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SOME STUDIES ON JORDAN $(\alpha, 1)^*$-BIDERIVATION IN RINGS WITH INVOLUTION

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Abstract. Let R be a ring with involution. In the present paper, we characterize biadditive mappings which satisfies some functional identities related to symmetric Jordan $(\alpha, 1)^*$-biderivation of prime rings with involution. In particular, we prove that on a 2-torsion free prime ring with involution, every symmetric Jordan triple $(\alpha, 1)^*$-biderivation is a symmetric Jordan $(\alpha, 1)^*$-biderivation.

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1. Introduction

A famous result due to Herstein [10], states that a Jordan derivation of a prime ring of characteristic not 2 must be a derivation. Cusack [8] and Bresar [7] are extended the above results to 2-torsion free semiprime rings and also states that any Jordan triple derivation is a derivation. In [24], Vukman prove that every Jordan derivation on a 6-torsion free semiprime $*$-ring is a Jordan triple derivation of R. Ashraf Ali [2] and Rehman et.al. (see [21]& [18]) prove that on a 2-torsion free semiprime ring every (α, β)-derivation is a Jordan triple (α, β)-derivation. In [17] Liu and Shiue are prove that the converse of the above statement is true. Fosner and Ilisevic [9] generalized above mentioned result for 2-torsion free semiprime ring. Following [1], every $(\alpha, \beta)^*$-derivation $*$-ring is a Jordan triple $(\alpha, \beta)^*$ derivation but the converse is in general not true. Recently, in [4] Ali Shakir et.al. prove that every Jordan triple $(\alpha, \beta)^*$-derivation is a Jordan $(\alpha, \beta)^*$-derivation on a 6-torsion free semiprime $*$-ring R and also the first

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author in [2] improved this result by removing 3-torsion free restriction. More related results has also been obtained in [3] to [17] where further references can be found. Following [20], the concept of $(\alpha,1)$-derivations introduced and studied centralizing properties of $(\alpha,1)$-derivations in semiprimerings by Shobhalatha. Motivated by this Jaya Subba Reddy et.al. introduced the concept of $(\alpha,1)$-reverse derivations and Generalized $(\alpha,1)$-reverse derivations on rings. Recently, author can be extended results on Ideals and $(\alpha,1)$-derivations (resp. $(\alpha,1)$-reverse derivations) in rings, Symmetric $(\alpha,1)$ Biderivations (resp. Symmetric reverse $(\alpha,1)$-Biderivations) and Symmetric Generalized $(\alpha,1)$-Biderivations in rings (See in [10] -[16]). The concept of symmetric biderivations was introduced by Maksa [18], [13]. In [5], Ali and Dar introduced the concept of symmetric Jordan $*$-biderivation and symmetric Jordan triple $*$-biderivation. Recently, first author together with [23] proved that on a 2-torsion free prime $*$-ring R, every Jordan triple $(\alpha,\beta)^*$-derivation is a Jordan $(\alpha,\beta)^*$-derivation. In the present paper, our aim is to establish a set of conditions under which every symmetric Jordan triple $(\alpha,1)^*$-biderivation on a ring with involution is a symmetric Jordan $(\alpha,1)^*$-biderivation. More precisely, we prove that on a 2 -torsion free prime ring with involution, every symmetric Jordan triple $(\alpha,1)^*$-biderivation is a symmetric Jordan $(\alpha,1)^*$-biderivation.

2. Priliminaries

Throughout this paper, R will represent an associative ring having at least two elements. However, R may not have unity. A ring R is said to be prime if $xRy=0$ implies that either $x=0$ or $y=0$ and semiprime if $xRx=0$ implies that $x=0$, where $x,y\in R$. For any $x,y\in R$, the symbol $[x,y]$ stands for the commutator $xy-yx$ and the symbol (x,y) stands for the anti-commutator $xy+yx$. An additive mapping $x\to x^*$ satisfying $(xy)^*=y^*x^*$ and $(x^*)^*=x$ for all $x,y\in R$, is called an involution on R. A ring R equipped with an involution is called *-ring or ring with involution. If $d:R\to R$ is an additive and if α be endomorphism of R, then d is said to be an $(\alpha,1)$-derivation (resp. Jordan $(\alpha,1)$-derivation) of R when for all $x,y\in R$, $d(xy)=d(x)\alpha(y)+xd(y)$ (resp. $d\left(x^2\right)=d(x)\alpha(x)+xd(x)\big)$. An additive mapping $d:R\to R$ is called Jordan triple $(\alpha,1)$-derivation if $d(xyx)=d(x)\alpha(yx)+xd(y)\alpha(x)+xyd(x)$ holds for all $x,y\in R$. An additive mapping $d:R\to R$ is called $(\alpha,1)^*$-derivation (resp. Jordan $(\alpha,1)^*$-derivation) if $d(xy)=d(x)\alpha\left(y^*\right)+xd(y)$ (resp. $d\left(x^2\right)=d(x)\alpha\left(x^*\right)+xd(x)\big)$ holds for all $x,y\in R$, where R is a ring with involution. One can easily prove that every Jordan $(\alpha,1)^*$-derivation $*$-ring is a Jordan triple $(\alpha,1)^*$-derivation. A symmetric biadditive mapping $B:R\times R\to R$ is said to be a symmetric $(\alpha,1)$-biderivation on R if $B(xy,z)=B(x,z)\alpha(y)+xB(y,z)$, for all $x,y,z\in R$. Following [3], a symmetric biadditive map $B:R\times R\to R$ is called a symmetric *-biderivation if $B(xy,z)=B(x,z)y^*+xB(y,z)$ holds for all $x,y,z\in R$, where R is a ring with involution. A symmetric biadditive map $d:R\times R\to R$ is said to be a symmetric Jordan $*$-biderivation if

$d\left(x^{2}, z\right)=d(x, z) x^{*}+x d(x, z)$ holds for all $x, z \in R$. A symmetric biadditive map $B: R \times R \rightarrow R$ is called a symmetric Jordan triple $*$-biderivation if $d(x y x, z)=d(x, z) y^{*} x^{*}+x d(y, z) x^{*}+x y d(x, z)$ holds for all $x, y, z \in R$. The previous work on biderivations motivates us to define symmetric Jordan $(\alpha, 1)^{*}$ - biderivation and symmetric Jordan triple $(\alpha, 1)^{*}$-biderivation. A symmetric biadditive map $d: R \times R \rightarrow R$ is said to be a symmetric Jordan $(\alpha, 1)^{*}$ - biderivation if $d\left(x^{2}, z\right)=d(x, z) \alpha\left(x^{*}\right)+x d(x, z)$ holds for all $x, z \in R$. A symmetric biadditive map $B: R \times R \rightarrow R$ is called a symmetric Jordan triple $(\alpha, 1)^{*}$-biderivation if $d(x y x, z)=d(x, z) \alpha\left(y^{*} x^{*}\right)+x d(y, z) \alpha\left(x^{*}\right)+x y d(x, z)$ holds for all $x, y, z \in R$.

In order to prove our main result we need to prove the following key lemmas:

Lemma 2.1. *Let* R *be a prime ring with involution and* α *be an automorphism of* R*. For* $a \in \mathrm{R}$*, if* $\alpha(\mathrm{x}) a \mathrm{x}^{*}=0$ *for all* $x \in R$*, then* $a=0$*.*

Proof.

$$\text{We have } \alpha(x) a x^{*}=0 \text{ for all } x \in R. \tag{1}$$

Replacing x by $x^{*}+y$ in (1), we get

$$\alpha(y) a x+\alpha\left(x^{*}\right) a y^{*}=0, \text{for all } x, y \in R. \tag{2}$$

This can be further written as

$$\alpha(y) a x=-\alpha\left(x^{*}\right) a y^{*}, \text{for all } x, y \in R. \tag{3}$$

Using the applications of (1) and (3), we obtain that

$$\begin{aligned}
a \alpha(x) a \alpha(\mathrm{z}) a x a & =a(\alpha(x) a \alpha(\mathrm{z})) a x a \\
& =-a \alpha\left(z^{*}\right) a \alpha\left(x^{*}\right) a x a \\
& =-a \alpha\left(z^{*}\right) a\left(\alpha\left(x^{*}\right) a x\right) a=0, \text{ for all } x, z \in R
\end{aligned}$$

Which means that $a \alpha(x) a \,\mathrm{Raxa}=(0)$, for all $x \in R$.
By the primeness of R forces that either $a \alpha(x) a=0$ or $a x a=(0)$, for all $x \in R$. Since α automorphism of R, so we conclude that $a x a=(0)$. Hence, $a=0$. □

Lemma 2.2. *Let* R *be a 2-torsion free ring with involution and* α *be endomorphism of* R*. If* $\mathrm{d}: R \times R \rightarrow R$ *is a symmetric Jordan* $(\alpha, 1)^{*}-$ *biderivation of* R*, then the following hold:*

$$\begin{aligned}
(i)\ d(x y+y x, z)= & d(x, z) \alpha\left(y^{*}\right)+d(y, z) \alpha\left(x^{*}\right)+x d(y, z) \\
& +y d(x, z), \textit{ for all } x, y, z \in R; \\
(ii)\ d(x y x, z)= & d(x, z) \alpha\left(y^{*} x^{*}\right)+x d(y, z) \alpha\left(x^{*}\right)+x y d(x, z), \\
& \textit{for all } x, y, z \in R. \\
(iii)\ d(x y t+t y x, z)= & d(x, z) \alpha\left(y^{*} t^{*}\right)+x d(y, z) \alpha\left(t^{*}\right)+x y d(t, z) \\
& +d(t, z) \alpha\left(y^{*} x^{*}\right)+t d(y, z) \alpha\left(x^{*}\right)+t y d(x, z), \\
& \textit{for all } t, x, y, z \in R.
\end{aligned}$$

Proof. (i) Given that $d : R \times R \to R$ is a symmetric Jordan $(\alpha, 1)^*$-biderivation of R.

That is

$$d\left(x^2, z\right) = d(x,z)\alpha\left(x^*\right) + xd(x,z) \text{ holds for all } x, z \in R. \tag{4}$$

Replacing x by $x+y$ in equation (4) we obtain

$$\begin{aligned} d\left((x+y)^2, z\right) =& d(x,z)\alpha\left(x^*\right) + d(x,z)\alpha\left(y^*\right) + d(y,z)\alpha\left(x^*\right) \\ &+ d(y,z)\alpha\left(y^*\right) + xd(x,z) + yd(x,z) + xd(y,z) + yd(y,z), \\ &\text{for all } x, y, z \in R. \end{aligned} \tag{5}$$

On the other hand, we have

$$\begin{aligned} d\left((x+y)^2, z\right) = d(xy+yx,z) + d(x,z)\alpha\left(x^*\right) + d(y,z)\alpha\left(y^*\right) + xd(x,z) \\ + yd(y,z), \text{ for all } x,y,z \in R. \end{aligned} \tag{6}$$

Comparing (5) and (6) relations, we get

$$\begin{aligned} d(xy+yx,z) =& d(x,z)\alpha\left(y^*\right) + d(y,z)\alpha\left(x^*\right) + xd(y,z) + yd(x,z), \\ &\text{for all} x,y,z \in R. \end{aligned}$$

(ii) Replacing y by $xy+yx$ in (i), we get

$$\begin{aligned} & d(x(xy+yx)+(xy+yx)x,z) \\ & = d(xy+yx,z)\alpha\left(x^*\right) + d(x,z)\alpha\left(x^*y^* + y^*x^*\right) + xd(xy+yx,z) \\ & + (xy+yx)d(x,z) \\ & = d(x,z)\alpha\left(x^*\right) + d(yx,z)\alpha\left(x^*\right) + d(x,z)\alpha\left(x^*y^*\right) + d(x,z)\alpha\left(y^*x^*\right) \\ & + xd(xy,z) + xd(yx,z) + xyd(x,z) + yxd(x,z) \\ & = d(x,z)\alpha\left(y^*x^*\right) + d(x,z)\alpha\left(x^*y^*\right) + d(x,z)\alpha\left(y^*x^*\right) + d(x,z)\alpha\left(\left(x^*\right)^2\right) \\ & + xd(y,z)\alpha\left(x^*\right) + yd(x,z)\alpha\left(x^*\right) + xd(x,z)\alpha\left(y^*\right) + xd(y,z)\alpha\left(x^*\right) \\ & + x^2d(y,z) + xyd(x,z) + yxd(x,z), \text{ for all } x,y,z \in R. \end{aligned} \tag{7}$$

On the other hand, we have

$$\begin{aligned} & d(x(xy+yx)+(xy+yx)x,z) = d\left(x^2y + yx^2, z\right) + 2d(xyx,z) \\ & = d(x,z)\alpha\left(x^*y^*\right) + xd(x,z)\alpha\left(y^*\right) + d(y,z)\alpha\left(\left(x^*\right)^2\right) + x^2d(y,z) \\ & + yd(x,z)\alpha\left(x^*\right) + xd(x,z)\alpha\left(y^*\right) + xd(y,z)\alpha\left(x^*\right) + yxd(x,z) \\ & + 2d(xyx,z)), \text{for all } x,y,z \in R. \end{aligned} \tag{8}$$

Comparing (7) and (8) relations, we get

$2d(xyx,z) = 2\left\{d(x,z)\alpha\left(y^*x^*\right) + xd(y,z)\alpha\left(x^*\right) + xyd(x,z)\right\}$,
for all $x, y, z \in R$.

Since R is 2-torsion free ring, the last expression yields the required result.

(iii) Replacing x by $x+\mathrm{t}$ in (ii), we get

$$\begin{aligned}
&d((x+t)y(x+t), z)=d(x+t, z)\alpha\left(y^*\right)\alpha\left(x^*+t^*\right)\\
&+(x+t)d(y, z)\alpha\left(x^*+t^*\right)+(x+t)yd(x+t, z)\\
&=d(x, z)\alpha\left(y^*x^*\right)+d(x, z)\alpha\left(y^*t^*\right)+d(t, z)\alpha\left(y^*x^*\right)+d(t, z)\alpha\left(y^*t^*\right)\\
&+xd(y, z)\alpha\left(x^*\right)+xd(y, z)\alpha\left(t^*\right)+td(y, z)\alpha\left(x^*\right)+td(y, z)\alpha\left(t^*\right)\\
&+xyd(x, z)+xyd(t, z)+\mathrm{tyd}(x, z)+\mathrm{tyd}(t, z), \text{for all } t, x, y, z \in R.
\end{aligned} \tag{9}$$

On the other hand, we have

$$\begin{aligned}
&d((x+t)y(x+t), z)=d(xyx, z)+d(tyt, z)+d(xyt+tyx, z)\\
&=d(x, z)\alpha\left(y^*x^*\right)+xd(y, z)\alpha\left(x^*\right)+xyd(x, z)+d(t, z)\alpha\left(y^*t^*\right)\\
&+tyd(t, z)+td(y, z)\alpha\left(t^*\right)+d(xyt+tyx, z), \text{for all } t, x, y, z \in R.
\end{aligned} \tag{10}$$

Comparing (9) and (10) relations, we get the required result.

This completes the proof.

We are now have enough information's to prove our main theorem: □

3. Main results

Theorem 3.1. *Let* R *be a prime ring with involution such that* char(R) $\neq 2$ *and* α *be automorphism of* R. *Then any symmetric Jordan triple* $(\alpha, 1)^*$ *biderivation* $d: R\times R\to R$ *is a symmetric Jordan* $(\alpha, 1)^*$*-biderivation.*

Proof. Assume that d is a symmetric Jordan triple $(\alpha, 1)^*$-biderivation of R. i.e.,

$$d(xyx, z)=d(x, z)\alpha\left(y^*x^*\right)+xd(y, z)\alpha\left(x^*\right)+xyd(x, z). \tag{11}$$

In view of Lemma 2.2 (iii), we have

$d(xyt+tyx, z)=d(x, z)\alpha\left(y^*t^*\right)+xd(y, z)\alpha\left(t^*\right)+xyd(t, z)$
$+d(t, z)\alpha\left(y^*x^*\right)+td(y, z)\alpha\left(x^*\right)+tyd(x, z)$, for all $t, x, y, z \in R$.

Thus, we obtain $d\left((xy)^2, z\right)=d(xyxy, z)$
$=d\left(xy(xy)+(xy)yx-xy^2x, z\right)$
$=d(xy(xy)+(xy)yx, z)-d\left(xy^2x, z\right)$
$=d(x, z)\alpha\left(\left(y^*\right)^2\right)\alpha\left(x^*\right)+xd(y, z)\alpha\left(y^*x^*\right)+xyd(xy, z)$
$+d(xy, z)\alpha\left(y^*x^*\right)+xyd(y, z)\alpha\left(x^*\right)+\left(xy^2\right)d(x, z)$
$-d(x, z)\alpha\left(\left(y^*\right)^2\right)\alpha\left(x^*\right)-xd\left(y^2, z\right)\alpha\left(x^*\right)-\left(xy^2\right)d(x, z)$, for $x, y, z \in R$.

This implies that

$$\begin{aligned}
&0=d\left((xy)^2, z\right)-d(xy, z)\alpha\left(y^*x^*\right)-xyd(xy, z)\\
&+\left(d\left(y^2, z\right)-d(y, z)\alpha\left(y^*\right)-yd(y, z)\right)\alpha\left(x^*\right), \text{for all } x, y, z \in R.
\end{aligned} \tag{12}$$

Thus the relation (12) can be written in the following form

$$\Delta(xy)+y\Delta(y)\alpha\left(x^*\right)=0, \text{for all } x, y \in R. \tag{13}$$

Where $\Delta(x)=d\left(x^2, z\right)-d(x, z)\alpha\left(x^*\right)-xd(x, z)$, for all $x, z \in R$.

Application of relation (13) yields that

$$
\begin{aligned}
2(ty)\Delta(x)\alpha\left(y^{*}\mathrm{t}^{*}\right) &= (ty)\Delta(x)\alpha\left(y^{*}\mathrm{t}^{*}\right)+(ty)\Delta(x)\alpha\left(y^{*}\mathrm{t}^{*}\right)\\
&= -ty\Delta(yx)\alpha\left(\mathrm{t}^{*}\right)-\Delta((\mathrm{t}y)x)\\
&= -ty\Delta(yx)\alpha\left(\mathrm{t}^{*}\right)-\Delta(\mathrm{t}yx)\\
&= \Delta(\mathrm{ty}\,x)-\Delta(\mathrm{t}yx)\\
&= 0,\ \text{for all } x,y,t\in R.
\end{aligned}
$$

Thus $2(ty)\Delta(x)\alpha\left(y^{*}t^{*}\right)=0$, for all $x,y,t\in R$.

Since char(R) $\neq 2$, the last expression yields that $(ty)\Delta(x)\alpha\left(y^{*}t^{*}\right)=0$, for all $x,y,t\in R$. Using the application of Lemma (2.1) twice in the above relation, we obtain that $\Delta(x)=0$, for all $x\in R$.

That is $d\left(x^{2},z\right)-d(x,z)\alpha\left(x^{*}\right)-xd(x,z)=0$, for all $x,z\in R$.

Hence, d is a symmetric Jordan $(\alpha,1)^{*}$-biderivation of R. This completes the proof the theorem. □

From the above theorem, we now deduce immediate the following corollary:

Corollary 3.2. *Let* R *be a prime ring with involution such that* char(R) $\neq 2$. *Then every symmetric Jordan triple* $*$*-biderivation* $d: R\times R\rightarrow R$ *is a symmetric Jordan* $*$*-biderivation.*

4. Discussion

The present manuscript is primarily addressed to algedraists whose exploration is connected with maps of rings (algebras) having some fresh properties for illustration Lie and Jordan derivations, generalized derivations, automorphsims and direct defenders etc. Specifically, the paper is addressed to ring proponents dealing with polynomial (algebraic) identities and their concepts. These identities have turned out to be applicable to certain problems in some other fine areas similar as in operator theory, functional analysis and Lattice theory. May be one might find some farther connections away. We remark that at least at the position of introductory descriptions the theory of identities admits some parallels with that of algebraic functions. Thus, some region of the paper may be of some interest not only ring proponents but observers.

The usual aim in the study of identities is to find the form of the maps involved, or, when this isn't possible, to determine the structure of the ring. More precisely the commutative structure of rings. One has to find all the trivial results of the proposed problems, that is, the results which don't depend on some structural properties of the ring but are simply consequences of a formal computation. We call them standard results. So, as a limitation of our investigation idea, the eventual actuality of a nonstandard result implies that the ring has a veritably special structure.

In the present paper our conclusion includes the characterization of biadditive mappings which satisfies some functional identities related to symmetric Jordan $(\alpha,1)^{*}$-biderivation of prime rings with involution. The results we proved that on a 2 -torsion prime ring with involution, every symmetric Jordan triple $(\alpha, 1)^*$-biderivation is a symmetric Jordan $(\alpha, 1)^*$ biderivation.

Conflicts of interest : The authors declare no conflict of interest.

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