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GENERALIZED SMARANDACHE CURVES WITH FRENET-TYPE FRAME

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Abstract. In this study, we investigate Smarandache curves with Frenet-type frame in Myller configuration for Euclidean 3-space E_3. Also, we introduce some characterizations and invariants of them. Then, we construct a numerical example with respect to these special Smarandache curves in order to understand the obtained materials.

1. Introduction

The theory of curves has quite an importance and applications in several work-frames such as; mathematics, architecture, engineering, etc., and also attracts a lot of researchers. It is one of the most important concepts of classical differential geometry in mathematics, as well. Additionally, one of the most popular examples of the special curves is Smarandache curves, which are determined as the regular curve constructed by these vectors, when the Frenet vectors of the unit speed regular curve are taken as position vectors [2,29].

On the other hand, the Frenet-Serret frame, which was founded by two researchers Frenet [8] and Serret [18], was a milestone for classical differential geometry. Then, several researchers investigated some new types of moving frames like Darboux frame [6]. In [16], the concepts of the geometry of Myller configurations $\mathfrak{M}(C, \overline{\xi}, \pi)$ and tangent Myller configuration $\mathfrak{M}_t(C, \overline{\xi}, \pi)$ are examined. As a generalization, a versor field (namely a unit vector field) and a plane field are denoted by $(C, \overline{\xi})$ and (C, π), respectively. A couple $\{(C, \overline{\xi}), (C, \pi)\}$ where $\overline{\xi} \in \pi$ is called a Myller configuration and denoted by $\mathfrak{M}(C, \overline{\xi}, \pi)$. If the planes π are tangent to C, it is called a tangent Myller configuration $\mathfrak{M}_t(C, \overline{\xi}, \pi)$ [13,15,16]. Additionally, if C is a curve on the surface S, the geometry of the field $(C, \overline{\xi})$ on surface S is the geometry of the associated Myller configurations $\mathfrak{M}_t(C, \overline{\xi}, \pi)$. Moreover, the Darboux frame is investigated for a Myller configuration $\mathfrak{M}(C, \overline{\xi}, \pi)$. Miron studied the Myller configuration in some studies [16,17]. We want to refer to the book [16] detailed terminology

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concerning the concept of Myller configuration.

In the existing literature, Myller configuration is examined in several studies. Macsim et al. scrutinized the special curves in a Myller configuration and also their properties [14]. Moreover, rectifying curves [13], Bertrand curves [15] in Myller configuration, and also Myller configuration for 4-dimensional Lorentz spaces [9] were determined. Additionally, İşbilir and Tosun introduced the osculating-type curves in Myller configuration in Euclidean 3-space E_3 [11], rectifying-type curves in Myller configuration for Euclidean 4-space E_4 in [10].

When the literature is examined, several researchers combined the Smarandache curves and different types special frames and special curves. Ali introduced the Smarandache curves with Frenet frame [1] and Taşköprü and Tosun investigated the Smarandache curves related to Sabban frame [28]. Furthermore, Alıç and Yılmaz studied the Smarandache curves with NCW-frame [2]. Çetin et al. investigated the Smarandache curves with respect to Bishop frame [5]. Then, Yılmaz and Savcı introduced Smarandache curves for type-2 Bishop frame [30]. Bektaş and Yüce determined the Smarandache curves with the Darboux frame in Euclidean 3-space [4]. Additionally, in [21], an application for Smarandache curves according to Frenet frame was given by Şenyurt and Sivas. Şenyurt and Öztürk studied the Salkowski and anti-Salkowski curves concerning the Smarandache curves with Frenet frame in [19, 20], respectively. Then, Şenyurt et al. studied the Smarandache curves with Flc frame [26]. Also, some studies had been completed in Minkowski space such as; Bayrak et al. [3] studied the Smarandache curves according to Frenet frame in Minkowski space, and Turgut and Yılmaz introduced the Smarandache curves with Frenet frame in Minkowski 4-space in [29]. Şenyurt and Eren gave the Smarandache curves of spacelike Salkowski curves with a spacelike principal normal related to Frenet frame [22] and Smarandache curves of spacelike anti-Salkowski curves with a timelike principal normal related to the Frenet frame [23]. Also, Şenyurt and Eren introduced Smarandache curves established from the spacelike anti-Salkowski curve with a spacelike principal normal according to Frenet frame [24] and Smarandache curves established from spacelike Salkowski curves with timelike principal normal according to the Frenet frame [25]. In addition to these, Karaman [12] and Demircan [7] studied the Smarandache curves in their master thesis.

In this paper, we investigate the Smarandache curves with Frenet-type frame in Myller configuration for Euclidean 3-space E_3. Due to the geometry of versor fields along a curve with Frenet-type frame in Myller configuration for Euclidean 3-space E_3 is a generalization of the theory of curves in classical Euclidean 3-space[1], we construct a generalization for Smarandache curves. After giving these special and generalized Smarandache curves, we obtain some characterizations and invariants of them. We also give relations between the

[1]See for generalization [9, 16].

Smarandache curves with Frenet-type frame in Myller configuration for Euclidean 3-space and Frenet frame in Euclidean 3-space. Moreover, we construct a numerical example with respect to these special Smarandache curves to understand the obtained materials.

2. Frenet-Type Frame in Myller Configuration

In this section, we remind some backgrounds with respect to the Frenet-type frame in Myller configuration for 3-dimensional Euclidean space.

Let $(C, \overline{\xi})$ be a versor field and let $\overline{r}(s)$ be a position vector of the curve C, where s is the arc-length on the curve C. For Frenet-type frame $\mathcal{R}_F = \{P, \overline{\xi}_1, \overline{\xi}_2, \overline{\xi}_3\}$ of versor field, then we can write:

$$\overline{r}'(s) = a_1(s)\overline{\xi}_1(s) + a_2(s)\overline{\xi}_2(s) + a_3(s)\overline{\xi}_3(s), \tag{1}$$

where $a_1^2(s) + a_2^2(s) + a_3^2(s) = 1$. Also, the following equations are satisfied:

$$\begin{cases} \overline{\xi}_1'(s) = K_1(s)\overline{\xi}_2(s), \\ \overline{\xi}_2'(s) = -K_1(s)\overline{\xi}_1(s) + K_2(s)\overline{\xi}_3(s), \\ \overline{\xi}_3'(s) = -K_2(s)\overline{\xi}_2(s), \end{cases} \tag{2}$$

where $K_1 > 0$. K_1-curvature and K_2-torsion have the same geometrical interpretation as the curvature and torsion of a curve in E_3. It should be noted that if $a_1(s) = 1$, $a_2(s) = 0$, and $a_3(s) = 0$, then we get the Frenet equations of a regular curve in 3-dimensional Euclidean space E_3 [8,16,18]. The fundamental theorem of invariants for versor field $(C, \overline{\xi})$ is expressed as follows [16]:

Theorem 2.1 ([16])**.** *If the invariants $K_1(s), K_2(s), a_1(s), a_2(s), a_3(s)$, with $a_1^2(s) + a_2^2(s) + a_3^2(s) = 1$ are smooth functions for $s \in [a, b]$, then there exist a curve $C : [a, b] \to E_3$ parametrized by arc-length s and a versor field $\overline{\xi}(s)$, $s \in [a, b]$, whose curvature, torsion and the functions $a_i(s)$ are $K_1(s), K_2(s)$ and $a_i(s), i = 1, 2, 3$, respectively. Any two such versor fields $(C, \overline{\xi})$ differ by a proper Euclidean motion.*

3. Smarandache Curves in Myller Configuration

In this section, we determine the Smarandache curves, which can be called also generalized Smarandache curves, with Frenet-type frame in Myller configuration for Euclidean 3-space E_3. We also give the invariants of them and present relations between the Frenet-type frame in Myller configuration Euclidean 3-space and the Frenet frame in Euclidean 3-space. Then, we give a numerical example with respect to the Smarandache curves with Frenet-type frame in Myller configuration for Euclidean 3-space E_3.

It should be noted that we use the following notation $\overline{r}(s) = \overline{r}$, $\overline{r}_i^*(s^*) = \overline{r}_i^*$, $\overline{\xi}_i(s) = \overline{\xi}_i$, $\overline{\xi}_i^*(s^*) = \overline{\xi}_i^*$, $K_i(s) = K_i$, $K_i^*(s^*) = K_i^*$, $a_i(s) = a_i$ and $a_i^*(s^*) = a_i^*$ for $i = 1,2,3$, for the sake of brevity.

3.1. Generalized $\overline{\xi}_1\overline{\xi}_2$-Smarandache curves

Definition 3.1. *Let $\overline{r} : I \to E_3$ for the arc-length parameter $s \in I$ be a regular curve with the Frenet-type frame $\mathcal{R}_F = \{\overline{\xi}_1, \overline{\xi}_2, \overline{\xi}_3, K_1, K_2\}$ in Myller configuration for Euclidean 3-space E_3. The curve $\overline{r}_1^* : J \to E_3$ for the arc-length parameter $s^* \in J$ is determined as follows:*

$$\overline{r}_1^* = \frac{1}{\sqrt{2}}\left(\overline{\xi}_1 + \overline{\xi}_2\right), \tag{3}$$

which is called $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve.

Now, let us introduce the invariants of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curves with the Frenet-type frame in Myller configuration for E_3 related to the curve $\overline{r}$. By taking the derivative of the equation (3) with respect to the parameter s, then we get:

$$(\overline{r}_1^*)' = \frac{d\overline{r}_1^*}{ds^*}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\left(\overline{\xi}_1' + \overline{\xi}_2'\right) \tag{4}$$

and

$$\left(a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^*\right)\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\left(-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3\right), \tag{5}$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{2K_1^2 + K_2^2}{2}}.$$

By using the equations (4) and (5), we have:

$$a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^* = \frac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{2K_1^2 + K_2^2}},$$

and then

$$\begin{cases} a_1^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{2K_1^2 + K_2^2}}, \overline{\xi}_1^* \right\rangle, \\ a_2^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{2K_1^2 + K_2^2}}, \overline{\xi}_2^* \right\rangle, \\ a_3^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{2K_1^2 + K_2^2}}, \overline{\xi}_3^* \right\rangle. \end{cases}$$

We get the first versor field $\overline{\xi}_1^*$ of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve as follows:

$$\overline{\xi}_1^* = \frac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{a_1^*\sqrt{2K_1^2+K_2^2}} - \frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*, \tag{6}$$

where $a_1^* \neq 0$. Differentiating the equation (6) with respect to the parameter s, we have:

$$\begin{aligned}
\frac{d\overline{\xi}_1^*}{ds} =& \frac{d\overline{\xi}_1^*}{ds^*}\frac{ds^*}{ds} \\
=& \dot{\overline{\xi}}_1^*\frac{ds^*}{ds} \\
=& \frac{\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3}{(a_1^*)^2\left(2K_1^2+K_2^2\right)^{3/2}} - \left(\frac{\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^*}{(a_1^*)^2}\right)\frac{ds^*}{ds}\overline{\xi}_2^* \\
& - \frac{a_2^*}{a_1^*}\left(-K_1^*\overline{\xi}_1^* + K_2^*\overline{\xi}_3^*\right)\frac{ds^*}{ds} - \left(\frac{\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^*}{(a_1^*)^2}\right)\frac{ds^*}{ds}\overline{\xi}_3^* \\
& + \left(\frac{a_3^*K_2^*}{a_1^*}\right)\frac{ds^*}{ds}\overline{\xi}_2^*,
\end{aligned}$$

where

$$\begin{cases}
\mu =a_1^*\left(-K_1' - K_1^2\right)\left(2K_1^2+K_2^2\right) \\
\qquad + K_1\left[\dot{a_1^*}\frac{ds^*}{ds}\left(2K_1^2+K_2^2\right) + a_1^*\left(2K_1K_1' + K_2K_2'\right)\right], \\
\eta =a_1^*\left(-K_1^2 + K_1' - K_2^2\right)\left(2K_1^2+K_2^2\right) \\
\qquad - K_1\left[\dot{a_1^*}\frac{ds^*}{ds}\left(2K_1^2+K_2^2\right) + a_1^*\left(2K_1K_1' + K_2K_2'\right)\right], \\
\rho =a_1^*\left(K_1K_2 + K_2'\right)\left(2K_1^2+K_2^2\right) \\
\qquad - K_2\left[\dot{a_1^*}\frac{ds^*}{ds}\left(2K_1^2+K_2^2\right) + a_1^*\left(2K_1K_1' + K_2K_2'\right)\right].
\end{cases}$$

Note that, $\dot{a_1^*} = \dfrac{da_1^*}{ds^*}$, $\dot{a_2^*} = \dfrac{da_2^*}{ds^*}$, and $\dot{a_3^*} = \dfrac{da_3^*}{ds^*}\cdot$ Therefore, we obtain the following:

$$\begin{aligned}
\dot{\overline{\xi}_1^*} =& \frac{d\overline{\xi}_1^*}{ds^*} \\
=& \frac{\sqrt{2}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right)}{\left(a_1^*\right)^2\left(2K_1^2 + K_2^2\right)^2} + \left(\frac{a_2^* K_1^*}{a_1^*}\right)\overline{\xi}_1^* - \left(\frac{\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^* - a_1^* a_3^* K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_2^* \\
& - \left(\frac{\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^* a_2^* K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_3^*.
\end{aligned}$$

Then, we get the K_1^*-curvature of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve:

$$K_1^* = \left\|\frac{d\overline{\xi}_1^*}{ds^*}\right\| = \frac{\psi}{\left(a_1^*\right)^2\left(2K_1^2 + K_2^2\right)^2},$$

where

$$\psi = \sqrt{2\left(\mu^2 + \eta^2 + \rho^2\right) + \begin{pmatrix} \left(a_1^* a_2^* K_1^*\right)^2 \\ + \left(\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^* - a_1^* a_3^* K_2^*\right)^2 \\ + \left(\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^* a_2^* K_2^*\right)^2 \end{pmatrix}\left(2K_1^2 + K_2^2\right)^4}\cdot$$

The versor field $\overline{\xi}_2^*$ of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve is expressed as follows:

$$\overline{\xi}_2^* = \frac{\sqrt{2}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right) + \begin{pmatrix} a_1^* a_2^* K_1^* \overline{\xi}_1^* \\ - \left(\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^* - a_1^* a_3^* K_2^*\right)\overline{\xi}_2^* \\ - \left(\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^* a_2^* K_2^*\right)\overline{\xi}_3^* \end{pmatrix}\left(2K_1^2 + K_2^2\right)^2}{\psi}\cdot$$

The versor field $\overline{\xi}_3^* = \overline{\xi}_1^* \times \overline{\xi}_2^*$ of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve is written with the help of the vector product as follows:

$$\begin{aligned}
\overline{\xi}_3^* =& \frac{\sqrt{2}\left((K_1\rho - K_2\eta)\,\overline{\xi}_1 + (K_1\rho + K_2\mu)\,\overline{\xi}_2 - K_1(\mu+\eta)\,\overline{\xi}_3\right)}{\psi a_1^*\sqrt{2K_1^2+K_2^2}} \\
&+ \frac{\left(2K_1^2+K_2^2\right)^2}{\psi}\left[\begin{array}{l}\left(\begin{array}{l}\dfrac{a_2^*}{a_1^*}\left(\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^*a_2^*K_2^*\right) \\ -\dfrac{a_3^*}{a_1^*}\left(\dot{a_2^*}a_1^* - a_1^*a_2^* - a_1^*a_3^*K_2^*\right)\end{array}\right)\overline{\xi}_1^* \\ -a_2^*a_3^*K_1^*\overline{\xi}_2^* + (a_2^*)^2K_1^*\overline{\xi}_3^*\end{array}\right] \\
&+ \left[\begin{array}{l}\left(\dfrac{-K_1\overline{\xi}_1 + K_1\overline{\xi}_2 + K_2\overline{\xi}_3}{a_1^*\sqrt{2K_1^2+K_2^2}}\right) \\ \times\dfrac{1}{\psi}\left(\begin{array}{l}a_1^*a_2^*K_1^*\overline{\xi}_1^* - \left(\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^* - a_1^*a_3^*K_2^*\right)\overline{\xi}_2^* \\ -\left(\dot{a_3}^*a_1^* - \dot{a_1^*}a_3^* + a_1^*a_2^*K_2^*\right)\overline{\xi}_3^*\end{array}\right)\left(2K_1^2+K_2^2\right)^2\end{array}\right] \\
&+ \left[\left(-\frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*\right) \times \frac{1}{\psi}\left(\sqrt{2}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right)\right)\right],
\end{aligned}$$

and also the K_2^*-torsion of the $\overline{\xi}_1\overline{\xi}_2$-Smarandache curve can be calculated as $K_2^*(s^*) = -\dfrac{\dot{\overline{\xi}_3^*}}{\overline{\xi}_2^*}\cdot$

Special Case 3.2. *Because of the fact that the geometry of versor fields along a curve with Myller configuration in Euclidean 3-space E_3 is a generalization of the usual theory of curves in E_3, we can get the following special case:*

- *If we take $a_1 = a_1^* = 1, a_2 = a_2^* = a_3 = a_3^* = 0$ in the written equations in this subsection for $\overline{\xi}_1\overline{\xi}_2$-Smarandache curves with Frenet-type frame in Myller configuration for E_3, then we get the TN-Smarandache curves with Frenet frame in E_3 (see [1, 7, 12]).*

3.2. Generalized $\overline{\xi}_1\overline{\xi}_3$-Smarandache curves

Definition 3.3. *Let $\overline{r} : I \to E_3$ for the arc-length parameter $s \in I$ be a regular curve with Frenet-type frame $\mathcal{R}_F = \{\overline{\xi}_1, \overline{\xi}_2, \overline{\xi}_3, K_1, K_2\}$ in Myller configuration for Euclidean 3-space E_3. The curve $\overline{r}_2^* : J \to E_3$ for the arc-length parameter $s^* \in J$ is determined as follows:*

$$\overline{r}_2^* = \frac{1}{\sqrt{2}}\left(\overline{\xi}_1 + \overline{\xi}_3\right), \tag{7}$$

which is called $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve.

Let us give the invariants of the Frenet-type frame in Myller configuration for $\overline{\xi}_1\overline{\xi}_3$-Smarandache curves related to the curve $\overline{r}$. Differentiating the equation (7) with respect to the parameter s, we have:

$$(\overline{r}_2^*)' = \frac{d\overline{r}_2^*}{ds^*}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\left(\overline{\xi}_1' + \overline{\xi}_3'\right) \tag{8}$$

and

$$\left(a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^*\right)\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}\left(K_1\overline{\xi}_2 - K_2\overline{\xi}_2\right)\,, \tag{9}$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{(K_1-K_2)^2}{2}} = \frac{\sqrt{(K_1-K_2)^2}}{\sqrt{2}} = \frac{|\,K_1-K_2\,|}{\sqrt{2}}.$$

With the help of the equations (8) and (9), we get:

$$a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^* = \frac{(K_1-K_2)\,\overline{\xi}_2}{\sqrt{(K_1-K_2)^2}} = \frac{(K_1-K_2)\,\overline{\xi}_2}{|\,K_1-K_2\,|},$$

and also

$$\begin{cases} a_1^* = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{\sqrt{(K_1-K_2)^2}}, \overline{\xi}_1^* \right\rangle = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{|\,K_1-K_2\,|}, \overline{\xi}_1^* \right\rangle, \\ a_2^* = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{\sqrt{(K_1-K_2)^2}}, \overline{\xi}_2^* \right\rangle = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{|\,K_1-K_2\,|}, \overline{\xi}_2^* \right\rangle, \\ a_3^* = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{\sqrt{(K_1-K_2)^2}}, \overline{\xi}_3^* \right\rangle = \left\langle \dfrac{(K_1-K_2)\,\overline{\xi}_2}{|\,K_1-K_2\,|}, \overline{\xi}_3^* \right\rangle, \end{cases}$$

where $K_1 \neq K_2$. We can write the versor field $\overline{\xi}_1^*$ of the $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve as:

$$\overline{\xi}_1^* = \frac{(K_1-K_2)\,\overline{\xi}_2}{a_1^*\sqrt{(K_1-K_2)^2}} - \frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*, \tag{10}$$

where $a_1^* \neq 0$. Taking differentiation of the equation (10) according to the parameter s, we obtain:

$$\begin{aligned} \frac{d\overline{\xi}_1^*}{ds} &= \frac{d\overline{\xi}_1^*}{ds^*}\frac{ds^*}{ds} \\ &= \dot{\overline{\xi}}_1^*\frac{ds^*}{ds} \\ &= \frac{\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3}{(a_1^*)^2(K_1-K_2)^2\sqrt{(K_1-K_2)^2}} - \left(\frac{\dot{a}_2^*a_1^* - \dot{a}_1^*a_2^*}{(a_1^*)^2}\right)\frac{ds^*}{ds}\overline{\xi}_2^* \end{aligned}$$

$$-\frac{a_2^*}{a_1^*}\left(-K_1^*\overline{\xi}_1^*+K_2^*\overline{\xi}_3^*\right)\frac{ds^*}{ds}-\left(\frac{\dot{a}_3^*a_1^*-\dot{a}_1^*a_3^*}{(a_1^*)^2}\right)\frac{ds^*}{ds}\overline{\xi}_3^*$$
$$+\left(\frac{a_3^*K_2^*}{a_1^*}\right)\frac{ds^*}{ds}\overline{\xi}_2^*,$$

where

$$\begin{cases}\mu=-\,a_1^*K_1\left(K_1-K_2\right)^3\,,\\ \eta=a_1^*\left(K_1'-K_2'\right)\left(K_1-K_2\right)^2\\ \qquad-\left(K_1-K_2\right)^2\left(\dot{a}_1^*\dfrac{ds^*}{ds}\left(K_1-K_2\right)+a_1^*\left(K_1'-K_2'\right)\right),\\ \rho=a_1^*K_2\left(K_1-K_2\right)^3\,.\end{cases}$$

Hence, we can write the followings after some calculations:

$$\begin{aligned}\dot{\overline{\xi}}_1^*&=\frac{d\overline{\xi}_1^*}{ds^*}\\&=\frac{\sqrt{2}\left(\mu\overline{\xi}_1+\eta\overline{\xi}_2+\rho\overline{\xi}_3\right)}{\left(a_1^*\right)^2\left(K_1-K_2\right)^4}+\left(\frac{a_2^*K_1^*}{a_1^*}\right)\overline{\xi}_1^*-\left(\frac{\dot{a}_2^*a_1^*-\dot{a}_1^*a_2^*-a_1^*a_3^*K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_2^*\\&\quad-\left(\frac{\dot{a}_3^*a_1^*-\dot{a}_1^*a_3^*+a_1^*a_2^*K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_3^*.\end{aligned}$$

Also, we have the K_1^*-curvature of the $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve as follows:

$$K_1^*=\left\|\frac{d\overline{\xi}_1^*}{ds^*}\right\|=\frac{\psi}{(a_1^*)^2\left(K_1-K_2\right)^4},$$

where

$$\psi=\sqrt{2\left(\mu^2+\eta^2+\rho^2\right)+\begin{pmatrix}\left(a_1^*a_2^*K_1^*\right)^2\\+\left(\dot{a}_2^*a_1^*-\dot{a}_1^*a_2^*-a_1^*a_3^*K_2^*\right)^2\\+\left(\dot{a}_3^*a_1^*-\dot{a}_1^*a_3^*+a_1^*a_2^*K_2^*\right)^2\end{pmatrix}\left(K_1-K_2\right)^8}\,.$$

Additionally, the versor field $\overline{\xi}_2^*$ of the $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve is given as:

$$\overline{\xi}_2^*=\frac{\sqrt{2}\left(\mu\overline{\xi}_1+\eta\overline{\xi}_2+\rho\overline{\xi}_3\right)+\begin{pmatrix}a_1^*a_2^*K_1^*\overline{\xi}_1^*\\-\left(\dot{a}_2^*a_1^*-\dot{a}_1^*a_2^*-a_1^*a_3^*K_2^*\right)\overline{\xi}_2^*\\-\left(\dot{a}_3^*a_1^*-\dot{a}_1^*a_3^*+a_1^*a_2^*K_2^*\right)\overline{\xi}_3^*\end{pmatrix}\left(K_1-K_2\right)^4}{\psi}.$$

The versor field $\overline{\xi}_3^* = \overline{\xi}_1^* \times \overline{\xi}_2^*$ of the $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve is presented as:

$$\begin{aligned}\overline{\xi}_3^* =& \frac{\sqrt{2}\left(K_1 - K_2\right)\left(\rho\overline{\xi}_1 - \mu\overline{\xi}_3\right)}{\psi a_1^* \sqrt{K_1 - K_2}^2} \\ &+ \frac{\left(K_1 - K_2\right)^4}{\psi}\left[\begin{array}{l}\left(\begin{array}{l}\left(\frac{a_2^*}{a_1^*}\right)\left(\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^* a_2^* K_2^*\right) \\ -\frac{a_3^*}{a_1^*}\left(\dot{a_2^*}a_1^* - a_1^* a_2^* - a_1^* a_3^* K_2^*\right)\end{array}\right)\overline{\xi}_1^* \\ - a_2^* a_3^* K_1^* \overline{\xi}_2^* + (a_2^*)^2 K_1^* \overline{\xi}_3^*\end{array}\right] \\ &+ \left[\begin{array}{l}\left(\frac{\left(K_1 - K_2\right)\overline{\xi}_2}{a_1^* \sqrt{\left(K_1 - K_2\right)^2}}\right) \\ \times \frac{\left(K_1 - K_2\right)^4}{\psi}\left(\begin{array}{l}a_1^* a_2^* K_1^* \overline{\xi}_1^* - \left(\dot{a_2^*}a_1^* - \dot{a_1^*}a_2^* - a_1^* a_3^* K_2^*\right)\overline{\xi}_2^* \\ - \left(\dot{a_3^*}a_1^* - \dot{a_1^*}a_3^* + a_1^* a_2^* K_2^*\right)\overline{\xi}_3^*\end{array}\right)\end{array}\right] \\ &+ \left[\left(-\frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*\right) \times \frac{1}{\psi}\left(\sqrt{2}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right)\right)\right],\end{aligned}$$

and also the K_2^*-torsion of the $\overline{\xi}_1\overline{\xi}_3$-Smarandache curve can be calculated as $K_2^* = -\frac{\dot{\overline{\xi}_3^*}}{\overline{\xi}_2^*}\cdot$

Special Case 3.4. *Due to the geometry of versor fields along a curve with Myller configuration in Euclidean 3-space E_3 is a generalization of the usual theory of curves in E_3, we can write the following special case:*

- *If we take $a_1 = a_1^* = 1, a_2 = a_2^* = a_3 = a_3^* = 0$ in the written equations in this subsection for $\overline{\xi}_1\overline{\xi}_3$-Smarandache curves with Frenet-type frame in Myller configuration for E_3, then we have the TB-Smarandache curves with Frenet frame in E_3 (see [1, 7, 12]).*

3.3. Generalized $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curves

Definition 3.5. *Let $\overline{r} : I \to E_3$ for the arc-length parameter $s \in I$ be a regular curve with the Frenet-type frame $\mathcal{R}_F = \{\overline{\xi}_1, \overline{\xi}_2, \overline{\xi}_3, K_1, K_2\}$ in Myller configuration for Euclidean 3-space E_3. The curve $\overline{r}_3^* : J \to E_3$ for the arc-length parameter $s^* \in J$ is determined as follows:*

$$\overline{r}_3^* = \frac{1}{\sqrt{2}}\left(\overline{\xi}_1 + \overline{\xi}_2 + \overline{\xi}_3\right), \tag{11}$$

which is called $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curve.

Let us examine the invariants of the Frenet-type frame in Myller configuration for $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curves according to the curve $\overline{r}$. Differentiating the equation (11) related to the parameter s, we get:

$$(\overline{r}_3^*)' = \frac{d\overline{r}_3^*}{ds^*}\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}\left(\overline{\xi}_1' + \overline{\xi}_2' + \overline{\xi}_3'\right) \tag{12}$$

and

$$\left(a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^*\right)\frac{ds^*}{ds} = \frac{1}{\sqrt{3}}\left(-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3\right), \tag{13}$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{K_1^2 + (K_1 - K_2)^2 + K_2^2}{3}}.$$

By means of the equations (12) and (13), we can obtain:

$$a_1^*\overline{\xi}_1^* + a_2^*\overline{\xi}_2^* + a_3^*\overline{\xi}_3^* = \frac{-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{K_1^2 + (K_1 - K_2)^2 + K_2^2}},$$

and we have

$$\begin{cases} a_1^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{K_1^2 + (K_1 - K_2)^2 + K_2^2}}, \overline{\xi}_1^* \right\rangle, \\ a_2^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{K_1^2 + (K_1 - K_2)^2 + K_2^2}}, \overline{\xi}_2^* \right\rangle, \\ a_3^* = \left\langle \dfrac{-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3}{\sqrt{K_1^2 + (K_1 - K_2)^2 + K_2^2}}, \overline{\xi}_3^* \right\rangle. \end{cases}$$

In that case, we get the versor field $\overline{\xi}_1^*$ of the $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curve as follows:

$$\overline{\xi}_1^* = \frac{-K_1\overline{\xi}_1 + (K_1 - K_2)\,\overline{\xi}_2 + K_2\overline{\xi}_3}{a_1^*\sqrt{K_1^2 + (K_1 - K_2)^2 + K_2^2}} - \frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*, \tag{14}$$

where $a_1^* \neq 0$. By taking the derivative of the equation (14) according to the parameter s, we get:

$$\begin{aligned} \frac{d\overline{\xi}_1^*}{ds} &= \frac{d\overline{\xi}_1^*}{ds^*}\frac{ds^*}{ds} \\ &= \dot{\overline{\xi}}_1^*\frac{ds^*}{ds} \\ &= \frac{\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3}{(a_1^*)^2\left(K_1^2 + (K_1 - K_2)^2 + K_2^2\right)^{3/2}} - \left(\frac{\dot{a}_2^*a_1^* - \dot{a}_1^*a_2^*}{(a_1^*)^2}\right)\frac{ds^*}{ds}\overline{\xi}_2^* \end{aligned}$$

$$
-\frac{a_2^*}{a_1^*}\left(-K_1^*\overline{\xi}_1^*+K_2^*\overline{\xi}_3^*\right)\frac{ds^*}{ds}-\left(\frac{\dot{a_3^*}a_1^*-\dot{a_1^*}a_3^*}{\left(a_1^*\right)^2}\right)\frac{ds^*}{ds}\overline{\xi}_3^*
+\left(\frac{a_3^*K_2^*}{a_1^*}\right)\frac{ds^*}{ds}\overline{\xi}_2^*,
$$

where

$$
\begin{cases}
\mu = a_1^*\left(-K_1'+K_1\left(K_2-K_1\right)\right)\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\
\quad + K_1\begin{bmatrix}\dot{a_1^*}\dfrac{ds^*}{ds}\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\ +a_1^*\left(K_1K_1'+\left(K_1-K_2\right)\left(K_1'-K_2'\right)+K_2K_2'\right)\end{bmatrix}, \\
\eta = a_1^*\left(-K_1^2+K_1'-K_2'-K_2^2\right)\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\
\quad - \left(K_1-K_2\right)\begin{bmatrix}\dot{a_1^*}\dfrac{ds^*}{ds}\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\ +a_1^*\left(K_1K_1'+\left(K_1-K_2\right)\left(K_1'-K_2'\right)+K_2K_2'\right)\end{bmatrix}, \\
\rho = a_1^*\left(K_2'+K_2\left(K_1-K_2\right)\right)\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\
\quad - K_2\begin{bmatrix}\dot{a_1^*}\dfrac{ds^*}{ds}\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right) \\ +a_1^*\left(K_1K_1'+\left(K_1-K_2\right)\left(K_1'-K_2'\right)+K_2K_2'\right)\end{bmatrix}.
\end{cases}
$$

After that, we have the following:

$$
\begin{aligned}
\dot{\overline{\xi}_1^*} &= \frac{d\overline{\xi}_1^*}{ds^*} \\
&= \frac{\sqrt{3}\left(\mu\overline{\xi}_1+\eta\overline{\xi}_2+\rho\overline{\xi}_3\right)}{\left(a_1^*\right)^2\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right)^2}+\left(\frac{a_2^*K_1^*}{a_1^*}\right)\overline{\xi}_1^*-\left(\frac{\dot{a_2^*}a_1^*-\dot{a_1^*}a_2^*-a_1^*a_3^*K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_2^* \\
&\quad -\left(\frac{\dot{a_3^*}a_1^*-\dot{a_1^*}a_3^*+a_1^*a_2^*K_2^*}{\left(a_1^*\right)^2}\right)\overline{\xi}_3^*.
\end{aligned}
$$

Then, we get the K_1^*-curvature of the $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curve as follows:

$$
K_1^*=\left\|\frac{d\overline{\xi}_1^*}{ds^*}\right\|=\frac{\psi}{\left(a_1^*\right)^2\left(K_1^2+\left(K_1-K_2\right)^2+K_2^2\right)^2},
$$

where

$$\psi = \sqrt{\begin{array}{l} 3\left(\mu^2+\eta^2+\rho^2\right) \\ + \begin{pmatrix} \left(a_1^* a_2^* K_1^*\right)^2 \\ + \left(\dot{a_2^*} a_1^* - \dot{a_1^*} a_2^* - a_1^* a_3^* K_2^*\right)^2 \\ + \left(\dot{a_3^*} a_1^* - \dot{a_1^*} a_3^* + a_1^* a_2^* K_2^*\right)^2 \end{pmatrix} \left(K_1^2 + (K_1-K_2)^2 + K_2^2\right)^4 \end{array}}.$$

The versor field $\overline{\xi}_2^*$ of the $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curve is given as:

$$\overline{\xi}_2^* = \frac{\begin{array}{l} \sqrt{3}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right) \\ + \begin{pmatrix} a_1^* a_2^* K_1^* \overline{\xi}_1^* \\ - \left(\dot{a_2^*} a_1^* - \dot{a_1^*} a_2^* - a_1^* a_3^* K_2^*\right)\overline{\xi}_2^* \\ - \left(\dot{a_3^*} a_1^* - \dot{a_1^*} a_3^* + a_1^* a_2^* K_2^*\right)\overline{\xi}_3^* \end{pmatrix} \left(K_1^2 + (K_1-K_2)^2 + K_2^2\right)^2 \end{array}}{\psi}.$$

In addition to these, the versor field $\overline{\xi}_3^* = \overline{\xi}_1^* \times \overline{\xi}_2^*$ is obtained as:

$$\begin{aligned} \overline{\xi}_3^* = & \frac{\sqrt{3}\left(\left(-K_2\eta + (K_1-K_2)\rho\right)\overline{\xi}_1 + (K_1\rho + K_2\mu)\overline{\xi}_2 - \left((K_1-K_2)\mu + K_1\eta\right)\overline{\xi}_3\right)}{\psi a_1^* \sqrt{K_1^2 + (K_1-K_2)^2 + K_2^2}} \\ & + \frac{\left(K_1^2 + (K_1-K_2)^2 + K_2^2\right)^2}{\psi} \left[\begin{array}{l} \begin{pmatrix} \frac{a_2^*}{a_1^*}\left(\dot{a_3^*} a_1^* - \dot{a_1^*} a_3^* + a_1^* a_2^* K_2^*\right) \\ - \frac{a_3^*}{a_1^*}\left(\dot{a_2^*} a_1^* - a_1^* a_2^* - a_1^* a_3^* K_2^*\right) \end{pmatrix} \overline{\xi}_1^* \\ - a_2^* a_3^* K_1^* \overline{\xi}_2^* + \left(a_2^*\right)^2 K_1^* \overline{\xi}_3^* \end{array}\right] \\ & + \left[\begin{array}{l} \left(\dfrac{-K_1\overline{\xi}_1 + (K_1-K_2)\overline{\xi}_2 + K_2\overline{\xi}_3}{a_1^* \sqrt{K_1^2 + (K_1-K_2)^2 + K_2^2}}\right) \\ \times \dfrac{1}{\psi} \begin{pmatrix} a_1^* a_2^* K_1^* \overline{\xi}_1^* - \left(\dot{a_2^*} a_1^* \right. \\ \left. -\dot{a_1^*} a_2^* - a_1^* a_3^* K_2^*\right)\overline{\xi}_2^* \\ - \left(\dot{a_3^*} a_1^* - \dot{a_1^*} a_3^* + a_1^* a_2^* K_2^*\right)\overline{\xi}_3^* \end{pmatrix} \left(K_1^2 + (K_1-K_2)^2 + K_2^2\right)^2 \end{array}\right] \\ & + \left[\left(-\frac{a_2^*}{a_1^*}\overline{\xi}_2^* - \frac{a_3^*}{a_1^*}\overline{\xi}_3^*\right) \times \frac{1}{\psi}\left(\sqrt{3}\left(\mu\overline{\xi}_1 + \eta\overline{\xi}_2 + \rho\overline{\xi}_3\right)\right)\right]. \end{aligned}$$

Then, the K_2^*-torsion can be given as $K_2^*(s^*) = -\dfrac{\dot{\overline{\xi}_3^*}}{\overline{\xi}_2^*}$, as well.

Special Case 3.6. *Since the geometry of versor fields along a curve with Myller configuration in Euclidean 3-space E_3 is a generalization of the usual theory of curves in E_3, we have the following special case:*

- *If we take $a_1 = a_1^* = 1, a_2 = a_2^* = a_3 = a_3^* = 0$ in the written equations in this subsection for $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-Smarandache curves with Frenet-type frame in Myller configuration for E_3, then we have the TNB-Smarandache curves with Frenet frame in E_3 (see [1, 7, 12]).*

Now, let us obtain an illustrative numerical example with respect to the generalized Smarandache curves in Myller configuration. Thanks to the study [27], we construct our example as follows:

Example 3.7. *Let us consider the following versor fields and invariants as:*

$$\begin{cases} \overline{\xi}_1(s) = \left(-\frac{8}{10}\sin s - \cos s, \frac{6}{10}\sin s\right), \\ \overline{\xi}_2(s) = \left(-\frac{8}{10}\cos s, \sin s, \frac{6}{10}\cos s\right), \\ \overline{\xi}_3(s) = \left(-\frac{6}{10}, 0, -\frac{8}{10}\right), \end{cases} \quad \text{and} \quad \begin{cases} K_1(s) = 1, \\ K_2(s) = 0, \end{cases}$$

and let us choose $a_1(s) = \sin s$, $a_2(s) = \cos s, a_3(s) = 0$, we have:

$$\overline{r}(s) = \left(-\frac{8s}{10}, 1, \frac{6s}{10}\right) \tag{15}$$

and

$$\begin{aligned} \frac{d\overline{r}}{ds} &= \left(-\frac{8}{10}, 0, \frac{6}{10}\right) \\ &= \sin s\left(-\frac{8}{10}\sin s, -\cos s, \frac{6}{10}\sin s\right) + \cos s\left(-\frac{8}{10}\cos s, \sin s, \frac{6}{10}\cos s\right) \\ &= a_1\overline{\xi}_1 + a_2\overline{\xi}_2 \,. \end{aligned}$$

Let us write the following Smarandache curves as follows:

- $\overline{\xi}_1\overline{\xi}_2$*-Smarandache curve:*

$$\begin{aligned} \overline{r}_1^* &= \frac{1}{\sqrt{2}}\left(\overline{\xi}_1 + \overline{\xi}_2\right) \\ &= \frac{1}{\sqrt{2}}\left(\frac{-8}{10}(\sin s + \cos s), -\cos s + \sin s, \frac{6}{10}(\sin s + \cos s)\right) . \end{aligned}$$

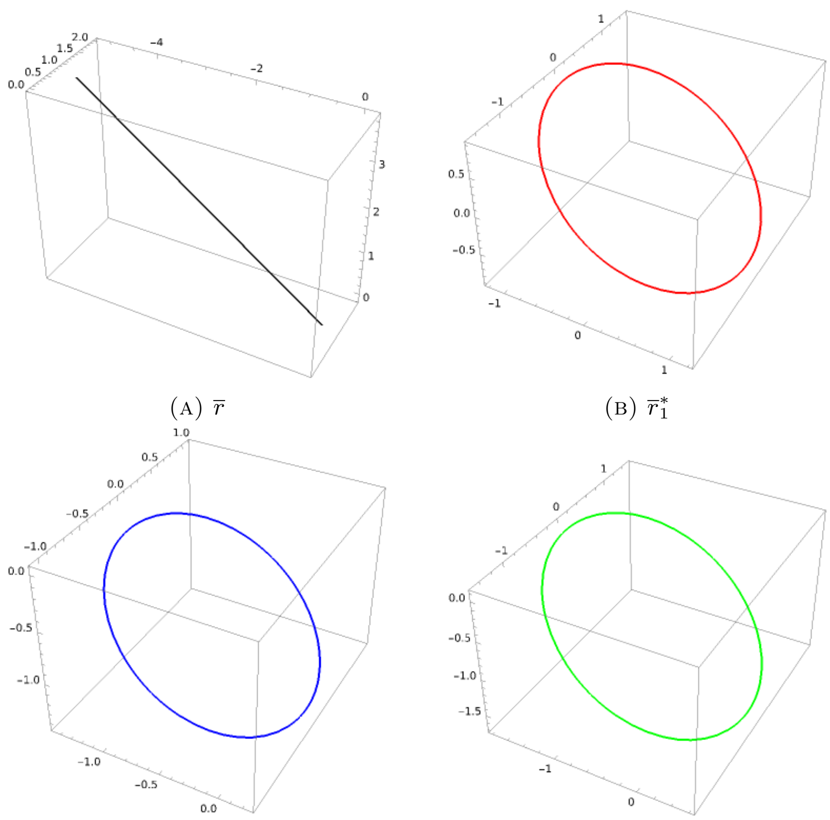
- $\overline{\xi}_1\overline{\xi}_3$*-Smarandache curve:*

$$\begin{aligned} \overline{r}_2^* &= \frac{1}{\sqrt{2}}\left(\overline{\xi}_1 + \overline{\xi}_3\right) \\ &= \frac{1}{\sqrt{2}}\left(\frac{-8}{10}\sin s - \frac{6}{10}\cos s, -\cos s, \frac{6}{10}\sin s + \frac{8}{10}\right), \end{aligned}$$

- $\overline{\xi}_1\overline{\xi}_2\overline{\xi}_3$-*Smarandache curve:*

$$\begin{aligned}\overline{r}_3^* &= \frac{1}{\sqrt{3}}\left(\overline{\xi}_1+\overline{\xi}_2+\overline{\xi}_3\right)\\ &= \frac{1}{\sqrt{3}}\left(\frac{-8}{10}(\sin s+\cos s)-\frac{6}{10}, -\cos s+\sin s, \frac{6}{10}(\sin s+\cos s)-\frac{8}{10}\right).\end{aligned}$$

In the following Figure 1, we can examine the special Smarandache curves with Frenet-type frame in Myller configuration for Euclidean space E_3:



(A) $\overline{r}$ (B) $\overline{r}_1^*$ (C) $\overline{r}_2^*$ (D) $\overline{r}_3^*$

Figure 1. The Curve $\overline{r}$ and Some Special Smarandache Curves of $\overline{r}$

It should be noted that Figure 1 is drawn by Wolfram Mathematica (Wolfram Cloud).

4. Conclusions

In this study, we introduced one of the most important and special curves; namely the Smarandache curves with Frenet-type frame in Myller configuration for E_3. We gave also some relations between the Smarandache curves with Frenet-type frame in Myller configuration for E_3 and Smarandache curves with Frenet frame in E_3. Then, we determined the invariants and some special cases. Additionally, we give an example with respect to them to support given materials.

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