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SOME BOUNDS FOR ZEROS OF A POLYNOMIAL WITH RESTRICTED COEFFICIENTS

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Abstract. For a Polynomial $P(z) = \sum_{j=0}^{n} a_j z^j$ with $a_j \geq a_{j-1}$, $a_0 > 0$ $(j = 1, 2, ..., n)$, a classical result of Enestrom-Kakeya says that all the zeros of $P(z)$ lie in $|z| \leq 1$. This result was generalized by A. Joyal et al. [3] where they relaxed the non-negative condition on the coefficents. This result was further generalized by Dewan and Bidkham [9] by relaxing the monotonicity of the coefficients. In this paper, we use some techniques to obtain some more generalizations of the results [3], [8], [9].

1. Introduction

The fundamental theorem of Algebra gives the guarantee for existence of as many zeros of a polynomial as its degree in the complex plane. But the impossibility of solving algebraically a polynomial equation of degree greater than 4 is an important problem in the history of Mathematics. Thus Mathematicians studied to identify the regions in a complex plane containing some or all the zeros of a given polynomial. The first result concerning the location of zeros of a polynomial was probably due to Gauss [1]. However, Enestrom and Kakeya independently put condition on the coefficients of a polynomial and proved the following elegant result [1] which is well known in the theory of distribution of the zeros of polynomials.

Theorem 1.1. *If* $P(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0$ *is a polynomial of degree* n *such that* $a_n \geq a_{n-1} \geq ... \geq a_1 \geq a_0 > 0$, *then all the zeros of P(z) lie in* $|z| \leq 1$.

This result is limited in scope as the hypothesis is very restrictive and the result does not hold for the polynomials with non-negative coefficients. There are many

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extensions and generalizations of Enestrom-Kakeya theorem in literature [7, 4, 6]. In this direction, Joyal et al. [3] proved the following result for polynomials with coefficients monotone but not positive.

Theorem 1.2. *Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0$ be a polynomial of degree n such that $a_n \geq a_{n-1} \geq ... \geq a_1 \geq a_0$, then all the zeros of P(z) lie in the disk $|z| \leq \frac{a_n - a_0 + |a_0|}{|a_n|}$.*

For a more general class of polynomials, Aziz and Zargar [8] proved the following generalization of Theorem 1.2.

Theorem 1.3. *If $P(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0$ is a polynomial of degree n such that for some $k \geq 1$, $ka_n \geq a_{n-1} \geq ... \geq a_1 \geq a_0$, then P(z) has all its zeros in $|z + k - 1| \leq \frac{ka_n - a_0 + |a_0|}{|a_n|}$.*

However, in this direction Dewan and Bidkham [9] obtained an interesting result by relaxing the condition of monotonicity of the coefficients. Infact, they proved the following result.

Theorem 1.4. *If $P(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0$ is a polynomial of degree n such that $a_n \leq a_{n-1} \leq ... \leq a_\lambda \geq a_{\lambda-1} \geq ... \geq a_1 \geq a_0$, then P(z) has all its zeros in $|z| \leq \frac{1}{|a_n|} \{2a_\lambda - a_n - a_0 + |a_0|\}$.*

The aim of this paper is to prove some generalizations of Theorem 1.3 and Theorem 1.4, which are also extensions of Theorem 1.1.

2. MAIN THEOREMS

Theorem 2.1. *Let $P(z) = \sum_{j=0}^{n} a_j z^j$ be a polynomial of degree n such that for some $k_1 \geq 1$, $k_2 \geq 1$, $k_1 a_n \geq k_2 a_{n-1} \geq a_{n-2} \geq ... \geq a_1 \geq a_0$. Then all the zeros of P(z) lie in*

$$\left| z + \frac{(k_1 - 1)\, a_n - (k_2 - 1)\, a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \{k_1 a_n - (k_2 - 1)\, a_{n-1} - a_0 + |a_0|\}.$$

Remark 2.1. Taking $k_1 = k$, $k_2 = 1$ in Theorem 2.1, we obtain Theorem 1.3.

Theorem 2.2. *Let $P(z) = \sum_{j=0}^{n} a_j z^j$ be a polynomial of degree n such that for some $k_1 \geq 1$, $k_2 \geq 1$, $k_1 a_n \leq k_2 a_{n-1} \leq ... \leq a_\lambda \geq a_{\lambda-1} \geq ... \geq a_1 \geq a_0$ $(0 \leq \lambda \leq n-1)$.*

Then all the zeros of P(z) lie in

$$\left|z+\frac{(k_1-1)\,a_n-(k_2-1)\,a_{n-1}}{a_n}\right| \le \frac{1}{|a_n|}\left\{\begin{array}{c} 2a_\lambda - k_1a_n + (k_2-1)a_{n-1} - a_0 \\ +|a_0| \end{array}\right\}.$$

Remark 2.2. By taking $k_1, k_2 = 1$, we obtain Theorem 1.4 and by taking $k_1 = k$, $k_2 = 1$ we obtain a result due to Shah and Liman [2] and for $\lambda = 0$ we get Theorem 1.3.

Theorem 2.3. *Let* $P(z) = \sum_{j=0}^{n} a_j z^j$ *be a polynomial of degree n such that for some* $k_\lambda \ge 1,\ \lambda = 1,2,...,n,\ k_1a_n \ge k_2a_{n-1} \ge ... \ge k_{\lambda-1}a_{n-\lambda+2} \ge k_\lambda a_{n-\lambda+1} \ge a_{n-\lambda} \ge ... \ge a_1 \ge a_0$. *Then all the zeros of P(z) lie in*

$$\left|z+\frac{(k_1-1)\,a_n-(k_2-1)\,a_{n-1}}{a_n}\right| \le \frac{1}{|a_n|}\left\{\begin{array}{c} k_1a_n-(k_2-1)\,|a_{n-1}| \\ +2\sum\limits_{j=2}^{\lambda}(k_j-1)\,|a_{n-j+1}| - a_0 \\ +\,|a_0| \end{array}\right\}.$$

Remark 2.3. Theorem 2.3 is a generalization of a result due to Shah and Liman [2] for $k_1 = k$ and $k_j = 1$; $j = 2, ..., \lambda$. Theorem 2.3 is also an extended generalization of a result due to Joyal et al. [3].

Proof of Main Theorem

Proof of Theorem 2.1. Consider a polynomial

$$\begin{aligned} F(z) &= (1-z)P(z) \\ &= \left[\begin{array}{c} -a_nz^{n+1} + (a_n-a_{n-1})\,z^n + (a_{n-1}-a_{n-2})\,z^{n-1} + ... \\ +\,(a_1-a_0)\,z + a_0 \end{array}\right] \\ &= \left[\begin{array}{c} -a_nz^{n+1} - \{(k_1-1)\,a_n - (k_2-1)\,a_{n-1}\}\,z^n \\ +\left\{\begin{array}{c} (k_1a_n - k_2a_{n-1})\,z^n + (a_{n-1}-a_{n-2})\,z^{n-1} + \\ ... + (a_1-a_0)\,z + a_0 \end{array}\right\} \end{array}\right] \end{aligned}$$

which implies

$$|F(z)| \ge \left[\begin{array}{c} |z|^n\,|a_nz + (k_1-1)a_n - (k_2-1)a_{n-1}| \\ -\,|(k_1a_n - k_2a_{n-1})z^n + ... + a_0| \end{array}\right].$$

Let $|z| > 1$ so that $\frac{1}{|z|} < 1$, which implies $\frac{1}{|z|^{n-j}} < 1; j = 0, 1, 2, ..., n-1$, therefore we have

$$|F(z)| \ge |z|^n \left[\begin{array}{c} |a_nz + (k_1-1)a_n - (k_2-1)a_{n-1}| \\ -\left\{\begin{array}{c} |k_1a_n - k_2a_{n-1}| + |a_{n-1}-a_{n-2}|\,\frac{1}{|z|} + \\ ... + |a_1-a_0|\,\frac{1}{|z|^{n-1}} + |a_0|\,\frac{1}{|z|^n} \end{array}\right\} \end{array}\right].$$

$$
\begin{aligned}
&\geq |z|^n \left[\begin{array}{c} |a_n z + (k_1-1)a_n - (k_2-1)a_{n-1}| \\ -\left\{ \begin{array}{c} |k_1 a_n - k_2 a_{n-1}| + |a_{n-1} - a_{n-2}| + \\ ... + |a_1 - a_0| + |a_0| \end{array} \right\} \end{array} \right] \\
&= |z|^n \big[|a_n z + (k_1-1)a_n - (k_2-1)a_{n-1}| \\
&\quad - k_1 a_n - (k_2-1)a_{n-1} - a_0 + |a_0| \big] > 0,
\end{aligned}
$$

if

$$
\left| z + \frac{(k_1-1)a_n - (k_2-1)a_{n-1}}{a_n} \right| > \frac{1}{|a_n|} \{k_1 a_n - (k_2-1)a_{n-1} - a_0 + |a_0|\}
$$

Therefore, it follows that the zeros of $F(z)$ with $|z| > 1$ lie in

$$
\left| z + \frac{(k_1-1)\,a_n - (k_2-1)\,a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \{k_1 a_n - (k_2-1)\,a_{n-1} - a_0 + |a_0|\}.
$$

But the zeros of $F(z)$ having modulus less than or equal to 1 already lie in this region and since all the zeros of $F(z)$ are also the zeros of $P(z)$, it follows that all the zeros of $P(z)$ lie in the disk

$$
\left| z + \frac{(k_1-1)\,a_n - (k_2-1)\,a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \{k_1 a_n - (k_2-1)\,a_{n-1} - a_0 + |a_0|\}.
$$

This completes the proof of Theorem 2.1. □

Example 2.1. Theorem 2.1 gives better result than that obtained by Theorem 1.3 and Theorem 1.4. Consider the polynomial $P(z) = z^7 + 3z^6 + 3z^5 + 2z^4 + 2z^3 + 2z^2 + 2z + 1$ of degree 7. By taking $k_1 = 5$, $k_2 = 1$ in Theorem 1.3 and Theorem 1.4, the zeros of $P(z)$ lie in $|z + 4| \leq 5$. However, by taking $k_1 = 5$, $k_2 = 3$ in Theorem 2.1, all the zeros of $P(z)$ lie in the circle $|z + 1| \leq 2$.

Proof of Theorem 2.2. Consider a polynomial $F(z) = (1 - z)P(z)$, then

$$
\begin{aligned}
F(z) &= (1-z)P(z) \\
&= \begin{array}{c} -a_n z^{n+1} + (a_n - a_{n-1})\,z^n + (a_{n-1} - a_{n-2})\,z^{n-1} + ... + (a_1 - a_0)\,z \\ +a_0 \end{array} \\
&= \left[\begin{array}{c} -a_n z^{n+1} - \{(k_1-1)\,a_n - (k_2-1)\,a_{n-1}\}\,z^n + (k_1 a_n - k_2 a_{n-1})\,z^n \\ + \big\{(a_{n-1} - a_{n-2})\,z^{n-1} + ... + (a_1 - a_0)\,z + a_0\big\} \end{array} \right] \\
&= \left[\begin{array}{c} -z^n\,[a_n z + (k_1-1)\,a_n - (k_2-1)\,a_{n-1}] + (k_1 a_n - k_2 a_{n-1})\,z^n \\ \big\{+ (a_{n-1} - a_{n-2})\,z^{n-1} + ... + (a_1 - a_0)\,z + a_0\big\} \end{array} \right]
\end{aligned}
$$

Let $|z| > 1$, then $\frac{1}{|z|} < 1$, therefore $\frac{1}{|z|^j} < 1$; $j = 0, 1, 2, ..., n-1$. Hence, we have

$$\begin{aligned} |F(z)| &\geq |z|^n \left[\begin{array}{c} |a_n z + (k_1 - 1) a_n - (k_2 - 1) a_{n-1}| \\ - \left\{ \begin{array}{c} |k_1 a_n - k_2 a_{n-1}| + |a_{n-1} - a_{n-2}| \frac{1}{|z|} + ... \\ + |a_{\lambda+1} - a_\lambda| \frac{1}{|z|^{n-\lambda-1}} + |a_\lambda - a_{\lambda-1}| \frac{1}{|z|^{n-\lambda}} + ... \\ + |a_1 - a_0| \frac{1}{|z|^{n-1}} + |a_0|^n \end{array} \right\} \end{array} \right] \\ |F(z)| &\geq |z|^n \left[\begin{array}{c} |a_n z + (k_1 - 1) a_n - (k_2 - 1) a_{n-1}| \\ - \left\{ \begin{array}{c} |k_1 a_n - k_2 a_{n-1}| + |a_{n-1} - a_{n-2}| + ... \\ + |a_{\lambda+1} - a_\lambda| + |a_\lambda - a_{\lambda-1}| + ... \\ + |a_1 - a_0| + |a_0| \end{array} \right\} \end{array} \right] \\ &= |z|^n \left[\begin{array}{c} |a_n z + (k_1 - 1) a_n - (k_2 - 1) a_{n-1}| \\ - \{2a_\lambda - k_1 a_n + (k_2 - 1) a_{n-1} - a_0 + |a_0|\} \end{array} \right] \\ &> 0, \end{aligned}$$

if

$$\left| z + \frac{(k_1 - 1) a_n - (k_2 - 1) a_{n-1}}{a_n} \right| > \frac{1}{|a_n|} \{2a_\lambda - k_1 a_n + (k_2 - 1) a_{n-1} - a_0 + |a_0|\}.$$

Hence all the zeros of $F(z)$ with $|z| > 1$ lie in

$$\left| z + \frac{(k_1 - 1) a_n - (k_2 - 1) a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \left\{ \begin{array}{c} 2a_\lambda - k_1 a_n \\ +(k_2 - 1) a_{n-1} - a_0 + |a_0| \end{array} \right\}.$$

Since all the zeros of $F(z)$ with $|z| \leq 1$ already lie in this region and since all the zeros of $F(z)$ are also the zeros of $P(z)$, it follows that all the zeros of $P(z)$ lie in the region

$$\left| z + \frac{(k_1 - 1) a_n - (k_2 - 1) a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \{2a_\lambda - k_1 a_n + (k_2 - 1) a_{n-1} - a_0 + |a_0|\}.$$

This completes the proof of Theorem 2.2. □

Example 2.2. Theorem 2.2 gives a better result than Theorem 1.4. Consider a polynomial $P(z) = 2z^6 + 3z^5 + 7z^4 + 9z^3 + 6z^2 + 3z + 1$ of degree 6, then by applying Theorem 1.4, it follows that the zeros of $P(z)$ lie in $|z| \leq 8$. However, by applying Theorem 2.2 for $k_1 = 3$ and $k_2 = 2$, the zeros of $P(z)$ lie in $\left|z + \frac{1}{2}\right| \leq \frac{9}{2}$.

Proof of Theorem 2.3. Consider a polynomial

$$\begin{aligned} F(z) &= (1 - z) P(z) \\ &= -a_n z^{n+1} + \{(a_n - a_{n-1}) z^n + ... + (a_1 - a_0) z + a_0\} \end{aligned}$$

$$= a_n z^{n+1} + z^n \left[\begin{array}{c} (a_n - a_{n-1}) + (a_{n-1} - a_{n-2})\frac{1}{z} + ... \\ +(a_{n-\lambda+2} - a_{n-\lambda+1})\frac{1}{z^{\lambda-2}} + (a_{n-\lambda+1} - a_{n-\lambda})\frac{1}{z^{\lambda-1}} \\ +(a_{n-\lambda} - a_{n-\lambda-1})\frac{1}{z^{\lambda}} + ... + (a_1 - a_0)\frac{1}{z^{n-1}} + a_0\frac{1}{z^n} \end{array} \right]$$

$$= -a_n z^{n+1}$$

$$+z^n \left[\begin{array}{c} \{-(k_1-1)a_n + (k_2-1)a_{n-1} + (k_1 a_n - k_2 a_{n-1})\} \\ +\{-(k_2-1)a_{n-1} + (k_3-1)a_{n-2} + (k_2 a_{n-1} - k_3 a_{n-2})\}\frac{1}{z} \\ +... + \left\{ \begin{array}{c} -(k_{\lambda-1}-1)a_{n-\lambda+2} + (k_\lambda - 1)a_{n-\lambda+1} \\ +(k_{\lambda-1}a_{n-\lambda+2} - k_\lambda a_{n-\lambda+1}) \end{array} \right\} \frac{1}{z^{\lambda-2}} \\ +\{-(k_\lambda-1)a_{n-\lambda+1} + (k_\lambda a_{n-\lambda+1} - a_{n-\lambda})\}\frac{1}{z^{\lambda-1}} \\ +\left\{(a_{n-\lambda} - a_{n-\lambda-1})\frac{1}{z^\lambda} + ... + (a_1 - a_0)\frac{1}{z^{n-1}} + a_0\frac{1}{z^n}\right\} \end{array} \right]$$

$$= \left[\begin{array}{c} -a_n z^{n+1} + z^n\{-(k-1)a_n + (k_2-1)a_{n-1}\} + (k_1 a_n - k_2 a_{n-1}) \\ +\{-(k_2-1)a_{n-1} + (k_3-1)a_{n-2} + (k_2 a_{n-1} - k_3 a_{n-2})\}\frac{1}{z} + \\ ... + \left\{ \begin{array}{c} -(k_{\lambda-1}-1)a_{n-\lambda+2} + (k_\lambda-1)a_{n-\lambda+1} \\ +(k_{\lambda-1}a_{n-\lambda+2} - k_\lambda a_{n-\lambda+1}) \end{array} \right\} \frac{1}{z^{\lambda-2}} + \\ \{-(k_\lambda-1)a_{n-\lambda+1} + (k_\lambda a_{n-\lambda+1} - a_{n-\lambda})\}\frac{1}{z^{\lambda-1}} \\ +\left\{(a_{n-\lambda} - a_{n-\lambda-1})\frac{1}{z^\lambda} + ... + (a_1 - a_0)\frac{1}{z^{n-1}} + a_0\frac{1}{z^n}\right\} \end{array} \right]$$

Let $|z| > 1$, then $\frac{1}{|z|} < 1$ and hence $\frac{1}{|z|^j} < 1$; $j = 0, 1, 2, ..., n-1$.
Therefore

$$|F(z)| \geq |z|^n \left[\begin{array}{c} |a_n z + (k-1)a_n - (k_2-1)a_{n-1}| \\ -\left\{ \begin{array}{c} |k_1 a_n - k_2 a_{n-1}| + (k_2-1)|a_{n-1}| + (k_3-1)|a_{n-2}| \\ +|k_2 a_{n-1} - k_3 a_{n-2}| + ... + (k_{\lambda-1}-1)|a_{n-\lambda+2}| \\ +(k_\lambda-1)|a_{n-\lambda+1}| + |k_{\lambda-1}a_{n-\lambda+2} - k_\lambda a_{n-\lambda+1}| \\ +(k_\lambda-1)|a_{n-\lambda+1}| + |k_\lambda a_{n-\lambda+1} - a_{n-\lambda}| \\ +|a_{n-\lambda} - a_{n-\lambda-1}| + ... + |a_1 - a_0| + |a_0| \end{array} \right\} \end{array} \right]$$

$$\geq |z|^n \left[\begin{array}{c} |a_n z + (k_1-1)a_n - (k_2-1)a_{n-1}| \\ -\left\{ \begin{array}{c} k_1 a_n - (k_2-1)|a_{n-1}| \\ +2\sum_{j=2}^{\lambda}(k_j-1)|a_{n-\lambda+1}| - a_0 + |a_0| \end{array} \right\} \end{array} \right]$$

$$> 0,$$

if

$$\left| z + \frac{(k_1-1)a_n - (k_2-1)a_{n-1}}{a_n} \right| > \frac{1}{|a_n|} \left\{ \begin{array}{c} k_1 a_n - (k_2-1)|a_{n-1}| + \\ +2\sum_{j=2}^{\lambda}(k_j-1)|a_{n-\lambda+1}| - a_0 + |a_0| \end{array} \right\}.$$

Hence all the zeros of $F(z)$ with $|z| > 1$ lie in

$$\left| z + \frac{(k_1-1)a_n - (k_2-1)a_{n-1}}{a_n} \right| \leq \frac{1}{|a_n|} \left\{ \begin{array}{c} k_1 a_n - (k_2-1)|a_{n-1}| + \\ +2\sum_{j=2}^{\lambda}(k_j-1)|a_{n-\lambda+1}| - a_0 + |a_0| \end{array} \right\}.$$

Since all the zeros of $F(z)$ with $|z| \leq 1$ already lie in this region and since all the zeros of $F(z)$ are also the zeros of $P(z)$, therefore it follows that all the zeros of the polynomial $P(z)$ lie in

$$\left|z + \frac{(k_1 - 1)\, a_n - (k_2 - 1)\, a_{n-1}}{a_n}\right| \leq \frac{1}{|a_n|} \left\{ \begin{array}{c} k_1 a_n - (k_2 - 1)\, |a_{n-1}| + \\ 2 \sum\limits_{j=2}^{\lambda} (k_j - 1)\, |a_{n-\lambda+1}| - a_0 + |a_0| \end{array} \right\}.$$

□

3. Conclusion

The results obtained in this paper give better bounds for the zeros of a polynomial as compared to the results available in literature.The applicability of our results has been demonstrated by examples. These results can be further extended for polynomials with Quaternionic variables and to other fields, hence has very good scope for further research. Besides, the zero bounds of polynomial has applications in various subjects like Algebraic Number Theory, Hilbert Space Theory, Computer Science, Cryptography, Engineering etc.

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