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A DECISION-MAKER CONFIDENCE LEVEL BASED MULTI-CHOICE BEST-WORST METHOD: AN MCDM APPROACH

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ABSTRACT. In real life, a decision-maker can assign multiple values for pairwise comparison with a certain confidence level. Studies incorporating multi-choice parameters in multi-criteria decision-making methods are lacking in the literature. So, In this work, an extension of the Best-Worst Method (BWM) with multi-choice pairwise comparisons and multi-choice confidence parameters has been proposed. This work incorporates an extension to the original BWM with multi-choice uncertainty and confidence level. The BWM presumes the Decision-Maker to be fully confident about preference criteria vectors best to others & others to worst. In the proposed work, we consider uncertainty by giving decision-makers freedom to have multiple choices for preference comparison and having a corresponding confidence degree for each choice. This adds one more parameter corresponding to the degree of confidence of each choice to the already existing MCDM, i.e. multi-choice BWM and yields acceptable results similar to other studies. Also, the consistency ratio remained low within the acceptable range. Two real-life case studies are presented to validate our study on proposed models.

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1. Introduction

We are residing in a complex environment where there are various conceivable criteria with various options for decision vulnerability. In our day-to-day routines, we generally gauge these conceivable criteria implicitly and are satisfied with the outcomes made in light of instinct. When the stakes are high,

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addressing the issue appropriately and unequivocally, through conceivable criteria in shaping significant decisions, is vital. Furthermore, organizing complex issues well and considering different plausible criteria (numerous rules) expressly prompts better choices.

Multi-Criteria Decision Making (MCDM) or Multi-Criteria Decision Analysis (MCDA) is a part of Operations Research that handles such situations. A typical outgrowth of decision-making science is multi-criteria decision-making (MCDM). Numerous MCDM strategies are available to help formulate and resolve decision-making problems involving multiple criteria. No technique is the finest, as each has its unique physiologies. The appropriate MCDM technique must be chosen per the problem's structure. It is acknowledged that a particular solution to a multi-criteria decision-making problem can only be found by integrating preference data.

The best-worst method (BWM) is one of the MCDM methods, which Jafar Rezaei introduced in 2015[1]. The BWM depends on a methodical pairwise examination of these criteria. The remarkable feature of the BWM is that it utilizes less number of pairwise examinations, which prompts less data collection and reliable results. BWM has arisen as a productive method for accounting for multi-criteria decision-making problems. Further endeavors are also concurrently made to deploy this strategy under uncertainty. Better approaches for displaying dubious information in decision-making have emerged and been applied.

In BWM, uncertainty may occur in the input data. Although uncertain data is frequently used in decision-making, early research denoting BWM favoured exact values as input data. Such a need complicates the decision-making process because exact values are typically difficult. As a result, uncertain input data must frequently be called using specific theories, namely the Fuzzy set theory, Possibility theory, Probability theory, or The theory of evident reasoning. Diagnosing an approach to coping with uncertainty is crucial. Over and above that, knowing which technique to use directly impacts the decision-making process and the outcome.

The layout of this paper is as follows: Section 2 is the Literature review; Section 3 is the motivation and Research objectives; Section 4 is about the preliminaries; Section 5 is about the Proposed models; Section 6 is the detailed Experimental study. Two real-life examples are set forth to validate the application of the developed approach; finally, Section 7 discusses the conclusions and future scope of this study.

2. Literature Review

The inheritance of uncertainty in many decision-making processes inspired decision-makers to explore new methods to find solutions. Literature review shows that most ambiguous real-world problems fit well in the frame of a multi-criteria decision-making framework. The application of MCDM lies in almost all domains, such as engineering, management, social science,[2, 3, 4, 5, 9, 10] etc. In addition, the best-worst method being one of the most convenient and efficient MCDM methods, many researchers outset exploring appropriate techniques with BWM in an undetermined situation. Consequently, much work has been done on BWM under such environments. The research field garnered an abundance of literature employing diverse techniques such as the Fuzzy set theory[6, 7, 8, 9, 10], Intuitionistic fuzzy theory [11, 12, 13, 14], Neutrosophic theory[11] [15, 16, 17, 18], Belief theory[19], Probability theory[20, 21, 22, 23, 24], Grey number theory [25, 26, 27], etc. Most of the earliest research manipulated BWM with exact values; Nevertheless, as times progressed, BWM has been practiced regularly with uncertainties along with exact values.

Masoomi et al. [28] used the fuzzy theory and assessed a group of strategic suppliers, primarily based on their green capabilities. Those as mentioned above were achieved by combining the Complex Proportional Assessment of Alternatives and Weighted Aggregated Sum Product Assessment approaches with the Fuzzy best-worst method (FBWM). An improved version of large-scale group decision-making established on the BWM using hesitant fuzzy information was offered to highlight the barriers to employing blockchain in supply chain management by Heidary et al.[29]. Govindan et al.[30] developed a mix of Fuzzy best–worst method, Fuzzy decision-making trial, Evaluation Laboratory, and Supermatrix structure for prioritization of circular economy adoption barriers for a cable and wire industry in Iran.

Mostafaeipour et al. [6] investigated barriers facing solar energy development in Iran's Alborz Province, manoeuvring the Fuzzy Best-Worst method and determining the gravity of the identified criteria and sub-criteria. The paper stated that the primary barriers to solar energy development are uncertain economic conditions, including the sanctions against Iran. In order to confirm the applicability of the suggested BWM alpha-cut approach, a real-world example of a Serbian home appliance company is explored. Working in a fuzzy environment, Amiri et al.[31] modelled sustainable supplier selection (SSS) for uncertain situations employing the BWM and alpha-cut methods. Regarding SSS, the problem contains features of multi-criteria decision-making. Therefore, BWM fits very well with the situation. The proposed method and the required solution also determine the degree of uncertainty and decision-makers' gratification by providing an alpha value. The decision-maker is more satisfied and, therefore, more confident in the decision-making measure when the alpha level is higher (closer to 1), yet uncertainty is greater if the alpha level is lower (closer to zero). This makes the decision-making process more reliable. Khan et al.[10] identified uncertainty as the risk element of the supply chain industry in the volatile globalized market and presented their proposal for operating the halal supply chain. In the study, forty-two halal supply chain risk essentials are recognized and prioritized using fuzzy BWM. The major finding of this study is the extensive list of risk essentials to the halal supply chain and their ranking towards the powerful administration of the halal supply chain.

Karimi et al.[9] used Fuzzy numbers in their total capacity and presented the Fuzzy best-worst method. The results of numerical trials in the hospital maintenance field demonstrate the fully Fuzzy BWM's excellent efficiency and acceptable performance. High reliability in the solutions is accomplished. Ecer et al.[32] proposed a framework that evaluates suppliers using fuzzy BWM regarding economic, social, and environmental sustainability in the presence of ambiguities in the decision-making process arising from a lack of quantitative information.

Wan et al.[11] treated uncertainties alias hesitancy in the reference comparisons using intuitionistic fuzzy values and presented a method called the novel extension of the fuzzy best-worst method. The method presented four linear programming models to realize the optimal Intuitionistic Fuzzy weights based on the proposed mathematical programming model for the optimistic, pessimistic, and neutral Decision-Maker. Majumder et al.[14] handled uncertainty using intuitionistic BWM and presented a paper to identify the important leading indicator for the efficiency of the water treatment plant. The proposal of this method is the hybrid of intuitionistic BWM and analytic hierarchy process. Wang et al.[12] incorporated the interval-valued Intuitionistic fuzzy technique along BWM to develop the importance of weight for meta-evaluation theory. Meta-evaluation theory and methods were adopted to appraise the reviewers of science and technology projects. The study presented a comprehensive model integrating dozens of pre-existing meta-evaluation criteria.

Vafadarnikjoo et al.[15] proposed a neutrosophic enrichment to the original BWM by recommending two new parameters of the Decision Makers (DMs') self-confusion in the best-to-others choices and the DMs' confusion in the others-to-worst choices. Abdel Basset et al.[18] adopted neutrosophic numbers, unravelling the supply chain problem with the best-worst method based on a novel Plithogenic model. The paper focused on two areas of the supply chain problem i.e. -Warehouse location and Plant evaluation, which is based on several criteria. They proposed a union study between plithogenic aggregation operations and the best-worst method. The purpose of this combination is to aggregate the decision-maker's opinions to apply the BWM to find the optimal weight of each criterion. Another study using neutrosophic numbers was done by Yucesan et al.[16] for failure prioritization. In the proposal, neutrosophic sets incorporate real-life indeterminate and inconsistent information. Afterwards, BWM is used to identify potential failures and their effects, quantifying their priorities.

Haqbin et al.[33] in their work focused on the recovery of five categories, namely operations, marketing, human resources, financial and customer relations in tourism SMEs after covid-19 and prioritized the recovery solution through a rough best-worst method. The modified grey decision model of BWM is used by Celikbelik et al.[34] to improve the quality of the transport system, and the result is validated through actual data within the capital of Hungary, Budapest. Bayesian best-worst Method of Munim et al.[35] assessing blockchain adoption strategies- single use, localization, substitution, and transformation established the infrastructure to apprehend the essential factors that need development to boost up the blockchain era adoption technique. Liming et al.[36] modified the best-worst technique in q-ROF environments, embodying WASPAS to model unsure human expressions to rank manufacturers in customized product development. The study presented by [37] adopted the Bayesian best worst method with measurement of alternatives and rankings according to compromise solution (MARCOS) facilitating the suitability-feasibility-acceptability (SFA) strategy to determine the best destination to relocate lithium battery plant after the COVID 19. Based on the Literature review done, the motivation and research objectives for this study are explained in the next section.

3. Motivation and Research Objective

The BWM originally proposed by Rezai[1] presumes the Decision Maker (DM) to be fully confident about preference criteria vectors: best to others and others to worst, which may not be valid in all situations in the view of Vafadarnikjoo et al.[15]. In their paper, Vafadarnikjoo et al. [15] recognized decision makers' confusion (in estimating the preference criteria) as an uncertain value in the decision-making process. They proposed a Neutrosophic upgrade to the original BWM by presenting two new parameters: the trust in the best-to-others preference and the trust in the others-to-worst preference. They presented two cases to validate their advocated Neutrosophic upgraded BWM by accounting for the certainty rating levels of the decision-maker accordingly named DMs' confidence levels.

Recently, Hasan et al. [38] suggested a different approach: extending the best-worst method inculcating pairwise comparisons as multi-choice parameters. They have shown that incorporating multi-choice parameters makes the real-life situation more favourably handled and solved for better inconsistency. Encouraged by Vafadarnikjoo et al.[15] and Hasan et al.'s[38] work, we propound an enhancement of BWM with multi-choice pairwise comparisons[38] coupled with multi-choice confidence parameters.

In place of the confidence level of the best to others vector and others to worst vector [15], the confidence level for each pairwise comparison is more rational. Since multi-pairwise comparisons can occur in reality, multi-confidence levels may vary for each choice of these pairwise comparisons. Thus, this work aims to develop a framework to solve MCDM problems— using BWM in particular— that incorporates multi-choice pairwise comparisons and corresponding confidence levels under realistic assumptions. Another objective is to validate and study the application of the developed framework. In the next section, the preliminaries required are explained.

4. Preliminaries

The section briefs the few essential preliminaries required for this study.

4.1. Multi-choice mathematical programming. A Multi-choice mathematical programming problem [38] is a type of mathematical programming problem wherein the requirement is to pick an alternative from a set of alternatives having several possible combinations of parameters to optimize an objective, limited by a fixed number of constraints. Initially, a typical multi-choice linear programming problem's right-hand side's goals of some constraints have 'multi-choice' parameters. There are numerous goals for each constraint, only one of which must be selected. The choice of goals should be made so that the options available for each constraint best serve the objective function. More than one arrangement may yield the best result[39]. The mathematical model of multi-choice linear programming is generally expressed as[40]

Find the solution as the variable $X = \{x_1, x_2, ...x_n\}$ to maximize Z

$$\text{where } Z = \sum c_j x_j,\ j = 1, 2, 3....n \tag{1}$$

$$\text{subject to } \sum a_{ij} x_j \leq \{{b_i}^1, {b_i}^2, {b_i}^3......{b_i}^{k_i}\}\ \forall i = 1, 2, 3....m \tag{2}$$

$$x_j \geq 0\ \forall j \tag{3}$$

The multi-choice programming approach became popular in sundry areas of real-life circumstances because of its potential to include multi-choice uncertainty.

4.2. Lagrange Interpolation. The polynomial P(x) with degree $\leq$ (n-1) is the Lagrange interpolating polynomial[41] that passes through the n points $(x_1, y_1 = f(x_1)), (x_2, y_2 = f(x_2)), ..., (x_n, y_n = f(x_n))$, and is given by

$$P(x) = \frac{(x - x_2)(x - x_3)....(x - x_n)}{(x_1 - x_2)(x_{-}x_3)...(x_1 - x_n)} y_1 + \frac{(x - x_1)(x - x_3)......(x - x_n)}{(x_2 - x_1)(x_2 - x_3)....(x_2 - x_n)} y_2.....$$

$$+\frac{(x-x_1)(x-x_2)......(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)....(x_n-x_{n-1})}y_n \tag{4}$$

The chief importance of this formula, Lagrange interpolation polynomial, is the use of given discrete information with ordered pairs as a continuous function of a variable. Waring first introduced the formula in 1779[42], which was then used by Euler in 1783 and re-published by Lagrange in 1795 [41].

TABLE 1. Symbols used in this research .

Symbol	Description
C_B	Best criterion
C_W	Worst criterion
C_j	j^{th}Criterion
O_B	Best to Others comparison vector
O_W	Others to Worst comparison vector
o_{Bj}	Pairwise comparison of best to the jth criterion
o_{jW}	Pairwise comparison of the jth to the worst criterion
W_B	Weight of the best criterion
W_W	Weight of the worst criterion
W_j	Weight of the jth criterion
ρ^+_{Bj}	Confidence level attached to best to jth comparison
ρ^-_{jW}	Confidence level attached to jth to worst comparison

4.3. Confidence level. For multi-criteria mathematical programming, confidence levels[15] is described as one of the parameters the decision-maker utilizes to expose his confidence in deciding a level of criteria preference using pairwise comparison. The idea was given more attention as human nature is clouded with doubts and confusion. There is always a degree to which humans feel dubious, although their large accommodation of sufficient confidence. Being wholly confident calls for a realistic experience of one's competencies and feeling stable. Embodying confidence facilitates credibility and dealing with uncertainty and challenges. Contextually, the lower the confidence, the stronger the conservation towards a specific preference. In this work, we considered seven confidence levels ranging from "No confidence" to "Absolutely high confidence"; see table-3.

4.4. Multi-choice BWM. Multi-criteria decision-making is based on the relative comparison of preference. The situation is complicated further if the decision-maker is imprecise about electing values for pairwise comparisons. Hasan et al. [38] nominated a solution for this situation by proposing a multi-choice parameter for pairwise comparison. Their work combined BWM with multi-choice parameters. They followed steps to reach relevant and reliable results, as mentioned below.

Step 1. At first, the most vital criterion (known as the best criterion, C_B) and the least vital criterion (known as the worst criterion, C_W) from the set of n criteria $\{C_1, C_2, ..., C_n\}$ are selected by the decision maker.

Step 2. Subsequently, appropriate values are appointed for reference comparison vectors: best to others and others to worst. The vector (O_B) is the best-to-others vector with k number of choices is shown in the equation (5) as:

$$O_B = \Big(o_{B1}, o_{B2}, o_{B3}, ..., o_{Bn}\Big), \text{ where } o_{Bj} = \{o_{Bj}^{1_j}, o_{Bj}^{2_j}, o_{Bj}^{3_j}3, ..., o_{Bj}^{k_j}\} \ \forall j \quad (5)$$

Each element of the vector O_B has more than one value as indeterminate and inconsistent parameters are proposed as multi-choice.
Likewise, vector (O_W) is the others-to-worst vector with k' number of choices given in equation (6) as:

$$O_W = \Big(o_{1W}, o_{2W}, o_{3W}, ..., o_{nW}\Big), \text{ where } o_{jW} = \{o_{jW}^{1_j}, o_{jW}^{2_j}, o_{jW}^{3}, ..., o_{jW}^{k'_j}\} \ \forall j \quad (6)$$

Step 3. After that, the optimum weights are deduced $(W_1^*, W_2^*, ..., W_n^*)$ to rank the criteria using following mathematical programming model:

$$\textbf{Model:} \ \min \max_j \left\{ \left| \frac{W_B}{W_j} - \{o_{Bj}^1, o_{Bj}^2, ..., o_{Bj}^k\} \right|, \left| \frac{W_j}{W_W} - \{o_{jW}^1, o_{jW}^2, ..., o_{jW}^{k'}\} \right| \right\} \quad (7)$$

subject to

$$\sum_{j=1}^{n} W_j = 1, \quad W_j \geq 0, \quad j = 1, 2, ..., n$$

Step 4. Post-constructing the model, the Lagrange Interpolating Polynomial formula, provided in equation 4, is deployed to simplify the model. The variables z^{Bj} and z^{jW} are adopted to represent node points whose values are $(0, 1, 2, ..., k_{j-1})$ with respect to o_{Bj}, and $(0, 1, 2, ..., k'_{j-1})$ with respect to o_{jW}. Thus, $\left\{o_{Bj}^1, o_{Bj}^2, o_{Bj}^3, ..., o_{Bj}^{k_j}\right\}$ and $\left\{o_{jW}^1, o_{jW}^2, o_{jW}^3, ..., o_{jW}^{k'_j}\right\}$ are the associated functional values of the interpolating polynomials at k and k' node points as shown in table 2.

Step 5. Conclusively, the simplified mathematical model of Step 3 is resolved to obtain the optimum solution. The mentioned method handled ambiguity by considering all multi-choice parameters.

Table 2. Node points and its functional values

Polynomial	Descriptions					
$P_{k_j-1}(z^{Bj})$	z^{Bj}	0	1	2	...	k_j-1
	$f(z^{Bj})$	o^1_{Bj}	o^2_{Bj}	o^3_{Bj}	...	$o^{k_j}_{Bj}$
$P_{k_j-1}(z^{jW})$	z^{jW}	0	1	2	...	k'_j-1
	$f(z^{jW})$	o^1_{jW}	o^2_{jW}	o^3_{jW}	...	$o^{k'_j}_{jW}$

5. Proposed models: MCBWM with multi-choice confidence levels

Hasan et al. [38] presented multi-choice in pairwise comparisons accommodating the unlikely situation of the natural world. Vafadarnikjoo et al. [15] accommodated the dubiety by attaching DM's confidence as a Neutrosophic number. Perceiving the idea of Vafadarnikjoo et al. [15], along with of Hasan et al. [38], the paper could be realized as if there is a multi-choice for pairwise comparison and the decision-maker is equally confident about all the choices expressed. In this paper, the authors attempt to integrate dubiety at two levels, one by considering multi-choice comparisons and assigning multi-choice confidence for each pairwise comparison. All the multi-parameters are considered real numbers. The steps for our method are as follows:

Step 1. As usual, at first, the most imperative criterion, the best criterion (C_B), and the least imperative criterion, the worst criterion (C_W), are selected from the set of criteria n criteria $\{C_1, C_2, ..., C_n\}$ by the decision-maker.

Step 2. Assigning suitable values for reference comparison vectors: best to others and others to worst. The vector (O_B) is the best-to-others vector with k number of choices is shown in equation (8) as:

$$O_B = \Big(o_{B1}, o_{B2}, o_{B3}, ..., o_{Bn}\Big), \text{ where } o_{Bj} = \{o^{1_j}_{Bj}, o^{2_j}_{Bj}, o^{3_j}_{Bj}3, ..., o^{k_j}_{Bj}\} \; \forall j \tag{8}$$

Each vector element O_B has multiple values to represent k_j multiple choices. Likewise, vector O_W is the others-to-worst vector with k'_j number of choices of its elements given in equation (9), as:

$$O_W = \Big(o_{1W}, o_{2W}, o_{3W}, ..., o_{nW}\Big), \text{ where } o_{jW} = \{o^{1_j}_{jW}, o^{2_j}_{jW}, o^{3}_{jW}, ..., o^{k'_j}_{jW}\} \; \forall j \tag{9}$$

Step 3. Afterwards, designate a number between 0 and 1, illustrating the decision-makers confidence in every element of the two vectors: best to others and others to worst. The possible linguistic terms with their numeric value for confidence levels are shown in table 3. The model recognizes $\rho^{+}_{Bj} = \{\rho^{+1_j}_{Bj}, \rho^{+2_j}_{Bj}$ $.., \rho^{+k_j}_{Bj}\}$ as confidence associated with O_B best to others vector i.e. o_{Bj}. Naturally, the term $\rho^{-}_{jW} = \{\rho^{-1_j}_{jW}, \rho^{-2_j}_{jW}, ...\rho^{-k'_j}_{jW}\}$, identifies as confidence affiliated with O_W others to the worst vector i.e. o_{jW}. Where ρ^{+}_{Bj} represents the multi-choice confidence attached to the comparison best to j^{th} and ρ^{-}_{jW} represents the multi-choice confidence attached to the comparison, worst to j^{th}. The following table shows some of the chosen values from Vafadarnikjoo et al. [15] for our proposed study.

Table 3. Confidence level linguistic terms and their Numeric values.

Confidence Level Linguistic Terms	Numeric value
No confidence	0.0
Low confidence	0.26
fairly low confidence	0.38
Medium confidence	0.50
Fairly high confidence	0.68
High confidence	0.90
Absolutely high confidence	1.00

Step 4. Now, here, two different models are proposed to determine the best weights for all the n criteria selected.

Model 1: $Min\ \{\xi * (\phi + \psi)\}$ (10)

$$\text{where } \phi = Max\Big(1/\rho^{+}_{B1}, 1/\rho^{+}_{B2}, ..., 1/\rho^{+}_{Bj}\Big) \quad \forall j \in J.$$

$$\text{and } \psi = Max\Big(1/\rho^{-}_{1W}, 1/\rho^{-}_{2W}, ..., 1/\rho^{-}_{jW}\Big) \ \forall j \in J.$$

s. t.

$$\Big|W_B - W_j \times \{o^{1_j}_{Bj} ...., o^{k_j}_{Bj}\}\Big| \leq \xi/\{\rho^{+}{}^{1_j}_{Bj}, ...\rho^{+}{}^{k_j}_{Bj}\},\ \forall j$$

$$\Big|W_j - W_W \times \{o^{1_j}_{jW}, ...o^{k_j}_{jW}\}\Big| \leq \xi/\{\rho^{-}{}^{1_j}_{jW}, ..., \rho^{-}{}^{k'_j}_{jW}\},\ \forall j$$

$$\xi \geq 0,\ \psi \geq 0,\ \phi \geq 0$$

$$\sum W_j = 1, W_j \geq 0,\ \forall j$$

Model 2: $Min\ \{\xi * (\phi + \psi)\}$ (11)

$$\text{where } \phi = \Big(1/\rho^{+}_{B1} + 1/\rho^{+}_{B2} + ... + 1/\rho^{+}_{Bj}\Big) \quad \forall j \in J.$$

$$\text{and } \psi = \Big(1/\rho^-_{1W} + 1/\rho^-_{2W} + ... + 1/\rho^-_{jW}\Big)\ \forall j \in J.$$

s. t.

$$\begin{aligned}
&\Big|W_B - W_j \times \{o^{1_j}_{Bj}...., o^{k_j}_{Bj}\}\Big| \leq \xi/\{\rho^{+1_j}_{Bj}, ...\rho^{+k_j}_{Bj}\},\ \forall j\\
&\Big|W_j - W_W \times \{o^{1_j}_{jW}, ...o^{k_j}_{jW}\}\Big| \leq \xi/\{\rho^{-1_j}_{jW}, ..., \rho^{-k'_j}_{jW}\},\ \forall j\\
&\xi \geq 0,\ \psi \geq 0,\ \phi \geq 0\\
&\sum W_j = 1, W_j \geq 0,\ \forall j
\end{aligned}$$

Where, W_j represents the weight of j^{th} criteria. The dissimilarity in the two models is the value of ψ and ϕ. Model 1 prefers the maximum among all the $1/\rho^+_{Bj}$ and $1/\rho^-_{jW}$ $\forall$ j. In model 2, ψ and ϕ are the sum of all $1/\rho^+_{Bj}$ and $1/\rho^-_{jW}$ $\forall$ j. The sum of the reciprocals is considered in model 2 with the notion of having the contribution from all the data values.

Step 5. Both models are decidedly simplified via the Lagrange Interpolating polynomial formula given by equation 4. Below are the Lagrange Polynomials for best to others[12] and others to worst[13].

$$\begin{aligned}
P_{k_j-1}(z^{Bj}) =& \frac{(z^{Bj}-1)(z^{Bj}-2)....(z^{Bj}-k+1)}{(-1)^{k-1}(k-1)!} o^1_{Bj}\\
&+ \frac{z^{Bj}(z^{Bj}-2)....(z^{Bj}-k+1)}{(-1)^{k-2}(k-2)!} o^2_{Bj}\\
&+ \frac{z^{Bj}(z^{Bj}-1)(z^{Bj}-3)....(z^{Bj}-k+1)}{(-1)^{k-3}(k-3)!2!} o^3_{Bj}\\
&+ ... + \frac{z^{Bj}(z^{Bj}-1)(z^{Bj}-2)....(z^{Bj}-k+2)}{(k-1)!} o^k_{Bj} \forall\ j
\end{aligned} \tag{12}$$

$$\begin{aligned}
P_{k'_j-1}(z^{jW}) =& \frac{(z^{jW}-1)(z^{jW}-2)....(z^{jW}-k'+1)}{(-1)^{k'-1}(k'-1)!} o^1_{jW}\\
&+ \frac{z^{jW}(z^{jW}-2)....(z^{jW}-k'+1)}{(-1)^{k'-2}(k'-2)!} o^2_{jW}\\
&+ \frac{z^{jW}(z^{jW}-1)(z^{jW}-3)....(z^{jW}-k'+1)}{(-1)^{k'-3}(k'-3)!2!} o^3_{jW}\\
&+ ... + \frac{z^{jW}(z^{jW}-1)(z^{jW}-2)....(z^{jW}-k'+2)}{(k'-1)!} o^{k'}_{jW} \forall\ j
\end{aligned} \tag{13}$$

Similarly for $\rho^+_{Bj} = \{\rho^{+1_j}_{Bj}, \rho^{+2_j}_{Bj}, ..., \rho^{+k_j}_{Bj}\}$ and $\rho^-_{jW} = \{\rho^{-1_j}_{jW}, \rho^{-2_j}_{jW}, ..., \rho^{-k'_j}_{jW}\}$.

Step 6. In this step, models 1 and 2 are simplified by consolidating the polynomials in place of the choice set of o_{Bj}, O_{jw}, and the confidence associated i.e. $\rho^{+}_{Bj} = \{\rho^{+1_j}_{Bj}, \rho^{+2_j}_{Bj} .., \rho^{+k_j}_{Bj}\}$ and $\rho^{-}_{jW} = \{\rho^{-1_j}_{jW}, \rho^{-2_j}_{jW}, ...\rho^{-k'_j}_{jW}\}$. Now, we can solve the simplified model from step 5 using appropriate software to obtain the optimum value of weights and ξ. The optimum value of ξ i.e. ξ^*, is further used to evaluate the consistency ratio.

5.1. Consistency Ratio. The best-worst method is based on the pairwise comparison, and the solution provided by this method will be reliable if the comparisons are consistent[1]. if the equation $a_{Bj} * a_{jW} = a_{BW}$ holds for all $j's$, then the comparison is labelled as perfectly consistent. However, although it might hold for some js, it might not be accurate for the remaining js. The Consistency Ratio (CR) is calculated to determine the comparisons' reliability. It is computed using the ξ^* value. The smaller the value of the CR, the more consistent the comparison. CR is calculated using the formula given by Rezai [1].

$$Consistency\ Ratio = \frac{\xi^*}{CI} \tag{14}$$

CI stands for Consistency Index, which is pre-established for different values of a_{BW}. If the value of pairwise comparison changes, the value of CI changes. The value of CI is calculated as the maximum possible root of the quadratic equation, provided below by Vafadarnikjoo et al. [15].

$$\Big(\frac{1}{\rho^+\rho^-}\Big)\xi^2 - \Big(\frac{O_{BW}(\rho^+ + \rho^-) + \rho^+ + \rho^-}{\rho^+\rho^-}\Big)\xi + (O^2_{BW} - O_{BW}) = 0 \tag{15}$$

In our proposed model of multi-choice confidence parameter for ρ^+ and ρ^-, the sum of reciprocals of all multi-choices of ρ^+ and sum of reciprocals of all multi-choices for ρ^-, respectively are used in the above equation (15) to calculate the consistency index.

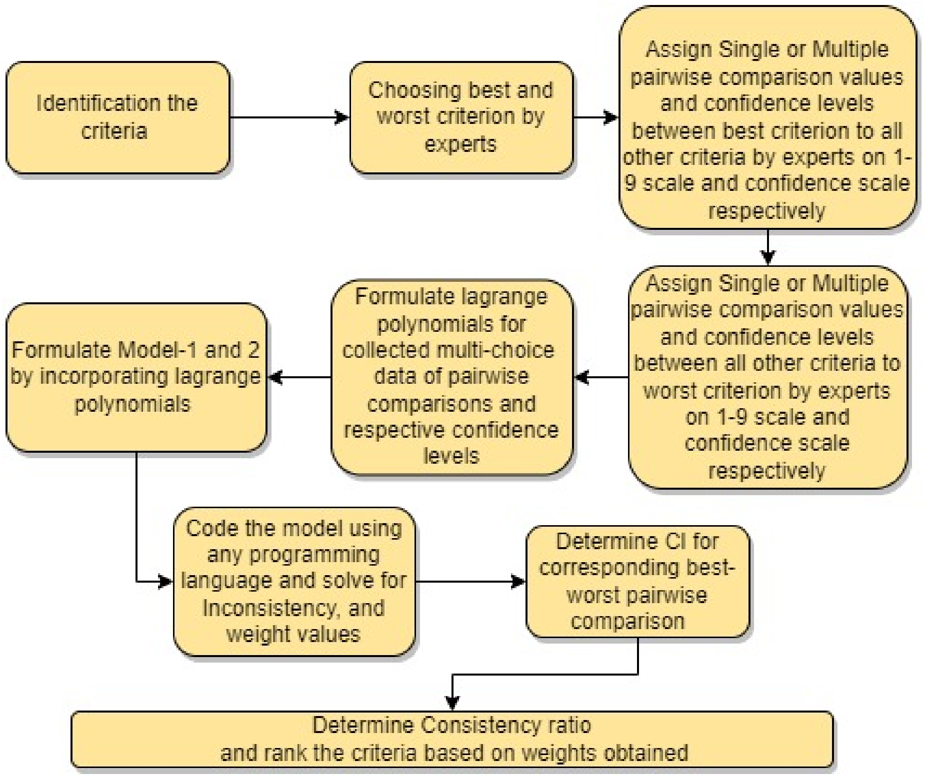

Figure 1. Methodology

The step-by-step methodology of solving problems with model 1 and model 2, where the multi-choice pairwise comparisons and respective confidence levels exist, is presented in figure 1.

6. Experimental Studies

To validate the application of the proposed models, we have exercised them on two case studies. The data has been simulated concerning the assumption of multi-choice parameters, i.e., pairwise comparisons and confidence levels. The pursuing subsections present detailed case studies. All the mathematical models are coded using AMPL[43] and solved using NEOS platform[44].

6.1. Case study-1. In case study 1, we worked over the same example of the transportation management innovation described in [1], but in a modified manner. The organization wishes to select the most desirable means of transport to deliver the product to its stores. There are many factors to pay attention to when dealing with transport. However, in this case study, only three criteria strategies are adopted to address the issue. The three criteria are named as Load Flexibility (LO), Reachability (RE), and Cost (CO). LO is designated as the worst criterion, and CO is granted as the best criterion. Pairwise multiple-choice comparisons of the best criteria to all other criteria and all other criteria to the worst criteria are displayed in the tables. The best criterion, CO, has two alternatives to LO: from Very Important or Intermediate of Very and Extremely Important, provided by the DM's opinion. The assigned number shown in the table, corresponding to the linguistic term opinion, is derived from Hasan et al.'s paper [38]. Similarly, when prioritizing the RE criterion over the worst criterion LO, the decision maker can choose between "Important" and "Very Important." After attaching multi-choice values for each pairwise comparison, the DM attaches one of the possible confidences exhibited in the following table as his/her confidence with each comparison.

Table 4. Multi-choice pairwise comparisons for the best criterion in Case Study-1.

Criteria	$LO(1)$	$RE(2)$	$CO(3)$
Best criteria: $CO(3)$	$\{7, 8\}$	2	1
DM's Confidence ρ^+ :	$\{0.68, 0.80\}$	0.5	1

Table 5. Multi-choice pairwise comparisons for the worst criterion in Case Study-1.

Criteria	Worst criteria: $LO(1)$	DM's Confidence:ρ^-
$LO(1)$	1	1
$RE(2)$	$\{5, 7\}$	$\{0.50, 0.68\}$
$CO(3)$	$\{7, 8\}$	$\{0.68, 0.80\}$

The proposed model 1 for case study 1 is as follows.

$$\textbf{Model: } Min\ \{\xi * (\phi + \psi)\} \tag{16}$$

$$\text{where } \phi = Max(1/\rho_{3j}^{+1_j}, 1/\rho_{3j}^{+2_j}, ..., 1/\rho_{3j}^{+k_j})\ \forall j,\ \forall k.$$

$$\text{and } \psi = Max(1/\rho_{j1}^{-1_j}, 1/\rho_{j1}^{-2_j}, ...1/\rho_{j1}^{-k'_j})\ \forall j,\ \forall k'.$$

s. t.

$$\left|W_3 - W_j \times \{o_{3j}^{k_j}\}\right| \leq \xi/\rho^{+}{}_{3j}^{k_j},\ \forall j$$

$$\left|W_j - W_1 \times \{o_{j1}^{k'_j}\}\right| \leq \xi/\rho^{-}{}_{j1}^{k'_j},\ \forall j$$

$$\xi \geq 0,\ \psi \geq 0,\ \phi \geq 0$$

$$\sum W_j = 1, W_j \geq 0,\ \forall j$$

The multi-choices for DM's confidence and preference criteria mentioned in the model are streamlined using Lagrange Interpolating Polynomials. Theoretically, if there are two choices: $k_j = 2$ i.e. $o_{Bj} = (o_{Bj}^1, o_{Bj}^2)$

$$P_1(z^{Bj}) = -(z^{Bj} - 1)o_{Bj}^1 + z^{Bj} o_{Bj}^2 \tag{17}$$

$$\begin{aligned} P_1(z^{31}) &= -(z^{31} - 1)o_{31}^1 + z^{31} o_{31}^2 \\ &= -(z^{31} - 1)7 + z^{31}8 \\ &= z^{31}(8 - 7) + 7 \\ &= z^{31} + 7 \end{aligned} \tag{18}$$

Likewise, the Lagrange polynomial for multi-choice for DM's confidence of best to others will be uncovered using the same formula. For two choices i.e. $k_j = 2$ i.e. $\rho_{Bj}^{+} = (\rho_{Bj}^{+1},\ \rho_{Bj}^{+2})$.

$$P_1'(z^{Bj}) = -(z^{Bj} - 1)\rho_{Bj}^{+1} + z^{Bj} \rho_{Bj}^{+2} \tag{19}$$

$$\begin{aligned} P_1'(z^{31}) &= -(z^{31} - 1)\rho_{31}^{+1} + z^{31} \rho_{31}^{+2} \\ &= -(z^{31} - 1) * 0.68 + z^{31} * 0.80 \\ &= z^{31}(0.80 - 0.68) + 0.68 \\ &= 0.12 * z^{31} + 0.68 \end{aligned} \tag{20}$$

For two choices of others to worst criteria: $k_j' = 2$ i.e. $o_{jW} = \{o_{jW}^1, o_{jW}^2\}$

$$P_1(z^{jW}) = -(z^{jW} - 1)o_{jW}^1 + z^{jW} o_{jW}^2 \tag{21}$$

$$\begin{aligned} P_1(z^{21}) &= -(z^{21} - 1)o_{21}^1 + z^{21} o_{21}^2 \\ &= -(z^{21} - 1) * 5 + z^{21} * 7 \\ &= z^{21} * (7 - 5) + 5 \\ &= 2 * z^{21} + 5 \end{aligned} \tag{22}$$

For two choices of DM's confidence for others to worst criteria: $k_j' = 2$ i.e. $\rho_{jW}^{-} = \{\rho_{jW}^{-1}, \rho_{jW}^{-2}\}$

$$P_1'(z^{jW}) = -(z^{jW} - 1)\rho_{jW}^{-1} + z^{jW} \rho_{jW}^{-2} \tag{23}$$

$$\begin{aligned} P_1'(z^{21}) &= -(z^{21} - 1)\rho_{21}^{-1} + z^{21} \rho_{21}^{-2} \\ &= -(z^{21} - 1) * 0.5 + z^{21} * 0.68 \\ &= z^{21} * (0.68 - 0.50) + 0.05 \\ &= 0.18 * z^{21} + 0.50 \end{aligned} \tag{24}$$

Adopting all these polynomials given by equations, the model (16) becomes as such.

$$\textbf{Model: } Min\ \{\xi * (\phi + \psi)\} \tag{25}$$

$$\text{where } \phi = Max(1/\rho_{3j}^{+1_j}, 1/\rho_{3j}^{+2_j}, ..., 1/\rho_{3j}^{+k_j})\ \forall j,\ \forall k.$$

$$\text{and } \psi = Max(1/\rho_{j1}^{-1_j}, 1/\rho_{j1}^{-2_j}, ...1/\rho_{j1}^{-k'_j})\ \forall j,\ \forall k'.$$

s. t.

$$\left|W_3 - W_1 \times (o_{31}^1 + z^{31})\right| \leq \xi/(\rho_{31}^{+1} + 0.12 * z^{31}),\ \forall j$$

$$\left|W_3 - W_2 \times (o_{32})\right| \leq \xi/\rho^+{}_{32},\ \forall j$$

$$\left|W_2 - W_1 \times (o_{21}^1 + 2 * z^{21})\right| \leq \xi/(\rho_{21}^{-1} + 0.18 * z^{21}),\ \forall j$$

$$\xi \geq 0,\ \psi \geq 0,\ \phi \geq 0$$

$$\sum W_j = 1, W_j \geq 0,\ \forall j$$

The solution of the above case study using model 1 provided the optimum values as $W_1* = W_{LO}{}^* = 0.0719$, $W_2* = W_{RE}{}^* = 0.3401$, $W_3* = W_{CO}{}^* = 0.5878$ representing the weights for load flexibility, reachability, and cost respectively with $W_{LO}{}^* < W_{RE}{}^* < W_{CO}{}^*$. The optimal value of ξ^* is 0.1359 and the node points are obtained as $z^{31} = 1$ and $z^{21} = 0$. Therefore, the suited choice for $o_{31} = 8$ and $\rho^+ = 0.80$ as $z^{31} = 1$ and for $o_{21} = 5$ and $\rho^- = 0.50$ as $z^{21} = 0$.

Unravelling model 2 with the same set of constraints offered the same optimum solution. The value of the consistency index for this specific situation, computed using equation (14) given in Vafadarnikjoo et al. [15], is 2.8236, Vafadarnikjoo et al.[15] used single values for ρ^+ and ρ^- whereas we proposed multi choice parameters for ρ^+ and ρ^-. To utilize the equation, we alternated ρ^+ by harmonic means of all the ρ^+ and alternated ρ^- by harmonic means of all the ρ^- in the equation 15. Considering the consistency index 2.8236, consistency ratio CR is computed as $CR = \frac{0.1359}{2.8236} = 0.0481$.

Table 6. Case Study-1 results for optimum weights and ranking

Methods	$W_1 = W_{LO}$	$W_2 = W_{RE}$	$W_3 = W_{CO}$	Ordering	CR
Rezaei[1]	0.0714	0.3387	0.5899	$W_{LO}{}^* < W_{RE}{}^* < W_{CO}{}^*$	0.058
Guo & Zhao [45]	0.1431	0.3496	0.5073	$W_{LO}{}^* < W_{RE}{}^* < W_{CO}{}^*$	0.0559
Proposed Model 1	0.0719	0.3401	0.5878	$W_{LO}{}^* < W_{RE}{}^* < W_{CO}{}^*$	0.0481
Proposed Model 2	0.0719	0.3401	0.5878	$W_{LO}{}^* < W_{RE}{}^* < W_{CO}{}^*$	0.0481

6.2. Case study-2. Similar to the first case study, the second case study is also by Rezaei et al.[1]. The case study utilized BWM to investigate supplier development issues. It assessed suppliers' eight recognized capability criteria and determined their weightings. The eight capabilities mentioned were: C1=Vendor capability (PR), C2=Product Quality capability (PQ), C3=Delivery capability (DE), C4 = Intangible capability (IN), C5=Service capability (SE), C6=Financial capability (FI), and C7=Sustainable capability(SU), and C8= Organizational capability (OR). The best capability criterion is C2=Product Quality (PQ), and the worst is C8=Organizational Competence (OR). Moreover, the value given to the best others pairwise comparison is 9. We delved into the same case study with multi-choice data for pairwise comparison and tied additional multi-choice confidence with each pairwise comparison. Table 7 and 8 show multi-choice pairwise comparisons and corresponding confidence levels.

Table 7. Multi-choice pairwise comparisons for the best criterion in Case Study-2.

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Best criteria: C_2	$\{5, 6\}$	1	$\{2, 3\}$	8	$\{4, 5.6\}$	3	$\{4, 6\}$	9
DM's Confidence ρ^+:	$\{0.38, 0.5\}$	1	$\{0.5, 0.68\}$	0.68	$\{0.38, 0.5, 0.68\}$	0.68	$\{0.50, 0.68\}$	0.68

Table 8. Multi-choice pairwise comparisons for the worst criterion in Case Study-2.

Criteria	Worst criteria: C_8	DM's Confidence: ρ^-
C_1	$\{2, 3\}$	$\{0.26, 0.68\}$
C_2	9	0.68
C_3	8	0.50
C_4	2	0.26
C_5	$\{2, 3, 4\}$	$\{0.26, 0.38, 0.68\}$
C_6	$\{5, 6\}$	$\{0.38, 0.68\}$
C_7	4	0.50
C_8	1	1

Model: $Min\ \{\xi * (\phi + \psi)\}$ (26)

where $\phi = Max(1/\rho_{2j}^{+1_j}, 1/\rho_{2j}^{+2_j}, ..., 1/\rho_{2j}^{+k_j})\ \forall j,\ \forall k.$

and $\psi = Max(1/\rho_{j8}^{-1_j}, 1/\rho_{j8}^{-2_j}, ...1/\rho_{j8}^{-k'_j})\ \forall j,\ \forall k'.$

s. t.

$$\left| W_2 - W_j \times \{o_{2j}^{k_j}\} \right| \leq \xi / \rho^{+}{}_{2j}^{k_j},\ \forall j$$

$$\left|W_j - W_8 \times \{o_{j8}^{k'_j}\}\right| \le \xi/\rho^{-k'_j}_{j8},\ \forall j$$
$$\xi \ge 0,\ \psi \ge 0,\ \phi \ge 0$$
$$\sum W_j = 1, W_j \ge 0,\ \forall j$$

To handle the multi-choice parameters in pairwise comparisons and respective confidence levels, the Lagrange polynomials will be used to redefine the above mathematical programming model. The Lagrange polynomial for two choices is already mentioned in case study 1 by equations (17), (19), (21), (23).

For $o_{21} = (o_{21}^1, o_{21}^2)$, $P_1(z^{21})$ will be

$$\begin{aligned} P_1(z^{Bj}) &= -(z^{Bj}-1)o_{Bj}^1 + z^{Bj}o_{Bj}^2 \\ P_1(z^{21}) &= -(z^{21}-1)o_{21}^1 + z^{21}o_{21}^2 \\ &= -(z^{21}-1)5 + z_{21}6 \\ &= z^{21}(6-5)+5 \\ &= z^{21}+5 \end{aligned} \tag{27}$$

For $o_{23} = (o_{23}^1, o_{23}^2)$, $P_1(z^{23})$ will be

$$\begin{aligned} P_1(z^{Bj}) &= -(z^{Bj}-1)o_{Bj}^1 + z^{Bj}o_{Bj}^2 \\ P_1(z^{23}) &= -(z^{23}-1)o_{23}^1 + z^{23}o_{23}^2 \\ &= -(z^{23}-1)2 + z^{21}3 \\ &= z^{23}(3-2)+3 \\ &= z^{23}+3 \end{aligned} \tag{28}$$

For $o_{27} = (o_{27}^1, o_{27}^2)$, $P_1(z^{27})$ will be

$$\begin{aligned} P_1(z^{Bj}) &= -(z^{Bj}-1)o_{Bj}^1 + z^{Bj}o_{Bj}^2 \\ P_1(z^{27}) &= -(z^{27}-1)o_{27}^1 + z^{27}o_{27}^2 \\ &= -(z^{27}-1)4 + z^{27}6 \\ &= z^{27}(6-4)+4 \\ &= z^{27}+4 \end{aligned} \tag{29}$$

For three choices, the Lagrange polynomial is defined as follows

$$\begin{aligned} &P_2(z^{Bj}) \\ &= \frac{(z^{Bj}-1)(z^{Bj}-2)....(z^{Bj}-k+1)}{(-1)^{k-1}(k-1)!}o_{Bj}^1 + \frac{z^{Bj}(z^{Bj}-2)....(z^{Bj}-k+1)}{(-1)^{k-2}(k-2)!}o_{Bj}^2 \\ &+ \frac{z^{Bj}(z^{Bj}-1)(z^{Bj}-3)....(z^{Bj}-k+1)}{(-1)^{k-3}(k-3)!2!}o_{Bj}^3 \forall\ j \end{aligned} \tag{30}$$

In this case study, $o_{25} = (o_{25}^1, o_{25}^2, o_{25}^3)$, $P_2(z^{25})$ will be

$$\begin{aligned} P_2(z^{25}) =& \frac{(z^{25}-1)(z^{25}-2)}{2!} o_{25}^1 - z^{Bj}(z^{25}-2)o_{25}^2 + z^{25} \\ =& \, z^{25} + 4 \end{aligned} \tag{31}$$

Since there are three choices for the DM's confidence of best to others, the polynomials to handle these choices for the confidence will be given as

$$\begin{aligned} P_1'(z^{21}) &= 0.12 * z^{21} + 0.38 \\ P_1'(z^{23}) &= 0.18 * z^{23} + 0.5 \\ P_2'(z^{25}) &= 0.03 * (z^{25})^2 + 0.09 * z^{25} + 0.38 \\ P_1'(z^{27}) &= 0.18 * z^{27} + 0.50 \end{aligned} \tag{32}$$

Considering the constraints regarding others to the worst, multi-choices are handled using the following equation,

$$\begin{aligned} P_{k_j-1}(z^{jW}) =& \frac{(z^{jW}-1)(z^{jW}-2)....(z^{jW}-k+1)}{(-1)^{k-1}(k-1)!} o_{jW}^1 \\ &+ \frac{z^{jW}(z^{jW}-2)....(z^{jW}-k+1)}{(-1)^{k-2}(k-2)!} o_{jW}^2 \\ &+ \frac{z^{ijW}(z^{jW}-1)(z^{jW}-3)....(z^{jW}-k+1)}{(-1)^{k-3}(k-3)!2!} o_{jW}^3 \\ &+ ... + \frac{z^{jW}(z^{jW}-1)(z^{jW}-2)....(z^{jW}-k+2)}{(k-1)!} o_{jW}^k \forall\ j \end{aligned} \tag{33}$$

For $o_{18} = (o_{18}^1, o_{18}^2)$, $P_1(z^{18})$ is given as follows

$$\begin{aligned} P_1(z^{jW}) &= -(z^{jW}-1)o_{jW}^1 + z^{jW} o_{jW}^2 \\ P_1(z^{18}) &= -(z^{18}-1)o_{18}^1 + z^{18} o_{18}^2 \\ &= -(z^{18}-1)2 + z^{18}3 \\ &= z^{18}(3-2) + 2 \\ &= z^{18} + 2 \end{aligned} \tag{34}$$

For $o_{58} = (o_{58}^1, o_{58}^2, o_{58}^3)$, $P_2(z^{58})$ is given as follows

$$\begin{aligned} P_2(z^{58}) =& \frac{(z^{58}-1)(z^{58}-2)}{2!} o_{58}^1 - z^{58}(z^{58}-2)o_{58}^2 + z^{58} \\ =& \, z^{58} + 2 \end{aligned} \tag{35}$$

For $o_{68} = (o_{68}^1, o_{68}^2)$, $P_1(z^{68})$, the polynomial is as follows

$$P_1(z^{68}) = z^{68} + 2 \tag{36}$$

Using the same interpolating formula for the Lagrange polynomial, the multi-choices for DM's confidence are managed by the equations given below

$$
\begin{aligned}
&P_1'(z^{18}) = 0.42 * z^{18} + 0.26\\
&P_2'(z^{58}) = -0.09 * (z^{58})^2 + 0.03 * z^{58} + 0.26\\
&P_1'(z^{68}) = 0.3 * z^{68} + 0.38 \qquad (37)
\end{aligned}
$$

Employing all the polynomials for multi choice the model in equation 26 becomes

$$
\begin{aligned}
&Min\ \{\xi * (\phi + \psi)\} \qquad (38)\\
&\text{where } \phi = Max(1/\rho_{3j}^{+1_j}, 1/\rho_{3j}^{+2_j}, ..., 1/\rho_{3j}^{+k_j})\ \forall j,\ \forall k.\\
&\text{and } \psi = Max(1/\rho_{j1}^{-1_j}, 1/\rho_{j1}^{-2_j}, ...1/\rho_{j1}^{-k_j'})\ \forall j,\ \forall k'.
\end{aligned}
$$

s. t.

$$
\begin{aligned}
&\left|W_2 - W_1 \times (o_{21}^1 + z^{21})\right| \le \xi/(\rho^{+}{}_{21}^{1} + 0.12 * z^{21})\\
&\left|W_2 - W_3 \times (o_{23}^1 + z^{23})\right| \le \xi/(\rho^{+}{}_{23}^{1} + 0.18 * z^{23})\\
&\left|W_2 - W_4 \times o_{24}\right| \le \xi/\rho^{+}{}_{24}\\
&\left|W_2 - W_5 \times (o_{25}^1 + z^{25})\right| \le \xi/(\rho^{+}{}_{25}^{1} + 0.09 * z^{25} + -0.03 * (z^{25})^2)\\
&\left|W_2 - W_6 \times o_{26}\right| \le \xi/\rho^{+}{}_{26}\\
&\left|W_2 - W_7 \times (o_{27}^1 + 2 * z^{27})\right| \le \xi/(\rho^{+}{}_{27}^{1} + 0.18 * z^{27})\\
&\left|W_2 - W_8 \times o_{28}\right| \le \xi/\rho^{+}{}_{28}\\
&\left|W_1 - W_8 \times (o_{18}^1 + z^{18})\right| \le \xi/(\rho^{-}{}_{18}^{1} + 0.42 * z^{18})\\
&\left|W_3 - W_8 \times o_{38}\right| \le \xi/\rho^{-}{}_{38}\\
&\left|W_4 - W_8 \times o_{48}\right| \le \xi/\rho^{-}{}_{48}\\
&\left|W_5 - W_8 \times (o_{58}^1 + z^{58})\right| \le \xi/(\rho^{-}{}_{58}^{1} + 0.03 * z^{58} - 0.09 * (z^{58})^2)\\
&\left|W_6 - W_8 \times (o_{68}^1 + z^{68})\right| \le \xi/(\rho^{-}{}_{68}^{1} + 0.30 * z^{68})\\
&\left|W_6 - W_8 \times o_{68}\right| \le \xi/\rho^{-}{}_{68}\\
&\xi \ge 0,\ \psi \ge 0,\ \phi \ge 0\\
&\sum W_j = 1, W_j \ge 0,\ \forall j
\end{aligned}
$$

Model 1 delivered the optimum solution as $W_1^* = W_{VE}^* = 0.0542$, $W_2^* = W_{PR}^* = 0.3231$,$W_3^* = W_{DE}^* = 0.2521$, $W_4^* = W_{IN}^* = 0.04047$, $W_5^* = W_{SE}^* = 0.0588$, $W_6^* = W_{FI}^* = 0.1337$, $W_7^* = W_{SU}^* = 0.1039$, $W_8^* = W_{OR}^* = 0.0334$,

$\xi^* = 0.4462$, $CI = 2.1955$, $CR = \frac{0.4462}{2.1955} = 0.2032$.

Model 2 came out as a better-proposed model as its consistency ratio is less than Vafadarnikjoo et al. [15] paper, but it got little difference in the order of the weights. Model 2 produces the optimum solution as $W_1^* = W_{VE}^* = 0.0753$, $W_2^* = W_{PR}^* = 0.2983$, $W_3^* = W_{DE}^* = 0.2344$, $W_4^* = W_{IN}^* = 0.0367$, $W_5^* = W_{SE}^* = 0.1047$, $W_6^* = W_{FI}^* = 0.1233$, $W_7^* = W_{SU}^* = 0.0.0960$, $W_8^* = W_{OR}^* = 0.0308$, $\xi^* = 0.4462$. $CI = 2.4630$, $CR = \frac{0.4462}{2.4630} = 0.1811$.

The solution of the above case study using two different models provided slightly different results. Using model 1, the solution maintained the same order as Vafadarnikjoo et al.[15] with the allowed consistency ratio.

Table 9. Case Study-2 results for optimum weights

Methods	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8
Proposed Model 1	0.0542	0.3231	0.2521	0.0404	0.0588	0.1337	0.1039	0.0334
Proposed Model 2	0.0753	0.2983	0.2344	0.0367	0.1047	0.1233	0.0960	0.0308

Table 10. Case Study-2 results for ranking of weights and Consistency ratio

Methods	ρ^+	ρ^-	Ordering	CR
Vafadarnikjoo et al.[15]	0.6210	0.3876	${W_2}^* > {W_3}^* > {W_6}^* > {W_7}^* > {W_5}^* > {W_1}^* > {W_4}^* > {W_8}^*$	0.1920
Vafadarnikjoo et al.[15]	0.5451	0.5035	${W_2}^* > {W_3}^* > {W_6}^* > {W_7}^* > {W_5}^* > {W_1}^* > {W_4}^* > {W_8}^*$	0.1837
Proposed Model 1	0.6210	0.3876	${W_2}^* > {W_3}^* > {W_6}^* > {W_7}^* > {W_5}^* > {W_1}^* > {W_4}^* > {W_8}^*$	0.2032
Proposed Model 2	0.5451	0.5035	${W_2}^* > {W_3}^* > {W_6}^* > {W_5}^* > {W_7}^* > {W_1}^* > {W_4}^* > {W_8}^*$	0.1811

7. Conclusions and Future study

This work has presented a multi-choice best-worst method where the decision maker can provide a multi-choice confidence level to the multi-choice pairwise comparison values. So, We proposed an extension of BWM with multi-choice pairwise comparisons and multi- choice confidence parameters. This study's main contribution is extending the original BWM with multi-choice uncertainty and a level of confidence. In real life, a decision-maker often faces situations where multiple choices exist. We consider a situation where DM can provide multiple choice for preference comparison and have a corresponding confidence degree for each choice. We have proposed two mathematical programming-based models. To show the managerial implications, the proposed models are validated over two real-life case studies. The models can be applied to any decision-making problem for prioritization of factors. It has added the assumption of multi-choice in real-world MCDM problems. Finally, a comparison has been made with the existing study. The proposed models performed well and can be used for solving

mcdm-based real-life problems.
There are some limitations in the proposed models. The proposed model uses only the harmonic mean for finding the CR. In future studies, an analysis of all averages can be done. The model is very much mathematical and needs some coding expertise to solve problems. In future studies, the limitations of this work can be rectified. This study can also be extended to group decision-making. Also, by incorporating uncertainty such as fuzzy, intuitionistic fuzzy, etc., we can extend the models and solve more complex problems.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : The datasets used are mentioned in the experimental study section of the Manuscript.

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