Journal of information and communication convergence engineering
http://jicce.org

J. Inf. Commun. Converg. Eng. 22(2): 127-132, Jun. 2024 Regular paper

Particle Swarm Optimization based Haptic Localization of Plates with Electrostatic Vibration Actuators

Gwanghyun Jo[1], Tae-Heon Yang[2]*, and Seong-Yoon Shin[3]*, *Member, KIICE*

[1]Department of Mathematical Data Analysis, Hanyang University ERICA, Ansan Gyeonggi-do, Republic of Korea
[2]Department of Mechanical Engineering, Konkuk University, Seoul, Republic of Korea
[3]School of Computer Science and Engineering, Kunsan National University, Guansan-si, Republic of Korea

Abstract

Haptic actuators for large display panels play an important role in bridging the gap between the digital and physical world by generating interactive feedback for users. However, the generation of meaningful haptic feedback is challenging for large display panels. There are dead zones with low haptic sensations when a small number of actuators are applied. In contrast, it is important to control the traveling wave generated by the actuators in the presence of multiple actuators. In this study, we propose a particle swarm optimization (PSO)-based algorithm for the haptic localization of plates with electrostatic vibration actuators. We modeled the transverse displacement of a plate under the effect of actuators by employing the Kirchhoff-Love plate theory. In addition, starting with twenty randomly generated particles containing the actuator parameters, we searched for the optimal actuator parameters using a stochastic process to yield localization. The capability of the proposed PSO algorithm is reported and the transverse displacement has a high magnitude only in the targeted region.

Index Terms: Haptic, Large display panel, Localization, Particle swarm optimization

I. INTRODUCTION

Haptic actuators for large display panels play an important role in bridging the gap between the digital and physical world by generating interactive feedback for users. For example, large touchscreen displays with haptic sensations can be utilized effectively in interactive education for children [1], tabletop medical training [2], and digital musical instruments [3]. Unlike small touchscreen displays in mobile devices, there are technical issues with generating meaningful haptic feedback for large display panels [4]. For example, if only a few actuators are employed, dead zones arise where no haptic feedback is sensed [5]. In contrast, localizing tactile feedback in the subregions of large display panels is challenging in the presence of multiple actuators as one must control the traveling wave generated by each actuator [6].

Attempts have been made to localize the subregions of the display by adjusting the parameters of each actuator such as the magnitude and phase. Studies on haptic localization evaluation have been empirical and experimental [7]. However, it is difficult to find the global maximization using an empirical approach because there are infinitely many choices of parameters for the actuators. Therefore, there is a need to precisely evaluate haptic feedback and optimize algorithms for localization.

In this paper, we propose a new algorithm for haptic actuator localization on a large display based on an optimization-type algorithm to determine the maximizer of the localization factor (LF). The difficulty lies in the fact that it is almost impossible to find the partial derivatives of the LF

Received 24 October 2023, **Revised** 23 December 2023, **Accepted** 23 December 2023
***Corresponding Author** Tae-Heon Yang (E-mail: thyang3572@gmail.com) Department of Mechanical Engineering, Konkuk University
Seong-Yoon Shin (E-mail: s3397220@kunsan.ac.kr) School of Software, Kunsan National University

Open Access https://doi.org/10.56977/jicce.2024.22.2.127 print ISSN: 2234-8255 online ISSN: 2234-8883

with respect to the actuator parameters such as the magnitude or phase. Therefore, we employed particle swarm optimization (PSO) because it involves a stochastic process without a gradient [8-11]. For convenience, electrostatic actuators were considered in the analysis because it is easy to control the magnitude and phases of the excitations using actuators [12]. In addition, the transverse displacement under the effect of excitation was modeled using the Kirchhoff-Love (KL) plate theory [13-14].

To evaluate the LF, the KL solutions were reconstructed using a finite difference method (FDM) algorithm, where the forward Euler scheme was used for time discretization. To determine the maximizer of the LF, random samples were generated for the actuator parameters. Then, using the PSO concept, the location of each particle was updated to find better positions for a higher LF.

The remainder of this paper is organized as follows. In Section 2, we propose a PSO-based optimization algorithm to determine the maximizer of the LF. Section 3 presents the results using the proposed algorithm. Finally, the conclusions are presented.

II. METHODS

In this section, we describe the PSO algorithm to localize electrostatic vibration actuators. The model equation is described in Subsection A. Next, the FDM-based solution reconstruction algorithm is presented in Subsection B. Finally, the PSO algorithm is proposed in Subsection C.

A. Modeling of the Display Panel Under the Effects of Electrostatic Vibration Actuators

To analyze the vibrating plates, we employ the KL plate theory [11-12]. We consider a rectangular-shaped plate $\Omega = [0, L] \subset \mathbb{R}^2$ under the excitation $f(x, t)$ by the electrostatic vibration actuators. Then, the transverse displacement denoted by $u(x, t)$: $\Omega\times\mathbb{R}\to\mathbb{R}$ is calculated using the governing equation

$$D\nabla^2\nabla^2 u(\mathbf{x},t) + \rho H \frac{\partial^2(\mathbf{x},t)}{\partial^2 t} + c\frac{\partial(\mathbf{x},t)}{\partial t} = f(\mathbf{x},t) \qquad (1)$$

Here, $D = EH^3/12(1-\nu^2)$ is the flexural rigidity, where E is the modulus of elasticity, ρ is the density, H is the thickness of the plate, c is the damping parameter of the plate and ν is the Young's modulus. We consider four actuators attached to the plate at Ω_i, ($i = 1, ..., 4$) having different magnitudes and phases (see Fig. 1). Hence, the excitation generated by the actuator systems of frequency f is described as

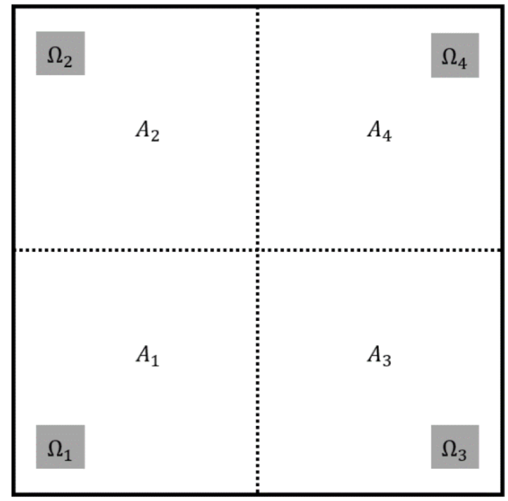


Fig. 1. Illustration of a large display with four actuators. The actuators are attached to the display on Ω_i (i = 1, ..., 4). The quadrants of the plate divided by mid-lines (dashed) are denoted by A_1, ..., A_4.

$$f(\mathbf{x},t) = \begin{cases} w_1\cos(2\pi ft - \phi_1), & if\ \mathbf{x}\in\Omega_1 \\ w_2\cos(2\pi ft - \phi_2), & if\ \mathbf{x}\in\Omega_2 \\ w_3\cos(2\pi ft - \phi_3), & if\ \mathbf{x}\in\Omega_3 \\ w_4\cos(2\pi ft - \phi_4), & if\ \mathbf{x}\in\Omega_4 \\ 0, & otherwise \end{cases} \qquad (2)$$

Here, the parameters w_i's and ϕ_i's are the magnitudes and phases of the actuators, respectively, which can be determined by the PSO optimization algorithm. For a well-posed system, the (homogeneous) boundary and initial conditions are imposed as:

(Boundary condition) $f(\mathbf{x}, t)=0,\ \mathbf{x}\in\partial\Omega$, (3)
(Initial condition) $f(\mathbf{x}, 0)=0$. (4)

Then, the governing equation (1) together with (3-4) yields a system with a unique solution.

The goal of this study is to localize the effects of actuators on the subregions of the display (see Fig. 1), which is denoted by A_i (i = 1, ..., 4). The haptic LF in region A_i (i = 1, ..., 4) is defined by the relative kinetic energies of the subregion. Here, the kinetic energy on subset $D\subset\mathbb{R}$ at time t is defined by

$$E(t) = \frac{1}{2}\rho H\int_D \left(\frac{\partial u}{\partial t}\right)^2 d\mathbf{x}$$

In summary, the LF on A_i (i = 1, ..., 4) is expressed mathematically as

$$LF_i = \frac{\int_{T_0}^{T}\int_{A_i} E(t)dAdt}{\int_{T_0}^{T}\int_{\Omega} E(t)dAdt} = \frac{\int_{T}^{T}\int_{A_i}\left(\frac{\partial u}{\partial t}\right)^2 d\mathbf{x}dt}{\int_{T_0}^{T}\int_{\Omega}\left(\frac{\partial u}{\partial t}\right)^2 d\mathbf{x}dt} \qquad (5)$$

From equation (5), it is clear that $0 \le LF_i \le 1$. Here, time integration is performed on $[T, T_0]$ to obtain the average kinetic energy, where the values T and T_0 are determined as presented in the Results section. Without loss of generality, we can fix $i = 1$ and drop the sub-index i in LF_i. Our optimization algorithm can be applied to the other subregions in the same manner.

Now, the optimization problem for localization can be stated as:

Problem 1. *Find* w_i, ϕ_i $(i = 1, ..., 4)$ *that maximizes the LF.*

B. FDM based Prediction of Solution

As it is difficult to obtain analytical solutions of equation (1), we employed the FDM algorithm to solve it. The domain is triangulated by uniform rectangles of size h to yield $N \times N$ nodes, i.e.,

$0 = x_1 < x_2 < ... < x_n = \mathrm{L}$,
$0 = y_1 < y_2 < .. .< y_n = \mathrm{L}$.

The trial function in the discretized space is denoted by u_h. The forward Euler approach is employed for time discretization. The time step is denoted by Δt. The nodal values $u_h (x_i y_j, n\Delta t)$ are denoted by $u^n_{i,j}$. Similarly, $f(x_i y_j, n\Delta T)$ is denoted by $f^n_{i,j}$. When there is no worry of confusion, we drop superscript n in $u^n_{i,j}$.

Now, we need to approximate the operator $\nabla^2\nabla^2 u_h = u_{xxxx} + 2u_{xx,yy} + u_{yyyy}$ in equation (1) on the internal points of the domain. The fourth order derivatives and $\nabla^2\nabla^2 u_h$ at $(x_i y_j)$ can be numerically approximated as in [15]:

$$
\begin{aligned}
u_{xxxx}|_{i,j} &= \frac{u_{i-2,j} - 4u_{i-1,j} + 6u_{i,j} - 4u_{i+1,j} + u_{i,j}}{h^4}, \\
u_{yyyy}|_{i,j} &= \frac{1}{h^4}\frac{u_{i,j-2} - 4u_{i,j-1} + 6u_{i,j} - 4u_{i,j+1} + u_{i,j+2}}{h^4}, \\
u_{xx,yy}|_{i,j} &= \frac{u_{i-1,j-1} - 2u_{i,j-1} + u_{i+1,j-1} - 2u_{i-1,j}}{h^4} \\
&+ \frac{+4u_{i,j} - 2u_{i+1,j} + u_{i-1,j} - 2u_{i,j+1} + u_{i+1,j}}{h^4}, \\
\nabla^2\nabla^2 u_h|_{i,j} &= u_{xxxx}|_{i,j} + 2u_{xx,yy}|_{i,j} + u_{yyyy}|_{i,j}
\end{aligned}
$$

Now, the forward FDM algorithm is expressed as

$$
\begin{aligned}
u^n_{i,j} = 2u^{n-1}_{i,j} - u^n_{i,j} - \frac{c\Delta T}{\rho H}\left(u^{n-1}_{i,j} - u^{n-2}_{i,j}\right) \\
+ \frac{(\Delta t)^2}{\rho H}\left(D\nabla^2\nabla^2 u|_{i,j} + f^n_{i,j}\right)
\end{aligned} \tag{6}
$$

Equation (6) is repeated with increasing n indexes up to N to produce numerical solutions up to the target time $T^{target} = N\Delta T$. Once the solution is generated by the FDM, the LF can be numerically evaluated by equation (5), which is denoted as $LF_{wi,\phi i}$ to emphasize the dependency with respect to the parameters.

C. Particle Swarm Optimization-Based Haptic Localization Algorithm

To solve optimization **Problem 1**, we employ particle swarm optimization (PSO) [8,9]. While the dimension of the parameters in **Problem 1** is eight (four weights and four phases), we can reduce the dimension to six by fixing $w_i = 1$ and $\phi_1 = 0$. This is valid because 1) the governing equation (1) is linear in u, and 2) the excitation function in (2) is periodic with respect to t.

Although an objective of this study is to determine the parameters w_2, w_3, w_4, ϕ_2, ϕ_3, ϕ_4 to maximize the LF, the PSO algorithm is a minimization-type algorithm. Therefore, we modify the optimization problem (with six parameters) slightly as:

Problem 2. *Find* w_i, ϕ_i $(i = 2, ..., 4)$ *that minimizes the loss function* $\Lambda(w_i, \phi_i) = 1 - LF_{wi,\phi i}$.

Regarding this new objective, we provide a brief remark regarding the equivalence of **Problem 1** and **Problem 2**:

Remark.

As $0 \le LF_{wi,\phi i} \le 1$, minimizing $\Lambda(w_i, \phi_i)$ in the interval [0,1] is equivalent to maximizing $LF_{wi,\phi i}$.

The PSO algorithm for **Problem 2** is as follows. First, n randomly generated particles ($\mathbf{X}^i_1$, $i = 1, ..., n$) and their velocities ($\mathbf{V}^i_1$, $i = 1, ..., n$) are defined. Here, the particle $\mathbf{X}^i_1$ contains parameters $(w^i_2, w^i_3, w^i_4, \phi^i_2, \phi^i_3, \phi^i_4)$ and $\mathbf{V}^i_1$ that are sampled from the uniform distribution in [-2,2]. The main idea of PSO is to evolve the locations of the particles to determine the best maximizing position for the LF. Once random sampling is generated, the personal best vectors (*Pbests*) are initialized with these particles, i.e.,

$$\mathbf{P}^i_1 = \mathbf{X}^i_1$$

Now, each particle is substituted in the loss function to find the particle with the lowest loss function (*Gbest*) among n particles, i.e.,

$$\mathbf{G}_1 = \operatorname{argmin}_{X^i_0} \Lambda(\mathbf{X}^i_0).$$

Then, the particles and velocities are updated recursively as $(l = 1, 2, ...)$

$$\mathbf{V}^i_{\ell+1} = w\kappa^{\ell}\mathbf{V}^i_{\ell} + c_1 U(0,1)\mathbf{P}^i_{\ell} + c_2 U(0,1)\mathbf{G}^i_{\ell} \tag{7}$$

$$\mathbf{X}^i_{\ell+1} = \mathbf{X}^i_{\ell} + \mathbf{V}^i_{\ell+1} \tag{8}$$

Equation (8) determines the updated locations of the particles with respect to the velocities. The velocities are updated using equation (7), where *Pbests* and *Gbest* are used to correct the current velocities with cognitive ($c_1 > 0$) and social weights ($c_2 > 0$). Here, $U(0, 1)$ represents a uniform distribution in the interval [0,1]. Also, the so-called inertia parameter $x > 0$ and decaying parameter $0 < \kappa < 1$ are used to prevent sudden change of velocities in iterations. At the end of each iteration, *Pbests* and *Gbest* are updated to save the best position of each particle and the global best position, respectively.

We state the optimization algorithm for haptic localization with *Nit* number of iterations:

Algorithm. PSO

Set $\ell = 1$.

1. Initialization.

1) Generate $\mathbf{X}_1^i$ and $\mathbf{V}_1^i$ $(\mathrm{i} = 1, \ldots, \mathrm{n})$.

2) Set *Pbest* vectors

$$\mathbf{P}_1^{\mathrm{i}} = \mathbf{X}_1^i$$

3) Perform FDM simulations to obtain $\Lambda(\mathbf{X}_0^i)$.

4) Set the *Gbest* vector

$$\mathbf{G}_1 = \mathrm{argmin}_{\mathbf{X}_0^i} \Lambda(\mathbf{X}_0^i).$$

2. Repeat the following process until $\ell < Nit$.

1) Update velocities $(\mathrm{i} = 1, \ldots, \mathrm{n})$

$$\mathbf{V}_{\ell+1}^i = w\kappa^\ell \mathbf{V}_\ell^i + c_1 U(0,1)\mathbf{P}_\ell^i + c_2 U(0,1)\mathbf{G}_\ell^i,$$

2) Update positions $(i = 1, \ldots, n)$

$$\mathbf{X}_{\ell+1}^i = \mathbf{X}_\ell^i + \mathbf{V}_{\ell+1}^i.$$

3) Perform FDM simulations to obtain $\Lambda(\mathbf{X}_\ell^i)$.

4) Update *Pbest* for all particles $(\mathrm{i} = 1, \ldots, \mathrm{n})$

$$\mathbf{P}_{\ell+1}^{\mathrm{i}} = \mathrm{argmin}_{\mathbf{X}_\ell^i} \Lambda(\mathbf{X}_{\ell+1}^i),$$

5) Update *Gbest*

$$\mathbf{G}_{\ell+1} = \mathrm{argmin}_{\theta \in \{\mathbf{P}_\ell^1, \ldots, \mathbf{P}_\ell^n, \mathbf{G}_\ell\}} \Lambda(\theta),$$

6) Set $\ell = \ell + 1$.

III. Results

In this section, the capability of the PSO algorithm is demonstrated for haptic localization. First, the parameters of the governing equations are described. The domain is $\Omega = [0, 0.2\ \mathrm{m}]^2$ and the locations of the actuators are

$\Omega_1 = [0.03\ \mathrm{m}, 0.04\ \mathrm{m}] \times [0.03\ \mathrm{m}, 0.042\ \mathrm{m}]$
$\Omega_2 = [0.03\ \mathrm{m}, 0.042\ \mathrm{m}] \times [0.158\ \mathrm{m}, 0.17\ \mathrm{m}]$
$\Omega_3 = [0.158\ \mathrm{m}, 0.17\ \mathrm{m}] \times [0.03\ \mathrm{m}, 0.042\ \mathrm{m}]$
$\Omega_4 = [0.158\ \mathrm{m}, 0.17\ \mathrm{m}] \times [0.158\ \mathrm{m}, 0.17\ \mathrm{m}]$

The physical properties of the plates are $E = 1$ G Pas, $\rho = 10^3$ kg/m^3, $H = 0.5$ mm, $\nu = 0.33$, and $f = 290$ Hz. For the FDM simulations, 27×27 nodal points were used with a time-step $\Delta T = 8 \cdot 10^{-5}$ The time parameters in equation (5) were T = 0.1s with $T_0 = 0.05$s. Finally, we used twenty particles for the PSO algorithm with *Nit* = 80. The inertial, cognitive and social weights were set to $w = 4$, $c_1 = c_2 = 2$ with decaying parameter $\kappa = 0.99$.

For comparison, let us consider parameters with the same magnitudes and phases assigned to each actuator, i.e.,

$w_i = 1,\ \phi_i = 0,\ i = 1, \ldots, 4$

The graphs of u_h obtained from the FDM simulations with the above choices are shown in Fig. 2(a). It can be observed that the displacements are symmetrical. As expected, the *LF* is 0.25, indicating that all subregions vibrate equally.

Now, we intend to localize region A_1 by controlling the actuator parameters. A naïve method is to activate an actuator only in region A_1, i.e.,

$w_1 = 1,\ w_2 = w_3 = w_4 = 0.$

As shown in Fig. 2 (b), the bottom-left side has a high magnitude for u_h whereas the other regions have a small magnitude. In this case, the *LF* is 63.67%, indicating the need for an optimization process to enhance the localization.

Finally, we report the optimizing factors obtained by the PSO algorithm. The loss function with respect to the number of iterations is shown in Fig. 2. The cost function gradually decreases as the number of iterations increases. In the last iteration, the *LF* is 72%. Considering the area of A_1 (25% of the whole plate), the result of 72% of the kinetic energy being focused on A_1 is meaningful. The actuator parameters obtained by PSO are listed in Table 1.

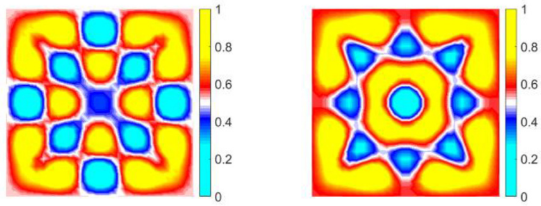
Table 1. Actuator parameters obtained by the PSO algorithm

w_1	1
w_2	0.221
w_3	0.221
w_4	0.074
ϕ_1	0 (radian)
ϕ_2	0.5π (radian)
ϕ_3	0.5π (radian)
ϕ_4	0.596π (radian)

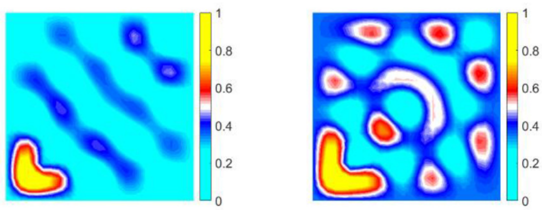
The graphs of u_h obtained using the PSO algorithm are plotted in Fig. 2(c). Compared with Fig. 2(b), we observe that the transverse displacement decreases on A_2, A_3, A_4, leading to localization on A_1. Therefore, the capability of the proposed algorithm to localize sensations using multiple actuators is verified.

IV. CONCLUSION

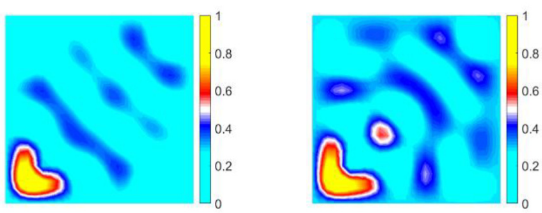
For a large display panel, a small number of actuators leads to the appearance of a dead zone with no haptic sensation. In contrast, the traveling wave generated by the actua-



(a) Simulation results with $w_i = 1, \phi_i = 0, i = 1, \ldots, 4$.



(b) Simulation results with $w_1 = 1, w_2 = w_3 = w_4 = 0$.



(c) Simulation results with parameters obtained using PSO.

Fig. 2. Graphs of u_h for the three cases. (a) $w_i = 1$, $\phi_i = 0$, $i = 1, \ldots, 4$, (b) $w_1 = 1$, $w_2 = w_3 = w_4 = 0$. (c) Parameters obtained by PSO. For all the cases, the left and right figures correspond to the solution at $t = 0.0464$ *s* and $t = 0.0696$ *s*, respectively.

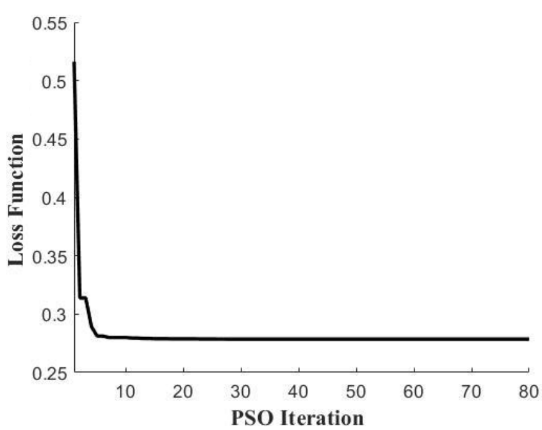


Fig. 3. A plot of the cost function with respect to PSO iterations.

tors must be controlled when there are multiple actuators to localize the sensations. In this study, we propose a PSO-based haptic localization method for large plates with four electrostatic vibration actuators of different magnitudes and phases. We modeled and simulated the behavior of the transverse displacement of the plates under the effects of actuators by employing a Kirchhoff-Love plate. The actuator parameters were controlled based on the parameters of the PSO algorithm. The transverse displacement generated by the actuators tuned by the PSO algorithm has a high amplitude only in the targeted quadrant, leading to the desired localization effect. Compared with previous studies (e.g., [7]), our algorithm robustly updates the locations of the actuators because the excitation function can be changed easily.

We believe that this PSO algorithm can be easily extended to various haptic localizations for other types of devices such as small panels in mobile devices or bar-shaped display panels. In addition, our methods can be extended to various localization-sensing applications such as interactive medical learning and interactive learning.

Acknowledgment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean Government (MSIT) (No. 2020R1C1C1A01005396).

REFERENCES

[1] I. Han, and J. B. Black, "Incorporating haptic feedback in simulation for learning physics," *Computers & Education*, vol. 57, no. 4, pp. 2281-2290, 2011. DOI: 10.1016/j.compedu.2011.06.012.

[2] U. von Zadow, S. Buron, T. Harms, F. Behringer, K. Sostmann, and R. Dachsel, "SimMed: combining simulation and interactive tabletops for medical education," in *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, pp. 1469-1478, 2013. DOI: 10.1145/2470654.2466196.

[3] C. H. Mejia, P. Germano, S. C. Echeverri, and Y. Perriard, "Artificial Neural Networks for Impact Position Detection in Haptic Surfaces," in *2019 IEEE International Ultrasonics Symposium (IUS)*, Aticle ID: 19242551, 2019. DOI: 10.1109/ULTSYM.2019.8925548.

[4] S. Park, D. Kim, and N. C. Park, "Rendering high-fidelity vibrotactile feedback on a plate via optimization of actuator driving signals," in *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, vol. 261, no. 6, pp. 548-555, 2020. DOI: 10.1109/ACCESS.2023.3247606.

[5] C. Hudin, J. Lozada, and V. Hayward, "Localized tactile feedback on a transparent surface through time-reversal wave focusing." *IEEE transactions on haptics*, vol. 8, no. 2 pp. 188-198, 2015. DOI: 10.1109/TOH.2015.2411267.

[6] T. W. Mason, "Design and testing of an electrostatic actuator with dual-electrodes for large touch display applications," Ph.D. dissertation, Miami University, 2021.

[7] Rajkumar, S. Mohan, et al. "Modeling and Experimental Evaluation of Haptic Localization Using Electrostatic Vibration Actuators." IEEE Access 11 (2023): 18582-18589.

[8] J. Kennedy, and R. Eberhart. "Particle swarm optimization." in *Proceedings of ICNN'95-International Conference on Neural Networks*. vol. 4, pp. 1942-1928, 1995. DOI: 10.1109/ICNN.1995.488968.

[9] W. Dongshu, D. Tan, and L. Liu. "Particle swarm optimization algorithm: an overview," *Soft computing* vol. 22, pp. 387-408, 2018. DOI: 10.1007/s00500-016-2474-6.

[10] T. M. Shami, A. A. El-Saleh; M. Alswaitti, Q. Al-Tashi, M. A.

Summakieh, and S. Mirjalili, “Particle Swarm Optimization: A Comprehensive Survey,” vol. 10, pp. 10031-10061, 2022. DOI: 10.1109/ACCESS.2022.3142859.

[11] J. Tian, M. Hou, H. Bian, and J. Li, “Variable surrogate model-based particle swarm optimization for high-dimensional expensive problems,” *Complex & Intelligent Systems*, vol. 9, pp. 3887-3935, 2023. DOI: 10.1007/s40747-022-00910-7.

[12] C. Basdogan, F, Giraud, V. Levesque, S. Choi, “A review of surface haptics: Enabling tactile effects on touch surfaces,” *IEEE transactions on haptics*, vol. 13 no. 3, pp. 450-470, 2020. DOI: 10.1109/TOH.2020.2990712.

[13] D. N. Arnold, A. L. Madureira, and S. Zhang, “On the range of applicability of the Reissner–Mindlin and Kirchhoff–Love plate bending models,” *Journal of elasticity and the physical science of solids*, vol. 67, no. 3, pp. 171-185, 2002. DOI: 10.1023/A:1024986427134.

[14] D. Kropiowska, L. Mikulski, and P. Szeptyński, “Optimal design of a Kirchhoff-Love plate of variable thickness by application of the minimum principle,” *Structural and Multidisciplinary Optimization*, vol. 59, no. 5, pp. 1581-1598, 2019. DOI: 10.1007/s00158-018-2143-3.

[15] R. K. Mohanty, “A fourth-order finite difference method for the general one-dimensional nonlinear biharmonic problems of first kind,” *Journal of computational and applied mathematics*, vol. 114, no. 2, pp. 275-290, 2000. DOI: 10.1007/s00158-018-2143-3.

Gwanghyun Jo

received his M.S. and Ph. D. degree from the Department of Mathematical Science, KAIST in 2013 and 2018, respectively. From 2019 to 2023, he was a faculty member in the Department of Mathematics at Kunsan University, Republic of Korea. From 2023 to present, he has been a faculty member in the Department of Mathematical Data Science, HanyangUniversity ERICA. His research interests include numerical analysis, computational fluid dynamics and machine learning.

Tae-Heon Yang

received the B.S. degree from Yonsei University, Republic of Korea in 2006, and M.S. and Ph.D. degrees from the Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology (KAIST) in 2008 and2012, respectively. From 2012 to 2017, he was a Senior Research Scientist at the Korea Research Institute of Standards and Science. From 2018 to 2023, he was with the Faculty of Electronics Engineering, Korea National University of Transportation. Since 2024, he has been a faculty of mechanical engineering, Konkuk University. His research interests include haptic sensors and actuators, medical simulators and human–computer interfaces.

Seong Yoon Shin

received his M.S. and Ph.D. degrees from the Dept. of Computer Information Engineering of Kunsan National University, Gunsan, Republic of Korea in 1997 and 2003, respectively. From 2006 to present, he has been a professor in the School of Computer Science and Engineering. His research interests include image processing, computer vision and virtual reality. He can be contacted at email: s3397220@kunsan.ac.kr