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ON ALMOST DEFERRED WEIGHTED CONVERGENCE†

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Abstract. This article introduces the notion of almost deferred weighted convergence, statistical deferred weighted almost convergence and almost deferred weighted statistical convergence for real valued sequences. Further, with the aid of interesting examples, we investigated some relationships among our proposed methods. Moreover, we prove a new type of approximation theorem and demonstrated that our theorem effectively extends and improves most of the earlier existing results. Finally, we have presented an example which proves that our theorem is a stronger than its classical versions.

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1. Introduction

The theory of sequence spaces is an essential measure for producing remarkable results with regard to summability techniques and it serves as the fundamental framework for several studies undertaken in the different fields of mathematics. By sequence space we refer a linear subspace of real or complex spaces. Let ℓ_∞, c and c_0 bounded, convergent and null sequence spaces respectively. These sequence spaces are Banach spaces with respect to the norm

$$\|z\|_\infty = \sup_k |z_k|.$$

The Hahn-Banach Extension Theorem is well acknowledged to have numerous advantages in the study of sequence spaces. One of these applications is the Banach limit [1], which gave rise to the idea of almost convergence. This means that the limit functional defined on c can be extended to the whole of ℓ_∞, and

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this extended functional is known as the Banach limit. In 1948, Lorentz [17] used the notion of a Banach limit to define almost convergence. A bounded sequence $z = (z_r)$ is said to be almost convergent to a number z iff

$$\lim_{m\to\infty} \frac{1}{m} \sum_{r=0}^{m} z_{k+r} = z,$$

exists uniformly in k. For more details about almost convergent (see [21], [25]). The other generalization of classical convergence is statistical convergence. In the same year 1951, Fast [7] and Steinhaus [24] independently introduced a different method for convergent sequences known as statistical convergence. Subsequently, this theory has been brought to a high degree of development by many researchers because of its wide applications in various fields of mathematics, such as in real analysis, measure theory and approximations theory and so on. Even in probability theory, statistical convergence has useful applications. In recent years, the concept of statistical convergence was explored in the context of some expansions of summability methods and approximation by linear positive operators. We extended the notions of almost convergence by using deferred weight in the current work. Weighted statistical convergence, which is based on weighted density, is a new variation of statistical convergence described by Karakaya and Chishti [15] in 2009. Mursaleen et al. [19] proposed a modified version of this idea in 2012. Belen and Mohiuddine [2] provided another idea in this area with its application in approximation findings, which they named weighted -statistically convergent. Our work in this paper is a refinement of weighted statistical convergence. The main focus of our current study is on different summability concepts of almost convergence, statistcal convergence in relation with almost convergence and demonstrates how these concepts and techniques produce a variety of approximation outcomes. Due to this, we establish several significant approximations results related to recently suggested methodologies that will significantly enhance all of the current outcomes. As an application to this theorem, an illustrative example is given using Bernstein polynomial.
A sequence (z_r) is called statistical convergent to number z, if for each $\epsilon > 0$, the set

$$P_\epsilon = \{r \in \mathbb{N} : |z_r - z| \geq \epsilon\}$$

has natural density zero i.e.,

$$\delta(P_\epsilon) = \lim_{m\to\infty} \frac{1}{m} |\{r \leq m : |z_r - z| \geq \epsilon\}| = 0.$$

To know more about the statistical convergence one may refer ([3], [8], [9], [10], [12], [23] and [22]). Let (q_m) and (p_m) be sequences of non-negative integers and it satisfies
(i) $q_m < p_m$ and
(ii) $\lim_{m\to\infty} p_m = +\infty$, $\forall\, m \in \mathbb{N}$.

Then the deferred Cesàro mean is defined as

$$s_m = \frac{1}{(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} z_r.$$

In [16] Küçükaslan and Yılmaztürk defined deferred density and deferred statistical convergence for sequences of real numbers. Let $K_d(m)= \{r : p_m < r \leq q_m, r \in K\}$, where K is a subset of $\mathbb{N}$. Then the deferred density of K is defined by

$$\delta_d(K) = \lim_{m\to\infty} \frac{1}{(p_m - q_m)} |K_d(m)|,$$

provided that the limit exists.
Note : Neither almost convergent implies statistical convergence nor statistical convergence implies almost convergence.
Let (ρ_r) be the sequence of non-negative real numbers and

$$R_m = \sum_{r=q_{m+1}}^{p_m} \rho_r.$$

Then, deferred Riesz mean is defined as follows

$$t_m = \frac{1}{R_m} \sum_{r=q_{m+1}}^{p_m} \rho_r z_r.$$

For more details about summability mean (see [6], [18]. Let us derive the product of deferred Cesaro and deferred Riesz means as

$$s_m t_m = \lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_r.$$

Also, $(s_m t_m)$ is said to be summable to z by deferred Cesaro and deferred Riesz(weighted) summability mean if

$$\lim_{m\to\infty} s_m t_m = z.$$

Assume the sequences (q_m) and (p_m) satisfies condition (i) and (ii) of deferred Cesaro. A real sequence (z_r) is said to be deferred weighted statistical convergent to z if $\forall\ \epsilon > 0$, the set

$$\{r \leq R_m(p_m - q_m) : |\rho_r z_r - l| \geq \epsilon\}$$

has zero deferred Cesaro and deferred Riesz (weighted) density, i.e.,

$$\lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)} \left| \left\{ r \leq R_m(p_m - q_m) : |\rho_r z_r - z| \geq \epsilon \right\} \right| = 0.$$

Essentially motivated by the aforementioned investigations and outcomes defined above, we provide an analysis of almost convergence. In second section of this paper we study almost deferred weighted convergence, statistical deferred weighted almost convergence and almost deferred weighted statistical convergence. Some interrelations between newly formed sequences are established. Further, by using the concept of almost deferred weighted convergence we establish Korovkin-type approximation theorem.

2. Main results

Definition 2.1. A bounded sequence (z_r) is said to be almost deferred weighted convergent to z if

$$\lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \left|\rho_r z_{k+r} - z\right| = 0, \tag{1}$$

uniformly in k, where $k \in \mathbb{N}$.

Definition 2.2. A bounded sequence (z_r) is said to be statistical deferred weighted almost convergent to z if $\forall\ \epsilon > 0$, the set

$$\{r \leq R_m(p_m - q_m) : |\frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_{k+r} - z| \geq \epsilon\}$$

has zero almost deferred weighted density i.e.;

$$\lim_{m\to\infty} \frac{1}{m}\left|\left\{r \leq R_m(p_m - q_m) : |\frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_{k+r} - z| \geq \epsilon\right\}\right| = 0,$$

uniformly in k.

Definition 2.3. A bounded sequence (z_r) is said to be almost deferred weighted statistical convergent to z if $\forall\ \epsilon > 0$, the set

$$\{r \leq R_m(p_m - q_m) : |\rho_r z_{k+r} - z| \geq \epsilon\}$$

has zero deferred Cesaro and deferred Riesz density, i.e.,

$$\lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)}\left|\left\{r \leq R_m(p_m - q_m) : |\rho_r z_{k+r} - z| \geq \epsilon\right\}\right| = 0,$$

uniformly in k.

Let us present an example of almost deferred weighted statistical convergent. Firstly suppose $q_m = 0$, $p_m = m$ and $\rho_r = 1$. Now let

$$z_r = \begin{cases} 1, & \text{for } n = r^2 \\ 0, & \text{for } n \neq r^2, \end{cases}$$

then (z_r) is almost deferred weighted statistical convergent to 0.

Remark 2.1. Every convergent sequence is almost deferred weighted convergent but converse need not true in general.
To show the converse part, we present the following example
Consider a sequence (z_r) as

$$z_r = \begin{cases} 1, & \text{for r is odd} \\ 0, & \text{otherwise.} \end{cases}$$

Clearly (z_r) is not convergent but it is almost deferred weighted statistical convergent to 0, because if we take $q_m = 0$, $p_m = m$ and $\rho_r = 1$

$$\lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)} \sum_{r=2m+1}^{4m} \rho_r z_{k+r} = 0,$$

uniformly in k. Thus (z_r) is almost deferred weighted convergent to 0.

Theorem 2.4. *If a sequence (z_r) is almost deferred weighted statistical convergent, then it is statistical deferred weighted almost convergent.*

Proof. Suppose (z_r) is almost deferred weighted statistical convergence to z. Then we have

$$\lim_{m\to\infty} \frac{1}{R_m(p_m - q_m)} \left| \left\{ r \le R_m(p_m - q_m) : |\rho_r z_{k+r} - z| \ge \epsilon \right\} \right| = 0.$$

uniformly in k. Let us fix two sets as:

$$K(\epsilon) = \left\{ r \le R_m(p_m - q_m) : |\rho_r z_{k+r} - z| \ge \epsilon \right\}$$

and

$$K^c(\epsilon) = \left\{ r \le R_m(p_m - q_m) : |\rho_r z_{k+r} - z| < \epsilon \right\}.$$

Then

$$\frac{1}{R_m(p_m - q_m)} |K(\epsilon)| = 0,$$

uniformly in k. We thus find that

$$\begin{aligned} \left| \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_{k+r} - z \right| &\le \left| \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r (z_{k+r} - z) \right| \\ &\quad + |z| \left| \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r - 1 \right| \\ &\le \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r\in\mathcal{K}(\epsilon))}}^{p_m} \rho_r \left| z_{k+r} - z \right| \end{aligned}$$

$$+ \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}^c(\epsilon))}}^{p_m} \rho_r \Big| z_{k+r} - z \Big| + \Big| \frac{1}{R_m} \sum_{r=q_{m+1}}^{p_m} \rho_r - 1 \Big|$$

$$\leq \frac{1}{R_m(p_m - q_m)} \Big| K(\epsilon) \Big| + \frac{1}{R_m(p_m - q_m)} \Big| K^c(\epsilon) \Big| + 0$$

$$\to 0 \quad \text{as} \quad m \to \infty,$$

uniformly in k. This implies $\Big| \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_{k+r} - z \Big| \to 0$. Thus (z_r) is deferred weighted almost convergent to z. This means that (z_r) is statistical deferred weighted almost convergent to z. □

For converse part, we present an example below.

Example 2.5. Consider a sequence (z_r) as

$$z_r = \begin{cases} \frac{1}{m}, & \text{for } r = m^3 - m^2, m^3 - m^2 + 1, \cdots, m^3 + 1; \\ -\frac{1}{m}, & \text{for } r = m^3; \\ 0, & \text{otherwise.} \end{cases}$$

Let $q_m = 0$, $p_m = m$ and $\rho_r = 1$. Then,

$$\lim_{m \to \infty} \frac{1}{m} \Big| \Big\{ r \leq R_m(p_m - q_m) : | \frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \rho_r z_{k+r} - 0 | \geq \epsilon \Big\} \Big| = 0.$$

This implies (z_r) is statistical deferred weighted almost convergent to 0. But it is not almost deferred weighted statistical convergent as

$$\lim_{m \to \infty} \frac{1}{R_m(p_m - q_m)} \Big| \Big\{ r \leq R_m(p_m - q_m) : |\rho_r z_{k+r} - z| \geq \epsilon \Big\} \Big| \neq 0.$$

Theorem 2.6. *If (z_r) is almost deferred weighted convergent to z then it is almost deferred weighted statistical convergent to z.*

Proof. Assume that (z_r) is almost deferred weighted convergent to z. Then

$$\frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} \Big| \rho_r z_{k+r} - z \Big| = 0.$$

Now,

$$\frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} |\rho_r z_{k+r} - z| \;\geq\; \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}(\epsilon))}}^{p_m} |\rho_r z_{k+r} - z|$$

$$\geq \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}(\epsilon))}}^{p_m} \epsilon$$
$$= \frac{\epsilon}{R_m(p_m - q_m)} |K(\epsilon)|,$$

uniformly in k. Thus, (z_r) is almost deferred weighted statistical convergent to z. □

Theorem 2.7. *If (z_r) is almost deferred weighted statistical convergent to z and satisfies the condition*

$$|\rho_r z_{k+r} - z| \leq C,$$

where $C \in \mathbb{R}^+$, then (z_r) is almost deferred weighted convergent to z.

Proof. Suppose (z_r) is almost deferred weighted statistical convergent to z and satisfies the condition $|\rho_r z_{k+r} - z| \leq C$. Then

$$\frac{1}{R_m(p_m - q_m)} \sum_{r=q_{m+1}}^{p_m} |\rho_r z_{k+r} - z|$$
$$= \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}(\epsilon))}}^{p_m} |\rho_r z_{k+r} - z| + \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}^c(\epsilon))}}^{p_m} |\rho_r z_{k+r} - z|$$
$$= E_1(m) + E_2(m),$$

where

$$E_1(m) = \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}(\epsilon))}}^{p_r} |\rho_r z_{k+r} - z|$$

and

$$E_2(m) = \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}^c(\epsilon))}}^{p_r} |\rho_r z_{k+r} - z|.$$

Now, if $r \in \mathcal{K}^c(\epsilon)$, then

$$E_2(m) = \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}^c(\epsilon))}}^{p_r} |\rho_r z_{k+r} - z| < \frac{\epsilon}{R_m(p_m - q_m)} |\mathcal{K}^c(\epsilon)|.$$

Also, if $r \in \mathcal{K}(\epsilon)$, then

$$E_1(m) = \frac{1}{R_m(p_m - q_m)} \sum_{\substack{r=q_{m+1} \\ (r \in \mathcal{K}(\epsilon))}}^{p_m} |\rho_r z_{k+r} - z| \leq \frac{C}{R_m(p_m - q_m)} |\mathcal{K}(\epsilon)|.$$

This implies that $\{E_1(m) + E_2(m)\} \to 0$ uniformly in k as $m \to \infty$. Hence, (z_r) is almost deferred weighted convergent to z. □

3. Korovkin - type Approximation Theorem for almost deferred weighted statistical convergent

Numerous mathematicians have studied Korovkin-type approximation theorems in a variety of contexts, including function spaces, Banach spaces etc. Kadak [14] recently studied Korovkin-type approximation theorems. Furthermore, Korovkin approximation theorem for Bernstein operator of rough statistical convergence of triple sequence was examined by Hazarika et al. [11]. For detailed study on Korovkin approximation theorems one may refer ([5], [4], [13] and [20]).

Theorem 3.1. *Let $C[0,1]$ be the space of all continuous function on $[0,1]$. Suppose $\mathcal{O}_s$ be a linear operator from $C[0,1] \to C[0,1]$, using $r = s-k$ in (2.1) and assume that*

$$\mathrm{Q_{k,m}} = \frac{1}{R_m(p_m - q_m)} \sum_{s=k+q_m+1}^{k+p_m} \rho_{s-k}\mathcal{O}_s(g,z),$$

satisfying

$$\lim_m \|\mathrm{Q_{k,m}}(1,z) - 1\|_\infty = 0, \textit{uniformly in } k \tag{2}$$

$$\lim_m \|\mathrm{Q_{k,m}}(t,z) - z\|_\infty = 0, \textit{uniformly in } k \tag{3}$$

$$\lim_m \|\mathrm{Q_{k,m}}(t^2,z) - z^2\|_\infty = 0, \textit{uniformly in } k. \tag{4}$$

Then for any $f \in C[0,1]$, we have

$$\lim_m \|\mathrm{Q_{k,m}}(g,z) - g(z)\|_\infty = 0, \textit{uniformly in } k.$$

Proof. Since $g \in C[0,1]$, $\exists$ a constant $\vartheta > 0$ such that

$$|g(z)| \le \vartheta$$
$$\Rightarrow \quad |g(t) - g(z)| \le 2\vartheta, \quad z \in [0,1]. \tag{5}$$

Also, as g is continuous on $[0,1]$ so for given $\epsilon > 0$, $\exists\, \delta > 0$ such that

$$|g(t) - g(z)| < \epsilon, \qquad \forall\ |t-z| < \delta. \tag{6}$$

Using (3.4) and (3.5), and putting $\mu(t) = (t-z)^2$, we have

$$|g(t) - g(z)| \quad < \quad \epsilon + \frac{2\vartheta}{\delta^2}\mu, \ \ \forall |t-z| < \delta.$$

This means that

$$-\epsilon - \frac{2\vartheta}{\delta^2}\mu < g(t) - g(z) \qquad < \epsilon + \frac{2\vartheta}{\delta^2}\mu.$$

Now we apply $\mathcal{O}_s(1,z)$ on above inequality and since $\mathcal{O}_s(1,z)$ is monotone and linear, we get

$$\mathcal{O}_s(1,z)\bigg(-\epsilon - \frac{2\vartheta}{\delta^2}\mu\bigg) < \mathcal{O}_s(1,z)(g(t) - g(z)) \qquad < \mathcal{O}_s(1,z)\bigg(\epsilon + \frac{2\vartheta}{\delta^2}\mu\bigg).$$

As z is fixed and so $g(z)$ is constant number, thus

$$-\epsilon\mathcal{O}_s(1,z) - \frac{2\vartheta}{\delta^2}\mathcal{O}_s(1,z) < \mathcal{O}_s(g,z) - \mathcal{O}_s(1,z)g(z) < \epsilon\mathcal{O}_s(1,z) + \frac{2\vartheta}{\delta^2}\mathcal{O}_s(1,z). \tag{7}$$

But

$$\begin{aligned}\mathcal{O}_s(g,z)-g(z) =&\mathcal{O}_s(g,z)-g(z)\mathcal{O}_s(1,z)+g(z)\mathcal{O}_s(1,z)-g(z)\\ &=[\mathcal{O}_s(g,z)-g(z)\mathcal{O}_s(1,z)]+g(z)[\mathcal{O}_s(1,z)-1]. \end{aligned} \tag{8}$$

Using (3.6) and (3.7), we get

$$\mathcal{O}_s(g,z)-g(z) <\epsilon\mathcal{O}_s(1,z)+\frac{2\vartheta}{\delta^2}\mu\mathcal{O}_s(1,z)+g(z)(\mathcal{O}_s(1,z)-1). \tag{9}$$

Now,

$$\begin{aligned}\mathcal{O}_s(\mu,z)-g(z) &= \mathcal{O}_s((t-z)^2,z)\\ &= \mathcal{O}_s((t^2-2tz+z^2,z)\\ &= \mathcal{O}_s((t^2,z)+2z\mathcal{O}_s(t,z)+z^2\mathcal{O}_s(1,z)\\ &= [\mathcal{O}_s((t^2,z)-z^2)]-2z[\mathcal{O}_s(t,z)-z]+z^2[\mathcal{O}_s(1,z)-1].\end{aligned}$$

Using (3.8), we get

$$\begin{aligned}&\mathcal{O}_s(g,z)-g(z)\\ &<\epsilon\mathcal{O}_s(1,z)+\frac{2\vartheta}{\delta^2}\{[\mathcal{O}_s(t^2,z)-z^2]+2z[\mathcal{O}_s(t,z)-z]+z^2[\mathcal{O}_s(1,z)-1]\}\\ &\quad+g(z)(\mathcal{O}_s(1,z)-1).\\ &=\epsilon[\mathcal{O}_s(1,z)-1]+\epsilon+\frac{2\vartheta}{\delta^2}\{[\mathcal{O}_s(t^2,z)-z^2]\\ &\quad+2z\mathcal{O}_s(t,z)-z+z^2[\mathcal{O}_s(1,z)-1]\}+g(z)\mathcal{O}_s(1,z)-1.\end{aligned}$$

Thus,

$$\begin{aligned}&|\mathcal{O}_s(g,z)-z|\\ &=\epsilon+(\epsilon+\vartheta)|\mathcal{O}_s(1,z)-1|+\frac{2\vartheta}{\delta^2}|z^2||\mathcal{O}_s(1,z)-1|+\frac{2\vartheta}{\delta^2}|\mathcal{O}_s(t^2,z)-z^2|\\ &\quad+\frac{4\vartheta}{\delta^2}|z||\mathcal{O}_s(t,z)-z|\\ &\le\epsilon+\left(\epsilon+\vartheta+\frac{2\vartheta}{\delta^2}\right)|\mathcal{O}_s(1,z)-1|+\frac{2\vartheta}{\delta^2}|\mathcal{O}_s(t^2,z)-z^2|+\frac{4\vartheta}{\delta^2}|z||\mathcal{O}_s(t,z)-z|\end{aligned}$$

Thus,

$$\|\mathcal{O}_s(g,z)-z\|_\infty\le\epsilon+\mathcal{K}(\|\mathcal{O}_s(1,z)-1\|_\infty+\|\mathcal{O}_s(t,z)-z\|_\infty+\|\mathcal{O}_s(t^2,z)-z^2\|_\infty), \tag{10}$$

where $\mathcal{K}=\max\{\epsilon+\vartheta+\frac{2\vartheta}{\delta^2},\frac{2\vartheta}{\delta^2},\frac{4\vartheta}{\delta^2}\}$.
Now, replacing $\mathcal{O}_s(g,z)$ by

$$\mathrm{Q_{k,m}}=\frac{1}{R_m(p_m-q_m)}\sum_{s=k+q_m+1}^{k+p_m}\rho_{s-k}\mathcal{O}_s(g,z),$$

in (3.9), using (3.1) − (3.3), we obtain

$$\lim_{m} \|\mathrm{Q}_{k,m}(g,z) - g(z)\|_{\infty} = 0,$$

uniformly in k. □

Now, we present an example of a sequence of positive linear operator satisfying the condition of Theorem 3.1 but does not satisfies the condition for classical korovkin type approximation.

Example 3.2. Consider the sequence of Bernstein Polynomial

$$B_s(g,z) = \sum_{m=0}^{s} g\left(\frac{m}{s}\right)\binom{s}{m} z^m (1-z)^{s-m} \; z \in [0,1]$$

Let the sequence $\mathcal{O}_s : C[0,1] \to C[0,1]$ with $\mathcal{O}_s(g,z) = (1+z_s)B_s(g,z)$, where (z_s) is defined as above in example 1. Then $B_s(1,z) = 1$, $B_s(t,z) = z$, $B_s(t^2,z) = z^2 + \frac{z-z^2}{s}$ and the sequence $\mathcal{O}_s$ satisfies the (3.1) − (3.3) conditions. Thus, we have

$$\lim_{s\to\infty} \|\mathcal{O}_s(g,z) - g(z)\|_{\infty} = 0.$$

On the other hand, we have $\mathcal{O}_s(g,0) = (1+z_s)g(0)$, since $B_s(g,0) = g(0)$, and hence

$$\begin{aligned} \|\mathcal{O}_s(g,z) - g(z)\|_{\infty} &\geq |\mathcal{O}_s(g,0) - g(0)| \\ &= z_s|g(0)|. \end{aligned}$$

We can see that $(\mathcal{O}_s)$ does not satisfies the classical Korovkin theorem, since $\limsup_{s\to\infty} z_s$ does not exist.

4. Conclusion

In this paper, we studied the concept of almost deferred weighted convergence, statistical deferred weighted almost convergence and almost deferred weighted statistical convergence. We also established some implication relations among newly formed sequence spaces. Finally, as an application point of view we proved Korovkin-type approximation theorem. The results presented in this article not only generalize the earlier works done by several mathematicians but also give a new outlook concerning the evolution of Korovkin-type approximation theorem. In further studies, almost lacunary statistical convergence by using double sequences can be studied.

Conflicts of interest : The authors declare that they have no conflict of interest.

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