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PAIR DIFFERENCE CORDIALITY OF CERTAIN SUBDIVISION GRAPHS

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ABSTRACT. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to $p-1$ elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate the pair difference cordial labeling behavior of subdivision of some graphs.

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1. Introduction

In this paper we consider finite, undirected and simple graphs only. A. Rosa was introduced the graceful labeling in [19] and harmonius labeling was introduced by R.L.Graham and N.J,A.Solane [11]. A weaker version of graceful and harmonious labeling called cordial labeling was introduced by Cachit[3]. Cordial labeling behaviour of several graphs studied in [1,2,4,5,6,7,8,9,10,13,14,15]. The notion of pair difference cordial labeling of a graph was introduced in [16]. The

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pair difference cordial labeling behavior of several graphs like path, cycle, star, wheel,helm, web, closed helm, closed web, parachute, fan , umbrella, (n,t)-kite gragh, butterfly have been investigated in [16,17,18,19,20,21,22,].In this paper we have investigate the pair difference cordiality of subdivision of some graphs

2. Preliminaries

Definition 2.1. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm1, \pm2, \pm3, \cdots, \pm\rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

Definition 2.2. [8]. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uvw.

Definition 2.3. [8]. The ladder L_n is obtained from two copies of the paths $u_1u_2 \cdots u_n$ and $v_1v_2 \cdots v_n$ by joining each u_i with $v_i, 1 \leq i \leq n$.Clearly L_n has $2n$ vertices and $3n - 2$ edges.

Definition 2.4. [8]. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.
Let $V(T_n) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n-1\}$ and $E(T_n) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i, v_iu_{i+1} : 1 \leq i \leq n-1\}$. Obviously T_n has $2n-1$ vertices and $3n-3$ edges.

Definition 2.5. The alternate triangular snake $A(T_n)$ is obtained from the path $u_1u_2 \cdots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.Now we define the vertex set and edge set of $A(T_n)$ as follows.

Type 1. The edge u_1u_2 lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle.In this case n is even.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2}\}$

and $E(A(T_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i}v_i, u_{2i-1}v_i : 1 \leq i \leq \frac{n}{2}\}$. It is easy to verify that in this case $A(T_n)$ has $\frac{3n}{2}$ vertices and $2n-1$ edges.

Type 2. The edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle.Here clearly n is even.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-2}{2}\}$ and $E(A(T_n)) = \{u_{2i}v_i, u_{2i+1}v_i : 1 \leq i \leq \frac{n-2}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$. Note that $A(T_n)$ has $\frac{3n-2}{2}$ vertices and $2n-3$ edges.

Type 3. The edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle.In this type n is odd.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-1}{2}\}$ and $E(A(T_n)) = \{u_{2i}v_i, u_{2i+1}v_i : 1 \leq i \leq \frac{n-1}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$. Clearly in this type $A(T_n)$ has $\frac{3n-1}{2}$ vertices and $2n-2$ edges.

Theorem 2.6. [16]. *The cycle C_n is pair difference cordial if and only if $n > 3$.*

Theorem 2.7. [16]. *The path P_n is pair difference cordial for all values of $n \neq 3$* .

3. Main Results

Theorem 3.1. *The subdivision of star $K_{1,n}, S(K_{1,n})$ is pair difference cordial if and only if $n \geq 2$.*

Proof. Let $V(S(K_{1,n})) = \{x, x_i, y_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n})) = \{xx_i, x_iy_i : 1 \leq i \leq n\}$.Clearly $S(K_{1,n})$ has $2n+1$ vertices and $2n$ edges.
When $n = 1$, $S(K_{1,1}) \cong P_3$ is not pair difference cordial proof follows from thorem 2.7.
Now consider $n > 1$. There are two cases arises.
Case 1. n is even .

Assign the label 2 to the vertex x. Next assign the labels $2, 4, 6, \cdots, n$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ and assign the labels $1, 3, 5, \cdots, n-1$ to the verices $y_1, y_2, y_3, \cdots, y_{\frac{n}{2}}$. Next assign the labels $-2, -4, -6, \cdots, -n$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots, x_n$ respectively. Next we consider the pendant vertices. Assign the labels $-1, -3, -5, \cdots, -n+1$ respectively to the vertices $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, y_{\frac{n+6}{2}}, \cdots, y_n$ respectively.

Case 2. n is odd .

As in case 1 , assign the labels to the vertices $x, x_i, y_i, 1 \leq i \leq n-1$. Finally assign the labels 3, -3 respectively to the vertices x_n, y_n. Since $\Delta_{f_1^c} = \Delta_{f_1} = n$, this vertex labeling establish that $S(K_{1,n})$. is a pair difference cordial graph.

□

Theorem 3.2. *The subdivision of bistar $B_{n,n}, S(B_{n,n})$ is pair difference cordial for all values of n.*

Proof. Let $V(S(B_{n,n})) = \{x, y, z, x_i, y_i, a_i, b_i : 1 \leq i \leq n\}$ and $E(S(B_{n,n})) = \{xx_i, x_i a_i, yy_i, y_i b_i : 1 \leq i \leq n\} \cup \{xz, yz\}$.Clearly $S(B_{n,n})$ has $4n + 3$ vertices and $4n + 2$ edges.
Case 1. $n = 2$.
Clearly $S(B_{1,1}) \cong P_7$.By theorem 2.7, $S(B_{1,1})$ is pair difference cordial.

Case 2. $n \geq 3$.

Assign the labels $1, -1, 2$ respectively to the vertices x, y, z. Now assign the labels $2, 4, 6, \cdots, 2n$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_n$ and assign the labels $-2, -4, -6 \cdots, -2n$ to the vertices $y_1, y_2, y_3, \cdots, y_n$ respectively. Now we consider the pendant vertices a_i. Assign the labels $3, 5, 7, \cdots, 2n + 1$ respectively to the vertices $a_1, a_2, a_3, \cdots, a_n$. We now consider the pendant vertices b_i. Assign the labels $-3, -5, -7, \cdots, -2n - 1$ to the vertices $b_1, b_2, b_3, \cdots, b_n$ respectively. Obviously,$\Delta f_1 = 2n, \Delta {f_1}^c = 2n + 1$, this vertex labeling gives that $S(B_{n,n})$ is pair difference cordial for all values of n.

□

Theorem 3.3. *$S(L_n)$ is pair difference cordial for all $n \geq 2$.*

Proof. Let $S(V(L_n)) = \{u_i, v_i, z_i : 1 \leq i \leq n\} \cup \{x_i, y_i : 1 \leq i \leq n - 1\}$ and $S(E(L_n)) = \{u_i x_i, x_i u_{i+1}, v_i y_i, y_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{u_i z_i, v_i z_i : 1 \leq i \leq n\}$.

Case 1. $n = 2$.
Clearly $S(L_2) \cong C_8$.By theorem 2.6, $S(L_2)$ is pair difference cordial.

Case 2. $n \geq 3$.
Thre are two cases arises.

Subcase 1. n is even.

Assign the labels $1, 3, 4$ respectively to the vertices u_1, u_2, u_3. Next assign the labels $8, 12, 16, \cdots, 2n - 4$ respectively to the vertices $u_5, u_7, u_9, \cdots, u_{n-1}$ and assign the labels $7, 11, 15, \cdots, 2n - 1$ to the vertices $u_4, u_6, u_8, \cdots, u_n$.Next consider the vertices x_i . Assign the labels $2, 6, 10, \cdots, 2n-2$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{n-1}$ also assign the labels $5, 9, 13, \cdots, 2n-3$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_{n-2}$.

Next we consider the vertices v_i. Assign the labels $-1, -3, -5, \cdots, -(2n-1)$ to the vertices $v_1, v_2, v_3, \cdots, v_n$ respectively and now conider the vertices y_i. Assign the labels $-2, -4, -6, \cdots, -(2n-2)$ to the vertices $y_1, y_2, y_3, \cdots, y_{n-1}$.Now we consider the vertices z_i. Assign the labels $2n, 2n+1, 2n+2, \cdots, \frac{5n-2}{2}$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n}{2}}$ also assign the labels $-2n, -(2n+1), -(2n+2), \cdots, -\frac{5n-2}{2}$ to the vertices $z_{\frac{n+2}{2}}, z_{\frac{n+4}{2}}, z_{\frac{n+6}{2}}, \cdots, z_n$.

Subcase 2. n is odd.

Assign the labels $1, 3, 4$ respectively to the vertices u_1, u_2, u_3. Next assign the labels $8, 12, 16, \cdots, 2n-2$ respectively to the vertices $u_5, u_7, u_9, \cdots, u_n$ and assign the labels $7, 11, 15, \cdots, 2n-3$ to the vertices $u_4, u_6, u_8, \cdots, u_{n-1}$.Next consider the vertices x_i . Assign the labels $2, 6, 10, \cdots, 2n-4$ respectively to the vertices $x_1, x_3, x_5, \cdots, x_{n-2}$ also assign the labels $5, 9, 13, \cdots, 2n-1$ respectively to the vertices $x_2, x_4, x_6, \cdots, x_{n-1}$.

Next we consider the vertices v_i. Assign the labels $-1, -3, -5, \cdots, -(2n-1)$ to the vertices $v_1, v_2, v_3, \cdots, v_n$ respectively. Now conider the vertices y_i. Assign the labels $-2, -4, -6, \cdots, -(2n-2)$ to the vertices $y_1, y_2, y_3, \cdots, y_{n-1}$.Now we move to the vertices z_i. Assign the labels $2n, 2n+1, 2n+2, \cdots, \frac{5n-3}{2}$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-1}{2}}$ also assign the labels $-2n, -(2n+1), -(2n+2), \cdots, -\frac{5n-3}{2}$ to the vertices $z_{\frac{n+1}{2}}, z_{\frac{n+3}{2}}, z_{\frac{n+5}{2}}, \cdots, z_{n-1}$. Finally assign the label 1 to the vertex z_n.

In both cases,clearly $\Delta f_1 = 3n-2 = \Delta {f_1}^c$. Therefore this vertex labeling gives that $S(L_n)$ is pair difference cordial graph for all n.

□

Theorem 3.4. *$S(T_n)$ is pair difference cordial for all values of $n \geq 3$.*

Proof. Let us take the vertex set and edge set of triangular snake T_n from the definition 2.4. Let the edge $u_iu_{i+1}, u_iv_i, u_{i+1}v_i$ be subdivided by the vertex x_i, y_i, z_i. Note that $S(T_n)$ has $5n-4$ vertices and $6n-6$ edges.There are three cases arises.

Case 1. $n = 2$.

Obviously $S(T_2) \cong C_6$. By theorem 2.6, $S(T_2)$ is pair difference cordial.

Case 2. $n = 3$.

Assign the labels $1, 3, -2$ to the vertices u_1, u_2, u_3 respectively.Next assign the labels $2, -1$ to the vertices x_1, x_2 respectively and assign the labels $3, -3$ to the vertices z_1, z_2 respectively.Finally assign the labels $5, -4$ to the vertices y_1, y_2 and assign the labels $4, -5$ respectively to the vertices v_1, v_2.

Case 3. $n > 3$.
There are two cases arises.

Subcase 1. n is even.

Assign the labels $1, -1$ to the vertices $u_1, u_{\frac{n+2}{2}}$. Next assign the labels $3, 8, 13, \cdots, \frac{5n-14}{2}$ respectively to the vertices $u_2, u_3, u_4, \cdots, u_{\frac{n}{2}}$ and assign the labels $-3, -8, -13, \cdots, -\frac{5n-14}{2}$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+8}{2}}, \cdots, u_n$ respectively. Now we consider the vertices x_i. Assign the labels $2, 7, 12, \cdots, \frac{5n-6}{2}$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ and assign the labels $-2, -7, -12, \cdots,$ $-\frac{5n-16}{2}$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots, x_{n-1}$ respectively.

Now move to the vertices y_i. Assign the labels $5, 10, 15, \cdots, \frac{5n-10}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-2}{2}}$ and assign the labels $-5, -10, -15, \cdots,$ $-\frac{5n-10}{2}$ to the vertices $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, y_{\frac{n+6}{2}}, \cdots, y_{n-1}$ respectively. Next we consider the vertices v_i. Assign the labels $6, 11, 16, \cdots, \frac{5n-8}{2}$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{2}}$ and assign the labels $-6, -11, -16, \cdots, -\frac{5n-8}{2}$ to the vertices $v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}, v_{\frac{n+6}{2}}, \cdots, v_{n-1}$ respectively.Now we move to the vertices z_i. Assign the labels $4, 9, 14, \cdots, \frac{5n-12}{2}$ respectively to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-2}{2}}$ and assign the labels $-4, -9, -14, \cdots, -\frac{5n-12}{2}$ to the vertices $z_{\frac{n+2}{2}}, z_{\frac{n+4}{2}}, z_{\frac{n+6}{2}}, \cdots,$ z_{n-1} respectively.

Next assign the labels $-\frac{5n-4}{2}, -\frac{5n-6}{2}, \frac{5n-4}{2}$ respectively to the vertices $y_{\frac{n}{2}}, v_{\frac{n}{2}},$ $z_{\frac{n}{2}}$.

Subcase 2. n is odd.

Assign the labels $1, -1$ to the vertices $u_1, u_{\frac{n+1}{2}}$. Next assign the labels $3, 8, 13, \cdots, \frac{5n-17}{2}$ respectively to the vertices $u_2, u_3, u_4, \cdots, u_{\frac{n-1}{2}}$ and assign the labels $-3, -8, -13, \cdots, -\frac{5n-17}{2}$ to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+7}{2}}, \cdots, u_{n-1}$ respectively. Now we consider the vertices x_i. Assign the labels $2, 7, 12, \cdots, \frac{5n-11}{2}$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-1}{2}}$ and assign the labels $-2, -7, -12,$ $\cdots, -\frac{5n-11}{2}$ to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+3}{2}}, x_{\frac{n+5}{2}}, \cdots, x_{n-1}$ respectively.

Now we consider the vertices y_i. Assign the labels $5, 10, 15, \cdots, \frac{5n-5}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-1}{2}}$ and assign the labels $-5, -10, -15, \cdots,$ $-\frac{5n-5}{2}$ to the vertices $y_{\frac{n+1}{2}}, y_{\frac{n+3}{2}}, y_{\frac{n+5}{2}}, \cdots, y_{n-1}$ respectively. Next we move to the vertices v_i. Assign the labels $6, 11, 16, \cdots, \frac{5n-13}{2}$ respectively to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-3}{2}}$ and assign the labels $-6, -11, -16, \cdots, -\frac{5n-13}{2}$ to the vertices $v_{\frac{n+1}{2}}, v_{\frac{n+3}{2}}, v_{\frac{n+5}{2}}, \cdots, v_{n-2}$ respectively and also assign the labels

$\frac{5n-7}{2}, -\frac{5n-7}{2}$ to the vertices $v_{\frac{n-1}{2}}, v_{n-1}$.

We now move to the vertices z_i. Assign the labels $4, 9, 14, \cdots, \frac{5n-15}{2}$ respectively to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-3}{2}}$ and assign the labels $-4, -9, -14, \cdots,$ $-\frac{5n-15}{2}$ to the vertices $z_{\frac{n+1}{2}}, z_{\frac{n+3}{2}}, z_{\frac{n+5}{2}}, \cdots, z_{n-2}$ respectively. Finally assign the labels $\frac{5n-7}{2}, -\frac{5n-9}{2}, -1$ to the vertices $z_{\frac{n-1}{2}}, z_{n-1}, u_n$.
In all the cases, clearly $\Delta f_1 = 3n-3 = \Delta {f_1}^c$.This vertex labeling gives that $S(T_n)$ is pair difference cordial graph for all values of n.

□

Theorem 3.5. *$S(A(T_n))$ is pair difference cordial if the edge u_1u_2 lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle for all even $n \geq 4$.*

Proof. The vertex set and edge set of alternate triangular snake $A(T_n)$ are taken from the definition 2.5.The edge u_iu_{i+1} subdivide by the vertex x_i, the edge $u_{2i-1}v_i$ subdivide by the vertex y_i and the edge $u_{2i}v_i$ subdivide by the vertex z_i. $S(A(T_n))$ has $\frac{7n-2}{2}$ vertices and $4n-2$ edges.There are two cases arises.

Case 1. $n \equiv 0 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-2}{2})-2$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-2}{2})$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n}{2}}$. Next assign the labels $-1, -7, -13,$ $\cdots, -(3(\frac{n-2}{2})-2)$ to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots, u_{n-1}$ and assign the labels $-3, -9, -15, \cdots, -3(\frac{n-2}{2})+6$ respectively to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}},$ $\cdots, u_{n-2}$.

Next consider the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-2}{2})-1$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots,$ $-(3(\frac{n-2}{2})-1)$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}}, x_{\frac{n+10}{2}}, \cdots, x_{n-1}$. Now assign the labels $3(\frac{n-2}{2})+4, 3(\frac{n-2}{2})+5, 3(\frac{n-2}{2})+6, \cdots, \frac{7n-4}{4}$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-4}{2}}$ and assign the labels $-(3(\frac{n-2}{2})+4), -(3(\frac{n-2}{2})+5), -(3(\frac{n-2}{2})+6), \cdots, -\frac{7n-4}{4}$ to the vertices $x_{\frac{n-2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots, x_{n-1}$.

We now consider the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-2}{2})+2$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-2}{2})+2)$ to the vertices $y_{\frac{n+4}{4}}, y_{\frac{n+8}{4}}, y_{\frac{n+12}{4}}, \cdots, y_{\frac{n}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-2}{2})+3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-2}{2})+3)$ to the vertices $v_{\frac{n+4}{4}}, v_{\frac{n+8}{4}}, v_{\frac{n+12}{4}},$ $\cdots, v_{\frac{n}{2}}$.

Next we move the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-2}{2}) + 1$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-2}{2}) + 1)$ to the vertices $z_{\frac{n+4}{4}}, z_{\frac{n+8}{4}}, z_{\frac{n+12}{4}}, \cdots, z_{\frac{n}{2}}$. Finally assign the labels $-\frac{7n-4}{4}$, $-(3(\frac{n-2}{2}) + 1), -(3\frac{n-2}{2})$ to the vertices $x_{n-2}, u_n, z_{\frac{n}{2}}$.

Case 2. $n \equiv 2 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-4}{2}) - 2$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-4}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-4}{2})$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$. Next assign the labels $-1, -7, -13, \cdots, -(3(\frac{n-4}{2}) - 2)$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}, \cdots, u_{n-1}$ and assign the labels $-3, -9, -15, \cdots, -3(\frac{n-4}{2})$ respectively to the vertices $u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, u_{\frac{n+14}{2}}, \cdots, u_n$.

Next we move to the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-2}{2}) + 2$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots, -(3(\frac{n-2}{2}) - 4)$ to the vertices $x_{\frac{n+4}{2}}, x_{\frac{n+8}{2}}, x_{\frac{n+12}{2}}, \cdots, x_{n-1}$. Now assign the labels $3(\frac{n-4}{2}) + 8, 3(\frac{n-2}{4}) + 9, 3(\frac{n-2}{2}) + 10, \cdots, \frac{7n-2}{4}$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-6}{4}}$ and assign the labels $-(3(\frac{n-2}{2}) + 6), -(3(\frac{n-2}{2}) + 7), -(3(\frac{n-2}{2}) + 8), \cdots, -\frac{7n-2}{4}$ to the vertices $x_{\frac{n+2}{4}}, x_{\frac{n+10}{4}}, x_{\frac{n+18}{4}}, \cdots, x_{n-2}$.

We now consider the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-4}{2}) + 2$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-2}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-2}{2}) + 2)$ to the vertices $y_{\frac{n+6}{4}}, y_{\frac{n+10}{4}}, y_{\frac{n+14}{4}}, \cdots, y_{\frac{n}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-4}{2}) + 3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-4}{2}) + 3)$ to the vertices $v_{\frac{n+6}{4}}, v_{\frac{n+10}{4}}, v_{\frac{n+14}{4}}, \cdots, v_{\frac{n}{2}}$.

Next we consider the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-4}{2}) + 1$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-2}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-4}{2}) + 1)$ to the vertices $z_{\frac{n+6}{4}}, z_{\frac{n+10}{4}}, z_{\frac{n+14}{4}}, \cdots, z_{\frac{n}{2}}$. Finally assign the labels $-\frac{7n-2}{4}$, $-(3(\frac{n-2}{2}) + 1), -(3\frac{n-2}{2})$ to the vertices $x_{n-2}, u_n, z_{\frac{n}{2}}$.

In both cases, obviously $\Delta_{f_1} = 2n - 1 = \Delta_{f_1^c}$. Therefore this vertex labeling gives that $S(AT_n)$ is pair difference cordial if the edge u_1u_2 lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle for all even $n \geq 4$.

□

Theorem 3.6. *$S(A(T_n))$ is pair difference cordial if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle for all even $n \geq 4$.*

Proof. Consider the vertex set and edge set of alternate triangular snake $A(T_n)$ as in definition 2.5. Let the edge $u_iu_{i+1}, u_{2i}v_i, u_{2i+1}v_i$ be subdivided by the vertex x_i, y_i, z_i. It is easy to verify that $S(A(T_n))$ has $\frac{7n-8}{2}$ vertices and $4n-6$ edges.
There are two cases arises.

Case 1. $n \equiv 0 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-2}{2})-2$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-2}{2})$ respectively to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n+2}{2}}$. Next assign the labels $-1, -7, -13, \cdots, -(3(\frac{n-2}{2})-8)$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}, \cdots, u_{n-2}$ and assign the labels $-3, -9, -15, \cdots, -3(\frac{n-2}{2})+6$ respectively to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}, \cdots, u_{n-1}$ and assign the labels $\frac{7n-8}{4}, -\frac{7n-8}{4}$ respectively to the vertices u_1, u_n.

Next consider the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-2}{2})-1$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots, -(3(\frac{n-2}{2})-1)$ to the vertices $x_{\frac{n+4}{2}}, x_{\frac{n+8}{2}}, x_{\frac{n+12}{2}}, \cdots, x_{n-2}$. Now assign the labels $3(\frac{n-2}{2})+1, 3(\frac{n-2}{2})+2, 3(\frac{n-2}{2})+3, \cdots, \frac{7n-12}{4}$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-2}{2}}$ and assign the labels $-(3(\frac{n-2}{2})+1), -(3(\frac{n-2}{2})+2), -(3(\frac{n-2}{2})+3), \cdots, -\frac{7n-12}{4}$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}}, x_{\frac{n+10}{2}}, \cdots, x_{n-1}$.

Now we move to the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-2}{2})-4$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-4}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-2}{2})-4)$ to the vertices $y_{\frac{n+4}{4}}, y_{\frac{n+8}{4}}, y_{\frac{n+12}{4}}, \cdots, y_{\frac{n-2}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-2}{2})-3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-4}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-2}{2})-3)$ to the vertices $v_{\frac{n+4}{4}}, v_{\frac{n+8}{4}}, v_{\frac{n+12}{4}}, \cdots, v_{\frac{n-2}{2}}$.

We now consider the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-2}{2})-5$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-4}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-2}{2})-5)$ to the vertices $z_{\frac{n+4}{4}}, z_{\frac{n+8}{4}}, z_{\frac{n+12}{4}}, \cdots, z_{\frac{n-2}{2}}$. Finally assign the labels $-(3(\frac{n-2}{4})-2), -(3(\frac{n-2}{2})+1), -(3(\frac{n-2}{2}))$ to the vertices $z_{\frac{n}{4}}, v_{\frac{n}{4}}, y_{\frac{n}{4}}$.

Case 2. $n \equiv 2 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-4}{2})-2$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-4}{2})$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n}{2}}$. Next assign the labels $-1, -7, -13,$

$\cdots, -(3(\frac{n-4}{2})-2)$ to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots, u_{n-2}$ and assign the labels $-3, -9, -15, \cdots, -3(\frac{n-4}{2})$ respectively to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}, \cdots,$ u_{n-1}.

Now we move to the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-4}{2})-1$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-2}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots,$ $-(3(\frac{n-4}{2})-1)$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+6}{2}}, x_{\frac{n+10}{2}}, \cdots, x_{n-2}$. Now assign the labels $3(\frac{n-4}{2})+4, 3(\frac{n-4}{4})+5, 3(\frac{n-4}{2})+6, \cdots, \frac{7n-10}{4}$ to the vertices $x_3, x_5, x_7, \cdots, x_{\frac{n-4}{4}}$ and assign the labels $3(\frac{n-4}{2})+4, 3(\frac{n-4}{4})+5, 3(\frac{n-4}{2})+6, \cdots, \frac{7n-10}{4}$ to the vertices $x_{\frac{n+2}{4}}, x_{\frac{n+10}{4}}, x_{\frac{n+18}{4}}, \cdots, x_{n-2}$.

We now consider the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-4}{2})+2$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-2}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-2}{2})+$ $2)$ to the vertices $y_{\frac{n+2}{4}}, y_{\frac{n+6}{4}}, y_{\frac{n+10}{4}}, \cdots, y_{\frac{n-2}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-4}{2})+3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-4}{2})+3)$ to the vertices $v_{\frac{n+2}{4}}, v_{\frac{n+6}{4}},$ $v_{\frac{n+10}{4}}, \cdots, v_{\frac{n-2}{2}}$.

Next we move to the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-4}{2})+1$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-2}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-4}{2})+$ $1)$ to the vertices $z_{\frac{n+2}{4}}, z_{\frac{n+6}{4}}, z_{\frac{n+10}{4}}, \cdots, z_{\frac{n-2}{2}}$. Finally assign the labels $-\frac{7n-6}{4}$, $-(\frac{7n-10}{4})$, $\frac{7n-10}{4}$ to the vertices u_1, u_n, x_1.

In all the cases, clearly $\Delta f_1 = 2n-3 = \Delta {f_1}^c$.This vertex labeling gives that $S(AT_n)$ is pair difference cordial graph if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle for all even $n \geq 4$.

□

Theorem 3.7. *$S(A(T_n))$ is pair difference cordial if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle for all odd $n \geq 3$.*

Proof. Consider the vertex set and edge set of alternate triangular snake $A(T_n)$ as in definition 2.5. Let the edges $u_iu_{i+1}, u_{2i}v_i, u_{2i+1}v_i$ be subdivided by the vertex x_i, y_i, z_i.Clearly $S(A(T_n))$ has $\frac{7n-7}{2}$ vertices and $4n-4$ edges.There are two cases arises.

Case 1. $n \equiv 1 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-3}{2})-2$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-1}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-3}{2})$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n+1}{2}}$. Next assign the labels $-1, -7,$ $-13, \cdots, -(3(\frac{n-3}{2})-2)$ to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}}, \cdots, u_{n-1}$ and assign the labels $-3, -9, -15, \cdots, -3(\frac{n-3}{2})$ respectively to the vertices $u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, u_{\frac{n+13}{2}}, \cdots, u_n$.

Next consider the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-3}{2}) - 1$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n-1}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots, -(3(\frac{n-3}{2})-1)$ to the vertices $x_{\frac{n+3}{2}}, x_{\frac{n+7}{2}}, x_{\frac{n+11}{2}}, \cdots, x_{n-1}$. Now assign the labels $3(\frac{n-3}{2}) + 4, 3(\frac{n-3}{2}) + 5, 3(\frac{n-3}{2}) + 6, \cdots, \frac{7n-7}{4}$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-3}{2}}$ and assign the labels $-3(\frac{n-3}{2}) + 4, -3(\frac{n-3}{2}) + 5, -3(\frac{n-3}{2}) + 6, \cdots, -\frac{7n-7}{4}$ to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+5}{2}}, x_{\frac{n+9}{2}}, \cdots, x_{n-2}$.

Now we move to the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-3}{2})+2$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-1}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-2}{2}) + 2)$ to the vertices $y_{\frac{n+3}{4}}, y_{\frac{n+7}{4}}, y_{\frac{n+11}{4}}, \cdots, y_{\frac{n-1}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-2}{2}) + 3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-1}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-2}{2})+3)$ to the vertices $v_{\frac{n+3}{4}}, v_{\frac{n+7}{4}}, v_{\frac{n+11}{4}}, \cdots, v_{\frac{n-1}{2}}$.

Next we consider the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-3}{2})+1$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-1}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-3}{2}) + 1)$ to the vertices $z_{\frac{n+3}{4}}, z_{\frac{n+7}{4}}, z_{\frac{n+11}{4}}, \cdots, z_{\frac{n-1}{2}}$. Finally assign the labels $-(\frac{7n-7}{4})$ to the vertices u_1.

Case 2. $n \equiv 3 \pmod 4$.

First we consider the vertices u_i. Assign the labels $1, 7, 13, \cdots, 3(\frac{n-1}{2})-2$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n+1}{2}}$ respectively and assign the labels $3, 9, 15, \cdots, 3(\frac{n-1}{2})$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n+3}{2}}$. Next assign the labels $-1, -7, -13, \cdots, -(3(\frac{n-1}{2}) - 8)$ to the vertices $u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, u_{\frac{n+13}{2}}, \cdots, u_{n-1}$ and assign the labels $-3, -9, -15, \cdots, -(3(\frac{n-1}{2})-6)$ respectively to the vertices $u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}}, u_{\frac{n+15}{2}}, \cdots, u_n$.

Next consider the vertices x_i. Assign the labels $2, 8, 14, \cdots, 3(\frac{n-1}{2}) - 1$ to the vertices $x_2, x_4, x_6, \cdots, x_{\frac{n+1}{2}}$ respectively and assign the labels $-2, -8, -14, \cdots, -(3(\frac{n-1}{2})-7)$ to the vertices $x_{\frac{n+5}{2}}, x_{\frac{n+9}{2}}, x_{\frac{n+13}{2}}, \cdots, x_{n-1}$. Now assign the labels $3(\frac{n-1}{2}) + 1, 3(\frac{n-1}{2}) + 2, 3(\frac{n-1}{2}) + 3, \cdots, \frac{7n-5}{4}$ to the vertices $x_1, x_3, x_5, \cdots, x_{\frac{n-1}{2}}$ and assign the labels $-3(\frac{n-1}{2}) + 1, -3(\frac{n-1}{2}) + 2, -3(\frac{n-1}{2}) + 3, \cdots, -\frac{7n-9}{4}$ to the vertices $x_{\frac{n+3}{2}}, x_{\frac{n+7}{2}}, x_{\frac{n+11}{2}}, \cdots, x_{n-1}$.

We now consider the vertices y_i. Assign the labels $5, 11, 17, \cdots, 3(\frac{n-3}{2})-1$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-3}{4}}$ and assign the labels $-5, -11, -17, \cdots, -(3(\frac{n-3}{2}) -$

1) to the vertices $y_{\frac{n+5}{4}}, y_{\frac{n+9}{4}}, y_{\frac{n+13}{4}}, \cdots, y_{\frac{n-1}{2}}$. Next we consider the vertices v_i. Assign the labels $6, 12, 18, \cdots, 3(\frac{n-3}{2})$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-3}{4}}$ and assign the labels $-6, -12, -18, \cdots, -(3(\frac{n-3}{2}))$ to the vertices $v_{\frac{n+5}{4}}, v_{\frac{n+9}{4}}, v_{\frac{n+13}{4}}, \cdots, v_{\frac{n-1}{2}}$.

Next we move to the vertices z_i. Assign the labels $4, 10, 16, \cdots, 3(\frac{n-3}{2})-2$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-3}{4}}$ and assign the labels $-4, -10, -16, \cdots, -(3(\frac{n-3}{2}) - 2)$ to the vertices $z_{\frac{n+5}{4}}, z_{\frac{n+9}{4}}, z_{\frac{n+13}{4}}, \cdots, z_{\frac{n-1}{2}}$. Finally assign the labels $-(\frac{7n-5}{4})$, $-(3(\frac{n-1}{2}))$ to the vertices $u_1, y_{\frac{n+1}{4}}$.

In both cases, obviously $\Delta f_1 = 2n-2 = \Delta {f_1}^c$. Therefore this vertex labeling gives that $S(A(T_n))$ is pair difference cordial if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle for all odd $n \geq 3$. □

4. Discussion

The concept of the pair sum labeling was introduced by Ponraj and Parthipan in [23]. Laterly Ponraj, Sathishnarayanan and Kala defined the difference cordial labeling of graphs in [24]. Motivated by these two concepts, we have introduced a new concept called pair difference cordial labeling of graphs. A consequential hypothesis arising in this context is that all graphs can be expected to possess the property of pair difference cordial labeling. Accordingly,the pair difference cordial labeling behaviour of subdivision of some graphs like $S(K_{1,n}), S(B_{n,n}), S(L_n), S(T_n), S(AT(_n))$ have been investigated in this paper.

5. Limitation of Research

Presently, it is difficult to investigate the pair difference cordial labeling behaviour of pyramid graph, swastik graph and circular lobster graph.

6. Future Research

The pair difference cordial labeling behaviour of cycle with a P_k chord, cycle with parallel P_k chord and cylinder graph are the possible future directions of research work.

7. Conclusion

In this paper , we have studied about the pair difference cordial labeling behaviour of subdivision of some graphs like $S(K_{1,n}), S(B_{n,n}), S(L_n), S(T_n)$, $S(AT(_n))$. The pair difference cordial labeling of subdivisions of some other graphs like $S(Q_n), S(AQ_n), S(DT(_n)), S(ADT(_n))$ are the open problems.

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