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A NEW WAY FOR SOLVING TRANSPORTATION ISSUES BASED ON THE EXPONENTIAL DISTRIBUTION AND THE CONTRAHARMONIC MEAN†

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Abstract. This study aims to determine the optimal solution to transportation problems. We proposed a novel approach for tackling the initial basic feasible solution. This is a critical step toward achieving an optimal or near-optimal solution. The transportation issue is an issue of distributing goods from several sources to several destinations. The literature demonstrates many ways to improve IBFS. In this work, to solve the IBFS, we suggested a new method based on the statistical formula called cumulative distribution function (CDF) in exponential distribution, and inverse contra-harmonic mean (ICHM). The spreadsheet converts transportation cost values into exponential cost cell values. The stepping-stone method is used to identify an optimum solution. The results are compared with other existing methodologies, the suggested method incorporates balanced, and unbalanced, maximizing the profits, random values, and case studies which produce more effective outcomes.

AMS Mathematics Subject Classification : 90C08, 90B06.
Key words and phrases : Transportation problem (TP), exponential distribution, inverse contra harmonic mean (ICHM), initial basic feasible solution, optimal solution.

1. Introduction

Transportation problems (TP) refer to the physical distribution of commodities from sources to destinations with the lowest overall transportation cost while fulfilling supply and demand restrictions [1]. Transportation costs are among the most important variables in every business's ability to optimize profits. In today's globalization and liberalization environment, it is often difficult to reduce

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the costs of raw materials, people, and equipment to reduce product costs. As a result, decreasing transportation expenses must be targeted at maximizing profit [2]. The transportation issue is a linear programming problem (LPP). There are many distinct types of transportation models, and the most fundamental transportation model was established by Hitchock in 1941 and 1949; Koopman developed it [3]. To address the transportation problem, a mathematical model for TP must be developed, and an IBFS that can be implemented using several methodologies must be discovered [4]. The well-known existing techniques for obtaining IBFS include Vogel's Approximation Method (VAM), the Northwest Corner Method, and the Least Cost Method (many ways for obtaining IBFS for TP have been developed) [5]. Transportation handles certain real-world business challenges such as capital investment, budget allocation, site analysis, assembly line balance, and production planning and scheduling [6].

Ekanayake, M.U.S.B.E., 2022 created a heuristic algorithm and an improved ant colony optimization algorithm to find an initial feasible solution (IFS) to a prohibitive transportation problem (PTP) [7]. Radhiah et al., 2023 discuss transportation cost optimization for fleet product distribution. (VAM) and (MODI) are used to distribute products from two sources to ten destinations. Using the transportation approach to solve transportation difficulties can reduce and optimize the cost of shipping a vehicle [8]. Kumar R, et al., propose a unique approach known as the direct sum approach (DSM). The outcome demonstrates that it is simple to compute and close to the optimal solution to the problem [9].

Jude et al., 2019 implemented a novel approach (Loop Product Difference) to improve the IBFS of a balanced transportation problem [10]. Ahmed, 2017 proposes a novel technique for finding an optimal solution to the transportation problem by calculating the maximum row cost difference (MRCD) and maximum column cost difference (MCCD). It has been discovered that the proposed method's performance is appropriate for solving transportation challenges [11]. As a first consideration to boost distribution cost reductions in the bread firm, Rusli et al., 2022 offered to achieve more optimal outcomes. To solve a transportation problem, employ the improved exponential distribution technique. This application's results are more accurate than the North-West approach [12].

Salah Alddin Sulaiman, 2019 proposed dispersion methods, some of which are used to discover the most basic feasible solution to transportation challenges. This metric yields better outcomes [13]. Munot & Ghadle, 2023 provide an innovative solution to tackling the transportation problem that lowers transportation costs by using modular arithmetic [14]. Sharma & Goel, 2022 employed a variety of strategies to identify IBFS for real-world issues for TP. The AMM (Arithmetic Mean Method) and the ASM (Assigning Shortest Minimax Method) are included [15]. Neetu & Ashok, 2016 proposed a novel North West Corner technique method using a statistical tool called the coefficient of range [16].

Hussein & Shiker, 2020 introduced a revolutionary VAM modification that will aid us in identifying an IBFS for the transportation issue that is almost as good as the optimal strategy [17]. Sahito, 2021 the author proposed to modify Vogel's approximation method for the optimality of transportation problems using a statistical approach (MVOTPST) [18]. Harmonic Mean was used to analyse the transportation issue by Stephen & Okeke directly from the source to the destination. The delivery schedule reduced the overall cost of the shipment [19]. Kadhim et al., 2021 suggested a novel strategy for tackling transportation difficulties based on the sort of maximization used to maximize earnings [20].

Azad & Hossain, 2017 devised an algorithm with an average row penalty (ARP) and an average column penalty (ACP), which are the discrepancies in cell values of each row and column [21]. Kaur et al., 2018 proposed the Maximum Difference Method (MDM) to obtain the best initial solutions to transportation issues [22]. Transportation problem techniques are to reduce the price or duration of transportation by using a new method named the minimum supply and demand method by the author Morade, 2017 proposed to find an initial basic feasible solution for the transportation problems [23]. Babu & Das, 2014 address this specific issue and develop a novel method for allocating zero supply or demand, preventing the formation of closed loops [24]. The transportation problem is explored using a novel approach. Gothi et al., 2021 apply the idea of the weighted arithmetic mean to find the initial basic, feasible, and optimum (or nearly optimum) solution to the TP [25]. (Exponential Distribution, 2023) [26]. (Contraharmonic Mean, 2023) [27]. Ahmed et al., 2016 propose an approach called the "Incessant Allocation Method" to achieve an initial feasible solution for transportation challenges. Several numerical problems are addressed to validate the approach [28]. Zabiba et al., 2023 implemented a new strategy for solving (TP) with an objective function of the sort of maximization. The new approach was developed by adding an important step to the procedure for determining the initial solution [29]. Hussein & Shiker, 2023 discovered a new technique called the matrix method developed for evaluating and determining an optimal solution to transportation problems [30]. Raval, 2023 demonstrates a novel way to determine the initial basic solution to a transportation problem that minimizes transportation costs [31]. Paul & Henry, 2023 proposed a unique approach for determining the IBFS by computing the Index of Dispersion [32].

This study aims to propose a new algorithm for an initial feasible solution that minimizes the cost of the transportation problem.

a) Cumulative exponential distribution function to convert the TP cost cell.
b) Inverse contra-harmonic mean for finding the penalties for allocating the cost.
c) This new algorithm can either reduce the overall cost or maximize the overall profit.

A statistical and probabilistic formula to calculate the IBFS of the TP is suggested in this research. In comparison to traditional procedures, the performance of the suggested technique is evaluated. This study contains many sections mathematical definition of TP, previously published techniques, relevant materials and techniques, a proposed technique, and the results are all described in the parts that follow.

2. Formulation of Transportation Problem

This Section explains the mathematical modeling of transportation problems in detail.
The TP is normally represented in a tabular form as illustrated below.

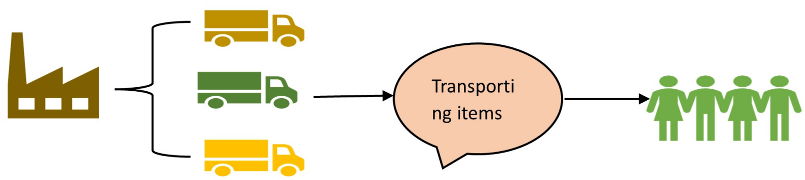

FIGURE 1. Transporting products from origin to destination

		Destination				
		1	2	...	n	Supply
Origin	1	X_{11} C_{11}	X_{12} C_{12}	... ...	X_{1n} C_{1n}	a_1
	2	X_{21} C_{21}	X_{22} C_{22}	... ...	X_{2n} C_{2n}	a_2
	$\vdots$	$\vdots$ $\vdots$	$\vdots$ $\vdots$	...	$\vdots$ $\vdots$	$\vdots$
	m	X_{m1} C_{m1}	X_{m2} C_{m2}	... ...	X_{mn} C_{mn}	a_m
	Demand	b_1	b_2	...	b_m	

TABLE 1. Basic Transportation Formulation

In Figure 1 transporting products is explained. In Table 1 a mathematical formulation is explained.
The problem can be represented mathematically as an LPP which is demonstrated below

$$\begin{aligned} Minimize\ Z \ = \ \sum\nolimits_{i=1}^{m}\sum\nolimits_{j=1}^{n} X_{ij}C_{ij} \\ subject\ to\ the\ constraints \\ \sum\nolimits_{j=1}^{n} x_{ij} = a_i;\ i \ = \ 1\ to\ m \\ \sum\nolimits_{i=1}^{m} x_{ij} = b_j;\ j \ = \ 1\ to\ n \\ x_{ij} \ \geq \ 0\ \forall\ i\ and\ j \\ \sum\nolimits_{i=1}^{m} a_i = \sum\nolimits_{j=1}^{n} b_j \\ Total\ supply = Total\ demand \end{aligned} \tag{1}$$

m : Product supply available in the origin
n : The number of goods required to reach the destination
$S_i(a_i)$ $(i = 1, 2, \ldots, m)$: Supply m units from sources to destinations
$D_j(b_j)$ $(j = 1, 2, \ldots, n)$: n Units of demand from origins to destinations
X_{ij} : Per unit quantity from origin to destination
C_{ij} : The cost of carrying one unit of commodity from origin to destination.

2.1. Exponential Distribution [26]**.** In probability theory and statistics, the exponential distribution, or negative exponential distribution, is utilized. The exponential distribution represents the probability of time between events in a Poisson point process, which is a process in which events occur continuously and independently at a constant average rate.
A Poisson point process is a form of a random mathematical object composed of points located at random in a mathematical space.
Because the transportation problem has a randomly generated cost that is independent of time, an exponential distribution is used.
The cdf function for the exponential distribution yields the probability of an observation from an exponential distribution with the scale parameter $\lambda \leq x$.
The cumulative distribution function (CDF) is given by

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x}\ ;\ x \geq 0 \\ 0\ ;\ x < 0 \end{cases} \tag{2}$$

In Excel solver to solve the Exponential distribution.

$$Exponential\ distribution = EXPON.DIST(X, lambda, True) \tag{3}$$

There are three parameters
$X-$ the cost of one unit of the commodity from source to destination

$Lambda - E(x) = \mu = 1/\lambda$
In this research, we consider $\lambda = \frac{1}{\mu}$

$$Cumulative- \begin{cases} The \textbf{ True } function\ indicates\ the \textbf{ Cumulative distribuon function}. \\ The \textbf{ False } function\ indicates\ the \textbf{ Probability density function} \end{cases}$$

2.2. Contra harmonic mean [27]**.** A contraharmonic mean is a function that is complementary to the harmonic mean. The arithmetic mean of the squares of a set of positive numbers divided by the arithmetic mean of the numbers is known as the contra-harmonic mean.

$$C\ (x_1, x_2,\ ...,\ x_n)\ =\ \left(\frac{{x_1}^2 + {x_2}^2 + ... + {x_n}^2}{x_1 + x_2 + ... + x_n}\right)^{-1} \tag{4}$$

Since we were determining the penalties in the transportation problem, we used one of the statistical formulas, a Contraharmonic mean. The Contraharmonic mean is greater than the arithmetic mean and the root mean square.

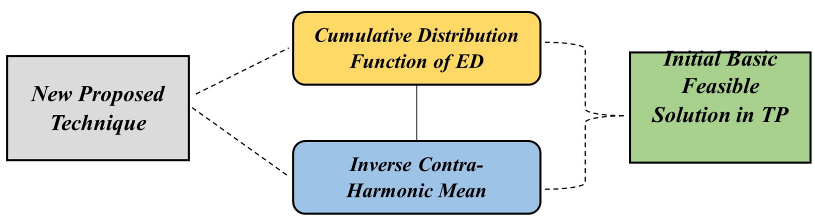

FIGURE 2. Transporting products from origin to destination

3. Methodology

This section includes a novel algorithm for discovering a new IBFS.

Step 1 Create a transportation issue.

- $\sum a_i = \sum b_j$ must be balanced.
- $\sum a_i \neq \sum b_j$ it is unbalanced TP. By adding a fake row or column with zero cost. To produce a balanced TP.

Step 2 Using equation 3, convert transportation cost cell values to exponential cost cell values using row-wise.

Step 3 The table now includes an exponential cost cell value. Equation 4 is used to calculate the penalty.

Step 4 Determine which penalty has the lowest value in each row or column. We take a look at the following row or column:

Step 5 We select the lowest-cost cell in that row or column in the subsequent row or column. Assign as many units of supply-demand values as the lowest-cost cell allows.

Step 6 If the condition is met, remove that row or column.

Step 7 When two columns or rows are filled at the same time. Only one column or row has been crossed off, while the other has zero demand (or supply).
Step 8 Redo steps 4,5,6, and 7. until supply and demand are satisfied.
Step 9 Calculate the initial basic feasible approach for the transportation issue using equation 1.
Step 10 To minimize IBFS's optimality, a generic stepping-stone technique is necessary.

3.1. Numerical Example for the Proposed Method. Consider the following transportation table showing production and transportation costs along with the supply and demand positions of Factories/distribution centers.

Distribution Centers				
Factories	M_1	M_2	M_3	Supply (a_i)
F_1	6	8	10	150
F_2	7	11	11	175
F_3	4	5	12	275
Demand (b_j)	200	100	300	

Table 2. General Transportation Problem [11]

Here in the Excel sheet, we have three parameters
$X-$ Every cell cost in the TP table
Mean = AVERAGE(6,8,10) is 8
$ExponentialDistribution = EXPON.DIST(6, 1/8, True)\ is$ 0.527
We will do the same for each cell

Distribution Centers				
Factories	M_1	M_2	M_3	Supply (a_i)
F_1	0.527	0.631	0.731	150
F_2	0.515	0.679	0.679	175
F_3	0.435	0.510	0.820	275
Demand (b_j)	200	100	300	

Table 3. Transportation cost cell values as Exponential distribution cost cell value

In Table 3 all the exponential distribution cost cell values are of positive numbers.
For calculating the penalty, we use Inverse contra-harmonic mean
i.e., $C(x_1, x_2, ..., x_n) = \left[\frac{(0.5270)^2+(0.631)^2+(0.731)^2}{(0.570)+(0.631)+(0.731)}\right]^{-1}$

In Tables 2-7 we perform the IBFS solution using equation 1 the proposed solution is **4525**.

Distribution Centers					
Factories	M_1	M_2	M_3	Supply (a_i)	(ICHM)-1
F_1	0.527	0.631	0.731	150	1.579
F_2	0.515	0.679	0.679_{175}	175	1.577
F_3	0.435	0.510	0.820	275	1.574
Demand (b_j)	200	100	300		
(ICHM)-1	2.017	1.625	[1.347]		

Table 4. First allocation for TP using (ICHM)-1

Distribution Centers					
Factories	M_1	M_2	M_3	Supply (a_i)	(ICHM)-2
F_1	0.527	0.631	0.731_{125}	150	1.579
F_3	0.435	0.510	0.820	275	1.574
Demand (b_j)	200	100	125		
(ICHM)-2	2.064	1.732	[1.299]		

Table 5. Second allocation for TP using (ICHM)-2

Distribution Centers				
Factories	M_1	M_2	Supply (a_i)	(ICHM)-2
F_1	0.527_{25}	0.631	25	[1.711]
F_3	0.435_{175}	0.510_{100}	275	2.104
Demand (b_j)	200	100		
(ICHM)-3	2.064	1.732		

Table 6. Third allocation for TP using (ICHM)-3

Distribution Centers				
Factories	M_1	M_2	M_3	Supply (a_i)
F_1	6_{25}	8	10_{125}	150
F_2	7	11	11_{175}	175
F_3	4_{175}	5_{100}	12	275
Demand (b_j)	200	100	300	

Table 7. Final Allocated Transportation Problem

3.2. Appendix. Some numerical examples for solving the initial solution are shown in Tables 8-10.

3.3. Random values problems for TP. 20. A manufacturer has distribution centres in locations X, Y, and Z. There are 12, 17, and 7 units of this product available at each of these centres, respectively. He needs 10, 10, and 14 units for

Balanced Transportation problem (BTP)	
Numerical Example-2 [1] $(C_{ij})_{4\times5} =$ $(4\ 4\ 9\ 10; 7\ 9\ 8\ 10\ 4; 9\ 3\ 7\ 10\ 6; 11\ 4\ 10\ 6\ 9)$ $(S_{ij})_{4\times1} = (100; 90; 80; 70)$ $(d_{ij})_{1\times5} = (60, 40, 90, 70, 80)$	Numerical Example-3 [3] $(C_{ij})_{3\times4} =$ $(4\ 19\ 22\ 11; 1\ 9\ 14\ 14; 6\ 6\ 16\ 14)$ $(S_{ij})_{3\times1} = (100; 30; 70)$ $(d_{ij})_{1\times4} = (40, 20, 60, 80)$
Numerical Example-4 [4] $(C_{ij})_{3\times4} =$ $(1\ 2\ 1\ 4; 3\ 3\ 2\ 1;\ 4\ 2\ 5\ )$ $(S_{ij})_{3\times1} = (30; 50; 20)$ $(d_{ij})_{1\times4} = (20, 40, 30, 10)$	Numerical Example-5 [11] $(C_{ij})_{3\times4} =$ $(4\ 6\ 8\ 8; 6\ 8\ 6\ 7; 5\ 7\ 6\ 8)$ $(S_{ij})_{3\times1} = (40; 60; 50)$ $(d_{ij})_{1\times4} = (20, 30, 50, 50)$
Numerical Example-6 [11] $(C_{ij})_{3\times5} =$ $(5\ 7\ 10\ 5\ 3;\ 8\ 6\ 9\ 9\ 12\ 14;\ 10\ 9\ 8\ 10\ 15)$ $(S_{ij})_{3\times1} = (5; 10; 10)$ $(d_{ij})_{1\times5} = (3, 3, 10, 5, 4)$	Numerical Example-7 [11] $(C_{ij})_{3\times5} =$ $(4\ 1\ 2\ 4\ 4;\ 2\ 3\ 2\ 2\ 3;\ 3\ 5\ 2\ 4\ \ )$ $(S_{ij})_{3\times1} = (60; 35; 40)$ $(d_{ij})_{1\times5} = (22, 45, 20, 18, 30)$
Numerical Example-8 [11] $(C_{ij})_{4\times6} =$ $(7\ 10\ 7\ 4\ 7\ 8;\ 5\ 1\ 5\ 5\ 3\ 3;\ 4\ 3\ 7\ 9\ 1\ 4;\ 4\ 6\ 9\ 0\ 0; 8)$ $(S_{ij})_{4\times1} = (5; 6; 2; 9)$ $(d_{ij})_{1\times6} = (4, 4, 6, 2, 4, 2)$	Numerical Example-9 [13] $(C_{ij})_{3\times2} =$ $(4\ 2; 7\ 5;\ 3;\ 10)$ $(S_{ij})_{3\times1} = (60; 40; 70)$ $(d_{ij})_{1\times2} = (105, 65)$
Numerical Example-10 [13] $(C_{ij})_{3\times4} =$ $(7\ 3\ 8\ 2; 5\ 6\ 11\ 12; 10\ 4\ 7\ 6;)$ $(S_{ij})_{3\times1} = (100; 200; 300)$ $(d_{ij})_{1\times4} = (80, 170, 190, 160)$	Numerical Example-11 [13] $(C_{ij})_{4\times4} =$ $(20\ 16; 14\ 20;\ 9\ 15\ 16\ 10;\ 8\ 13\ 5\ 9; 9\ 6\ 5\ 11)$ $(S_{ij})_{4\times1} = (9; 8; 7; 5)$ $(d_{ij})_{1\times4} = (5, 10, 5, 9)$
Numerical Example-12 [14] $(C_{ij})_{3\times4} =$ $(19\ 30\ 50\ 10; 70\ 30\ 40\ 60; 40\ 8\ 70\ 20;)$ $(S_{ij})_{3\times1} = (7; 9; 18)$ $(d_{ij})_{1\times4} = (5, 8, 7, 14)$	Numerical Example-13 [18] $(C_{ij})_{3\times4} =$ $(6\ 8; 10\ 9;\ 5\ 8\ 11\ 5;\ 6\ 9\ 12\ 5)$ $(S_{ij})_{3\times1} = (50; 75; 25)$ $(d_{ij})_{1\times4} = (20, 20, 50, 60)$
Numerical Example-14 [18] $(C_{ij})_{3\times4} =$ $(4\ 5\ 8\ 4; 6\ 2\ 8\ 1; 8\ 7\ 9\ 10;)$ $(S_{ij})_{3\times1} = (52; 57; 54)$ $(d_{ij})_{1\times4} = (60, 45, 8, 50)$	

Table 8. Some numerical examples of BTP

Unbalanced Transportation problem (UBTP)	
Numerical Example-15 [3] $(C_{ij})_{4\times3} =$ $(7\ 3\ 6;\ 4; 6\ 8;\ 5\ 8\ 4; 8\ 4\ 3)$ $(S_{ij})_{4\times1} = (5; 10; 7; 3)$ $(d_{ij})_{1\times3} = (5, 8, 10)$	Numerical Example-16 [11] $(C_{ij})_{4\times3} =$ $(3\ 4\ 6;\ 7\ 3; 8;\ 6\ 4\ 5; 7\ 5\ 2)$ $(S_{ij})_{4\times1} = (100; 80; 90; 120)$ $(d_{ij})_{1\times3} = (100, 110, 60)$
Numerical Example-17 [11] $(C_{ij})_{3\times5} =$ $(5\ 4\ 8\ 6\ 5; 4\ 5\ 4\ 3\ 2;\ 3\ 6\ 5\ 8\ 4)$ $(S_{ij})_{3\times1} = (600; 400; 1000)$ $(d_{ij})_{1\times5} = (450, 400, 200, 250, 300)$	Numerical Example-18 [28] $(C_{ij})_{4\times4} =$ $(12\ 10\ 6\ 13;\ 19\ 8\ 16\ 25;\ 17\ 15\ 15\ 20;\ 23\ 22\ 26\ 12)$ $(S_{ij})_{4\times1} = (150; 200; 600; 225)$ $(d_{ij})_{1\times4} = (300, 500, 75, 100)$

Table 9. Some numerical examples for UBTP [11]

his retail stores A, B, and C, respectively. The transportation cost (in rupees) per unit between each centre, the outlet is given. Determine the optimal distribution to minimize the cost of transportation.

In Table 11 by using equation 1 the proposed technique solution is **4,205**.

21. A company has three plants at sites A, B, and C that supply warehouses at D, E, F, and G. The monthly plant capacities are 30, 50, and 50 units,

Maximization
Numerical Example-19 [20]
$(C_{ij})_{3\times 3} = (24\ 28\ 21;\ 27\ 25\ 26;\ 23\ 22\ 29)$
$(S_{ij})_{3\times 1} = (65; 110; 75)$
$(d_{ij})_{1\times 3} = (100, 70, 80)$

TABLE 10. Profit maximization problem (MTP)

Retail outlet				
Distribution Centre	A	B	C	Availability (a_i)
X	135_2	71_{10}	260	12
Y	81_{10}	126	144_7	17
Z	170	189	201_7	7
Requires (b_j)	12	10	14	

TABLE 11. Randoms generated problem

respectively. The required number of units for a warehouse each month are 30, 30, 50, and 20. Unit transportation is indicated in rupees. To reduce the cost of transportation, determine the company's optimal distribution.

Warehouses					
Plants	D	E	F	G	Supply (a_i)
A	22	17_{10}	25	15_{20}	30
B	13_{30}	10_{20}	16	20	50
C	16	19	9_{50}	5	50
Requirements (b_j)	30	30	50	20	

TABLE 12. Randoms values included in the transportation problem

Equation 1 is used to solve the minimum solution for IBFS is **1,510** in Table 12.
22. The National Oil Company (NOC) has five depots and four refineries. Transportation cost per ton capacities and requirements are given below. Determine the optimum allocation of output.

In Table 13 by using the formula of equation 1 the overall proposed initial solution is **3,422**.

4. Results and Discussion

In the current study, we proposed a new algorithm for addressing the initial basic feasible solution for the transportation issue, which the materials are detailed in subsections 2.1-2.2 and also in Figure 2. In section 3-3.1 proposes a

Depots						
Refineries	D_1	D_2	D_3	D_4	D_5	Capacity (a_i)
R_1	20	14_{27}	34	40	42	27
R_2	10_7	41	13_{28}	20	40	35
R_3	23_4	41	36	21_{33}	46	37
R_4	33_{11}	20	41	37	43_{34}	45
Requirements (b_j)	22	27	28	33	34	

TABLE 13. Randoms values

novel technique that uses a probability and statistical formula to determine the lowest cost for the initial feasible solution and may also be utilized to maximize profit.

Case study

Iron ore is a popular commodity in two states, Karnataka and Orissa. A case study was obtained from an Indian railway freight secondary source, which is per unit cost of train load from Karnataka railway junction, Orissa, to Uttar Pradesh and Punjab (approximate cost). The supply and demand figures represent the train load cost of iron ore (approximate cost).

	Uttar Pradesh	Punjab	Supply
Karnataka	$3,436_{735}$	$3,569_{1,67,770}$	1,68,505
Orissa	$1,670_{1,18,105}$	2,900	1,18,105
Demand	1,18,840	1,67,770	

TABLE 14. Case study for TP

In Table 14 by equation 1 the minimum cost is **7,98,531,940**.

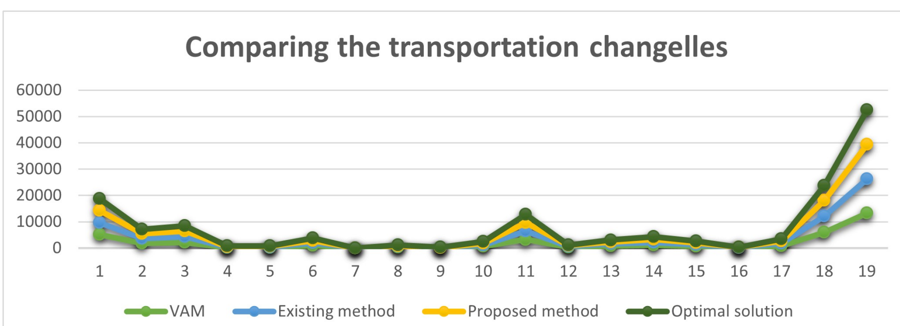

FIGURE 3. Comparison graph

No.	From Journal	Size	Status	VAM	Existing Method	Proposed Method	Optimal Solution
1	[11]	3x3	BTP	5125	4550	4525	4525
2	[2]	4x5	BTP	1820	1780	1780	1780
3	[3]	3x4	BTP	2170	2120	2100	2040
4	[4]	3x4	BTP	180	180	180	180
5	[11]	3x4	BTP	197	183	183	183
6	[11]	3x5	BTP	960	930	920	920
7	[11]	3x5	BTP	290	290	290	290
8	[11]	4x6	BTP	68	68	68	68
9	[13]	3x2	BTP	600	600	600	600
10	[13]	3x4	BTP	3210	3210	3210	3210
11	[13]	4x4	BTP	308	306	300	300
12	[15]	3x4	BTP	779	779	743	743
13	[18]	3x4	BTP	1100	1060	1060	1060
14	[18]	3x4	BTP	674	674	674	674
15	[3]	4x3	UBTP	90	90	90	90
16	[11]	4x3	UBTP	880	930	810	780
17	[11]	3x5	UBTP	6000	6050	6000	5600
18	[28]	4x4	UBTP	13,225	13,075	13,075	13,075
19	[20]	3x3	MTP	6950	6950	6950	6950

Table 15. Comparison study of TP

No.	Problems	VAM	Proposed	Optimal
20	Random problem 1	4,205	4,205	4,205
21	Random problem 2	1590	1510	1510
22	Random problem 3	3,485	3,422	3,302

Table 16. Comparison of the random values

In section 3.2 Table 8 comprises many journal papers; numerical examples are chosen and solved using the new suggested technique. We may look at the proposed method's solutions in Table 15 and Figure 3 with 19 numerical examples that include balanced, unbalanced, and maximizing case problems that reduce cost and maximize profit. Three random values in section 3.3 the solutions are provided in Table 16 and Figure 4 and a case study is in section 4.

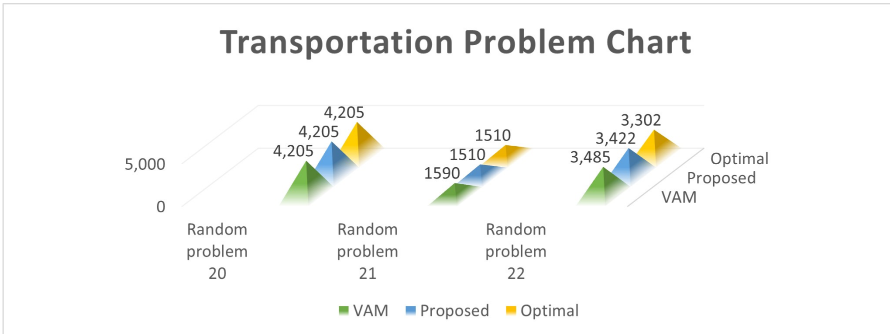

FIGURE 4. A chart for Random values solution

5. Conclusion and Future Scope

This manuscript aims to find the transportation problem's minimum shipping cost of products from several sources to several destinations. One of the key phases to arrive at an optimal solution for TP is an initial basic feasible solution (IBFS). For finding IBFS we have taken different journals numerical examples of BTP, UBTP, and MTP for solving transportation issues.

- By using the proposed algorithm using statistical formulas called cumulative distribution function in Exponential distribution and Contraharmonic mean.
- The results of our study found a better initial basic feasible solution for the transportation problems.
- A stepping-stone method is used for finding the optimal solution.
- The proposed approach is that it frequently yields an optimal or close to an optimal solution

This study is restricted to probability mass distribution in Exponential distribution and the cases of unbalanced transportation problems in total supply $\leq$ total demand.

Future research could be done in real-life applications and also used in Transshipment problems, Assignment problems, and Traveling salesman problems.

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