u = \frac{|u_i(y)|}{\sqrt{u_c^2(y) - u_i^2(y)}} = \frac{|0.2887|}{\sqrt{(0.6103)^2 - (0.2887)^2}} \cong 0.5369$$

And from Table 1 of Reference 9 we can get $k_{RN} = 1.96$ where $r_u = 0.5369$.

The expanded uncertainty U_p is

$$U_{p-Fotowicz} = k_{RN} u_c(y) = (1.96)\cdot(0.6103) = 1.1962$$

4.3.4. Modified IEC61888 method

Uncertainty was obtained by the SRSS method, and the channel uncertainty is obtained by multiplying λ to the uncertainty obtained by the SRSS method.

For this case, $r_{RN} \cong 0.66$ and corresponding λ is 1.06 from Table 1.

$$CU_{M-61888} = \lambda\cdot\left(RA_1, \overleftrightarrow{DR}_1, \overleftrightarrow{EE}_1, \ldots, TE_2, PS_2, MTE_2\right)_{SRSS} = 1.06\cdot\left(0.75, \left(\overleftrightarrow{0.95\cdot 0.2}\right), \left(\overleftrightarrow{0.95\cdot 0.5}\right), \ldots, 0.0075, 0.015, 0.375\right)_{SRSS} \cong 1.1936$$

Results are summarized in Table 3. The ISA67.04 method showed the largest uncertainty. The other methods show similar uncertainty values in Case 3.

Table 3
The channel uncertainty.

Evaluation method	Channel Uncertainty		
	Case 1	Case 2	Case 3
ISA67.04.01 Method	5.105	6.091	1.732
GUM95 Method	5.105	5.755	1.196
Fotowicz Method	5.105	4.933	1.196
Modified IEC61888 Method	5.105	4.955	1.194

5. Monte Carlo simulation

For the example in ISA-RP67.04.02, as shown in Table 3, the combined channel uncertainties are quite different. Therefore, Monte Carlo method was used to determine which method was most practical, that is, more applicable to the industry. In the calculation of combined uncertainty by error propagation (e.g., SRSS), the mean and the standard deviation those are the primary moment and the secondary moment, respectively, are conserved, but the probability density function (pdf) is unknown, whereas Monte Carlo simulation propagates the probability density function of the input elements, so we can obtain both a pdf of measurand y and the realistic combined uncertainty. The relationship between the measurand Y and the input elements x_i is as follows:

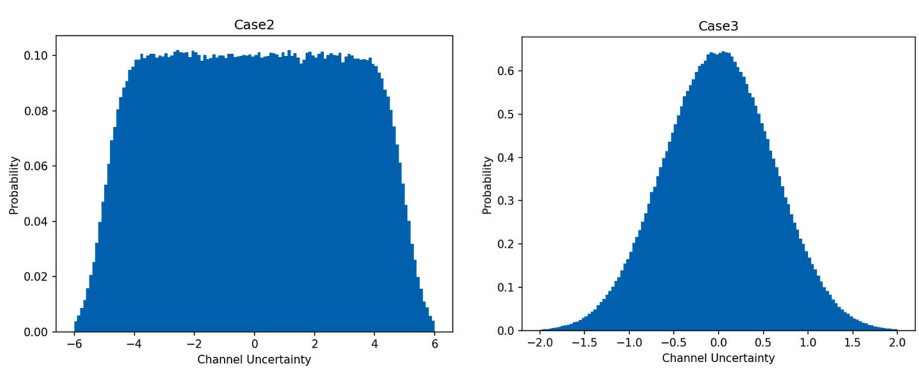
$$Y = RA_1 + DR_1 + EE_1 + PS_1 + MTE_1 + RA_2 + DR_2 + TE_2 + PS_2 + MTE_2 \quad \text{(Eq. 13)}$$

The mean of each input element is zero.

Monte Carlo simulation was performed using program written in Python language. Sampling was done using normal distribution function and rectangular distribution function in Python. The number of trials requires more than 200,000 times to maintain 95 % of confidence level (p = 0.95) in Eq. (14), so 10^6 trials were performed for this calculation [13].

$$N \geq \frac{10^4}{1-p} \quad \text{(Eq. 14)}$$

For Case 2, as a result of Monte Carlo simulation, the mean was evaluated as −0.0054 and the standard deviation was 2.9360. The percentage of samples outside the two standard deviations was calculated as 0.2 %, which means that the probability density function of the measurand is not a normal distribution as shown in Fig. 2. The uncertainty of the 95 % of confidence level was 4.932, and the coverage factor at this point was calculated as 1.680. The shape of probability density function shows the characteristics of the rectangular distribution rather than the normal distribution. This is because EE_1 which has significantly large uncertainty and assumed to be the rectangular distribution compared to the other input elements.



(a) Case2 (b) Case3

Fig. 2. Probability density function for Case 2 and Case 3.

Table 4
Results of channel uncertainty compared with Monte Carlo simulation.

Evaluation Method	Channel Uncertainty		
	Case 1 (All normal Distributions)	Case 2 (Normal + Large Rectangular Distribution)	Case 3 (Normal + Small Rectangular Distribution)
ISA67.04 Method	5.105[a]	6.091 (23.7 %)[b]	1.7317 (45.5 %)
GUM95 Method	5.105	5.755 (16.7 %)	1.1962 (0.5 %)
Fotowicz Method	5.105	4.933 (0.0 %)	1.1962 (0.5 %)
Modified IEC61888 Method	5.105	4.955 (0.5 %)	1.1936 (0.3 %)
Monte Carlo Simulation	–	4.932	1.1903

[a] Same as IEC 61888 result, all input element has normal distribution.
[b] Value in the parenthesis is percentage of difference compared with MCM result.

For Case 3, the mean was 0.0001 and the standard deviation was 0.611. The percentage of samples outside the two standard deviations was 4.4 %, and the uncertainty at 95 % confidence level was 1.190 and the coverage factor k was 1.951 which is similar to that of the normal distribution 1.96. Also, the probability density function looks like the normal distribution as shown in Fig. 2 (Case 3).

6. Results and discussion

For examples of pressure measuring devices shown in ISA-RP67.04.02, channel uncertainty was evaluated using IEC61888 (including ISA67.04.01), the GUM95 method recommended by BIMP, and the method proposed by P. Fotowicz. Case 1 assumed the normal distributions for all the input elements shown in ISA-RP67.04.02 example and channel uncertainty were evaluated for the different methods. As in Table 4, all methods showed the same results. If all input elements follow a normal distribution, the resultant distribution becomes a normal distribution. The combined uncertainty can be obtained by the SRSS method and the coverage factor at the 95 % confidence level is 1.96 for the normal distributions.

In Case 2, it assumed that three rectangular distributions for the input elements; one is environmental effect, EE_1 which has the largest uncertainty; two rectangular ones are the drifts, i.e., DR_1 and DR_2. The remaining distributions are assumed normal distribution. Only ISA67.04 method describes the special input elements which are not random and non-normally distributed elements, it treats them not in SRSS operation, but in algebraic sum operation. ISA67.04 method gives a largest uncertainty of 6.091 among that of the other methods; GUM95 method does 5.755. Since there is no analytical method for combining normal and rectangular distributions, Fotowicz proposed an approximation formula and provided a coverage factor table for *R*–N distribution. It gives the uncertainty of 4.933 while Monte Carlo method does that of 4.932; the modified IEC61888 method by authors does that of 4.933. The results show very large deviation depending on the method used to evaluate the channel uncertainty for Case 2.

In Case 3, the largest rectangular distribution only reduced to the one-tenth of EE_1 of the Case 2. The other two rectangular distributions and normal distributions remained the same as Case 2. Still ISA67.04 method gives a large uncertainty and other three methods do similar results.

GUM95 Supplement 1 [13] recommends Monte Carlo Simulation as an alternative to the SRSS method, where assumptions for the GUM95 are not applicable. Although Monte Carlo Method (MCM) gives accurate results, it needs computer programming. As a result of MCM, the channel uncertainty was evaluated as 4.932 and 1.1903 on both Case 2 and Case 3 in Table 5. MCM is not needed for Case 1 because the combined distribution is obviously normal.

Since channel uncertainty obtained by MCM can be interpreted realistic, ISA67.04 method is the most conservative and overestimates by 23.7 % for Case 2 and by 45.5 % for Case 3 compared with MCM. GUM95 method is less conservative than the ISA67.04 method, but still overestimate 16.7 % than the MCM for Case 2.

Table 5
Result of validation for compared with Monte Carlo simulation.

Evaluation method	U(y)	d_{low}	d_{high}	δ	Validated
Case 2					
ISA67.04 method	6.091	1.159	1.159	0.05	No
GUM95 method	5.755	0.832	0.832	0.05	No
Fotowicz method	4.933	0.001	0.001	0.05	Yes
Modified IEC61888 method	4.955	0.023	0.023	0.05	Yes
Monte Carlo Simulation	$y_{low} = -4.932$		$y_{high} = 4.932$		
Case 3					
ISA67.04 method	1.732	0.541	0.541	0.005	No
GUM95 method	1.196	0.006	0.006	0.005	No
Fotowicz method	1.196	0.006	0.006	0.005	No
Modified IEC61888 method	1.194	0.003	0.003	0.005	Yes
Monte Carlo Simulation	$y_{low} = -1.190$		$y_{high} = 1.190$		

The Fotowicz method and the modified IEC61888 method by authors give similar results as the Monte Carlo method. These two methods can calculate channel uncertainty easily without help of computer programming, regardless of the presence of input elements with rectangular distribution.

Reference 13 provides a procedure to validate the result calculated by the GUM95 method with the Monte Carlo method. First, d_{low} and d_{high} are calculated using Eq. (15) and Eq. (16).

$$d_{low} = |y - U(y) - y_{low}| \tag{Eq. 15}$$

$$d_{high} = |y + U(y) - y_{high}| \tag{Eq. 16}$$

Where, y is the expected value of the object to be measured, U(y) is the channel uncertainty calculated by other methods, and y_{low} and y_{high} are 2.5 % and 97.5 % values calculated by Monte Carlo simulation.

The numerical tolerance of the uncertainty, or the standard deviation, can be obtained by expressing the standard uncertainty as $c \times 10^l$, where c is an integer with a number of digits equal to the number of significant digits of the standard uncertainty and l is an integer. Then the numerical tolerance δ is expressed as:

$$\delta = \frac{1}{2}\, 10^l \tag{Eq. 17}$$

If d_{low} and d_{high} both are less than the numerical tolerance, then the result is validated and accepted. Otherwise, it is determined not validated. We have taken two significant digits from the standard uncertainty and the numerical tolerances turn out to be 0.05 and 0.005 for the Case 2 and Case 3, respectively.

The validity of various methods is summarized in Table 4. For Case 2 $y = 0$, $y_{low} = -4.932$ and $y_{high} = 4.932$ and $y = 0$, $y_{low} = -1.190$ and $y_{high} = 1.190$ for Case 3 from Monte Carlo simulation.

7. Conclusions

The reactor trip system of a nuclear power plant has trip setpoints that trigger the automatic reactor trip when the plant is in an abnormal state. The setpoint of the reactor trip system shall be set to consider the measurement uncertainty of the instrument channel and provides a reasonable margin between the analytical limit and the trip setpoint.

Channel uncertainty was evaluated for the pressure measurement in ISA-RP67.04.02 example by 1) ISA67.04 method, 2) GUM95 method recommended by BIMP, 3) Fotowicz method to consider the large rectangular distribution, and 4) IEC61888 method modified in this study.

The ISA67.04 method estimates the channel uncertainty appropriately when all input elements have normal distributions, but significantly overestimates the channel uncertainty when the uncertainty of element with rectangular distributions is much greater than that of elements with normal distributions. So ISA67.04 method is not recommended to use where the dominant input element has the rectangular distribution.

The GUM95 method also estimates the channel uncertainty appropriately when all input elements have normal distributions, but overestimates the channel uncertainty like ISA67.04.01 method when the uncertainty of element with rectangular distribution is dominant among the input elements. For this case, Fotowicz method with the modified coverage factor is one of the choices.

Existing methods give too conservative channel uncertainty when there are input elements with non-normal distributions. The proposed method (modified IEC61888 method) overcomes these shortcomings. All input element uncertainties are combined using SRSS regardless of their probability distribution functions, and compensation factor is introduced to relief the effect of non-normal distributions. The channel uncertainty by proposed method showed reasonable agreement with Monte Carlo simulation and the numerical tolerance is acceptable. The proposed method could be a practical method when input elements with the rectangular distribution exist without using a sophisticate computer software. The proposed method can be easily used by those who are not familiar with GUM95 method.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Derivation of rectangular distribution using the principle of maximum entropy

Suppose P is a continuous probability distribution. The entropy is defined as

$$H(P) = -\int_X^0 P(x) log P(x) dx \tag{Eq. A1}$$

with the following constraints.

(1) $P(x) \geq 0$
(2) $\int P(x)dx = 1$
(3) $\int P(x)\, r_i(x)\, dx = \alpha_i \; for \; 1 \leq i \leq m$

The first two constraints are characteristics of the probability density function. The third constraint is optional. There could be more than one constraint if m > 1.

Lagrange multiplier could be introduced for the constraints.

$$L(P, \lambda_0, \lambda_1, \cdots, \lambda_m) = -\int_X P(x) log P(x) dx + \lambda_0 \left(\int_X P(x)\, dx - 1 \right) + \sum_{i=1}^{m} \lambda_i \left(\int_X P(x)\, r_i(x) dx - \alpha_i \right) \tag{Eq. A2}$$

To maximize the entropy, we maximize the Lagrangian. To maximize the Lagrangian, take the derivative of the Lagrangian with respect to $P(x)$ and set the derivative 0.

$$\begin{aligned} \frac{\partial}{\partial P(x)} L(P, \lambda_0, \lambda_1, \cdots, \lambda_m) &= -\frac{\partial}{\partial P(x)} \int_X P(x)\, log P(x) dx + \lambda_0 \frac{\partial}{\partial P(x)} \left(\int_X P(x)\, dx - 1 \right) + \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial P(x)} \left(\int_X P(x)\, r_i(x) dx - \alpha_i \right) \\ &= -\int_X \frac{\partial}{\partial P(x)} [P(x)\, log P(x)] dx + \lambda_0 \frac{\partial}{\partial P(x)} \left(\int_X P(x)\, dx - 1 \right) + \sum_{i=1}^{m} \lambda_i \frac{\partial}{\partial P(x)} \left(\int_X P(x)\, r_i(x) dx - \alpha_i \right) \\ &= [-log P(x) - 1] + \lambda_0 + \sum_{i=1}^{m} \lambda_i\, r_i = 0 \end{aligned}$$

Therefore,

$$\begin{aligned} log P(x) &= -1 + \lambda_0 + \sum_{i=1}^{m} \lambda_i\, r_i \\ P(x) &= exp\left(-1 + \lambda_0 + \sum_{i=1}^{m} \lambda_i\, r_i \right) = \frac{exp\left(\sum_{i=1}^{m} \lambda_i\, r_i \right)}{exp\,(1 - \lambda_0)} \end{aligned} \tag{Eq. A3}$$

Because $\int P(x)dx = 1$ from constraint (2),

$$\int P(x)dx = \int exp\left(-1 + \lambda_0 + \sum_{i=1}^{m} \lambda_i\, r_i \right) dx = exp\,(-1 + \lambda_0) \int exp\left(\sum_{i=1}^{m} \lambda_i\, r_i \right) dx = 1$$

Therefore,

$$e^{1-\lambda_0} = \int \exp\left(\sum_{i=1}^{m} \lambda_i\, r_i \right) dx \tag{Eq. A4}$$

Substitute Eq. (A4) into Eq. (A3), then we can get $P(x)$ as

$$P(x) = \frac{\exp\left(\sum_{i=1}^{m}\lambda_i\, r_i\right)}{\exp\,(1-\lambda_0)} = \frac{\exp\left(\sum_{i=1}^{m}\lambda_i\, r_i\right)}{\int \exp\left(\sum_{i=1}^{m}\lambda_i\, r_i\right)dx} \tag{Eq. A5}$$

The only constraint on a distribution we have is $X = [a,b]$. So m = 0 which means that $\lambda_1 = \lambda_2 = \cdots = \lambda_m = 0$.
Then, we can get

$$\begin{aligned} P(x) &= \frac{\exp\left(\sum_{i=1}^{m}\lambda_i\, r_i\right)}{\int \exp\left(\sum_{i=1}^{m}\lambda_i\, r_i\right)dx} \\ &= \frac{e^0}{\int_a^b e^0 dx} = \frac{1}{b-a} \end{aligned} \tag{Eq. A6}$$

This means that the maximum entropy probability distribution $P(x)$ is the rectangular distribution.

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