Studying The Topic Of The Function Extremum Of Two Variables In The Conditions Of Remote Learning And Application Of Digital Technologies

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Summary

In contemporary education, the rapid advancement of digital technologies elevates demands for integrating the latest tools into the learning process. Mathematical analysis, as a discipline, benefits from computer mathematics in distance education, enhancing practical aspects and enabling individualized learning. This article addresses the integration of the Maple computer mathematics system into higher education, specifically in teaching "Mathematical Analysis." Emphasizing its role in distance learning, computer mathematics optimizes the educational environment, reducing the time required for knowledge acquisition. The article showcases the application of Maple in finding extremum points and introduces an educational software simulator, enabling students to practice the method. The simulator, developed within Maple, facilitates self-checking and enhances the study of functions. Conclusions drawn from the study highlight the positive impact of these tools on distance education, affirming Maple's role in enhancing professional training and information culture among higher education students.

Keywords:
Training Maple-simulator; digital technologies; system of computer mathematics; method of finding extremum of function of two variables; typical problems of mathematical analysis; Maple.

1. Introduction

The Problem Statement. In modern science, remote learning technologies based on the use of information and communication technologies (ICT) are steadily entering the practice of many training institutions of forms and levels. Since one of the main advantages of remote education is independence from a geographical location, from the distance between the teacher and the student, and it is also important that remote education reflects all the inherent components of the training process (goals, content, methods, organizational forms, means of learning) and is implemented by specific means of Internet technology or other means, implying interactivity [1].

Forced remote learning caused by the pandemic has become the only available form of learning, and on the other hand a challenge for all participants in the educational process. To organize quality learning using digital technology, to inspire and motivate students, to solve technical problems - these are the requirements of the time, new conditions of life, new and unified means of interaction and, finally, a new style of thinking.

Today, the use of digital technology in education is one of the most important and sustainable trends in the development of the global educational process. Digital technologies allow intensifying the educational process, increasing the speed and quality of perception, understanding and assimilation of knowledge.

Perspective direction for activation of educational activity of students is a change of a technique of training the mathematical analysis through introduction of digital technologies and systems of computer mathematics for strengthening of a role of independent work of the student.

Mathematics as an academic subject has great opportunities for the implementation of new means of information technology during remote learning, as the use of computer mathematics systems and computers gives the opportunity to strengthen the applied and practical orientation of the course of mathematics and creates conditions for the implementation of an individual approach to a qualitatively new level. Digital technology allows you to make the process of learning individual, differentiated and mobile. At the same time, digital technology does not replace the teacher, but complements him.

Analysis of recent research and publications. One of the conditions for improving mathematical training in higher education institutions is the active use of computer mathematical systems (CMS). Every year, the number of scientific and educational and methodological works devoted to the use of CMS in the process of higher mathematics training is increasing. The general questions

of using information technologies in mathematics teaching at secondary and high schools are investigated in the works of V. Y. Bykov [2] , M. I. Zhaldak [3], V. I. Klochko, S. G. Litvinov [4], V. M. Mikhalevich [5], S. A. Rakov, Y. S. Ramski, O. V. Spivakovsky, S. A. Semerikov, Y. V. Trius [6].

The system of computer mathematics gives an opportunity to enrich the subjects of natural-and-mathematical cycle, to extend their application, to influence significantly the mathematical activity. Therefore, the main content of mathematical training for students will be not mastering certain algorithms of problem solving (they, by the way, are quite effectively solved by computer), but mathematical competence, understanding, application of mathematical research methods [7]. All this should be taken into account in the development of methodological systems for teaching mathematically oriented subjects in higher education.

In this article, the scientist shares his experience of using the Maple computer algebra system in teaching mathematics to students majoring in technical studies. Authors says that the course of mathematics for engineering students should be focused on an applied approach to mastering the mathematical apparatus, rather than computer technology. This approach teaches future engineers to use that skill to solve their job tasks. The authors have used Maple computing environment in teaching mathematics for several years. This graphic system can produce curve and surfaces graphics specified in different coordinate systems quickly and accurately. The animation of basic mathematical notions help learners succeed in course without sticking to complex computations. Visualized mathematical knowledge via Maple allows improving education. [8]

In this paper [9] presents the use of Maple as a multimedia tool in classrooms, in particular, in mathematics and related science classes for undergraduate students. Author says the mathematical computations and manipulations can be extensive and tedious for students in solving large and complex problems. He employ interactive Maple sessions in many classrooms such as in calculus, applications of abstract algebra, combinatorial, and linear algebra to solve for solutions in either numeric form or symbolic form. With explanation of the computational process and time saving involved, students find the integration of this technology into regular lectures make a significant improvement in their learning process.

A study to explore lecturers' views regarding a newly developed integral calculus with Maple software module was conducted. Engineering technology students can attain a meaningful mathematics learning if they are allowed to actively participate in hands-on activities. Nine lecturers with at least eight years of teaching experience were involved in the evaluation of the module. As a result, a module which emphasized on student-centered learning based on conceptual and procedural understanding and metacognitive awareness teaching approach will be produced. This module will be used to enhance students' procedural and conceptual understanding in learning integral calculus at the university. [10]

The aim of the present paper [11] is to have an introduction trip focusing on the advantages of using powerful software like Maple in computational teaching. Some examples are focused on representing the objects and basic algebraic manipulation commands, very successfully used in teaching sciences. In this context, the resources, including the collection of online lessons plan, videos and other resources were created to provide a better understanding of CT. But the most important are software resources, because of their flexibility and wide application tools.

In the article [12] authors shares the experience of using Maple computer algebra system in teaching mathematics to students majoring in technical studies. Universities need to change classical teaching methods to educate top graduates properly to get knowledge accumulated by mankind and modern technologies. The authors have used Maple computing environment in teaching mathematics for several years. This graphic system can produce curve and surfaces graphics specified in different coordinate systems quickly and accurately. The animation of basic mathematical notions help learners succeed in course without sticking to complex computations. Visualized mathematical knowledge via Maple allows improving education. It motivates to acquire knowledge and trains better professionals. The experiment increased qualitative academic performance by 16,9% in the experimental group.

In our opinion, a promising use of computer mathematics systems is the development of computer simulators for solving problems of mathematical analysis with the implementation of symbolic calculations, which allow the student to get a step-by-step solution of problems.

Under the training Maple-simulators (TMS) we will understand the training simulators for solving the typical problems of higher mathematics, designed for automated reproduction of the step-by-step course of solving problems with the presence of text comments in the Ukrainian language, which are developed and function in the SCM Maple environment [5].

The purpose of the article. The purpose of the article is to develop and analyze the training Maple-simulator for the study of the function of two variables extremum as an element of activation of cognitive activity of students in terms of remote learning.

2. Theoretical Consideration

Theoretical and empirical methods of scientific research were used to perform the set tasks. In particular, the theoretical methods are: a study of legal framework, methodological and methodical works connected with the research problem, interpretation of literature sources, analysis and synthesis; empirical: questioning students, expert evaluation.

3. Experimental Consideration

Today, the number of academic hours allocated for the study of sections and topics of natural-and-mathematical cycle of subjects is aimed at increasing the proportion of independent work of the student: the share of hours allocated to lectures, practical and laboratory classes is reduced, but the amount of training resources and the number of academic hours that are put to independent study increases. Under such conditions, in order to improve the quality of education, it is natural to use digital technologies in the learning process of students, since the quality of independent work depends to a large extent on the level of implementation of ICT tools in the learning process. Studying the discipline "Mathematical analysis" and "Computer mathematics systems", future teachers form the skills in using information communication technologies in education. So, for example, when studying the course «Mathematical analysis», students master the section «Functions of many variables» - this is one of the important sections of mathematical analysis, which is used in physics, mechanics and other sciences in the creation of mathematical models and in solving various problems.

In this chapter, one of the topics is «Extremum of functions of two variables». Let's look at using the basic capabilities of Maple CMS to solve problems for finding extrema of functions of several variables. There are several commands in this system for investigating functions for extremum, which are included in the standard set of this program's library. In order to find the minimum and maximum of a function from one or many variables on a certain interval, you can use the following commands (Table 1):

Table 1: Standard commands of the Maple computer mathematics system

Minimum of a function	Maximum of a function
`f:= 4x^2+y^2+2:` `#minimize (f, vars, ranges, opts):` `minimize(f,x,y,x=-5..5,y=-5..5,location);` • `2, {[{x = 0, y = 0}, 2]}`	`f:=-4*x^2-(y-2)^2+4*x:` `#maximize(f, vars, ranges, opts):` `maximize(f,x,y,x=-5..5,y=-5..5,location);` • `1, {[{x = 1/2, y = 2}, 1]}`

where f is the function under study for extremum; vars is the list of variables by which minimum or maximum is searched for; ranges is the scope of variables of the form $x_1=a_1..b_1,\ x_2=a_2..b_2,\ ...,\ x_n=a_n..b_n$ for a function of n variables.

If you do not specify infinity... infinity, then the search for an extremum will be conducted only on the set of real numbers, opts is a list of optional parameters. For instance, if the parameter location (or location=true) is entered, the result is output in the extended form, the coordinates of points of minimum (maximum) are specified in curly brackets after the value of minimum (maximum). If the minimum (maximum) of the corresponding function doesn't exist or can't be found, the expression corresponding to the given function will be displayed, and if the parameter location is present the text location=false and an empty list will be displayed.

Let's analyze the work of the standard command built into the Maple CMS to find maxima and minima. As you can see, after calling this command we only got the maximum or minimum value of the function itself, and we should have known right away whether we were looking for the maximum or the minimum. In order to see the clear nature of the point and the behavior of the function, we should additionally plot the surface of the function using the plots3d package. Before accessing the commands of the package, you need to connect it using the with(plots) command (Fig. 1-2).
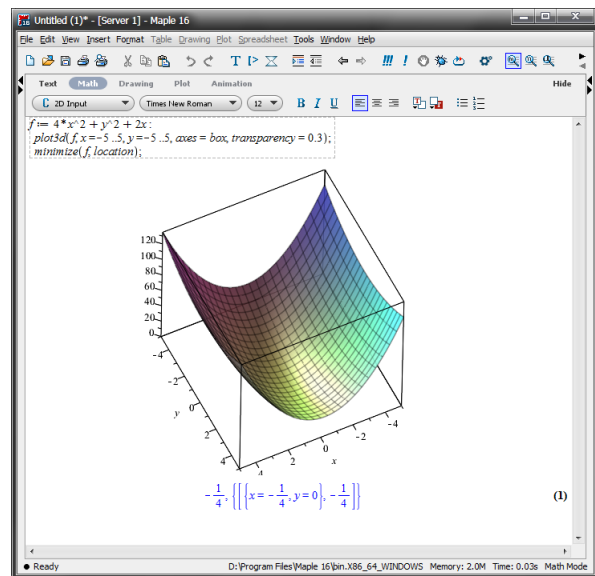

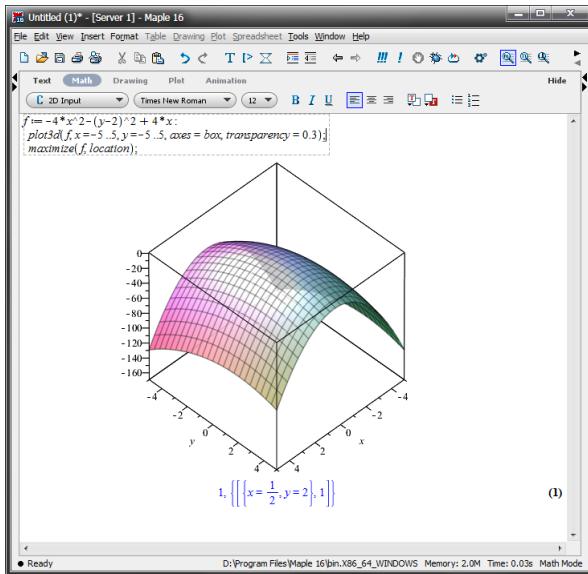Figure1. Minimum value of the function.

Figure2. Maximum value of the function.

As we can see from the work of the standard command, one of the main drawbacks of the Maple system of computer mathematics, on the way of its effective use in the training process, is the lack of adaptation of the system to the training purposes. The standard command does not provide a complete description of the search for a function extremum of two variables by classical mathematical rules and algorithms, which students learn during lectures and implement in practical classes, and in advance you need to know the nature of the point and the boundaries of the surface construction. As you know, this system was developed primarily for professional scientific and engineering activities. It does not demonstrate the step-by-step solution of examples, but only the final answer.

For effective work of students, in conditions of remote learning, and further independent work of students, adaptation of Maple CMS is offered by creation of author's training Maple-simulator, allowing not only to check the final result of a function extremum, but also to see each step of search of points of a maximum and minimum, and this, in turn, activates independent work of students and forms practical competences on the higher mathematics.

Two versions of the author's Maple-simulator have been developed: the most compact one, which can be implemented by a single programming line in the CMS Maple environment by connecting the corresponding author's library and the more advanced version, allowing students to reproduce the step-by-step process of solving the corresponding problems.

Let's demonstrate the operation of a compact Maple training simulator (Figure 3).
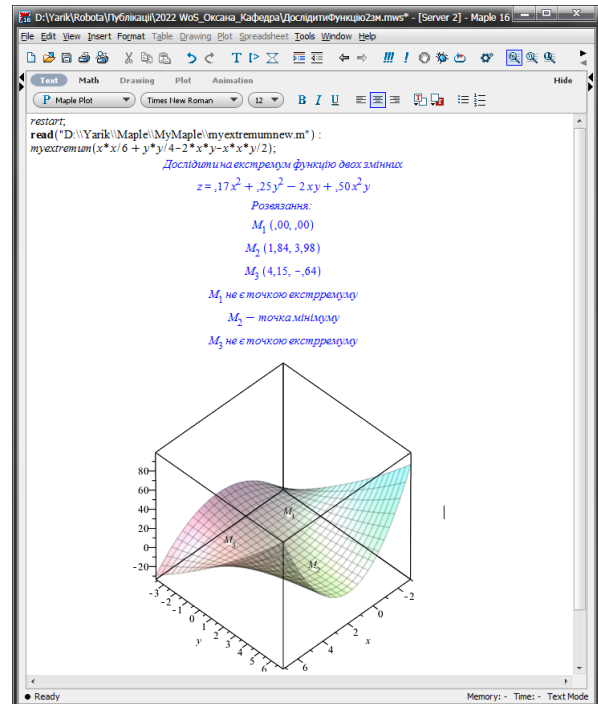

Figure 3. Compact variant of Maple training simulator.

Detailed version of the simulator and examples of its application. The program code of the learning Maple-simulator for the step-by-step search of points of extremum:

```
restart;
  with (Linear Algebra) : with (plots) : with
(plottools) :
  print(Дослідити на екстремум функцію двох
змінних);
    f:=x*x/6 +y*y/4-2*x*y+.5*x*x*y:
    z = f; z := f:
  print (Розвязання: );
  print (Знайдемо частинні похідні першого
порядку );
    `z'`[x]=diff(z,x);
    `z'`[y]=diff(z,y);
  print (Прирівняемо знайдені похідні до нуля,
та запишемо систему
    diff(z,x)=0;
    diff(z,y)=0;
  print    (Розвязавши    систему    матимемо
стаціонарні точки підозрілі екстремум);
    sp := [solve( {diff(z, x) = 0, diff(z, y) =0},
{x, y}, real) ] :
    for   i   from   1   to   nops   (sp)   do
print(M[i](rhs(sp[i][l]),rhs(sp[i][2]))  );
      xx[i]   :=   rhs(sp[i]   [  1])   :
yy[i] :=rhs(sp[i][2]):
    end do:
  print(Знаходимо відповідні частинні похідні
другого порядку);
    `z'`[xx]=diff(z,x) ` '`[x];
    `z''`[xy]=diff(z,x) ` '`[y];
```

```
    V[yy]=diff(^y)^[y];
  print(Одержані значення позначаємо A, B, C);
    A=`z''`[xx];
A:=diff(diff(z,x),x): A=A;
    B=`z''`[xy];
B:=diff(diff(z,x),y): B=B;
    C=`z''`[yy];
C:=diff(diff(z,y),y): C=C;
  print (Складемо вираз );
    Delta = Matrix(2, [[A, B], [B,C]]);
    Delta = AC-B2;
  print(Підставимо значення A, B, C в отриманий
вираз );
    Delta = Matrix(2, [[A, B], [B,C]]);
    DD := Determinant Matrix( 2, [ [ A, B], [B,
C] ])) :
    Delta = DD;
  print(Обчислимо значення параметрів A, B, C
та визначника для кожної знайденої точки і
проаналізуємо отримані результати);
    x1 := min(xx[iii]$iii= 1..nops(sp)) :
    x2 := max(xx[iii]$iii= 1..nops(sp)) :
    yl := min(yy[iii]$iii= 1..nops(sp)) :
    y2 := max(yy[iii]$iii= 1..nops(sp)):
  for i from 1 to nops (sp) do print (Для точки
 M [ i ] );
Al:=subs(x=rhs(sp[i][l]),y= rhs(sp[i][2]),A):
Bl:=subs(x=rhs(sp[i][l]),y = rhs(sp[i][2]),B):
C1:=subs(x=rhs(sp[i][l]),y = rhs(sp[i][2]),C):
DDl:=subs(x=rhs(sp[i][l]),y=rhs(sp[i][2]),DD):
    print( A = A1, B = B1, C = CI, Delta = DDl);
    if (DDl > O and Al > 0) then print(M[i] -
точка мінімуму); end if:
    if (DDl > O and Al < 0) then print(M[i] -
точка максимуму); end if:
    if (DD1 < 0) then print (M [ i ] не є точкою
екстрремуму ); end if:
    if (DDl = 0) then print( для зясування
характеру точки потрібні додаткові
дослідження ); end if:
    g[i]                                :=
pointplot3d( [rhs(sp[i][l]),rhs(sp[i][2]),su
bs(x=rhs( sp[ i ] [ 1 ]), y=rhs( sp[ i] [ 2]),
f) ], axes = box, symbol = sphere, colour=red) :
img[ i] := textplot3d( [rhs( sp[ i ] [ 1 ]),
rhs( sp[ i ] [ 2]), subs(x= rhs(sp[i][l]),y =
rhs(sp[i][2]),f) + 2, M[i]],axes = box):
    ww[i] := pointplot( [rhs( sp[i] [ 1 ]),
rhs( sp[ i] [2]) ]) :
  end do:
  otst := 3:
glO := plot3d(z, x = xl − otst ..x2 + otst, y
= yl − otst ..y2 + otst, axes= box, transparency
= 0.5):
display(gl0,   g[iii]$iii=   1   ..nops(sp),
img[ii]$ii = 1 ..nops(sp));
gl1 := contourplot( f, x = xl − otst ..x2 + otst,
y = yl − otst ..y2 + otst, filledregions = true,
coloring = [ "White", "DarkViolet", contours=
10]):
display( gl1, ww[ iii]$iii = 1..nops (sp));
```

```
gl2 := contourplot3d( z, x = xl − otst ..x2 +
otst, y = yl − otst ..y2 + otst, filledregions
= true):
display(gl2,  g[ iii]$iii  =  1  ..nops(sp),
img[ ii]$ii = 1 ..nops(sp));
```

The result of this simulator is shown in Figure 4-5.
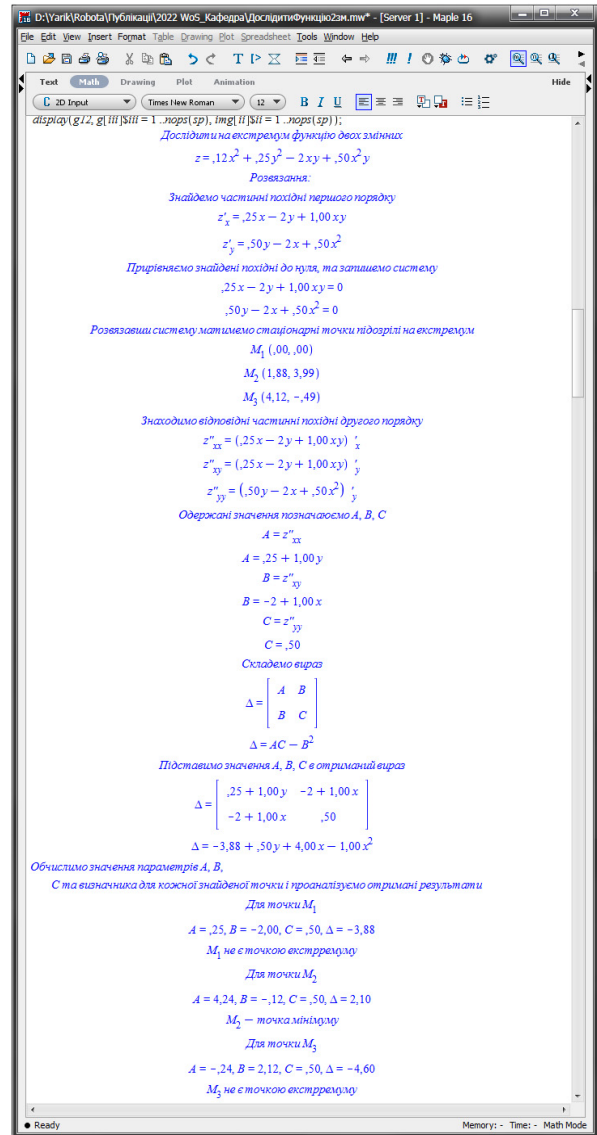

Figure 4. Expanded version of the training Maple-simulator.
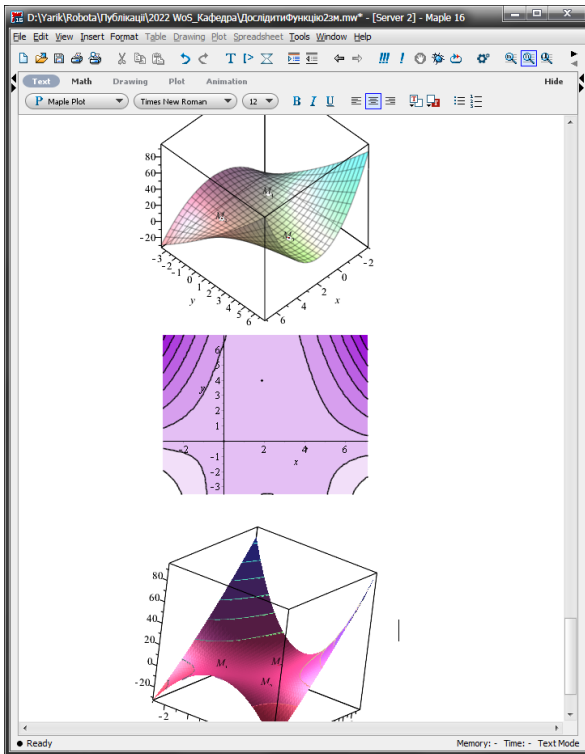
Figure 5. Expanded version of the training Maple-simulator.

If a function has both a maximum point and a minimum point, the disadvantage of the standard Maple CMS procedure is that it has no command indicating the nature of the point. The standard procedure is already designed for maximum or minimum, so you have to introduce a separate procedure to find the maximum point and a separate procedure to find the minimum point. The author's Maple-simulator is not bound to the nature and number of points of extremum, it makes it possible to obtain all possible points of extremum simultaneously and specifies their type, the student only needs to specify the function under study.

In the educational process the training Maple simulators have a variety of functions (informational and cognitive, managerial, didactic, motivational, etc.), so they can be used at "Mathematical Analysis" classes or outside normal hours. At classes the training Maple simulators can be used at different stages and for different purposes. In the beginning to update the background knowledge. For example, prior studying the topic "Gradient descent method," it is advisable to ask students to review the surface shape and critical points. When explaining new material - to provide interest and motivation to study the topic. At the end of the class - to consolidate the studied material, to form skills and to gain experience. Creation and use of such simulators should be treated with special attention. It should also be taken into account that students should analyze the solution of the problem.

During the individual work of the student the training Maple simulators can be used to revise the studied material or to organize mixed learning. During mixed learning, students are asked to work independently on the simulator on the topic "Extremum of a function of two variables", which will be considered in the practical class. Students review the step-by-step search for points of extremum at home, and a short and easy comprehension test can be performed in the classroom. In the practical class, students work with the instructor to review unclear points in the new material, solve problems to revise the topic studied, and work in pairs and groups. Consequently, after learning the theoretical material on the topic: "Extremum of a function of two variables" at home, the student comes to the class prepared and motivated. Such organization of teaching mathematics provides students with more opportunities for independent acquisition of knowledge and development of self-control. Students, while working with the simulator, have the opportunity to repeatedly change the condition and analyze the stages of the solution, if any questions arise, it is advisable to write them down in a notebook and consider them at a practical lesson.

To check the effectiveness of using the training Maple simulator in the student's individual work, a questionnaire survey was conducted among students who gave answers to questions about the use of the TMS.

When asked if they liked using the computer in learning, 95.4% of the students, among the 65 surveyed, gave a positive answer (Figure 6).
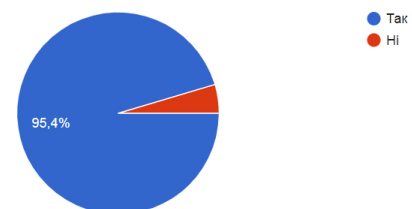

Figure 6. Using the computer in learning

The following answers prevailed in open-ended questions about the advantages of using the training Maple simulator: due to the possibility to get a step-by-step solution and to compare it with my own, in many cases I can find and understand my mistakes, or, if necessary, to see the next step without waiting for an opportunity to consult a teacher (87%); there is an opportunity to experiment with the problem's condition and to analyze changes in the step-by-step solution (39%); comparing my solution to that covered by the training Maple simulator, sometimes I get answers to important for me questions which I had before.

Among the disadvantages of using the training Maple simulator the following answers prevailed: I don't like working with a computer (4.6%); lack of a window - based interface (33%).

Three teachers of the Mathematics and Computer Science Department of Vinnytsia State Pedagogical University and one teacher from the Higher Mathematics Department of Vinnytsia National Technical University participated in the experiment on the use of the simulator. The teachers were interviewed about the use of the training Maple simulator at lectures and practical lessons. As a result of the discussion we came to the following conclusions: students work much more actively at classes using the computer than when only the chalk, board, and paper are used; there is significantly less time to provide advice on solving problems and the opportunity to pay attention to more in-depth issues; much faster and with less stress I can find and explain the error in the student's solution even if he could not find it himself using the training Maple simulator; it is easier to provide students with individual problems of appropriate level of complexity.

It should be noted that with this simulator, the student can check the correctness of his/her calculations as well as visually see the surface of the function and points of extremum. In addition, the training Maple-simulator provides the opportunity to conduct certain studies. For example, the student can change the condition and through repeated use of the NMT can better learn the material and learn how to use the method of finding points of extremum of a function of two variables.

4. Conclusion

Society in present-day is largely digitized and this tendency is gaining momentum. It ishighly competitive and this competitiveness will increase. Information competence of students and their striving for self-actualization will remain one of the basic requirements of the education system now and in the near future.

Thus, the use of information technology tools, in particular CMS and created on their basis intellectual author's simulators, significantly affects the content, methods, organizational forms of teaching methods of calculations of «Mathematical Analysis» and allows you to raise the level of professional training and informational culture of students. The availability of the extended version of the suggested NMT provides an opportunity for the student to perform self-monitoring and thus can be applied to a better and faster independent mastering of the material for the search for extremes of functions of several variables. However, both variants of the simulator deserve further research with the aim of their improvement and use both during classroom and remote work of students. The prospects for further research are support and development information communication technologies in the teaching of pedagogic students' skills in the use of computer mathematics system.

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