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PREVENTION STRATEGIES TO CONTROL AN EPIDEMIC USING A SEIQHRV MODEL

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Abstract. This study investigates the impact of precautionary measures, such as isolating exposed individuals, wearing masks, and maintaining physical distance, on preventing infectious disease. A deterministic SEIQHRV epidemic model is employed for this purpose. The model's positivity, boundedness, disease-free, and endemic equilibrium points are identified. A sensitivity test assesses the impact of preventive measures on infected classes. Results show that a basic reproduction number less than unity drives disease eradiction, while a higher unity value encourages the adoption of preventive measures.

1. Introduction

Infectious diseases like Covid-19, Monkeypox, AIDS, and Dengue pose global threats, leading to health concerns. Despite the importance of vaccinations, uncertainty and noncompliance with guidelines contribute to ongoing infections. Mathematical modeling, especially deterministic epidemic modeling, has emerged as a crucial tool for understanding and controlling the dynamics of infectious diseases. It aids in forecasting outbreaks, comprehending infection propagation across population classes, and guiding effective disease prevention strategies. Beinane et al. [1] proposed a fractional SEIQHR model, highlighting the enduring endemic nature of Covid-19 and underscoring the need for continuous preventive measures. Butt et al. [2] constructed a nonlinear SEIQHR fractional model incorporating the Atangana-Baleanu (ABC) derivative to address the dynamics of Covid-19.

Butt et al. [3] suggested that implemented strategies, particularly quarantining exposed individuals, effectively reduce the affected population and contribute to

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achieving herd immunity. Carcione et al. [4] considered a SEIR (susceptible, exposed, infected, and recovered) to study the infected population in Italy. Dwomoh et al.[6] analyzed the SEIQHRS model in the context of Ghana.

Manchein et al. [13] introduced a SEQIJR model, incorporating quarantine (Q) and isolation (J) for studying the impact of these measures on the Covid-19 outbreak.

Mandel et al. [14] developed a deterministic model with estimations for the three Indian states: Maharashtra, Delhi, and Tamil Nadu. James et al. [10] tested Covid-19's dynamics in the different countries United States, India, and Brazil.

Safi, M. [16] analyzed the role of quarantine and isolation in epidemiology in his thesis work. Safi and Gumell [17] have introduced the SEIQHRS model to assess the collective influence of quarantine measures targeting asymptomatic cases and isolation strategies for individuals displaying clinical symptoms in the context of mitigating the transmission of a communicable disease. They ([18], [19]) investigated a novel mathematical model delineating the transmission dynamics of a disease, considering the impact of quarantine for latent cases, isolation for symptomatic cases, and the presence of an imperfect vaccine. Soni et al. [22] estimated basic reproduction number and herd immunity for India to stop Covid-19. Soni et al. [23] considered the flattening of the logarithmic plot and discover that the preventative measures are effective. Sharma S. and Sharma P.K. [20] considered Holling type II incidence rate for a SIQR model. They [21] also analyzed the Stability of a SIR model with a saturated incidence rate and Holling functional type II treatment rate. Umbedkar et al. [24] analyzed an SEIR model with a modified saturated incidence rate and Holling type II treatment function. Memon et al. [15] exhibited the importance of isolation and quarantine strategies in the control of an epidemic.

2. Methodology

Inspired by the earlier work done by Kolebaje et al. [11] for the various countries of Africa, this research propose a deterministic SEIQHRV model (susceptible (S), exposed (E), infected severely (I_s), infected mildly (I_a), hospitalized (H), quarantined (Q), recovered (R) and vaccinated (V)). This study includes the vaccinated class in the previous model of O.T. Kolebaje et al. [11]. The Next-Generation matrix technique given by Diekmann et al. [5], is used to determine the basic reproduction number. The simulation is performed on seven highly correlated Indian states using the MATLAB software to verify the validity of the model. The sensitivity analyze

explore the importance of various parameters. The information for the states that would be the subject of the analysis came from the National Informatics Centre (NIC) of India's daily reports [9]. Figure 3.1 exhibits the transmission process of the infection among the different compartments of the population.

3. Formulation of Model

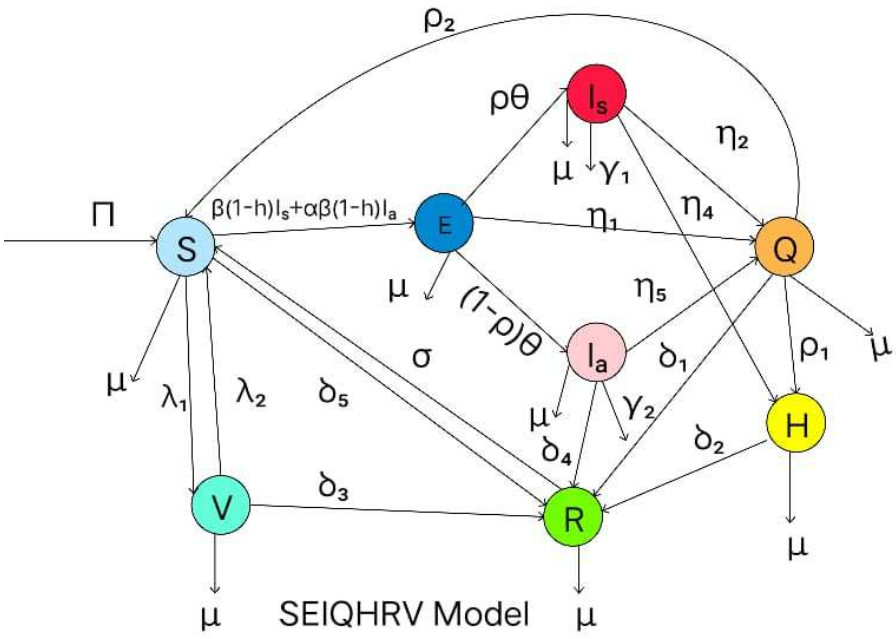

Figure 3.1: SEIQHRV model dynamics among the compartments.

3.1. Diagram of Model This model system runs on the following set of ordinary differential equations:

$$(1)\quad \frac{dS}{dt} = \pi - \beta(1-h)SI_s - \beta\alpha(1-h)SI_a - \lambda_2 S - \delta_5 S + \lambda_1 V + \rho_2 Q + \sigma R - \mu S$$

$$(2)\quad \frac{dE}{dt} = \beta(1-h)SI_s + \beta\alpha(1-h)SI_a - \theta E - \eta_1 E - \mu E$$

$$(3)\quad \frac{dI_s}{dt} = \rho\theta E - \gamma_1 I_s - \eta_2 I_s - \eta_4 I_s - \mu I_s$$

$$(4)\quad \frac{dI_a}{dt} = (1-\rho)\theta E - \delta_4 I_a - \gamma_2 I_a - \eta_3 I_a - \mu I_a$$

$$(5)\quad \frac{dQ}{dt} = \eta_2 I_s + \eta_3 I_a + \eta_1 E - \rho_2 Q - \delta_1 Q - \mu Q - \rho_1 Q$$

$$\frac{dH}{dt} = \rho_1 Q - \mu H - \gamma_3 H - \delta_2 H + \eta_4 I_s \tag{6}$$

$$\frac{dR}{dt} = \delta_2 H + \delta_1 Q + \delta_4 I_a + \delta_5 S + \delta_3 V - \mu R - \sigma R \tag{7}$$

$$\frac{dV}{dt} = \lambda_2 S - \lambda_1 V - \mu V - \delta_3 V \tag{8}$$

$$D(t) = \gamma_1 I_s(t) + \gamma_2 I_a(t) + \gamma_3 H(t) \tag{9}$$

$$C(t) = \eta_2 I_s + \eta_3 I_a(t) + \rho_1 Q + \eta_4 I_s \tag{10}$$

$$N(t) = S(t) + E(t) + I_s(t) + I_a(t) + Q(t) + H(t) + R(t) + V(t) \tag{11}$$

With the initial conditions, $S(0) \geq 0, E(0) \geq 0, I_s(0) \geq 0, I_a(0) \geq 0, Q(0) \geq 0, H(0) \geq 0, R(0) \geq 0, V(0) \geq 0$.
The parameter involved in this model system are described in Table 1.

3.2. Positivity of the solution

Theorem 3.1. *If the aforementioned initial circumstances are true, then the system (1)'s solution.* $(S(t), E(t), I_s(t), I_a(t), Q(t), H(t), R(t), V(t))$ *must be positive* $\forall t > 0$.

Proof. From the model system equations, we have

$$\frac{dS}{dt} = \pi + \lambda_1 V(t) + \rho_2 Q(t) + \sigma R(t) - M_1(t) S,$$
$$\text{where} \quad M_1(t) = \beta(1-h) I_s + \beta\alpha(1-h) I_a + \lambda_2 + \delta_5 + \mu.$$

$$\frac{dS}{dt} e^{\int_0^t M_1(\tau)d\tau} + M_1(t) S(t) e^{\int_0^t M_1(\tau)d\tau} = [(\pi + \lambda_1 V + \rho_2 Q + \sigma R)] e^{\int_0^t M_1(\tau)d\tau}$$

$$\frac{d}{dt}\left(S(t) e^{\int_0^t M_1(\tau)d\tau} \right) = [(\pi + \lambda_1 V + \rho_2 Q + \sigma R)] e^{\int_0^t M_1(\tau)d\tau}$$

$$S(t) e^{\int_0^t M_1(\tau)d\tau} - S(0) = \int_0^t (\pi + \lambda_1 V + \rho_2 Q + \sigma R) e^{\int_0^t M_1(\tau)d\tau} dt$$

(12)

$$S(t) = S(0) e^{-\int_0^t M_1(\tau)d\tau} + e^{-\int_0^t M_1(\tau)d\tau} \int_0^t (\pi + \lambda_1 V + \rho_2 Q + \sigma R) e^{\int_0^t M_1(\tau)d\tau} dt \geq 0.$$

Similarly

$$E(t) = E(0) e^{-\int_0^t M_2(\tau)d\tau} + e^{-\int_0^t M_2(\tau)d\tau} \int_0^t (\beta(1-h) S I_s + \beta\alpha(1-h) S I_a) e^{\int_0^t M_2(\tau)d\tau} dt \geq 0 \tag{13}$$

Table 1. Parameter and symbol with their description and value

Parameter	Description	Unit
π	Population growth rate	Person day^{-1}
β	Transmission rate of disease	$Person^{-1}day^{-1}$
h	Part of S(t) taking preventative precautions	limitless
μ	Natural death rate of population	Day^{-1}
a	I_a's relative contagiousness compared to I_s	limitless
σ	Rate at recovered loss their immunity	Day^{-1}
θ	$\frac{1}{\text{Incubation period}} = \text{infection rate}$	Day^{-1}
ρ	component of infections with symptoms	limitless
ρ_1	Rate at which quarantine hospitalized	Day^{-1}
ρ_2	Rate at which quarantine become susceptible	Day^{-1}
δ_1	Rate at which quarantine people recovered	Day^{-1}
δ_2	Rate at which hospitalized people recovered	Day^{-1}
δ_3	Rate at which vaccinated people recovered	Day^{-1}
δ_4	Rate at which asymptomatic people recovered	Day^{-1}
δ_5	Rate at which susceptible people recovered	Day^{-1}
η_1	Rate at which exposed people isolated	Day^{-1}
η_2	Rate at which symptomatic people isolated	Day^{-1}
η_3	Rate at which asymptomatic people isolated	Day^{-1}
η_4	Rate at which symptomatic people hospitalized	Day^{-1}
$\gamma_1, \gamma_2, \gamma_3$	COVID-19-related death rate	Day^{-1}
λ_1	Rate of vaccination for those who are susceptible	Day^{-1}
λ_2	Rate at which vaccinated become susceptible	Day^{-1}

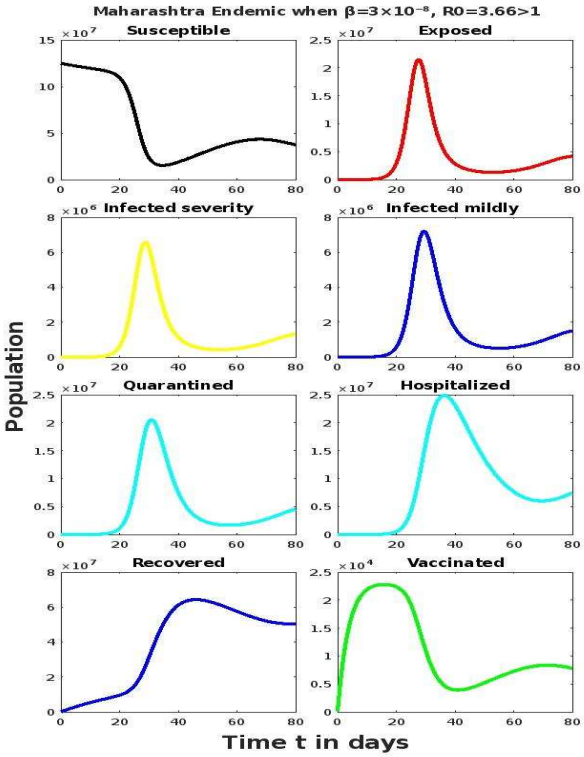

Figure 4.1: Endemic situation in Maharashtra

(14)

$$I_s(t) = I_s(0)e^{-\int_0^t M_3(\tau)d\tau} + e^{-\int_0^t M_3(\tau)d\tau}\int_0^t \rho\theta E e^{\int_0^t M_3(\tau)d\tau}dt \geq 0$$

(15)

$$I_a(t) = I_a(0)e^{-\int_0^t M_4(\tau)d\tau} + e^{-\int_0^t M_4(\tau)d\tau}\int_0^t (1-\rho)\theta E e^{\int_0^t M_4(\tau)d\tau}dt \geq 0$$

(16)

$$Q(t) = Q(0)e^{-\int_0^t M_5(\tau)d\tau} + e^{-\int_0^t M_5(\tau)d\tau}\int_0^t (\eta_1 E + \eta_2 I_s + \eta_3 I_a)e^{\int_0^t M_5(\tau)d\tau}dt \geq 0$$

(17)

$$H(t) = H(0)e^{-\int_0^t M_6(\tau)d\tau} + e^{-\int_0^t M_6(\tau)d\tau}\int_0^t (\rho_1 Q + \eta_4 I_s)e^{\int_0^t M_6(\tau)d\tau}dt \geq 0$$

(18)

$$R(t) = R(0)e^{-\int_0^t M_7(\tau)d\tau} + e^{-\int_0^t M_7(\tau)d\tau}\int_0^t (\delta_2 H + \delta_1 Q + \delta_4 I_a + \delta_5 S + \delta_3 V)e^{\int_0^t M_7(\tau)d\tau}dt \geq 0$$

(19)

$$V(t) = V(0)e^{-\int_0^t M_8(\tau)d\tau} + e^{-\int_0^t M_8(\tau)d\tau}\int_0^t (\lambda_2 S)e^{\int_0^t M_8(\tau)d\tau}dt \geq 0$$

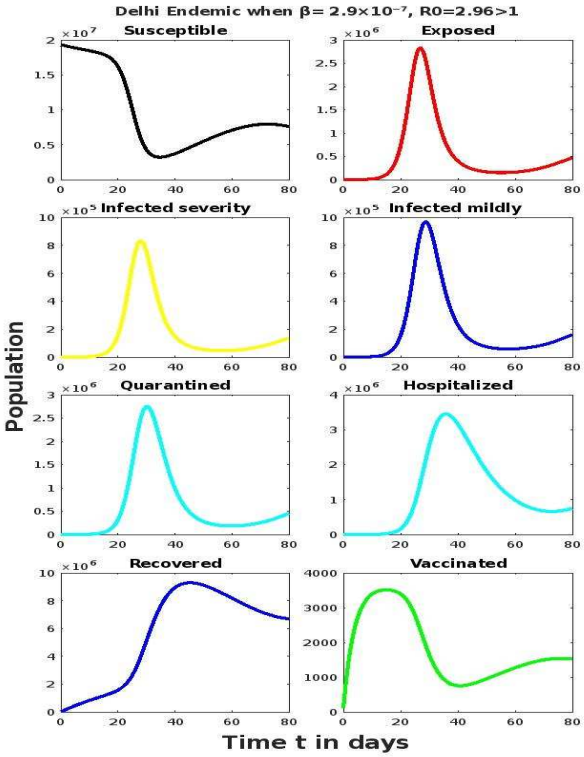

Figure 4.2: Endemic situation in Delhi

Where

$$M_2(\tau) = \theta + \eta_1 + \mu, M_3(\tau) = \gamma_1 + \eta_2 + \eta_4 + \mu, M_4(\tau) = \delta_4 + \gamma_2 + \eta_3 + \mu,$$
$$M_5(\tau) = \rho_2 + \delta_1 + \mu + \rho_1, M_6(\tau) = \mu + \gamma_3 + \delta_2, M_7(\tau) = \mu + \sigma,$$
$$M_8(\tau) = \delta_1 + \mu + \delta_3.$$

□

3.3. Boundedness

Theorem 3.2. *In the region* Δ*, the model system's solutions are uniformly bounded.*
$\Delta = \{(S, E, I_s, I_a, Q, H, R, V) \in \Re^8_+ : 0 \leq S + E + I_s + I_a + Q + H + R + V \leq \frac{\pi}{\mu}\}$.

Proof. Let $(S, E, I_s, I_a, Q, H, R, V)$ be any non negative solution of model system.
Let $N(0) = S(0) + E(0) + I_s(0) + I_a(0) + Q(0) + H(0) + R(0) + V(0) > 0$.
Now, addition of all above model provides,
$\frac{dN}{dt} = \pi - \mu(S + E + I_s + I_a + Q + H + R + V) - \gamma_1 I_s - \gamma_2 I_a - \gamma_3 H \leq \pi - \mu N$

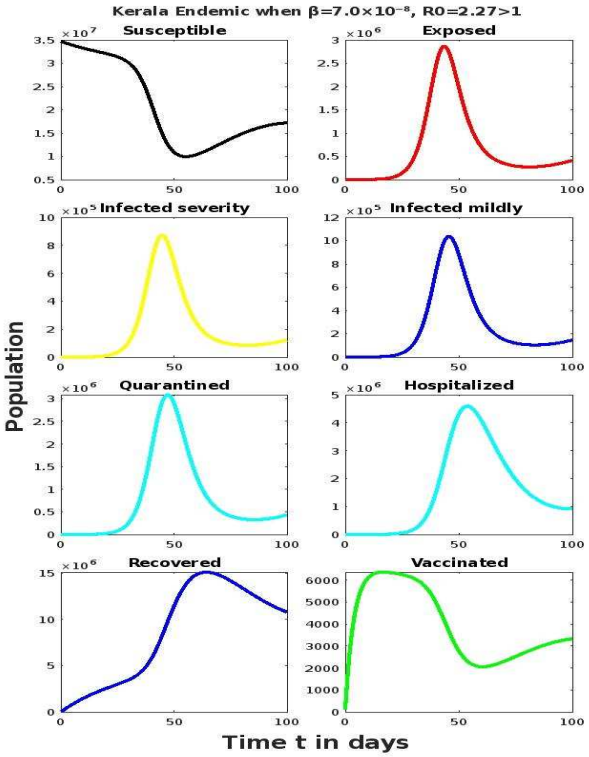

Figure 4.3: Endemic situation in Kerala

So by differential equation theory, we obtain
$N \leq N(0)e^{-\mu t} + \frac{\pi}{\mu}(1 - e^{-\mu t}), \quad \text{so if} \quad t \to \infty, 0 \leq N(t) \leq \frac{\pi}{\mu}.$
Thus, the region Δ is positively invariant, so that all solutions of model system with initial conditions in $\Re^8_+$. As a result, the initial value problem is well posed. □

3.4. Basic Reproduction Number This number represents the typical number of infectious individuals who are infected by a single infected individual throughout the course of their whole infectious period in a community that is entirely susceptible. The disease spread classes are E, I_a, I_s, Q.
The disease spread system is given by:

$$\frac{dE}{dt} = \beta(1-h)SI_s + \beta\alpha(1-h)SI_a - \theta E - \eta_1 E - \mu E \tag{20}$$

$$\frac{dI_s}{dt} = \rho\theta E - \gamma_1 I_s - \eta_2 I_s - \eta_4 I_s - \mu I_s \tag{21}$$

$$\frac{dI_a}{dt} = (1-\rho)\theta E - \delta_4 I_a - \gamma_2 I_a - \eta_3 I_a - \mu I_a \tag{22}$$

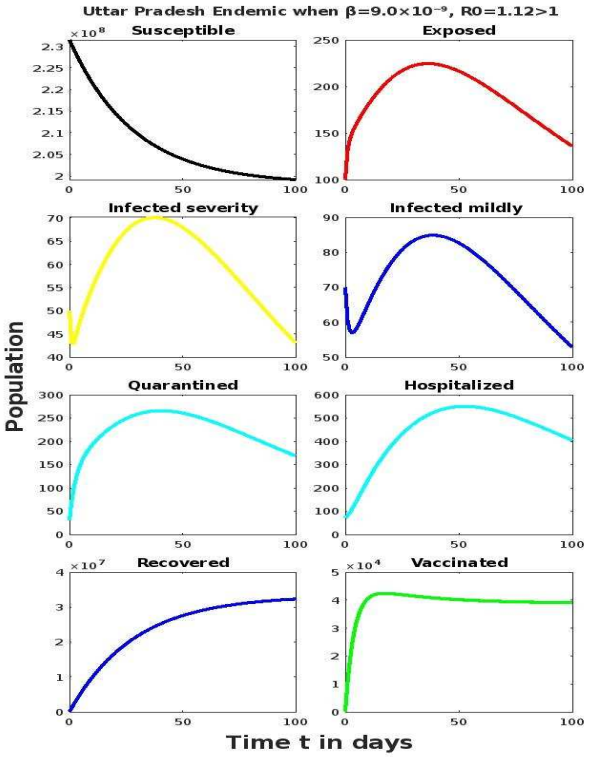

Figure 4.4: Endemic situation in Uttar Pradesh

$$\frac{dQ}{dt} = \eta_2 I_s + \eta_3 I_a + \eta_1 E - \rho_2 Q - \delta_1 Q - \mu Q - \rho_1 Q \tag{23}$$

The above system's (20-23) compact matrix form is given by

$$\frac{dX}{dt} = \mathbb{F}(X) - \mathbb{V}(X), \quad \text{where} \quad X = (E, I_s, I_a, Q)^T \tag{24}$$

where $\mathbb{F}$ and $\mathbb{V}$ are the new infectious matrix and transfer matrix between compartments respectively.

$$\mathbb{F} = \begin{bmatrix} \beta(1-h)SI_s + \beta\alpha(1-h)SI_a \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

$$\mathbb{V} = \begin{bmatrix} \mu E + \theta E + \eta_1 E \\ -\rho\theta E + (\mu + \gamma_1 + \eta_2 + \eta_4) I_s \\ -(1-\rho)\theta E + (\delta_4 + \mu + \gamma_2 + \eta_3) I_a \\ -\eta_1 E - \eta_2 I_s - \eta_3 I_a + (\rho_1 + \rho_2 + \delta_1 + \mu) Q \end{bmatrix} \tag{26}$$

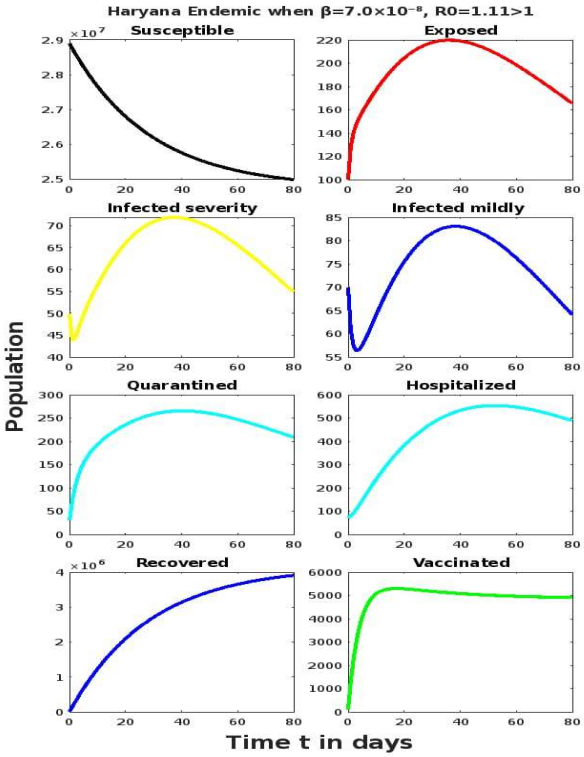

Figure 4.5: Endemic situation in Haryana

The partial derivatives of $\mathbb{F}$ and $\mathbb{V}$ with respect to E, I_s, I_a and Q at the disease free equilibrium point $(\frac{\pi}{\mu}, 0, 0, 0)^T$ provides the transition matrix K and L respectively.

$$K = \begin{bmatrix} 0 & \frac{\pi}{\mu}\beta(1-h) & \frac{\pi}{\mu}\alpha\beta(1-h) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{27}$$

$$L = \begin{bmatrix} \mu+\theta+\eta_1 & 0 & 0 & 0 \\ -\rho\theta & \mu+\gamma_1+\eta_2+\eta_4 & 0 & 0 \\ -(1-\rho)\theta & 0 & \mu+\gamma_2+\eta_3+\delta_4 & 0 \\ -\eta_1 & -\eta_2 & -\eta_3 & \rho_1+\rho_2+\delta_1+\mu \end{bmatrix} \tag{28}$$

We define the next generation matrix by KL^{-1}, and the basic reproduction number $\mathcal{R}_0$ is the radius of spectral KL^{-1}.

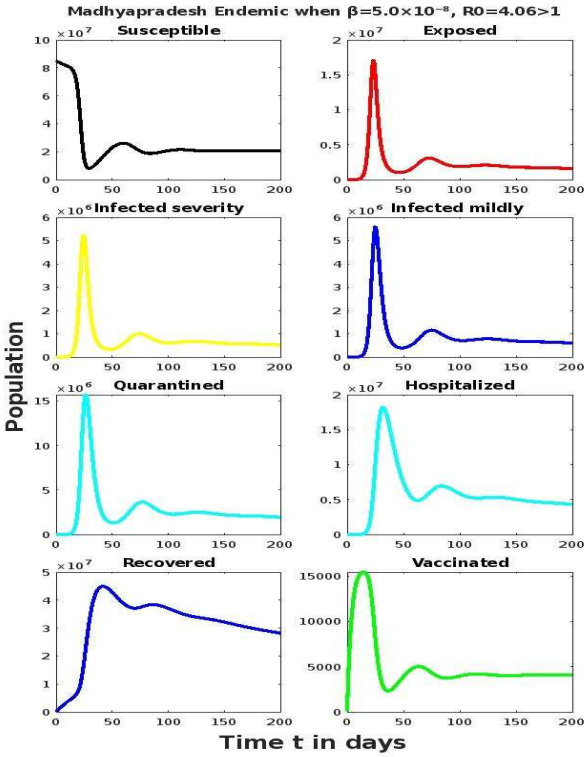

Figure 4.6: Endemic situation in Madhya Pradesh

(29)
$$KL^{-1} = \begin{bmatrix} \frac{\frac{\pi}{\mu}\beta(1-h)\rho\theta}{L_1L_2} + \frac{\frac{\pi}{\mu}\beta(1-h)(1-\rho)\alpha\theta}{L_1L_3} & \frac{\pi}{\mu L_2}\beta(1-h) & \frac{\pi}{\mu L_3}\beta(1-h) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $L_1 = \mu + \theta + \eta_1, L_2 = \mu + \gamma_1 + \eta_2 + \eta_4, L_3 = \mu + \gamma_2 + \eta_3 + \delta_4$.

(30)
$$\mathcal{R}_0 = \frac{\frac{\pi}{\mu}\beta(1-h)\rho\theta}{L_1L_2} + \frac{\frac{\pi}{\mu}\beta(1-h)(1-\rho)\alpha\theta}{L_1L_3}$$

(31)
$$\mathcal{R}_0 = \mathcal{R}_0^A + \mathcal{R}_0^B$$

$$\text{where} \quad \mathcal{R}_0^A = \frac{\frac{\pi}{\mu}\beta(1-h)\rho\theta}{L_1L_2}, \quad \mathcal{R}_0^B = \frac{\frac{\pi}{\mu}\beta(1-h)(1-\rho)\alpha\theta}{L_1L_3}$$

$\mathcal{R}_0^A$ denotes the number of secondary infections by an infected person during their time spent in the infected population. It is a measure of the number of the $\frac{(1-h)\pi}{\mu}$ susceptible population that are infected by $\rho\theta$ people in the infected group with a bilinear transmission rate β, with $\frac{1}{\mu+\gamma_1+\eta_2+\eta_4}$ and $\frac{1}{\mu+\theta+\eta_1}$ being the time an infected

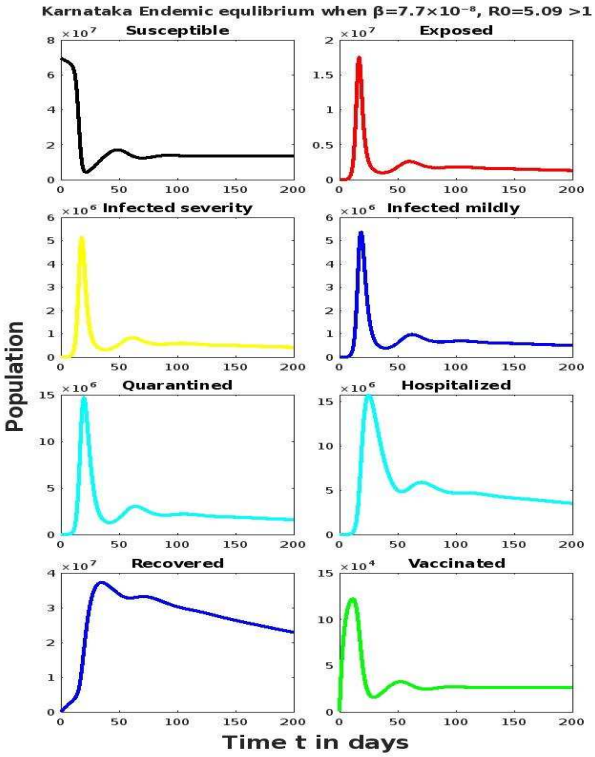

Figure 4.7: Endemic situation in Karnataka

individual remains in the infected and exposed compartment respectively. $\mathcal{R}_0^B$ denotes the number of secondary infections by an asymptotic infected person during their time spent in the asymptotic population. It represent the number of the $\frac{(1-h)\pi}{\mu}$ susceptible population that are infected by $(1-\rho)\theta$ people in the asymptomatic compartment with an enhanced transmission rate $\alpha\beta$ and $\frac{1}{\mu+\gamma_2+\eta_3+\delta_4}$ being the time an individual remains in the asymptomatic compartment.

3.5. Equilibrium Points Setting all model equations equal to zero yields the model system's equilibrium points. The disease free equilibrium point is $X_{DFE} = (\frac{\pi}{\mu}, 0, 0, 0, 0, 0, 0, 0)$ and the endemic equilibrium point is

$$X_{EE} = (S^*, E^*, I_s^*, I_a^*, Q^*, H^*, R^*, V^*,),$$

where

$$S^* = \frac{\pi}{\mu\mathcal{R}_0}, \tag{32}$$

$$I_s^* = K_1 E^*, \quad \text{where} \quad K_1 = \frac{\rho\theta}{\mu + \gamma_1 + \eta_2 + \eta_4}, \tag{33}$$

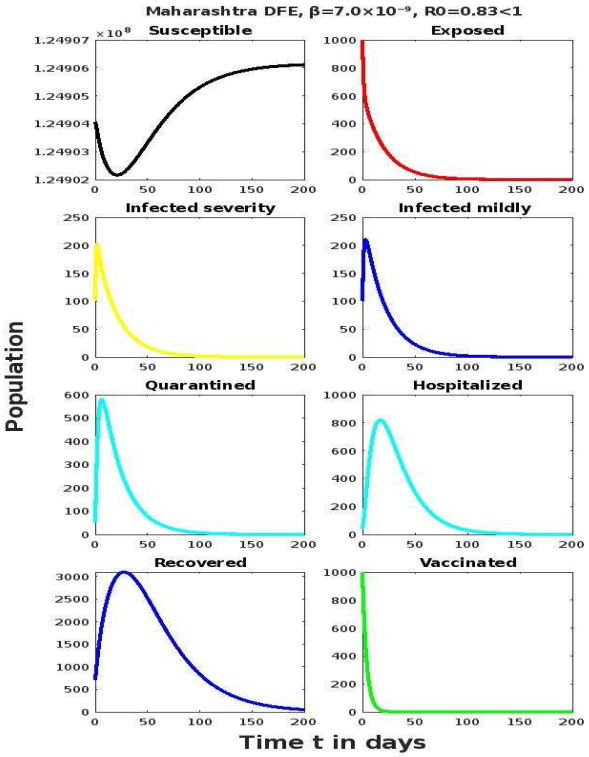

Figure 4.8: Endemic situation in Maharashtra

$$(34)\quad I_a^* = K_2 E^*, \quad \text{where} \quad K_2 = \frac{(1-\rho)\theta}{\delta_4+\mu+\gamma_2+\eta_3},$$

$$(35)\quad Q^* = (K_1\eta_2 + K_2\eta_3 + \eta_1)\frac{E^*}{\rho_1+\rho_2+\mu+\delta_1},$$

$$(36)\quad H^* = \frac{((\eta_2+\eta_4)K_1 + K_2\eta_3)\rho_1 E^*}{(\rho_1+\rho_2+\mu+\delta_1)(\mu+\gamma_3+\delta_2)},$$

$$(37)\quad R^* = \frac{1}{\mu+\sigma}\left[\left\{\frac{(\delta_1+\frac{\delta_2\rho_1}{\mu+\gamma_3+\delta_2})(K_1\eta_2+K_2\eta_3+\eta_1)}{\rho_1+\rho_2+\mu+\delta_1} + \frac{\eta_4\delta_2 K_1}{\mu+\gamma_3+\delta_2} + \delta_4 K_2\right\}E^* + \frac{(\delta_5+\frac{\delta_3\lambda_2}{\lambda_1+\mu+\delta_3})\pi}{\mu\mathcal{R}_0}\right],$$

$$(38)\quad V^* = \frac{\lambda_2\pi}{(\lambda_1+\mu+\delta_3)\mu\mathcal{R}_0},$$

$$(39)\quad E^* = \frac{\pi+\lambda_1 V^* - (\theta+\eta_1+\mu)S^*}{(\theta+\eta_1+\mu)-\rho_2 M_1 - \sigma M_2}$$

Where

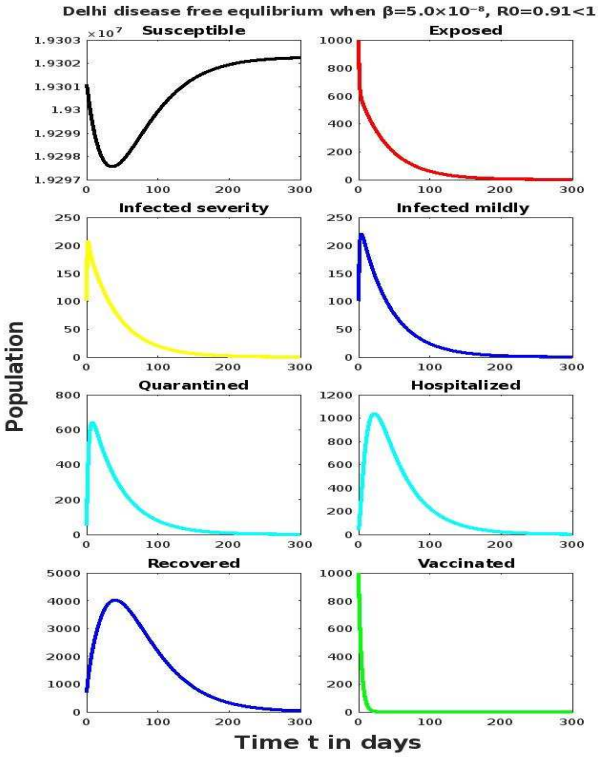

Figure 4.9: Endemic situation in Delhi

$$M_1 = \frac{\eta_1 + \eta_2(\frac{\rho\theta}{\mu+\gamma_1+\eta_2+\eta_4}) + \frac{\eta_3(1-\rho)\theta}{\mu+\gamma_2+\eta_3+\delta_4}}{\mu+\delta_1+\rho_1+\rho_2},$$

$$M_2 = \frac{\delta_1 M_1 + \delta_2 \frac{(\rho_1 M_1 + \eta_4 \frac{\rho\theta}{(\mu+\gamma_1+\eta_2+\eta_4)})}{\mu+\gamma_3+\delta_2} + \delta_4(\frac{(1-\rho)\theta}{\mu+\gamma_2+\eta_3+\delta_4}) + \delta_5(\frac{\rho\theta}{\mu+\gamma_1+\eta_2+\eta_4})}{\mu+\sigma}.$$

4. Stability at Equilibrium

4.1. Local Stability at Equilibrium The Jacobian matrix of the dynamic system is given by:

$$J = \begin{bmatrix} -\Delta_1-(\lambda_2+\delta_5+\mu) & 0 & -\beta(1-h)S & -\alpha\beta(1-h)S & \rho_2 & 0 & \sigma & \lambda_1 \\ \Delta_1 & -(\theta+\eta_1+\mu) & \beta(1-h)S & \alpha\beta(1-h)S & 0 & 0 & 0 & 0 \\ 0 & \rho\theta & -(\gamma_1+\eta_2+\eta_4+\mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\rho)\theta & 0 & -(\gamma_2+\eta_3+\delta_4+\mu) & 0 & 0 & 0 & 0 \\ 0 & \eta_1 & \eta_2 & \eta_3 & -(\rho_1+\rho_2+\delta_1+\mu) & 0 & 0 & 0 \\ 0 & 0 & \eta_4 & 0 & \rho_1 & -(\delta_2+\gamma_3+\mu) & 0 & 0 \\ \delta_5 & 0 & 0 & \delta_4 & \delta_2 & \delta_1 & -(\mu+\sigma) & \delta_3 \\ \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda_1+\delta_3+\mu) \end{bmatrix}$$

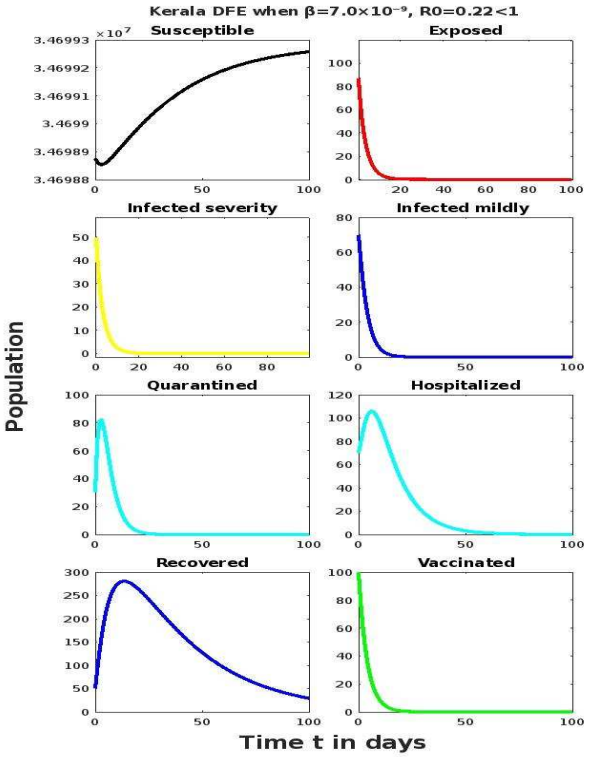

Figure 4.10: Endemic situation in Kerala

where $\Delta_1 = \beta(1-h)I_s + \alpha\beta(1-h)I_a$ At disease free equilibrium

$J_{DFE} =$

$$\begin{bmatrix} -(\lambda_2+\delta_5+\mu) & 0 & -\frac{\beta(1-h)\pi}{\mu} & -\frac{\alpha\beta(1-h)\pi}{\mu} & \rho_2 & 0 & \sigma & \lambda_1 \\ 0 & -(\theta+\eta_1+\mu) & \frac{\beta(1-h)\pi}{\mu} & \frac{\alpha\beta(1-h)\pi}{\mu} & 0 & 0 & 0 & 0 \\ 0 & \rho\theta & -(\gamma_1+\eta_2+\eta_4+\mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\rho)\theta & 0 & -(\gamma_2+\eta_3+\delta_4+\mu) & 0 & 0 & 0 & 0 \\ 0 & \eta_1 & \eta_2 & \eta_3 & -(\rho_1+\rho_2+\delta_1+\mu) & 0 & 0 & 0 \\ 0 & 0 & \eta_4 & 0 & \rho_1 & -(\delta_2+\gamma_3+\mu) & 0 & 0 \\ \delta_5 & 0 & 0 & \delta_4 & \delta_2 & \delta_1 & -(\mu+\sigma) & \delta_3 \\ \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda_1+\delta_3+\mu) \end{bmatrix}$$

At endemic equilibrium

$J =$

$$\begin{bmatrix} -\Delta_1^* -(\lambda_2+\delta_5+\mu) & 0 & -\beta(1-h)S^* & -\alpha\beta(1-h)S^* & \rho_2 & 0 & \sigma & \lambda_1 \\ \Delta_1^* & -(\theta+\eta_1+\mu) & \beta(1-h)S^* & \alpha\beta(1-h)S^* & 0 & 0 & 0 & 0 \\ 0 & \rho\theta & -(\gamma_1+\eta_2+\eta_4+\mu) & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\rho)\theta & 0 & -(\gamma_2+\eta_3+\delta_4+\mu) & 0 & 0 & 0 & 0 \\ 0 & \eta_1 & \eta_2 & \eta_3 & -(\rho_1+\rho_2+\delta_1+\mu) & 0 & 0 & 0 \\ 0 & 0 & \eta_4 & 0 & \rho_1 & -(\delta_2+\gamma_3+\mu) & 0 & 0 \\ \delta_5 & 0 & 0 & \delta_4 & \delta_2 & \delta_1 & -(\mu+\sigma) & \delta_3 \\ \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda_1+\delta_3+\mu) \end{bmatrix}$$

where $\Delta_1^* = \beta(1-h)I_s^* + \alpha\beta(1-h)I_a^*$ By employing MATLAB software and utilizing the values presented in Table 2, we conducted an assessment of the eigenvalues

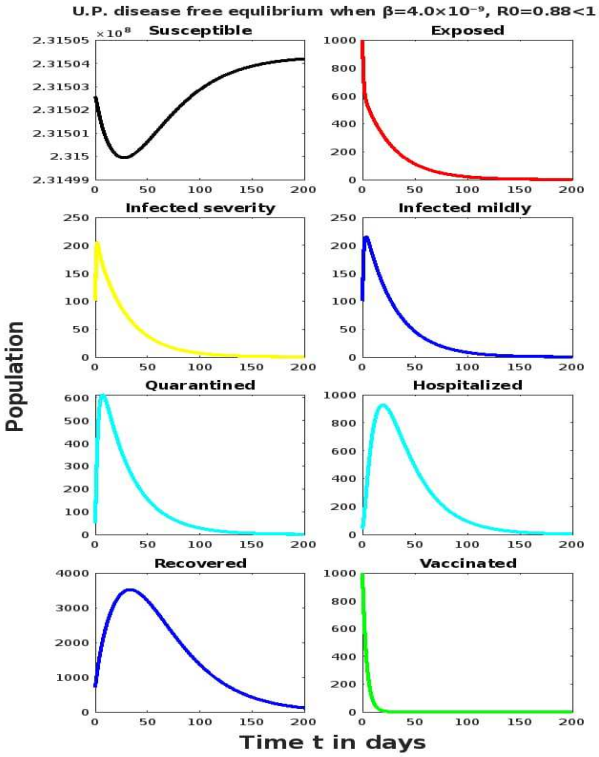

Figure 4.11: Endemic situation in Uttar Pradesh

corresponding to the disease-free and endemic equilibrium points for seven distinct Indian states. Notably, our findings indicate that at the disease-free equilibrium point, all eigenvalues possess negative real parts across the aforementioned states. This observation, in accordance with the Routh-Hurwitz criterion, establishes local asymptotic stability when the basic reproductive number ($\mathcal{R}_0$) is less than 1 and instability when $\mathcal{R}_0$ exceeds 1. Similarly, our analysis reveals that the endemic equilibrium point exhibits local asymptotic stability across all seven states when $\mathcal{R}_0$ surpasses 1, as evidenced by the negativity of the real parts of all eigenvalues.

4.2. Global Stability at Disease Free Equilibrium

Theorem 4.1. *The disease-free equilibrium is globally asymptotically stable if* $\mathcal{R}_0 < 1$ *and unstable if* $\mathcal{R}_0 > 1$.

Proof. Consider the Lyapunov function $L(E, I_s) = \kappa_1 E + \kappa_2 I_s$, where κ_1 and κ_2 are non-negative parameters. Obviously $L(E, I_s) \in C^1$. Moreover, at DFE $L(E, I_s) = 0$

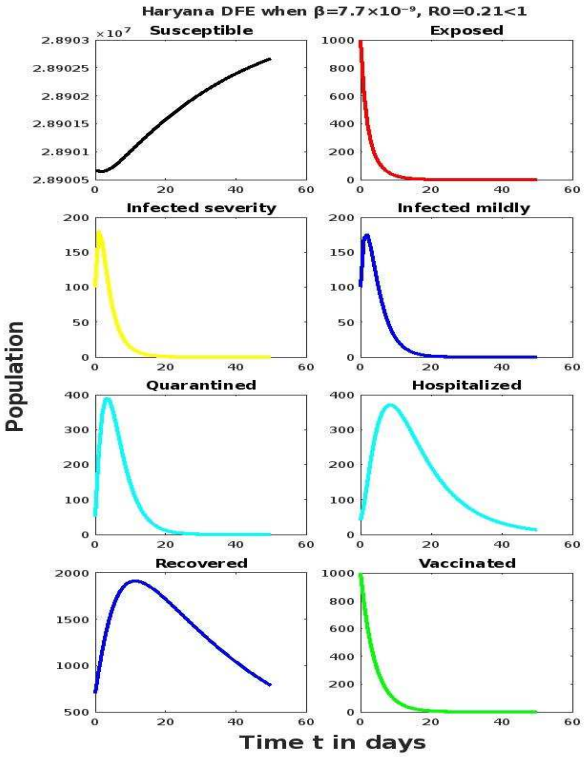

Figure 4.12: Endemic situation in Haryana

and it is positive definite $\forall(S, E, I_s, I_a, Q, H, R, V) \in \Re^{8+}$. Now,

$$\frac{dL}{dt} = \kappa_1 \frac{dE}{dt} + \kappa_2 \frac{dI_s}{dt}$$

$$\frac{dL}{dt} = \kappa_1(\beta(1-h)SI_s + \beta\alpha(1-h)SI_a - \theta E - \eta_1 E - \mu E) + \kappa_2(\rho\theta E - \gamma_1 I_s - \eta_2 I_s - \eta_4 I_s - \mu I_s)$$

$$\frac{dL}{dt} = \kappa_1(\beta(1-h)SI_s + \beta\alpha(1-h)SI_a) - \{\kappa_1(\theta + \eta_1 + \mu) - \kappa_2\rho\theta\}E - \kappa_2(\gamma_1 + \eta_2 + \eta_4 + \mu)I_s$$

Choosing $\kappa_1 = \rho\theta$, $\kappa_2 = (\theta + \eta_1 + \mu)$ and putting $S = \frac{\pi}{\mu}$, $I_a = 0$. We obtain, $\frac{dL}{dt} = \left\{\frac{\beta(1-h)\pi\rho\theta}{\mu} - (\theta + \eta_1 + \mu)(\gamma_1 + \eta_2 + \eta_4 + \mu)\right\}I_s$ $\frac{dL}{dt} = (\theta + \eta_1 + \mu)(\gamma_1 + \eta_2 + \eta_4 + \mu)(\mathcal{R}_0^A - 1)$ Since $\mathcal{R}_0^A < 1$ follows from $\mathcal{R}_0 < 1$, therefore it is clear that $\frac{dL}{dt} < 0$, when $\mathcal{R}_0 < 1$ and Moreover, $\frac{dL}{dt} = 0$, if $I_a = 0$. Hence, by LaSalle's Invariance Principle [12], the disease-free equilibrium point is globally asymptotically stable. □

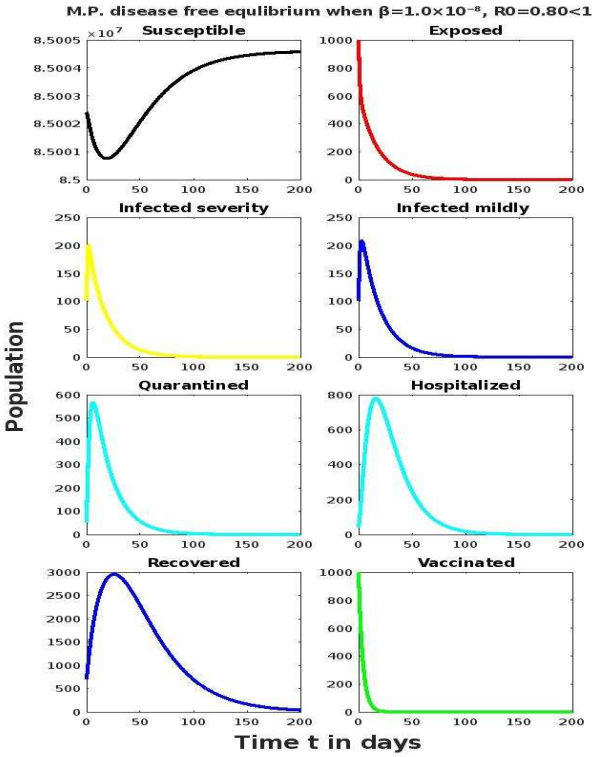

Figure 4.13: Endemic situation in Madhya Pradesh

4.3. Global Stability at Endemic Equilibrium

Theorem 4.2. *If $\mathcal{R}_0 > 1$, then there exist a disease- endemic equilibrium χ^1 and it is globally asymptotically stable in the interior of Δ.*

Proof. Given that $\mathcal{R}_0 > 1$, then the existence and local asymptotic stability of the disease- endemic equilibrium is guaranteed. Consider the Lyapunov function.

$$L(E, I_s, I_a, Q, H, R, V) = E - E^* - E^* \ln\left(\frac{E}{E^*}\right) + I_s - I_s^* - I_s^* \ln\left(\frac{I_s}{I_s^*}\right) + I_a - I_a^* - I_a^* \ln\left(\frac{I_a}{I_a^*}\right) + Q - Q^* - Q^* \ln\left(\frac{Q}{Q^*}\right) + H - H^* - H^* \ln\left(\frac{H}{H^*}\right) + V - V^* - V^* \ln\left(\frac{V}{V^*}\right)$$

$$\frac{dL}{dt} = \left(1 - \frac{E^*}{E}\right)\frac{dE}{dt} + \left(1 - \frac{I_s^*}{I_s}\right)\frac{dI_s}{dt} + \left(1 - \frac{I_a^*}{I_a}\right)\frac{dI_a}{dt} + \left(1 - \frac{Q^*}{Q}\right)\frac{dQ}{dt} + \left(1 - \frac{H^*}{H}\right)\frac{dH}{dt} + \left(1 - \frac{V^*}{V}\right)\frac{dV}{dt}$$

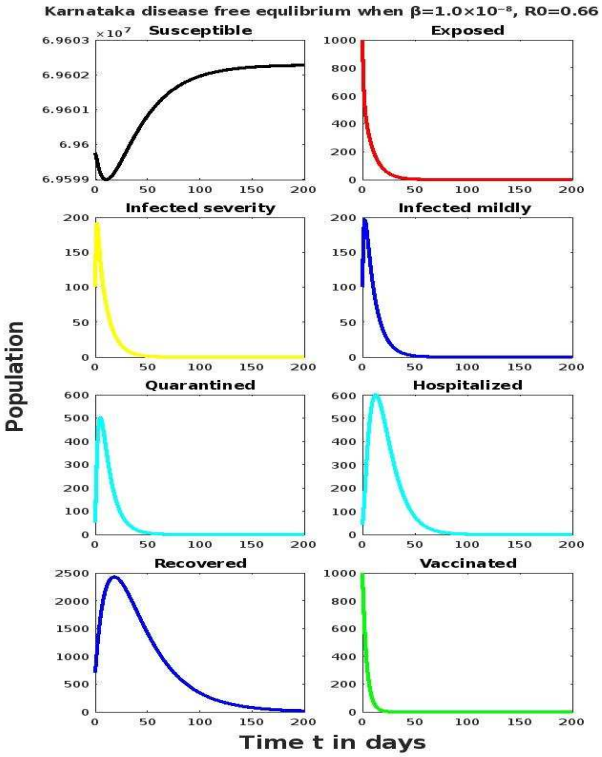

Figure 4.14: Endemic situation in Karnataka

$$\begin{aligned}\frac{dL}{dt} &= \left(1-\frac{E^*}{E}\right)[\beta(1-h)SI_s+\beta\alpha(1-h)SI_a-\theta E-\eta_1 E-\mu E]\\ &+\left(1-\frac{I_s^*}{I_s}\right)[\rho\theta E-\gamma_1 I_s-\eta_2 I_s-\eta_4 I_s-\mu I_s]\\ &+\left(1-\frac{I_a^*}{I_a}\right)[(1-\rho)\theta E-\delta_4 I_a-\gamma_2 I_a-\eta_3 I_a-\mu I_a]\\ &+\left(1-\frac{Q^*}{Q}\right)[\eta_2 I_s+\eta_3 I_a+\eta_1 E-\rho_2 Q-\delta_1 Q-\mu Q-\rho_1 Q]\\ &+\left(1-\frac{H^*}{H}\right)[\rho_1 Q-\mu H-\gamma_3 H-\delta_2 H+\eta_4 I_s]\\ &+\left(1-\frac{V^*}{V}\right)[\lambda_2 S-\lambda_1 V-\mu V-\delta_3 V]\end{aligned}$$

In endemic situation, $\beta(1-h)S^*I_s^*+\beta\alpha(1-h)S^*I_a^*=(\theta+\eta_1+\mu)E^*$, $\rho\theta E^*=(\gamma_1+\eta_2+\eta_4+\mu)I_s^*$, $(1-\rho)\theta E^*=(\delta_4+\gamma_2+\eta_3+\mu)I_a^*$, $\eta_2 I_s^*+\eta_3 I_a^*+\eta_1 E^*=(\rho_2+\delta_1+\mu+\rho_1)Q^*$, $\rho_1 Q^*+\eta_4 I_s^*=(\mu+\gamma_3+\delta_2)H^*$, $\lambda_2 S^*=(\lambda_1+\mu+\delta_3)V^*$. Using this we obtain,

$$\frac{dL}{dt} = \beta(1-h)SI_s\left(1-\frac{E^*}{E}\right) + \beta(1-h)S^*I_s^*\left(1-\frac{E}{E^*}\right)$$
$$+ \alpha\beta(1-h)SI_a\left(1-\frac{E^*}{E}\right) + \alpha\beta(1-h)S^*I_a^*\left(1-\frac{E}{E^*}\right)$$
$$+ \rho\theta E\left(1-\frac{I_s^*}{I_s}\right) + \rho\theta E^*\left(1-\frac{I_s}{I_s^*}\right)$$
$$+ (1-\rho)\theta E\left(1-\frac{I_a^*}{I_a}\right) + (1-\rho)\theta E^*\left(1-\frac{I_a}{I_a^*}\right)$$
$$+ (\eta_2 I_s + \eta_3 I_a + \eta_1 E)\left(1-\frac{Q^*}{Q}\right) + (\eta_2 I_s^* + \eta_3 I_a^* + \eta_1 E^*)\left(1-\frac{Q}{Q^*}\right)$$
$$+ (\rho_1 Q + \eta_4 I_s)\left(1-\frac{H^*}{H}\right) + (\rho_1 Q^* + \eta_4 I_s^*)\left(1-\frac{H}{H^*}\right)$$
$$+ \lambda_2 S\left(1-\frac{V^*}{V}\right) + \lambda_2 S^*\left(1-\frac{V}{V^*}\right).$$

Since $E \le E^*$, $I_s \le I_s^*$, $I_a \le I_a^*$, $Q \le Q^*$, $H \le H^*$ and $V \le V^*$.

$$\frac{dL}{dt} = \beta(1-h)S^*I_s^*\left(2-\frac{E^*}{E}-\frac{E}{E^*}\right) + \alpha\beta(1-h)S^*I_a^*\left(2-\frac{E^*}{E}-\frac{E}{E^*}\right)$$
$$+ \rho\theta E^*\left(2-\frac{I_s^*}{I_s}-\frac{I_s}{I_s^*}\right) + (1-\rho)\theta E^*\left(2-\frac{I_a^*}{I_a}-\frac{I_a}{I_a^*}\right)$$
$$+ (\eta_2 I_s^* + \eta_3 I_a^* + \eta_1 E^*)\left(2-\frac{Q^*}{Q}-\frac{Q}{Q^*}\right) + (\rho_1 Q^* + \eta_4 I_s^*)\left(2-\frac{H^*}{H}-\frac{H}{H^*}\right)$$
$$+ \lambda_2 S^*\left(2-\frac{V^*}{V}-\frac{V}{V^*}\right)$$

Since by arithmetic-geometric $\left(2-\frac{E^*}{E}-\frac{E}{E^*}\right) \le 0$, $\left(2-\frac{I_s^*}{I_s}-\frac{I_s}{I_s^*}\right) \le 0$, $\left(2-\frac{I_a^*}{I_a}-\frac{I_a}{I_a^*}\right) \le 0$, $\left(2-\frac{Q^*}{Q}-\frac{Q}{Q^*}\right) \le 0$, $\left(2-\frac{H^*}{H}-\frac{H}{H^*}\right) \le 0$, $\left(2-\frac{V^*}{V}-\frac{V}{V^*}\right) \le 0$.

$\Rightarrow \frac{dL}{dt} \le 0$ and $\frac{dL}{dt} = 0$, if $E = E^*$, $I_s = I_s^*$, $I_a = I_a^*$, $Q = Q^*$, $H = H^*$, $V = V^*$. Hence, by LaSalle's Invariance Principle [12], the endemic equilibrium χ^1 is globally asymptotically stable. □

4.4. Numerical Simulation We apply the model on the seven different states of India: Maharashtra, Delhi, Kerala, Uttar Pradesh, Haryana, Madhya Pradesh and Karnataka.

All other parameter values are same for both situation and values in bracket indicates the value of that parameter for the disease free equilibrium (DFE).

For example Maharashtra, the average death rate, (μ), used for the simulation will be $\frac{1}{71.9 \times 365} = 3.81 \times 10^{-5}$ per day with growth rate $\mu N_0 = 4,758.85$.

It is assumed that 60% of newly acquired infections are symptomatic and 40% of susceptible take preventative measures. α is 1.5 for the asymptomatic class's relative infectiousness. Quarantined population with symptoms moved into hospitalized class at a rate $\rho_1 = \frac{1}{7} = 0.143/day$, whereas without symptoms they are moved into the susceptible population at a rate $\rho_2 = \frac{1}{14} = 0.0714/day$. For people in the hospitalized and asymptomatic classifications, the average remission time is set at 14 days and 7 days, respectively. Hence, their correspondence recovery rates are $\delta_1 = 0.0714/day$ and $\delta_2 = 0.143/day$ respectively. The parameter values utilized in the simulations are provided in Table 2. For both the epidemic and disease-free conditions of the simulation, we presume the population's initial values in the various compartments to be as follows:

$S(0) = N_0$, $E(0) = 1000$, $I_s(0) = 100$, $I_a(0) = 100$, $Q(0) = 50$, $H(0) = 50$, $R(0) = 700$, $V(0) = 1000$. We plot the graph of the above mentioned Indian states for both the conditions (DFE and EE) using MATLAB software. Figures 4.1 to 4.7 illustrate the stability of endemic equilibrium point for the Indian states across various epidemiological classes, including susceptible, exposed, infected severity, infected mildly, quarantined, hospitalized, recovered, and vaccinated. One can easily observe that as $\mathcal{R}_0 > 1$, the epidemic curve approaches to its endemic equilibrium point globally as $t \to \infty$.

A comparative analysis reveals a consistent trend across all states within their respective classes. Notably, there is a discernible pattern wherein the infected compartments exhibit an incremental growth until reaching a saturation point, leading to the establishment of an endemic scenario.

Figures 4.8 to 4.14 delineate the disease-free equilibrium status for the Indian states of Maharashtra, Delhi, Kerala, Uttar Pradesh, Haryana, Madhya Pradesh, and Karnataka, respectively. Examination of the epidemic curves reveals that, with the exception of the susceptible class, all other compartments exhibit a zero inclination. Consequently, in this scenario, the disease is poised for eradication. One can easily observe that as $\mathcal{R}_0 < 1$, the epidemic curve approaches to its disease-free equilibrium point locally as well as globally as $t \to \infty$.

Table 2. Value of parameters

Parameter	Maharashtra	Delhi	Kerala	U.P.	Haryana	M.P.	Karnataka	Source
L.E.	71.9	73.5	73.5	66.9	69.4	66.7	69.7	[8]
N_0	124,904,071	19,301,096	34,698,876	231,502,578	28,900,667	85,002,417	69,599,762	[9]
π	4758.85	719.93	1294.27	9468.45	1141.58	3485.1	2735.27	Calculate
β_{EE}	3.0×10^{-8}	2.9×10^{-7}	7.0×10^{-8}	9.0×10^{-8}	7.0×10^{-7}	5.0×10^{-8}	7.7×10^{-8}	Assumed
β_{DFE}	7.0×10^{-9}	5×10^{-8}	7.0×10^{-9}	4.0×10^{-9}	7.7×10^{-9}	1.0×10^{-8}	1.0×10^{-8}	Assumed
h	0.4	0.4	0.4	0.4	0.4	0.4	0.4	Assumed
μ	3.81×10^{-5}	3.73×10^{-5}	3.73×10^{-5}	4.09×10^{-5}	3.95×10^{-5}	4.1×10^{-5}	3.93×10^{-5}	Calculate
a	1.5	1.5	1.5	1.5	1.5	1.5	1.5	Assumed
σ	0.03	0.03	0.03	0.03	0.03	0.03	0.03	Assumed
θ	0.438	0.438	0.438	0.438	0.438	0.438	0.438	Assumed
ρ	0.6	0.6	0.6	0.6	0.6	0.6	0.6	Assumed
ρ_1	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1/7 Assumed
ρ_2	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714	1/14 Assumed
δ_1	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1/7 Assumed
δ_2	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714	0.0714	1/14 Assumed
δ_3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	1/4 Assumed
δ_4	0.143	0.143	0.143	0.143	0.143	0.143	0.143	1/7 Assumed
δ_5	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	Assumed
η_1	0.123	0.123	0.123	0.123	0.123	0.123	0.123	Assumed
η_2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	Assumed
η_3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	Assumed
η_4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	Assumed
γ_1	0.018	0.0615	0.0515	0.0415	0.0034	0.048	0.001	Estimated
γ_2	0.02	0.02	0.02	0.02	0.02	0.02	0.02	Assumed
γ_3	0.02	0.02	0.02	0.02	0.02	0.02	0.02	Assumed
λ_1	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	0.005 (0)	Assumed
λ_2	0.00005 (0)	0.00005 (0)	0.00005 (0)	0.00005 (0)	0.00005 (0)	0.00005 (0)	0.00005 (0)	Assumed

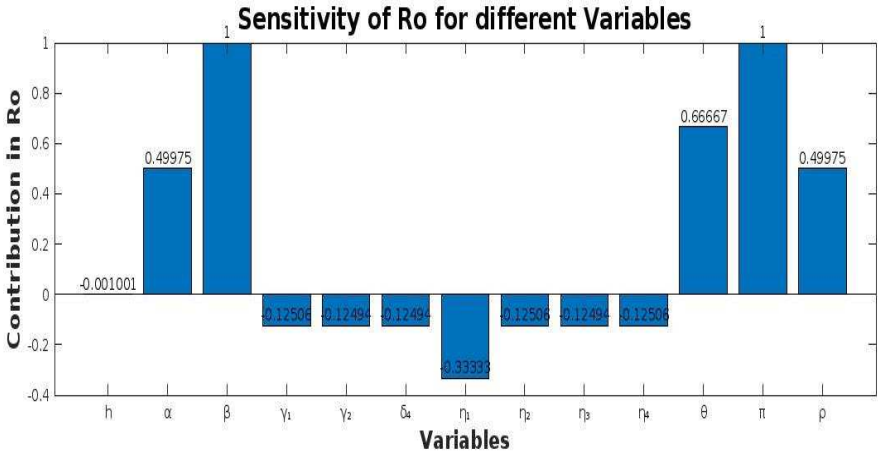

Figure 5.1: Various parameters' effects on basic reproduction number

5. Sensitivity Analysis

The direct differentiation method is used to do the sensitivity analysis. The sensitivity index $\psi_{\omega}^{\mathcal{R}_0}$ of a parameter ω is calculated by

$$\psi_{\omega}^{\mathcal{R}_0} = \frac{(d\mathcal{R}_0)}{d\omega} \times \frac{\omega}{\mathcal{R}_0}.$$

We take all variables values as 0.001 to examine $\mathcal{R}_0$'s sensitivity with regard to other factors. As compare to other variables the parameters $\alpha, \beta, \theta, \rho$ and π has a positive influence effect on the rate of reproduction, while remaining other parameters has negative influence (see Figure 5.1).

Positive influence means reducing the value of that parameter can lead to deduct the number of infective and ultimately the basic reproduction number. Whereas negative influence means increasing the value of that parameter can lead to reduce the infections.

Now, keeping all variables and parameters values fixed mentioned in Table 2 and let us change into one parameter on which we want to check the impact on different infected classes.

Furthermore, the impact of changing of the transmission rate β on exposed, asymptotically infected and infected compartments respectively can be seen in the following Figures 5.2 and 5.3.

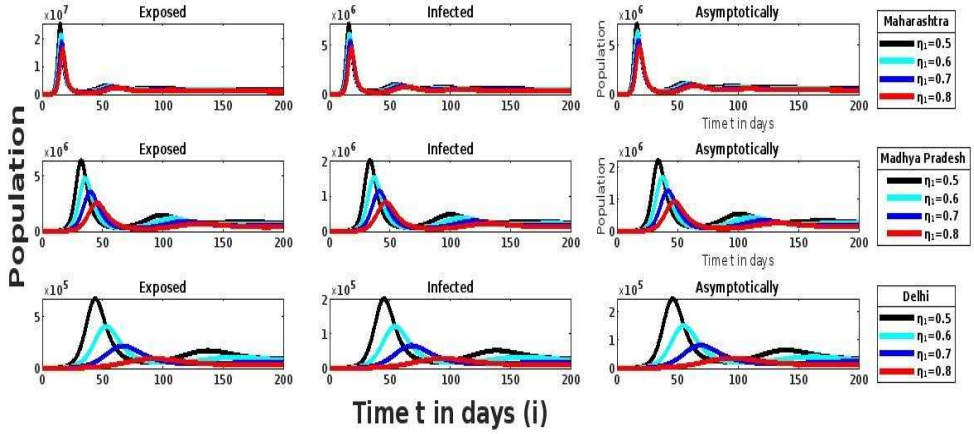

Figure 5.2: Effect of change transmission rate β rate in Maharashtra, Madhya Pradesh, Delhi

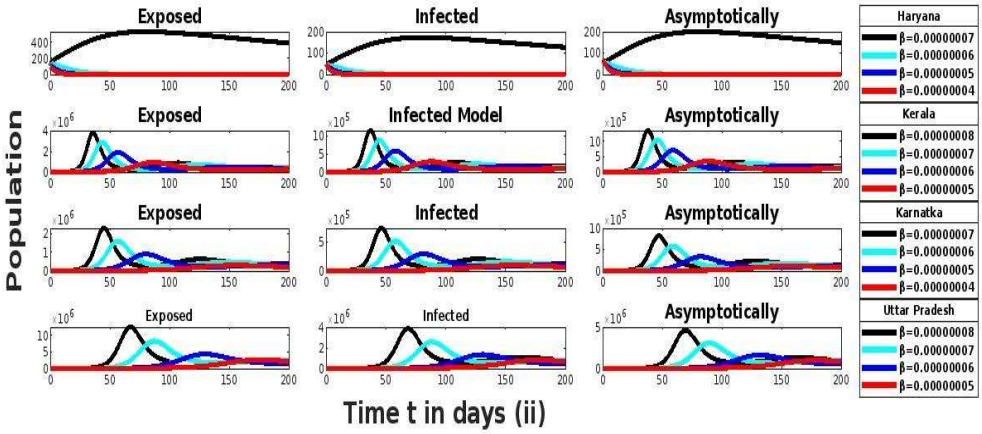

Figure 5.3: Effect of change transmission rate β rate in Haryana, Kerala, Karnataka and Uttar Pradesh

In the Figure 5.2 and Figure 5.3 one can easily observe that as the transmission rate β decreases through lockdown, social distancing etc. all of the infective compartments exhibits relatively decrease in infection.

Additionally, to check the effect of isolation of exposed population on exposed, infected and asymptotically infected compartments, since the rate of isolation η_1 of exposed population has negative influence, we increase its value to reduce the infection. The change and impact can be seen in the following Figure 5.4 and

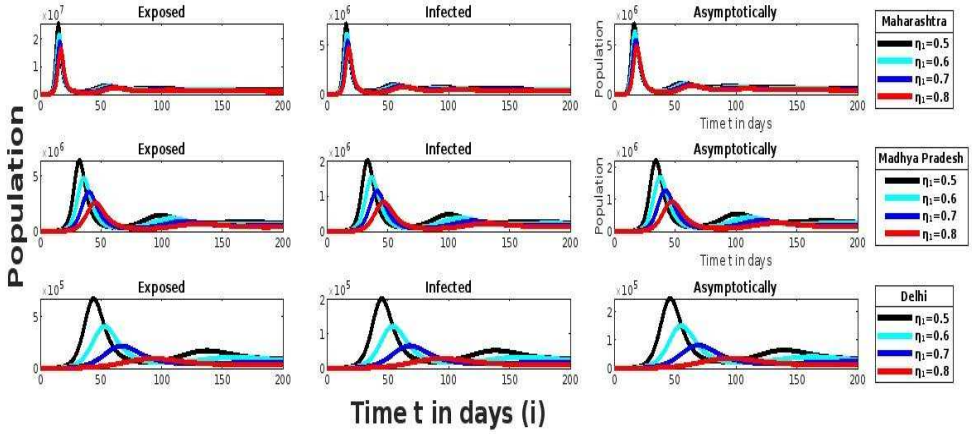

Figure 5.4: Effect of change in exposed isolation rate η_1 in Maharashtra, Madhya Pradesh, Delhi

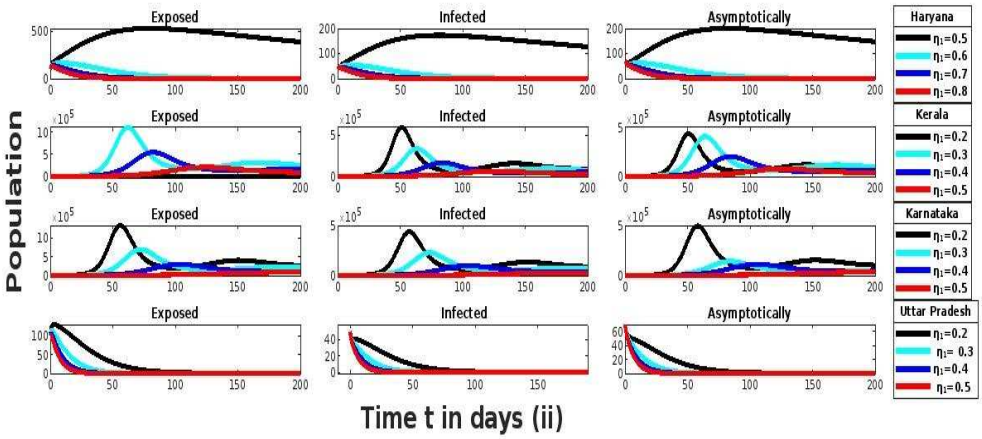

Figure 5.5: Effect of change in exposed isolation rate η_1 in Haryana, Kerala, Karnataka and Uttar Pradesh

Figure 5.5. In the Figure 5.4 and 5.5 one can easily observe that as the prevention parameter value η_1 increases, the number of asymptotically infected and infected individuals decreases significantly. Thus isolation at initial stage can reduce the infection significantly.

Since some of the parameter values are taken from other sources and some are assumed, as a future scope of this study, one can find the appropriate value of parameters to find the best fitting with real data.

6. Conclusion

Model simulations explore the fact that whenever $\mathcal{R}_0 > 1$, the disease spreads rapidly. Moreover, in a disease free condition ($i.e.\mathcal{R}_0 < 1$) , the disease is in under control. The paper concludes that these seven states display similar tendencies in both the conditions (DFE and EE) for all the corresponding compartments. To reduce the value of the basic reproduction number, the government should focus on implementing early stage prevention measures. This study demonstrates the importance of isolating the exposed population, the impact of vaccinations, and the impact of lowering the transmission rate on different infected groups. Hence, to control the epidemic's spread, the authorities need to use the same strategies and impose all possible prevention measures, such as lockdown, quarantine/isolation, use of sanitizers, and social distance, in all of the infected states.

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