pISSN 1229-3008 eISSN 2287-6251
Progress in Superconductivity and Cryogenics
Vol.26, No.2, (2024), pp.9~18 https://doi.org/10.9714/psac.2024.26.2.009

Investigation of the interaction between spin density wave and superconductivity in two band high temperature iron based superconductor $Ba_{1-x}Na_xFe_2As_2$

Teklie Lissanu Tegegne*

Injibara University, Department of Physics, P.O. Box 40, Injibara, Ethiopia.

(Received 22 November 2023; revised or reviewed 11 June 2024; accepted 12 June 2024)

Abstract

The current study deals with the possible interplay between superconductivity and spin density wave in two band model high temperature iron based superconductor (FeBSC) $Ba_{1-x}Na_xFe_2As_2$. The electron and hole bands in the presence of the inter-band interaction between the two bands is becoming a vital issue to deal with the high temperature physics of the iron-based superconductors. In this research work, a model Hamiltonian appropriate for the system under consideration has been developed and the temperature dependent Green's function technique has been employed to get the solution for the equations of motion constructed for the two band model high temperature FeBSC $Ba_{1-x}Na_xFe_2As_2$. By making use of the decoupling procedure, the equations of motion for the dependence of superconducting transition temperature ($\mathrm{T_C}$) on spin density wave(SDW) order parameter (Δ_{SDW}) in the electron intra-band ($\Delta_{sc(e)}$), hole intra-band ($\Delta_{sc(h)}$) and inter-band $\left(\Delta_{sc(eh)}\right)$ for $Ba_{1-x}Na_xFe_2As_2$ have been obtained. We have also obtained the expression for the dependence of spin density wave transition temperature ($\mathrm{T_{SDW}}$) on Δ_{SDW} for $Ba_{1-x}Na_xFe_2As_2$. Using some plausible approximations and appropriate experimental values for the parameters in the obtained equations of motion, phase diagrams of $\mathrm{T_C}$ versus $\Delta_{sc(e)}$, $\Delta_{sc(h)}$ and $\Delta_{sc(eh)}$ are plotted. Furthermore, a phase diagram of T_{SDW} versus Δ_{SDW} is plotted for the material under consideration. Finally, using the above mentioned phase diagrams, the interplay between superconductivity and spin density wave in the two band model high temperature FeBSC $Ba_{1-x}Na_xFe_2As_2$ has been demonstrated to be a very distinct possibility. The agreement of the current finding with the experimental observations is quite commendable.

Key words: interplay; iron based superconductor; spin density wave; superconducting order parameter; $Ba_{1-x}Na_xFe_2As_2$

1. INTRODUCTION

In high-temperature superconductors, the pairing mechanism of electrons and the capability of conducting of electricity without dissipation of energy is one of the most exciting phenomenon of condensed matter physics. The invention of high-temperature superconductors in a class of materials which are based on iron was amongst the most outstanding breakthroughs in Solid State Physics research [1]. This new invention has brought to an end the hegemony of cuprates as the only high temperature superconductors and had motivated Physicists to conduct wider researches on FeBSCs at ambient pressure. So far, these newly discovered FeBSCs have the highest superconducting transition temperature next to cuprates [2].

The FeBSCs have began flurry of activities as physicists attempt to discern the source of superconductivity in these materials and improve them for possible applications in some electrical appliance. These new high temperature iron based superconductors have motivating features from the basic physics point of view, which is the superconducting pairing mechanism possibly related to the interplay between magnetism in the phase diagram, which has massive impact on the world of superconductivity and revitalized worldwide interest in this area and opened a new frontier for the experimental and theoretical study of high temperature superconductors in general and FeBSCs in particular. It was reported that, a layered iron-based compound LaOFeAs undergoes superconducting transition by doping with F ions at the O site ($La(O_{1-x}F_x)FeAs$ ($x = 0.05-0.12$)).

The transition temperature (T_C) shows a trapezoid shape dependence on the F content, with the highest T_C of about 26 K [3]. Furthermore, though iron has been considered as venomous to superconductivity due to its strong local magnetic moment, a number of superconducting compounds containing iron have been known for quite a long period of time. This finding has sparked intense research activities on FeBSCs and up to now the superconducting transition temperature for the FeBSCs has been raised up to 56 K in $Gd_{1-x}Th_xFeAsO$ [4] and 57.3 K in $Sm_{0.95}La_{0.05}O_{0.85}F_{0.15}FeAs$ [5] under ambient pressure.

As is well known, one of the most extensively studied families of the iron-based superconductors and the first to be discovered was the 1111-family which crystallized in the ZrCuSiAs-type structure with a tetragonal P4/nmm space group [6]. The other typical type of iron based superconductors are the 122 families such as $BaFe_2As_2$, $CaFe_2As_2$, $EuFe_2As_2$ and $SrFe_2As_2$ which are becoming preferable to deal with than the other families of the FeBSCs and comprise of the heart of the FeBSCs where the best quality single crystals are obtained. The ion based

* Corresponding author: teklelisanu21@gmail.com

122 families of superconductors have ThCr2Si2-type structure with tetragonal space group type iron arsenide [7, 8].

Superconductivity can also be induced in undoped and underdoped compounds by applying pressure [9]. With increasing temperature, the overall electron concentration is enhanced while the total hole concentration is mitigated.

It shows that the electron-hole compensation, if it exists, can only occur in a narrow temperature range, and in most of the temperature range there is an electron-hole disparity. Therefore, in a given system the systematic studies with or without any impurities or charges in electronic structure have been extremely advantageous. The suitable and successful way to increase superconducting transition temperature in FeBSCs is by carrier doping or chemical substitution, which modulates the lattice parameters and introduces internal chemical pressure which has been proven to be a convenient and effective way to increase the superconducting transition temperature of the FeSCs. The invention of the magnetically-active superconductors, iron pnictides based on FeAs [1, 8] or Fe(Se,S,Te) [10, 11] has further enhanced the on-going researches about interplay between different ordered electronic states in materials [12-14]. In itinerant electrons systems, the interactions that lead to the establishment of superconducting (SC) and magnetic SDW orders, "pull" and "push" the same particles, and as a result influence each other. In particular, two orders may support each other and lead to similar local interplay between SC and SDW states; or one of them may completely suppress the other order, resulting in a state with spatially separated regions of "pure" SDW or SC orders. The phase transitions between various states may also be either continuous or abrupt. The result of this interplay depends critically on the properties of the interactions, such as symmetry of superconducting pairing, their relative strengths, and also on the properties of the Fermi surface (FS), such as its shape or the density of electronic states.

Upon electron- or hole doping in $Ba(Fe_{1-x}Co_x)_2As_2$ [15], or $Ba_{1-x}K_xFe_2As_2$[8] and $Ba_{1-x}Na_xFe_2As_2$[16] the antiferromagnetic(AF) order gets gradually suppressed and superconductivity emerges well before the magnetic order vanishes. In this under-doped regime the antiferromagnetism and superconductivity orders interplay and compete for the same low-energy electronic states [17-19]. With these doping, the superconducting critical temperature, superconducting parameters and the condensation energy are enhanced whereas the antiferromagnetic order parameter is reduced. Most of the parent compounds of FeBSCs are magnetically ordered.

These parent compounds experience a magnetic transition at a guaranteed spin density wave transition temperature leading to the formation of a SDW. The interplay between two or more order parameters in high temperature superconductors is fabulously decoder in condensed matter physics. Naturally, one order would inhibit the other phenomena, i.e. magnetism and superconductivity are mutually exclusive entities. Because of these relations, the spin density wave and superconducting order parameters are an essential issue in the present study on the FeBSCs. The interplay between the two orders has been the emphasis of many experimental and theoretical researches. Obviously, the, magnetic and structural transitions are detached and that magnetism and superconductivity interplay microscopically as depicted by spin rotation (SR) or nuclear magnetic resonance (NMR) measurements **[**20, 21**]**. Furthermore, the microscopic interplay between SDW and superconductivity in $Ba_{1-x}K_xFe_2As_2$ was recently demonstrated by neutron diffraction [22, 23]. Thus, it is vital to comprehend whether this is a common phenomenon for all hole doped 122 materials or that K substituted compounds perhaps constitute exceptional case due to the closely analogous ionic size of eight harmonized Ba and K. There are analogous outcomes for the alternative hole-doped $Ba_{1-x}Na_xFe_2As_2$ compound in which the sodium ions are smaller than barium ions and described the magnetic and superconducting behaviors of these specimens [24]. The observed magnetic ordering and an orthorhombic structural distortion for x = 0.35 and bulk superconductivity for specimens within the range of 0.4 $\leq x \leq 0.6$, with a maximum T_C of 34 K occurs at x= 0.4. The iron based superconductor $Ba_{1-x}Na_xFe_2As_2$ is currently one of the most interesting materials for its large critical current. The interplay between superconductivity and SDW takes place in a certain temperature range and doping levels in some specified compounds as depicted by some investigator [25]. Experimental and theoretical testimonies are present that exhibit the availability of incongruity instigated by SDW unsteadiness and electron-doping by F mitigates the SDW unsteadiness and recovers the superconducting order. It is believed that, the $La(O_{1-x}F_x)FeAs$ provides a thrilling novel compound depicting contending states in layered compounds [26]. Furthermore, SDW occurs in the FeBSCs due to the nesting of electron and hole Fermi surfaces [27,28]. In FeBSC $Ba_{1-x}Na_xFe_2As_2$, the superconductivity and spin density wave states interplay for $0.25 < x < 0.35$ [29].

In this research work, we propose a minimal two band model FeBSCs with inter-band interaction between the electron and hole bands. The essential characteristics of the minimal two-band unconventional FeBSCs is the presence of two energy gaps for electron (Δ_e) for hole (Δ_h) which both vanish at the same superconducting transition temperature (T_C). The two-band model permits for the existence of both intra-band and inter-band superconducting interactions that couple the two bands. According to the microscopic theory of superconductivity, the occurrence of the two energy gaps is clarified by specific intra-band interaction "i" with coupling potential(U_{ii}) in each band and the inter-band interaction with coupling potential(U_{ij}) between the two bands [27].

The occurrence of inter-band interaction potential enriches the pairing capacity in the high temperature iron-based compounds and also leads the electron and hole intra-band superconducting order parameters to vanish at identical critical temperature. The presence of two band models enables to capture the essential features of high temperature superconductivity and SDW ordering in $Ba_{1-x}Na_xFe_2As_2$. The experimental values for the zero temperature superconducting energy gaps for the two band model high temperature iron based superconductor $Ba_{1-x}Na_xFe_2As_2$ in the electron and hole Fermi surfaces are about 8.5meV and 2.6meV respectively [30].

2. SYSTEM OF HAMILTONIAN FORMATION IN TWO BAND MODEL FeBSC

To study the interplay between superconductivity and SDW of FeBSC $Ba_{1-x}Na_xFe_2As_2$, we consider a two-band model, which deals with the vital physics of the multi-band high temperature superconducting state and SDW order.

The decoupled mean field system Hamiltonian of a superconducting state by considering the effect of magnetic interaction in the two band model FeBSC $Ba_{1-x}Na_xFe_2As_2$ is given as [31-33],

$$\hat{H} = \hat{H}_{BCS(e)} + \hat{H}_{BCS(h)} + \hat{H}_{eh} + \hat{H}_{SDW} \quad (1)$$

where the first two terms are Hamiltonian for the electron and hole intra-bands respectively and the last two terms are the Hamiltonian inter-band interaction and the mean field Hamiltonian term for the SDW state respectively and are expressed as,

$$\hat{H}_e = \sum_{k,\sigma} \in_e (k)\, \hat{c}^+_{k\sigma}\hat{c}_{k\sigma} - \sum_{k,k'} U_e (k,k')\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow}\hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow} \quad (2)$$

$$\hat{H}_h = \sum_{k,\sigma} \in_h (k)\hat{d}^+_{k\sigma}\hat{d}_{k\sigma} - \sum_{k,k'} U_h(k,k')\hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow}\hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow} \quad (3)$$

$$\hat{H}_{eh} = -\sum_{k,k'} U_{eh}(k,k')(\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow}\hat{d}_{-k\prime\downarrow}\hat{d}_{k\prime\uparrow} + \hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow}\hat{c}_{-k\prime\downarrow}\hat{c}_{k\prime\uparrow} \quad (4)$$

$$\hat{H}_{SDW} = -\Delta_{SDW}\sum_k\left(\hat{c}^\dagger_{(k+q)\uparrow}\hat{d}_{-k\downarrow} + \hat{d}^\dagger_{(k+q)\uparrow}\hat{c}_{-k\downarrow}\right) \quad (5)$$

where

$$\begin{cases}\Delta_{SDW} = \sum_{k,Q} U_{SDW} \ll \hat{d}^+_{-k\downarrow}\hat{c}_{(k'+q)\uparrow} \gg \\ \Delta^*_{SDW} = \sum_{k,Q} U_{SDW} \ll \hat{c}^+_{(k+q)\uparrow}\hat{d}_{-k'\downarrow} \gg\end{cases}$$

Now, substituting (2-5) into (1) we get

$$\begin{aligned}\hat{H} = &\sum_{k,\sigma} \in_e (k)\hat{c}^+_{k\sigma}\hat{c}_{k\sigma} + \sum_{k,\sigma} \in_h (k)\hat{d}^+_{k\sigma}\hat{d}_{k\sigma} - \\ &\Delta_{sc(e)}\sum_{k,k'}(\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow} + \hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow}) \\ &-\Delta_{sc(h)}\sum_{k,k'}(\hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow} + \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow}) - \\ &\sum_{k,k'}(\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow} + \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow}) \\ &-\Delta_{sc(eh)}\sum_{k,k'}\left(\hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow} + \hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow}\right) - \\ &\Delta_{SDW}\sum_k\left(\hat{c}^\dagger_{(k+q)\uparrow}\hat{d}_{-k\downarrow} + \hat{d}^\dagger_{(k+q)\uparrow}\hat{c}_{-k\downarrow}\right)\end{aligned} \quad (6)$$

where, the first two terms are the energy of conduction electrons and holes respectively, the third and fourth terms involve superconductivity due to the intra-paring at the electron and hole Fermi surfaces respectively. The fifth and sixth are the terms involving superconductivity due to the inter-band between the two bands and the last term is a term involving SDW. $\hat{c}^\dagger_{k\uparrow}(\hat{c}_{-k\downarrow})$ and $\hat{d}^\dagger_{k\uparrow}(\hat{d}_{-k\downarrow})$ are the creation (annihilation) operators in the electron and hole bands respectively having the wave number k and spin ↑ or ↓. $\in_e$ (k) and $\in_h$ (k) denote the single particle energies measured with respect to the Fermi energy in the electron and hole Fermi surfaces respectively $\Delta_{sc(e)}$, $\Delta_{sc(h)}$, $\Delta_{sc(eh)}$ and Δ_{SDW} have retained the definitions given earlier.

2.1. EQUATIONS OF MOTION FOR INTRA AND INTER BANDS OF THE FeBSC $Ba_{1-x}Na_xFe_2As_2$

By using the double time temperature dependent Green's function technique, we determined the equation of motion for the superconducting correlation relation $\ll \hat{c}^\dagger_{k\uparrow}, \hat{c}^\dagger_{-k\downarrow} \gg$ in the electron band [34] which is expressed as,

$$\omega \ll \hat{c}^\dagger_{k\uparrow}, \hat{c}^\dagger_{-k\downarrow} \gg = \langle[\hat{c}^\dagger_{k\uparrow}, \hat{c}^\dagger_{-k\downarrow}]\rangle + \ll [\hat{c}^\dagger_{k\uparrow}, \hat{H}], \hat{c}^\dagger_{-k\downarrow} \gg$$

$$\begin{aligned}\omega \ll \hat{c}^\dagger_{k\uparrow}, \hat{c}^\dagger_{-k\downarrow} \gg = \ll &[\hat{c}^\dagger_{k\uparrow}, \hat{H}_{SC(e)}] + [\hat{c}^\dagger_{k\uparrow}, \hat{H}_h] + \\ &[\hat{c}^\dagger_{k\uparrow}, \hat{H}_{eh}] + [\hat{c}^\dagger_{k\uparrow}, \hat{H}_{SDW}], \hat{c}^\dagger_{-k\downarrow} \gg\end{aligned} \quad (7)$$

Solving the commutation relations for $[\hat{c}^\dagger_{k\uparrow}, \hat{H}_e]$, by applying the commutators and anticommutators rules we get,

$$\begin{aligned}[\hat{c}^+_{k\uparrow}, \hat{H}_e] = [\hat{c}^+_{k\uparrow}, &\sum_{k,\sigma} \in_e (k)\hat{c}^+_{k\sigma}\hat{c}_{k\sigma} \\ &- \Delta_{sc(e)}\sum_{k,k'}(\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow} + \hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow})]\end{aligned}$$

From which we obtain,

$$[\hat{c}^+_{k\uparrow}, \hat{H}_e] = -\in_e \hat{c}^+_{k\uparrow} + \Delta_e\hat{c}_{-k'\downarrow} \quad (8)$$

Similarly, the commutation relation for $[\hat{c}^+_{k\uparrow}, \hat{H}_h]$ becomes,

$$\begin{aligned}[\hat{c}^+_{k\uparrow}, \hat{H}_h] = [\hat{c}^+_{k\uparrow}, &\sum_{k,\sigma} \in_h (k)\hat{d}^+_{k\sigma}\hat{d}_{k\sigma} \\ &- \Delta_{sc(h)}\sum_{k,k'}(\hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow} + \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow})]\end{aligned}$$

$$\begin{aligned}[\hat{c}^+_{k\uparrow}, \hat{H}_h] = &\sum_{k,\sigma} \in_h (k)[\hat{c}^+_{k\uparrow}, \hat{d}^+_{k\sigma}\hat{d}_{k\sigma}] \\ &- \Delta_{sc(h)}\sum_{k,k'}[\hat{c}^+_{k\uparrow}, \hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow}] \\ &- \Delta_{sc(h)}\sum_{k,k'}[\hat{c}^+_{k\uparrow}, \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow}]\end{aligned}$$

Thus we get,

$$[\hat{c}^+_{k\uparrow}, \hat{H}_h] = 0 \quad (9)$$

In the same way, the commutation relation for $[\hat{c}^+_{k\uparrow}, \hat{H}_{eh}]$ is expressed as,

$$\begin{aligned}[\hat{c}^+_{k\uparrow}, \hat{H}_{eh}] = [\hat{c}^+_{k\uparrow}, -\Delta_{sc(eh)}\sum_{k,k'}(\hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow} + \\ \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow}) - \Delta_{sc(eh)}\sum_{k,k'}\left(\hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow} + \hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow}\right)]\end{aligned}$$

$$\begin{aligned}[\hat{c}^+_{k\uparrow}, \hat{H}_{eh}] = &-\Delta_{sc(eh)}\sum_{k,k'}([\hat{c}^+_{k\uparrow}, \hat{c}^+_{k\uparrow}\hat{c}^+_{-k\downarrow}] + \\ &[\hat{c}^+_{k\uparrow}, \hat{d}_{-k'\downarrow}\hat{d}_{k'\uparrow}]) - \Delta_{sc(eh)}\sum_{k,k'}\left([\hat{c}^+_{k\uparrow}, \hat{d}^+_{k\uparrow}\hat{d}^+_{-k\downarrow}] + \right. \\ &\left.[\hat{c}^+_{k\uparrow}, \hat{c}_{-k'\downarrow}\hat{c}_{k'\uparrow}]\right)\end{aligned}$$

Hence, we get,

$$[\hat{c}^+_{k\uparrow}, \hat{H}_{eh}] = \Delta_{sc(eh)}\hat{c}_{-k'\downarrow} \quad (10)$$

Moreover, the commutation relation for $[\hat{c}^{\dagger}_{k\uparrow},\hat{H}_{SDW}]$ becomes,

$$[\hat{c}^{\dagger}_{k\uparrow},\hat{H}_{SDW}] = [\hat{c}^{\dagger}_{k\uparrow}, -\Delta_{SDW}\sum_k(\hat{c}^{\dagger}_{(k+q)\uparrow}\hat{d}_{-k\downarrow} + \hat{d}^{\dagger}_{(k+q)\uparrow}\hat{c}_{-k\downarrow})]$$
$$[\hat{c}^{\dagger}_{k\uparrow},\hat{H}_{SDW}] = -\Delta_{SDW}\sum_k([\hat{c}^{\dagger}_{k\uparrow},\hat{c}^{\dagger}_{(k+q)\uparrow}]\hat{d}_{-k\downarrow} - \hat{c}^{\dagger}_{(k+q)\uparrow}[\hat{c}^{\dagger}_{k\uparrow},\hat{d}_{-k\downarrow}]) - \Delta_{SDW}\sum_k([\hat{c}^{\dagger}_{k\uparrow},\hat{d}^{\dagger}_{(k+q)\uparrow}]\hat{c}_{-k\downarrow} - \hat{d}^{\dagger}_{(k+q)\uparrow}[\hat{c}^{\dagger}_{k\uparrow},\hat{c}_{-k\downarrow}])$$

From which we get,

$$[\hat{c}^{\dagger}_{k\uparrow},\hat{H}_{SDW}] = \Delta_{SDW}\hat{c}^{\dagger}_{k\uparrow} \qquad (11)$$

Finally, using (8-11) into (7), the equation of motion for $\ll \hat{c}^{\dagger}_{k\uparrow},\hat{c}^{\dagger}_{-k\downarrow}\gg$ is expressed as,

$$\ll \hat{c}^{\dagger}_{k\uparrow},\hat{c}^{\dagger}_{-k\downarrow}\gg = \frac{\Delta_{sc(e)}+\Delta_{sc(eh)}}{\omega+\epsilon_e-\Delta_{SDW}} \ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg \qquad (12)$$

Similarly, the equation of motion for $\ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg$ in the electron intra band is given by,

$$\omega \ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg = \langle[\hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}]\rangle + + \ll [\hat{c}_{-k\downarrow},\hat{H}],\hat{c}^{\dagger}_{-k\downarrow}\gg$$
$$\omega \ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg = 1 + \ll [\hat{c}_{-k\downarrow},\hat{H}_e] + [\hat{c}_{-k\downarrow},\hat{H}_h] + [\hat{c}_{-k\downarrow},\hat{H}_{eh}] + [\hat{c}_{-k\downarrow},\hat{H}_{SDW}],\hat{c}^{\dagger}_{-k\downarrow}\gg \qquad (13)$$

So, solving the commutation relation of (13), we obtain,

$$[\hat{c}_{-k\downarrow},\hat{H}_e] = [\hat{c}_{-k\downarrow},\sum_{k,\sigma}\epsilon_e(k)\hat{c}^{\dagger}_{k,\sigma}\hat{c}_{-k,\sigma} - \Delta_{sc(e)}\sum_k(\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow})]$$
$$[\hat{c}_{-k\downarrow},\hat{H}_e] = \sum_{k,\sigma}\epsilon_e(k)[\hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{k,\sigma}\hat{c}_{-k,\sigma}] - \Delta_{sc(e)}\sum_k([\hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow}] + [\hat{c}_{-k\downarrow},\hat{c}_{-k\downarrow}\hat{c}_{k\uparrow}])$$

From which we get,

$$[\hat{c}_{-k\downarrow},\hat{H}_{BCS(e)}] = \epsilon_e(k)\hat{c}_{-k\downarrow} + \Delta_{sc(e)}\hat{c}^{\dagger}_{k\uparrow} \qquad (14)$$

The commutating relation for $[\hat{c}_{-k\downarrow},\hat{H}_h]$ yields,

$$[\hat{c}_{-k\downarrow},\hat{H}_h] = [\hat{c}_{-k\downarrow},\sum_{k,\sigma}\epsilon_h(k)\hat{d}^{\dagger}_{k,\sigma}\hat{d}_{-k,\sigma} - \Delta_{sc(h)}\sum_k(\hat{d}^{\dagger}_{k\uparrow}\hat{d}^{\dagger}_{-k\downarrow} + \hat{d}_{-k\downarrow}\hat{d}_{k\uparrow})]$$
$$[\hat{c}_{-k\downarrow},\hat{H}_h] = \sum_{k,\sigma}\epsilon_h(k)[\hat{c}_{-k\downarrow},\hat{d}^{\dagger}_{k,\sigma}\hat{d}_{-k,\sigma}] - \Delta_{sc(h)}\sum_k([\hat{c}_{-k\downarrow},\hat{d}^{\dagger}_{k\uparrow}\hat{d}^{\dagger}_{-k\downarrow}] + [\hat{c}_{-k\downarrow},\hat{d}_{-k\downarrow}\hat{d}_{k\uparrow}])$$

Thus we get,

$$[\hat{a}_{-k\downarrow},\hat{H}_h] = 0 \qquad (15)$$

Following similar procedure as above, commutation relation for $[\hat{c}_{-k\downarrow},\hat{H}_{eh}]$ is given by,

$$[\hat{c}_{-k\downarrow},\hat{H}_{eh}] = [\hat{c}_{-k\downarrow}, -\Delta_{sc(eh)}\sum_k(c^{\dagger}_{k\uparrow}\hat{c}^{\dagger}_{-k\downarrow} + \hat{d}_{-k\downarrow}\hat{d}_{k\uparrow}) - \Delta_{sc(eh)}\sum_k(d^{\dagger}_{k\uparrow}\hat{d}^{\dagger}_{-k\downarrow} + \hat{c}_{-k\downarrow}\hat{c}_{k\uparrow})]$$

From which we obtain,

$$[\hat{c}_{-k\downarrow},\hat{H}_{eh}] = \Delta_{sc(eh)}\hat{c}^{\dagger}_{k\uparrow} \qquad (16)$$

Moreover, the commutation relation for $[\hat{c}_{-k\downarrow},\hat{H}_{SDW}]$ is given by,

$$[\hat{c}_{-k\downarrow},\hat{H}_{SDW}] = [\hat{c}_{-k\downarrow}, -\Delta_{SDW}\sum_k(\hat{c}^{\dagger}_{(k+q)\uparrow}\hat{d}_{-k\downarrow} + \hat{d}^{\dagger}_{(k+q)\uparrow}\hat{c}_{-k\downarrow})]$$
$$[\hat{c}_{-k\downarrow},\hat{H}_{SDW}] = -\Delta_{SDW}\sum_k[\hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{(k+q)\uparrow}\hat{d}_{-k\downarrow}] - \Delta_{SDW}\sum_k[\hat{c}_{-k\downarrow},\hat{d}^{\dagger}_{(k+q)\uparrow}\hat{c}_{-k\downarrow}]$$

Hence we get,

$$[\hat{C}_{-k\downarrow},\hat{H}_{SDW}] = -\Delta_{SDW}\hat{c}_{-k\downarrow} \qquad (17)$$

Substituting (14-17) into (13), the equation of motion for $\ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg$ becomes,

$$\ll \hat{c}_{-k\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg = \frac{1}{\omega-\epsilon_e(k)+\Delta_{SDW}} + \frac{\Delta_{sc(e)}+\Delta_{sc(eh)}}{\omega-\epsilon_e(k)+\Delta_{SDW}} \ll \hat{c}^{\dagger}_{k\uparrow},\hat{c}^{\dagger}_{-k\downarrow}\gg \qquad (18)$$

Now, using (18) in (12) we get,

$$\ll \hat{c}^{\dagger}_{k\uparrow},\hat{c}^{\dagger}_{-k\downarrow}\gg = \frac{\Delta_{sc(e)}+\Delta_{sc(eh)}}{(\omega+\epsilon_e(k)-\Delta_{SDW})(\omega-\epsilon_e(k)+\Delta_{SDW})-(\Delta_{sc(e)}+\Delta_{sc(eh)})^2} \qquad (19)$$

Similarly, employing the same procedure as above, the equation of motion for the SDW correlation function $\ll \hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}\gg$ can be expressed as,

$$\omega \ll \hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}\gg = \langle[\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}]\rangle + \ll [\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}],\hat{d}^{\dagger}_{-k\downarrow}\gg$$
$$\omega \ll \hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}\gg = \ll [\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_e] + [\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_h] + [\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_{eh}] + [\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_{SDW}],\hat{d}^{\dagger}_{-k\downarrow}\gg \qquad (20)$$

From which we obtain,

$$[\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_e] = -\epsilon_e(k+q)\hat{c}^{\dagger}_{(k+q)\uparrow} + \Delta_{sc(e)}\hat{c}_{-(k+q)\downarrow} \qquad (21)$$
$$[\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_h] = 0 \qquad (22)$$
$$[\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_{eh}] = \Delta_{sc(eh)}\hat{a}_{-(k+q)\downarrow} \qquad (23)$$
$$[\hat{c}^{\dagger}_{(k+q)\uparrow},\hat{H}_{SDW}] = \Delta_{SDW}\hat{a}^{\dagger}_{(k+q)\uparrow} \qquad (24)$$

Using (21-24) in (20), the equation of motion for $\ll \hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}\gg$ is expressed as,

$$\ll \hat{c}^{\dagger}_{(k+q)\uparrow},\hat{d}^{\dagger}_{-k\downarrow}\gg = \frac{\Delta_{sc(e)}+\Delta_{sc(eh)}}{\omega+\epsilon_e(k)-\Delta_{SDW}} \ll \hat{c}_{-(k+q)\downarrow},\hat{d}^{\dagger}_{-k\downarrow}\gg \qquad (25)$$

In the same way, the equation of motion for the SDW correlation function $\ll \hat{C}_{-(k+q)\downarrow},\hat{c}^{\dagger}_{-k\downarrow}\gg$ can be expressed as,

$$\omega \ll \hat{c}_{-(k+q)\downarrow}, \hat{d}^{\dagger}_{-k\downarrow} \gg = \langle [\hat{c}_{-(k+q)\downarrow}, \hat{d}^{\dagger}_{-k\downarrow}] \rangle + \ll [\hat{c}_{-(k+q)\downarrow}, \hat{H}], \hat{c}^{\dagger}_{-k\downarrow} \gg$$

$$\omega \ll \hat{c}_{-(k+q)\downarrow}, \hat{d}^{\dagger}_{-k\downarrow} \gg = 1 + \ll [\hat{c}_{-(k+q)\downarrow}, \hat{H}_e] + [\hat{c}_{-(k+q)\downarrow}, \hat{H}_h] + [\hat{c}_{-(k+q)\downarrow}, \hat{H}_{eh}] + [\hat{c}_{-(k+q)\downarrow}, \hat{H}_{SDW}], \hat{c}^{\dagger}_{-k\downarrow} \gg \quad (26)$$

Furthermore, solving the following commutation relations we get,

$$[\hat{c}_{-(k+q)\downarrow}, \hat{H}_e] = \in_e (k+q)\hat{c}_{-(k+q)\downarrow} + \Delta_{sc(e)}\hat{c}^{\dagger}_{(k+q)\uparrow} \quad (27)$$

$$[\hat{c}_{-(k+q)\downarrow}, \hat{H}_h] = 0 \quad (28)$$

$$[\hat{c}_{-(k+q)\downarrow}, \hat{H}_{eh}] = \Delta_{sc(eh)}\hat{c}^{\dagger}_{(k+q)\uparrow} \quad (29)$$

$$[\hat{c}_{-(k+q)\downarrow}, \hat{H}_{SDW}] = -\Delta_{SDW}\hat{c}_{-(k+q)\downarrow} \quad (30)$$

Substituting (27-30) into (26), the equation of motion for $\ll c_{(k+q)\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{c}_{-(k+q)\downarrow}, \hat{d}^{\dagger}_{-k\downarrow} \gg = \frac{1}{\omega - \in_e(k) + \Delta_{SDW}} + \frac{\Delta_{sc(e)} + \Delta_{sc(eh)}}{\omega - \in_e(k) + \Delta_{SDW}} \ll \hat{c}^{\dagger}_{(k+q)\uparrow}, \hat{d}^{\dagger}_{-k\downarrow} \gg \quad (31)$$

2.2. SUPERCONDUCTING TRANSITION TEMPERATURE DEPENDANCE ON SPIN DENSITY WAVE ORDER PARAMETER FOR THE ELECTRON INTRA BAND

Now, if we consider the electron intra-band pairing interaction only, the expression for $\ll \hat{c}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg$ becomes,

$$\ll \hat{c}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg = -\frac{1}{2}\left(\frac{\Delta_{sc(e)} + \Delta_{SDW}}{\omega^2 - \in_e^2(k) - (\Delta_{sc(e)} + \Delta_{SDW})^2} + \frac{\Delta_{sc(e)} - \Delta_{SDW}}{\omega^2 - \in_e^2(k) - (\Delta_{sc(e)} - \Delta_{SDW})^2}\right) \quad (32)$$

The superconducting order parameters in the electron intra band is expressed as,

$$\Delta_{sc(e)} = \frac{U_e}{\beta}\sum_k \ll \hat{c}^{\dagger}_{k\uparrow}, \hat{c}^{\dagger}_{-k\downarrow} \gg \quad (33)$$

where U_e is the electron intra-band interaction potential, $\beta = \frac{1}{k_B T}$ and k_B is the Boltzmann constant.
Using (32) in (33) we get,

$$\Delta_{sc(e)} = -\frac{U_e}{2\beta}\sum_{k,n}\left(\frac{\Delta_{sc(e)} + \Delta_{SDW}}{\omega_n^2 - \in_e^2(k) - (\Delta_{sc(e)} + \Delta_{SDW})^2} + \frac{\Delta_{sc(e)} - \Delta_{SDW}}{\omega_n^2 - \in_e^2(k) - (\Delta_{sc(e)} - \Delta_{SDW})^2}\right) \quad (34)$$

Now, we use the expression $\omega \longrightarrow i\omega_n$ and the Matsubara frequency [35] is given by,

$$\omega_n = \frac{(2n+1)\pi}{\beta} \quad (35)$$

Substituting (35) into (34) and rearranging we get,

$$\Delta_e = \frac{U_e\beta}{2}\sum_{k,n}\left(\frac{\Delta_{sc(e)} + \Delta_{SDW}}{((2n+1)\pi)^2 + \beta^2\left(\in_e^2(k) + (\Delta_{sc(e)} + \Delta_{SDW})^2\right)} + \frac{\Delta_{sc(e)} - \Delta_{SDW}}{((2n+1)\pi)^2 + \beta^2\left(\in_e^2(k) + (\Delta_{sc(e)} - \Delta_{SDW})^2\right)}\right) \quad (36)$$

By changing summation into integration in the interval $-\hbar\omega_F < \in_e(k) < \hbar\omega_F$ and by introducing the density of state at the Fermi level, $D_e(0)$ we get,

$$\frac{2}{\lambda_e} = \int_0^{\hbar\omega_F}\left(\left(1 + \frac{\Delta_{SDW}}{\Delta_{sc(e)}}\right)\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + (\Delta_{sc(e)} + \Delta_{SDW})^2}\right)}{\sqrt{\in_e^2(k) + (\Delta_{sc(e)} + \Delta_{SDW})^2}} + \left(1 - \frac{\Delta_{SDW}}{\Delta_{sc(e)}}\right)\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + (\Delta_{sc(e)} - \Delta_{SDW})^2}\right)}{\sqrt{\in_e^2(k) + (\Delta_{sc(e)} - \Delta_{SDW})^2}}\right) d\in_e(k) \quad (37)$$

Where $\lambda_e = D_e(0)U_e$ and is the electron intra-band coupling parameter.
We know that, as $T \longrightarrow T_C$, $\Delta_{sc(e)} \longrightarrow 0$, then (37) becomes,

$$\frac{2}{\lambda_e} = I_1 + I_2 \quad (38)$$

Where I_1 and I_2 are the integrals defined by,

$$I_1 = \int_0^{\hbar\omega_F}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) + \int_0^{\hbar\omega_F}\frac{\Delta_{SDW}}{\Delta_{sc(e)}}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) \quad (39)$$

And

$$I_2 = \int_0^{\hbar\omega_F}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) - \int_0^{\hbar\omega_F}\frac{\Delta_{SDW}}{\Delta_{sc(e)}}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) \quad (40)$$

Now, applying the Laplace's Transform and after a couple of steps, the first integral in (39) yields,

$$\int_0^{\hbar\omega_F}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\xi(3)}{8\pi^2} \quad (41)$$

where ξ is the Xi function and $\xi(3) = 1.202$.
Using L'Hopital's rule and employing a couple of steps the second integral in (39) gives,

$$\int_0^{\hbar\omega_F}\lim_{\Delta_e \to 0}\frac{\Delta_{SDW}}{\Delta_{sc(e)}}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e(k) = -\frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_F + \Delta_{SDW}}{\hbar\omega_F - \Delta_{SDW}}\right) - \int_0^{\hbar\omega_F}\frac{\beta\Delta_{SDW}^2\tanh^2\left(\frac{\beta}{2}\sqrt{\in_e^2(k) + \Delta_{SDW}^2}\right)}{2\sqrt{\in_e^2(k) + \Delta_{SDW}^2}} d\in_e \quad (42)$$

Similarly, the integral in (40) becomes,

$$\int_0^{\hbar\omega_F} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\xi(3)}{8\pi^2} \quad (43)$$

And

$$\int_0^{\hbar\omega_F} \lim_{\Delta_e\to 0}\frac{\Delta_{SDW}}{\Delta_{sc(e)}} \frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) = -\frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right) - \int_0^{\hbar\omega_F} \frac{\beta\Delta_{SDW}^2\tanh^2\left(\frac{\beta}{2}\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}\right)}{2\sqrt{\epsilon_e^2(k)+\Delta_{SDW}^2}} d\epsilon_e(k) \quad (44)$$

Now, substituting (41-44) into (38) we get,

$$\frac{1}{\lambda_e} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_C}\right) - \frac{7\Delta_{SDW}^2\beta^2\xi(3)}{8\pi^2} - \frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right) \quad (45)$$

Now, if we consider small Δ_{SDW} we can ignore the Δ_{SDW}^2 term.
Therefore, (45) becomes,

$$\frac{1}{\lambda_e} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_C}\right) - \frac{\Delta_{SDW}}{4k_B T_C}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right) \quad (46)$$

Rearranging (46), the expression for T_C due to the electron intra-band is given by,

$$T_C = 1.14\frac{\hbar\omega_F}{k_B}\exp\left(-\frac{1}{\lambda_e} - \gamma_e\Delta_{SDW}\right) \quad (47)$$

Where

$\gamma_e = \frac{1}{4k_B T_C}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right)$, $D_e(0) = 0.0127\ (\mathrm{meV})^{-1}$, $U_e = 74.37$ meV and $\hbar\omega_F$ = 7meV[36].

2.3. SUPERCONDUCTING TRANSITION TEMPERATURE DEPENDANCE ON SPIN DENSITY WAVE ORDER PARAMETER FOR HOLE INTERA BAND

Furthermore, applying analogous procedure as for the electron intra-band, the expression for superconducting transition temperature for the hole intra-band is given as,

$$T_C = 1.14\frac{\hbar\omega_F}{k_B}\exp\left(-\frac{1}{\lambda_h} - \gamma_h\Delta_{SDW}\right) \quad (48)$$

Where $\gamma_h = \frac{1}{4k_B T_C}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right)$ and $\lambda_h = D_h(0)U_h$, $D_h(0) = 0.0127\ (\mathrm{meV})^{-1}$, $U_h = 41.15$ meV and $\hbar\omega_F$ = 7meV [36].

2.4. DEPENDANCE OF SUPERCONDUCTING TRANSITION TEMPERATURE ON SPIN DENSITY WAVE ORDER PARAMETER FOR THE INTER BAND BETWEEN THE ELECTRON AND HOLE BANDS

The inter-band between electron and hole superconducting order parameter can be related to the Green's function as,

$$\Delta_{sc(eh)} = \frac{U_{eh}}{2\beta}\sum_k\left(\ll \hat{c}_{k\uparrow}^\dagger, \hat{c}_{-k\downarrow}^\dagger \gg + \ll \hat{d}_{k\uparrow}^\dagger, \hat{d}_{-k\downarrow}^\dagger \gg\right)$$

where U_{eh} is the inter-band interaction potential between the two bands.

Applying a couple of steps, the transition temperature, T_C due to the inter-band interaction can be expressed as,

$$T_C = 1.14\frac{\hbar\omega_F}{k_B}\exp\left(-\frac{1}{\lambda_{eh}} - \gamma_{eh}\Delta_{SDW}\right) \quad (49)$$

Where $\gamma_{eh} = \frac{1}{4k_B T_C}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right)$ and $\lambda_{eh} = U_{eh}\sqrt{D_e(0)D_h(0)}$ is the inter-band coupling parameter and $D_e(0)$ and $D_h(0)$ are the density of states at the electron and hole Fermi levels respectively.

2.5. DEPENDANCE OF SPIND DENSITY WAVE TRANSITION TEMPERATURE ON SPIN DENSITY WAVE ORDER PARAMETER

Using (25) in (31) and rearranging, the Green's function correlation for the SDW becomes,

$$\ll \hat{c}_{-(k+q)\downarrow}, \hat{d}_{-k\downarrow}^\dagger \gg = -\frac{1}{2}\left(\frac{\Delta_{SC}+\Delta_{SDW}}{\omega^2-\epsilon_k^2-(\Delta_{SC}+\Delta_{SDW})^2} + \frac{\Delta_{SC}-\Delta_{SDW}}{\omega^2-\epsilon_k^2-(\Delta_{SC}-\Delta_{SDW})^2}\right) \quad (50)$$

The SDW order parameter can be related to Green's function as,

$$\Delta_{SDW} = \frac{V}{\beta}\sum_k \ll \hat{c}_{-(k+q)\downarrow}, \hat{d}_{-k\downarrow}^\dagger \gg \quad (51)$$

where V is the spin interaction between charge carriers of electron and hole bands.
Thus, we get,

$$\Delta_{SDW} = -\frac{V}{2\beta}\sum_{k,n}\left(\frac{\Delta_{SC}+\Delta_{SDW}}{(\omega_n^2-\epsilon_k^2)-(\Delta_{SC}+\Delta_{SDW})^2} + \frac{\Delta_{SDW}-\Delta_{SC}}{(\omega_n^2-\epsilon_k^2)-(\Delta_{SDW}-\Delta_{SC})^2}\right) \quad (52)$$

Using the same procedure as above, we obtain,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_{SDW}}\right) - \frac{7\Delta_{SDW}^2\beta^2\xi(3)}{8\pi^2} + \frac{\beta\Delta_{SDW}}{4}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right) \quad (53)$$

With small value of Δ_{SDW}, the Δ_{SDW}^2 term can be ignored. Hence, (53) becomes,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_{SDW}}\right) + \frac{\Delta_{SDW}}{4k_B T_C}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right) \quad (54)$$

From which we get the expression for T_{SDW} to be,

$$T_{SDW} = 1.14\frac{\hbar\omega_F}{k_B}\exp\left(-\frac{1}{\lambda_{SDW}} + \mu\Delta_{SDW}\right). \quad (55)$$

where $\mu = \frac{1}{4k_B T_{SDW}}\ln\left(\frac{\hbar\omega_F+\Delta_{SDW}}{\hbar\omega_F-\Delta_{SDW}}\right)$ and $\lambda_{SDW} = D(0)V$ and is the SDW coupling parameter.

2.6. DEPENDANCE OF SPIN DENSITY WAVE ORDER PARAMETER ON TEMPERATURE IN PURE SPIN DENSITY WAVE REGION

The superconducting state does not exist in pure SDW region which describes the magnetic state only. So, the expression for the SDW coupling parameter with $\Delta_{SC} = 0$ and $\Delta_{SDW} \neq 0$ can be expressed as,

$$\frac{1}{\lambda_{SDW}} = \int_0^{\hbar\omega_F}\frac{\tanh\left(\frac{\beta}{2}\sqrt{\epsilon_k^2+\Delta_{SDW}^2}\right)}{\sqrt{\epsilon_k^2+\Delta_{SDW}^2}}\,d\epsilon_k \quad (56)$$

Thus, after a couple of steps we get,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T}\right) - \Delta_{SDW}^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2(0.1065) \quad (57)$$

As $\Delta_{SDW} \to 0$, $T \to T_{SDW}$ in pure SDW region. Hence, (57) becomes,

$$\frac{1}{\lambda_{SDW}} = \ln\left(1.14\frac{\hbar\omega_F}{k_B T_{SDW}}\right) \quad (58)$$

Now, substituting (58) into (57) we get,

$$\ln\left(1.14\frac{\hbar\omega_F}{k_B T_{SDW}}\right) = \ln\left(1.14\frac{\hbar\omega_F}{k_B T}\right) - \Delta_{SDW}^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2(0.1065) \quad (59)$$

From which we get,

$$\ln\left(\frac{T}{T_{SDW}}\right) = -(\Delta_{SDW})^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2(0.1065)$$

Now, using the relation $\ln(1-x) = -x - \frac{x^2}{2} + \cdots$, Eq. (5.248) becomes,

$$\ln\left(\frac{T}{T_{SDW}}\right) = \ln\left(1-\left(1-\frac{T}{T_{SDW}}\right)\right)$$
$$= -\left(1-\frac{T}{T_{SDW}}\right) - \frac{\left(1-\frac{T}{T_{SDW}}\right)^2}{2}\ln\left(\frac{T}{T_{SDW}}\right)$$
$$= -\left(1-\frac{T}{T_{SDW}}\right) - \left(1-\frac{T}{T_{SDW}}\right)$$
$$= -(\Delta_{SDW})^2\left(\frac{1}{\pi k_B T_{SDW}}\right)^2(0.1065)$$

Finally we get,

$$\Delta_{SDW}(T) = 3.06k_B T_{SDW}\left(1-\frac{T}{T_{SDW}}\right)^2$$
$$\Delta_{SDW}(T) = 3.06k_B T_{SDW}\left(1-\frac{T}{T_{SDW}}\right)^{1/2} \quad (60)$$

where T_{SDW} = 134 K.

3. RESULT AND DISCUSIONS

In this section of the current research work, we present the results we obtained in the previous sections and the analysis we made for the theoretical study on the interplay between superconductivity and spin density waves in two-band model high temperature iron-based superconductor $Ba_{1-x}Na_xFe_2As_2$. By formulating a system Hamiltonian in two band model high temperature iron based superconductor and employing the temperature dependent Green's function technique, we got the equations of motion for the dependence of T_C and T_{SDW} on Δ_{SDW} for $Ba_{1-x}Na_xFe_2As_2$. Now, by using (47-49) and applying some convenient plausible approximations, the phase diagrams of T_C versus Δ_{SDW} for the electron and hole intra-bands and the inter-band between electron and hole bands are plotted for $Ba_{1-x}Na_xFe_2As_2$ as depicted in Fig. 1. As can be observed from the figure, T_C decreases with increasing Δ_{SDW} in each band for the material under consideration.

This indicates that, the existence of magnetic ordering has resulted in the suppression of T_C due to the presence of high spin scattering which flips the spins of the charge carriers and eradicates the singlet correlations which is accountable for the Cooper pairing interactions in $Ba_{1-x}Na_xFe_2As_2$.

Moreover, as shown in Fig. 2, the phase diagram of T_{SDW} versus Δ_{SDW} is plotted using (55) for $Ba_{1-x}Na_xFe_2As_2$. From the figure, it can be observed that, T_{SDW} increases as Δ_{SDW} increases the electron and hole intra bands and their inter-band interactions. Although the increment of T_{SDW} for lower values of Δ_{SDW} seems insignificant, its increment is significant for higher values of Δ_{SDW}. In any case, our results show enhancement of T_{SDW} as Δ_{SDW} increases.

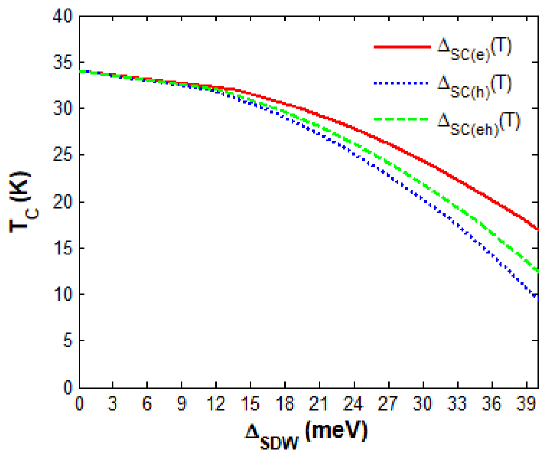

Fig. 1. T_C versus Δ_{SDW} for the electron and hole intra-bands and inter-band for the FeBSC $Ba_{1-x}Na_xFe_2As_2$.

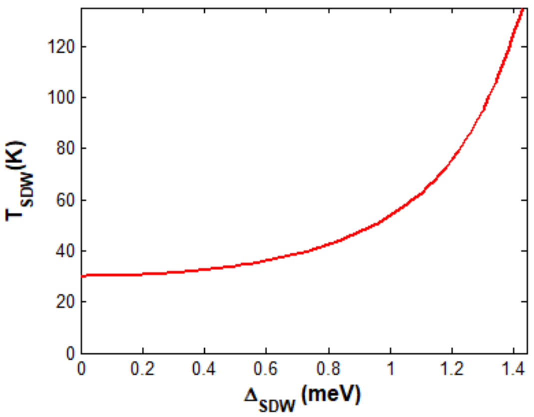

Fig. 2. T_{SDW} versus Δ_{SDW} for the iron based superconductor $Ba_{1-x}Na_xFe_2As_2$.

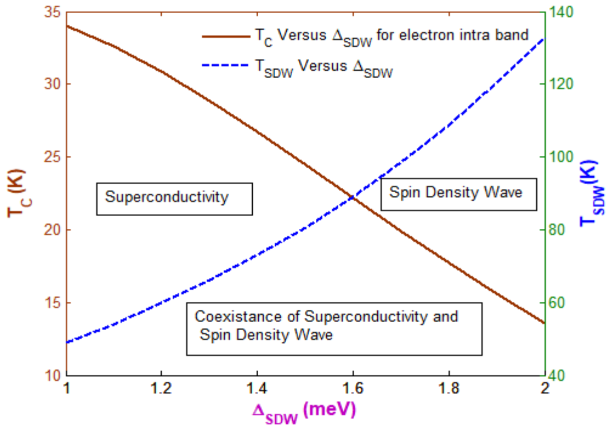

Fig. 3. Interplay between superconductivity and SDW due to electron intra-band interaction for $Ba_{1-x}Na_xFe_2As_2$.

Now, by merging Figures 1 (in the electron intra-band interaction) and 2, we depicted a region where both superconductivity and SDW interplay or coexist in the electron intra-band interaction as shown in Fig. 3 for the two band high temperature FeBSC $Ba_{1-x}Na_xFe_2As_2$. The figure demonstrates the strong competition between the superconductivity and SDW phases emphasizing the interplay between SDW and superconductivity at low magnetic order parameter. More precisely, an extended region of phase interplay or coexistence with Δ_{SDW} and the maximum transition temperature inside this region of ~23 K is observed. When the spin density wave order parameter (Δ_{SDW}) is further increased we obtained pure spin density wave phase.

Similarly, we have plotted the superconducting transition temperatures (T_C) versus Δ_{SDW} due to hole intra-band interaction. This is done by merging Figures 1(in the hole intra-band interaction) and 2 for $Ba_{1-x}Na_xFe_2As_2$ as shown Fig. 4. Here we find that, the interplay between SDW and superconductivity phases at low magnetic order parameters for $T_C < T_{SDW}$ [37]. In fact, the superconducting phase is dominant in the low spin density wave order parameter region whereas SDW phase is observed at large spin density wave order parameter values. The phase diagram shows the interplay between the SDW and superconducting phases in large region and the maximum T_C of 29 K.

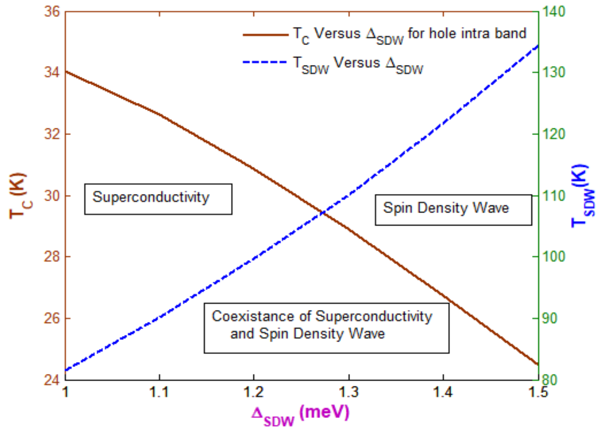

Fig. 4. Interplay between superconductivity and SDW due to hole intra-band interaction for $Ba_{1-x}Na_xFe_2As_2$.

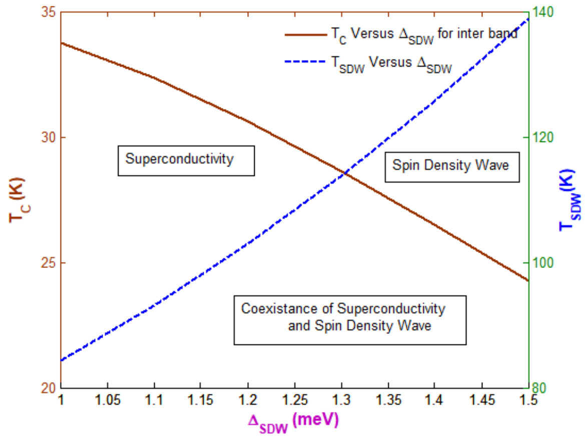

Fig. 5. Interplay between superconductivity and SDW due to inter-band interaction for $Ba_{1-x}Na_xFe_2As_2$.

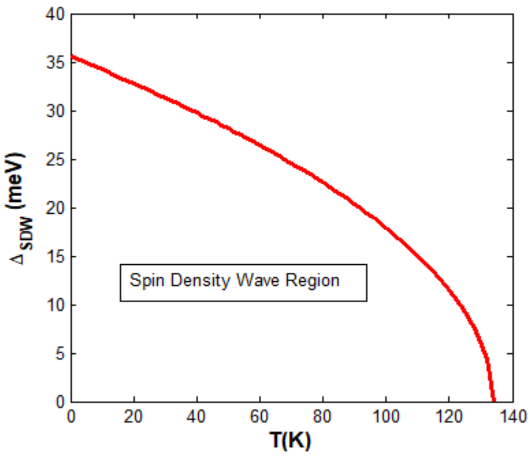

Fig. 6. Δ_{SDW} versus temperature in the pure SDW region for $Ba_{1-x}Na_xFe_2As_2$.

Furthermore, we have plotted T_C versus Δ_{SDW} due to inter-band interaction by merging Figures 1(in the inter-band interaction) and 2 for $Ba_{1-x}Na_xFe_2As_2$ as shown Fig. 5. Here we obtained that, the interplay between the spin density wave and superconductivity phases at low magnetic order parameters for $T_C < T_{SDW}$ [37]. In fact the superconducting phase is dominant in the low spin density wave order parameter region where as SDW phase is observed at large spin density wave order parameter values. More precisely, the graph shows the interplay of the SDW and SC phases in large region and the maximum T_C of 28 K.

Finally, using (60), we have plotted the phase diagrams for Δ_{SDW} versus temperature (T) for FeBSC $Ba_{1-x}Na_xFe_2As_2$ as shown in Fig. 6. As can be observed from the figure, the SDW order parameter decreases as temperature increases and vanishes at T_{SDW} [37].

4. CONCLUSIONS

In the current work, we have demonstrated the interplay between superconductivity and SDW in two band model high temperature FeBSC $Ba_{1-x}Na_xFe_2As_2$. By formulating a model Hamiltonian and using Green's function formalism, we got mathematical expressions which display the reliance of T_C on Δ_{SDW} for the electron and hole intra-bands and inter-band for $Ba_{1-x}Na_xFe_2As_2$. Using the obtained expressions, we have plotted phase diagrams of T_C versus Δ_{SDW} for the intra bands and inter band from which one can observe the suppression of T_C as Δ_{SDW} increases as shown in Fig. 1. The variation of T_{SDW} on Δ_{SDW} has been also plotted as depicted in Fig. 2 showing the enhancement of T_{SDW} as Δ_{SDW} increases for the system under consideration. Lastly, by merging the two phase diagrams we have demonstrated the interplay between superconductivity and SDW in the electron and hole intra-bands and the inter-band for the two band high temperature FeBSC $Ba_{1-x}Na_xFe_2As_2$ as shown in Figures 3-5. Our study shows that T_C decreases as Δ_{SDW} increases, whereas T_{SDW} increases as Δ_{SDW} increases. From these one can comprehend that, the SDW and superconducting phases resist each other whereas T_{SDW} and Δ_{SDW} reinforce each other. It is interesting that many of the superconductors with the highest T_C values possess density wave (DW) instabilities. These are systems with large electron–phonon or electron–electron interactions leading to many different types of instabilities of the Fermi surface in addition to superconductivity. In the current study, the presence of DWs is likely to limit T_C. This competition results in the appearance of the combined phases where DWs and superconductivity interplay and gives rise to a great many new and interesting phenomena regardless of the background microscopic instability mechanisms. The complexity of these competing instabilities lead to the wide diversity of non-trivial phenomena seen in many superconducting materials. We have also plotted the phase diagram of Δ_{SDW} versus temperature in the pure SDW region for $Ba_{1-x}Na_xFe_2As_2$ which shows the suppression of Δ_{SDW} as temperature increases as demonstrated in Fig. 6. Our investigation shows that the interplay between superconductivity and spin density wave is a possible phenomenon and is in agreement with previous observations [38].

FUNDING: This research received no external funding.

CONFLICTS OF INTEREST

There is no self-interested thinking on the part of any of the authors of this study regarding this publication, the research findings that have been published, the expenses related to conducting the research, obtaining and using its findings, or any non-financial personal ties.

REFERENCES

[1] Y. Kamihara, T. Watanabe, M. Hirano, and H. Hosono, "Iron-based layered superconductor $La[O_{1-x}F_x]FeAs$ (x = 0.05 - 0.12) with T_c = 26 K," *Journal of the American Chemical Society*, vol. 130, no. 11, pp. 3296-3297, 2008.

[2] X. Chen, P. Dai, D. Feng, T. Xiang, and F. C. Zhang, "Iron-based high transition temperature superconductors," *National Science Review*, vol. 1, no. 3, pp. 371-395, 2014.

[3] Y. Kamihara, H. Hiramatsu, M. Hirano, R. Kawamura, H. Yanagi, T. Kamiya, and H. Hosono, "Iron-based layered superconductor: LaOFeP," *Journal of the American Chemical Society*, vol. 128, no. 31, pp. 10012-10013, 2006.

[4] C. Wang, L. Li, S. Chi, Z. Zhu, Z. Ren, Y. Li, Y. Wang, X. Lin, Y. Luo, S. Jiang, and X. Xu, "Thorium-doping-induced superconductivity up to 56 K in $Gd_{1-x}Th_xFeAsO$," *Europhysics Letters*, vol. 83, no. 6, pp. 67006, 2008.

[5] A. Nuwal, S. Kakani, and S. L. Kakani, "Generalized multiple gap model for the superconductivity in Fe-As based superconductors," *SOP Transactions on theoretical Physics*, vol. 1, no. 2, pp. 7-25, 2014.

[6] H. Hosono, K. Tanabe, E. Takayama-Muromachi, H. Kageyama, S. Yamanaka, H. Kumakura, M. Nohara, H. Hiramatsu, and S. Fujitsu, "Exploration of new superconductors and functional materials, and fabrication of superconducting tapes and wires of iron pnictides," *Science and Technology of Advanced Materials*, vol. 16, no. 3, pp. 033503, 2015.

[7] Q. Huang, Y. Qiu, W. Bao, J. W. Lynn, M. A. Green, Y. Chen, T. Wu, G. Wu, and X. H. Chen, "Magnetic order in $BaFe_2As_2$, the parent compound of the FeAs based superconductors in a new structural family," *arXiv*:0806.2776, 2008.

[8] M. Rotter, M. Tegel, and D. Johrendt, "Superconductivity at 38 K in the iron arsenide $(Ba_{1-x}K_x)Fe_2As_2$," *Physical Review Letters*, vol. 101, no. 10, pp. 107006, 2008.

[9] M. Kumar, M. Nicklas, A. Jesche, N. Caroca-Canales, M. Schmitt, M. Hanfland, D. Kasinathan, U. Schwarz, H. Rosner, and C. Geibel, "Effect of pressure on the magnetostructural transition in $SrFe_2As_2$," *Physical Review B*, vol. 78, no. 18, pp. 184516, 2008.

[10] F. C. Hsu, J. Y. Luo, K. W. Yeh, T. K. Chen, T. W. Huang, P. M. Wu, Y. C. Lee, Y. L. Huang, Y. Y. Chu, D. C. Yan, and M. K. Wu, "Superconductivity in the PbO-type structure α-FeSe," *Proceedings of the National Academy of Sciences*, vol. 105, no. 38, pp. 14262-14264, 2008.

[11] A. Subedi, L. Zhang, D. J. Singh, and M. H. Du, "Density functional study of FeS, FeSe, and FeTe: Electronic structure, magnetism, phonons, and superconductivity," *Physical Review B*, vol. 78, no. 13, pp. 134514, 2008.

[12] L. N. Bulaevskii, A. I. Buzdin, M. L. Kulić, and S. V. Panjukov, "Coexistence of superconductivity and magnetism theoretical predictions and experimental results," *Advances in Physics*, vol. 34, no. 2, pp. 175-261, 1985.

[13] Y. Onuki, R. Settai, K. Sugiyama, T. Takeuchi, T. C. Kobayashi, Y. Haga, and E. Yamamoto, "Recent advances in the magnetism and superconductivity of heavy fermion systems," *Journal of the Physical Society of Japan*, vol. 73, no. 4, pp. 769-87, 2004.

[14] F. Steglich, "Superconductivity and magnetism: From antagonism to mutual interplay," *Physica C: Superconductivity and its applications*, vol. 460, pp. 7-12, 2007.

[15] A. S. Sefat, R. Jin, M. A. McGuire, B. C. Sales, D. J. Singh, and D. Mandrus, "Superconductivity at 22 K in Co-doped $BaFe_2As_2$ crystals," *Physical Review Letters*, vol. 101, no. 11, pp. 117004, 2008.

[16] L. Wang, F. Hardy, A. E. Böhmer, T. Wolf, P. Schweiss, and C. Meingast, "Complex phase diagram of $Ba_{1-x}Na_xFe_2As_2$: A multitude of phases striving for the electronic entropy," *Physical Review B*, vol. 93, no. 1, pp. 014514, 2016.

[17] D. K. Pratt, W. Tian, A. Kreyssig, J. L. Zarestky, S. Nandi, N. Ni, S. L. Bud'ko, P. C. Canfield, A. I. Goldman, and R. J. McQueeney, "Coexistence of competing antiferromagnetic and

superconducting phases in the underdoped $Ba(Fe_{0.953}Co_{0.047})_2As_2$ compound using x-ray and neutron scattering techniques," *Physical Review Letters*, vol. 103, no. 8, pp. 087001, 2009.
[18] P. Marsik, K. W. Kim, A. Dubroka, M. Rössle, V. K. Malik, L. Schulz, C. N. Wang, C. Niedermayer, A. J. Drew, M. Willis, and T. Wolf, "Coexistence and competition of magnetism and superconductivity on the nanometer scale in underdoped $BaFe_{1.89}Co_{0.11}As_2$," *Physical Review Letters*, vol. 105, no. 5, pp. 057001, 2010.
[19] H. Kim, M. A. Tanatar, W. E. Straszheim, K. Cho, J. Murphy, N. Spyrison, J. P. Reid, B. Shen, H. H. Wen, R. M. Fernandes, and R. Prozorov, "Competition between superconductivity and magnetic/nematic order as a source of anisotropic superconducting gap in underdoped $Ba_{1-x}K_xFe_2As_2$," *Physical Review B*, vol. 90, no. 1, pp. 014517, 2014.
[20] M. H. Julien, H. Mayaffre, M. Horvatić, C. Berthier, X. D. Zhang, W. Wu, G. F. Chen, N. L. Wang, and J. L. Luo, "Homogeneous vs. inhomogeneous coexistence of magnetic order and superconductivity probed by NMR in Co-and K-doped iron pnictides," *Europhysics Letters*, vol. 87, no. 3, pp. 37001, 2009.
[21] Y. Laplace, J. Bobroff, F. Rullier-Albenque, D. Colson, and A. Forget, "Atomic coexistence of superconductivity and incommensurate magnetic order in the pnictide Ba $(Fe_{1-x}Co_x)_2As_2$," *Physical Review B*, vol. 80, no. 14, pp. 140501, 2009.
[22] S. E. Avci, O. Chmaissem, E. A. Goremychkin, S. Rosenkranz, J. P. Castellan, D. Y. Chung, I. S. Todorov, J. A. Schlueter, H. Claus, M. G. Kanatzidis, and A. Daoud-Aladine, "Magnetoelastic coupling in the phase diagram of $Ba_{1-x}K_xFe_2As_2$ as seen via neutron diffraction," *Physical Review B*, vol. 83, no. 17, pp. 172503, 2011.
[23] S. Avci, O. Chmaissem, D. Y. Chung, S. Rosenkranz, E. A. Goremychkin, J. P. Castellan, I. S. Todorov, J. A. Schlueter, H. Claus, A. Daoud-Aladine, and D. D. Khalyavin, "Phase diagram of $Ba_{1-x}K_xFe_2As_2$," *Physical Review B*, vol. 85, no. 18, pp. 184507, 2012.
[24] R. Cortes-Gil, D. R. Parker, M. J. Pitcher, J. Hadermann, and S. J. Clarke, "Indifference of Superconductivity and Magnetism to Size-Mismatched Cations in the Layered Iron Arsenides $Ba_{1-x}Na_xFe_2As_2$," *Chemistry of Materials*, vol. 22, no. 14, pp. 4304-4311, 2010.
[25] Q. W. Wang, D. Y. Liu, Y. M. Quan, and L. J. Zou, "Coexistence and competition of spin-density-wave and superconducting order parameters in iron-based superconductors," *Physics Letters A*, vol. 380, no. 34, pp. 2685-2692, 2016.
[26] J. Dong, H. J. Zhang, G. Xu, Z. Li, G. Li, W. Z. Hu, D. Wu, G. F. Chen, X. Dai, JL. Luo, and Z. Fang, "Competing orders and spin-density-wave instability in $La(O_{1-x}F_x)FeAs$," *Europhysics Letters*, vol. 83, no. 2, pp. 27006, 2008.
[27] A. Chubukov, "Pairing mechanism in Fe-based superconductors. Annu," Rev. *Condens. Matter Phys*, vol. 3, no. 1, pp. 57-92, 2012.
[28] T. Yoshida, S. I. Ideta, T. Shimojima, W. Malaeb, K. Shinada, H. Suzuki, I. Nishi, A. Fujimori, K. Ishizaka, S. Shin, and Y. Nakashima, "Anisotropy of the superconducting gap in the iron-based superconductor $BaFe_2(As_{1-x}P_x)_2$," *Scientific Reports*, vol. 4, no. 1, pp. 7292, 2014.
[29] F. Waßer, A. Schneidewind, Y. Sidis, S. Wurmehl, S. Aswartham, B. Büchner, and M. Braden, "Spin reorientation in $Ba_{0.65}Na_{0.35}$ Fe_2As_2 studied by single-crystal neutron diffraction," *Physical Review B*, vol. 91, no. 6, pp. 060505, 2015.
[30] E. Sheveleva, B. Xu, P. Marsik, F. Lyzwa, B. P. Mallett, K. Willa, C. Meingast, T. Wolf, T. Shevtsova, Y. G. Pashkevich, and C. Bernhard, "Muon spin rotation and infrared spectroscopy study of $Ba_{1-x}Na_xFe_2As_2$," *Physical Review B*, vol. 101, no. 22, pp. 224515, 2020.
[31] Y. Bang and G. R. Stewart, "Superconducting properties of the s±-wave state: Fe-based superconductors," *Journal of Physics: Condensed Matter*, vol. 29, no. 12, pp. 123003, 2017.
[32] H. Maeter, G. Pascua, H. Luetkens, J. Knolle, S. Aswartham, S. Wurmehl, G. Behr, B. Büchner, Z. Shermadini, K. Sedlak, and A. Amato, "Strong Competition of Superconducting and Magnetic Order Parameters in $Ba_{1-x}Na_xFe_2As_2$," *arXiv*:1210.6881, 2012.
[33] S. Sen and H. Ghosh, "Intra-inter band pairing, order parameter symmetry in Fe-based superconductors: A model study," *Journal of Alloys and Compounds*, vol. 618, pp. 102-109, 2015.
[34] D. N. Zubarev, "Double-time Green functions in statistical physics," *Soviet Physics Uspekhi*, vol. 3, no. 3, pp. 320, 1960.
[35] A. Nieto, "Evaluating sums over the Matsubara frequencies," *Computer Physics Communications*, vol. 92, no. 1, pp. 54-64, 1995.
[36] Y. Bang and G. R. Stewart, "Anomalous scaling of ΔC versus T_C in the Fe-based superconductors: the-wave pairing state model," *New Journal of Physics*, vol. 18, no. 2, pp. 023017, 2016.
[37] A. B. Vorontsov, M. G. Vavilov, and A. V. Chubukov, "Interplay between magnetism and superconductivity in the iron pnictides," *Physical Review B*, vol. 79, no. 6, pp. 060508, 2009.
[38] S. E. Avci, J. M. Allred, O. Chmaissem, D. Y. Chung, S. Rosenkranz, J. A. Schlueter, H. Claus, A. Daoud-Aladine, D. D. Khalyavin, P. Manuel, and A. Llobet, "Structural, magnetic, and superconducting properties of $Ba_{1-x}Na_xFe_2$ As_2," *Physical Review B*, vol. 88, no. 9, pp. 094510, 2013.