Analysis of the Influence of Atmospheric Turbulence on the Ground Calibration of a Star Sensor

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Under the influence of atmospheric turbulence, a star’s point image will shake back and forth erratically, and after exposure the originally small star point will spread into a huge spot, which will affect the ground calibration of the star sensor. To analyze the impact of atmospheric turbulence on the positioning accuracy of the star’s center of mass, this paper simulates the atmospheric turbulence phase screen using a method based on a sparse spectrum. It is added to the static-star-simulation device to study the transmission characteristics of atmospheric turbulence in star-point simulation, and to analyze the changes in star points under different atmospheric refractive-index structural constants. The simulation results show that the structure function of the atmospheric turbulence phase screen simulated by the sparse spectral method has an average error of 6.8% compared to the theoretical value, while the classical Fourier-transform method can have an error of up to 23% at low frequencies. By including a simulation in which the phase screen would cause errors in the center-of-mass position of the star point, 100 consecutive images are selected and the average drift variance is obtained for each turbulence scenario; The stronger the turbulence, the larger the drift variance. This study can provide a basis for subsequent improvement of the ground-calibration accuracy of a star sensitizer, and for analyzing and evaluating the effect of atmospheric turbulence on the beam.

Keywords: Atmospheric turbulence, Centroid extraction, Liquid crystal spatial light modulator, Sparse spectrum, Star sensor

OCIS codes: (010.1330) Atmospheric turbulence; (120.6085) Space instrumentation

I. INTRODUCTION

Before the actual operation of a star sensitizer in orbit, it is necessary to complete the calibration of the star-sensitizer system error in the ground environment, and the quality of a star point simulated by the star simulator will directly affect the accuracy of the star sensitizer’s ground calibration. Stellar sensitizers are subjected to a variety of complex environmental challenges during their actual operation, such as stray light, space radiation, and atmospheric turbulence. Atmospheric turbulence, an important factor affecting the ground observation of stars, can cause changes in the refractive index of the atmosphere and thus change the wave front of the beam, causing a series of effects such as light-intensity scintillation, the angle of arrival of the crests and troughs, spot drift, and the receiving end of the spot aberrations [1, 2]. These problems can lead to drifting of star-point pixels, large errors in the center of mass of the star map, etc., which in turn affect the ground calibration of the star sensitizer.

Currently the main methods used to model atmospheric turbulence phase screens are the power-spectrum-inversion method, the Zernike-polynomial method, the covariance method, and the sparse-spectrum method. In 2013, Wang et
al. [3] used the Zernike-polynomial method to simulate and experimentally validate the atmospheric turbulence phase screen. In 2015, Cai et al. [4] analyzed for the first time the error of atmospheric turbulence phase screens based on sparse spectral models, which can be generated without cycle length according to practical needs. In addition, they realized the fast, high-precision simulation of the atmospheric turbulence phase screen by using the nonuniform-sampling power-spectrum-inversion method. With the gradual maturity of liquid-crystal spatial light modulator (LC-SLM) technology, the use of LC-SLMs to simulate the phase aberration caused by atmospheric turbulence has become a major experimental option. In 2018, Lin [5] implemented the generation of atmospheric turbulence phase screens using the Zernike method, and analyzed these light spots at the receiving end with a dynamic reconstruction method. In 2021, Iwano et al. [6] demonstrated that the mean value of the logarithm of the turbulent-dissipation rate increases with decreasing spatial altitude. In 2022, an experimental study on turbulence spectral characteristics and similarity laws in Qinghai-Tibet region was made by Yang et al. [7, 8].

The Zernike polynomials are severely missing from the higher-frequency components at lower orders, and although the higher-frequency components are enriched as the order increases, the computational effort will be enormous in the higher-order operations, leading to a large consumption of time [9–12]. In the subharmonic complementary discrete Fourier-transform method, although the computational efficiency is improved, the generated phase screen is periodic. On the other hand, the sparse spectral model adopts the sequence-summation method, which is capable of generating phase screens of any size without periodicity, preserving the scale parameters related to atmospheric turbulence with low computational effort [13–17].

In this paper, we propose to introduce an atmospheric turbulence simulation unit into a static star-simulator system, in which the atmospheric turbulence phase screen is simulated based on a sparse spectral model, and the simulated dynamic atmospheric turbulence phase screen is loaded into a star chart using a LC-SLM to investigate the atmospheric transport properties of star points in the star chart.

II. SYSTEM COMPOSITION AND WORKING PRINCIPLE

To assess the effect of atmospheric turbulence on the accuracy of star sensitizers when they perform stargazing calibrations, this study chooses to incorporate an atmospheric turbulence phase screen during the simulation of star points by a static star simulator. The atmospheric turbulence simulation system is mainly composed of a collimated optical system, LC-SLM, beam splitter mirror and convergent optical system. Among them, the collimating optical system is composed of a light source, a filter, and a star-point divider, to simulate infinitely distant star points. The overall structure of the system is shown in Fig. 1.

The light emitted by the source is modified by the filter light plate, where the output wavelength of the laser is 785 nm, which illuminates the star-point divider engraved with a light-transmitting microvia to form a star-point light source, and the star-point light source yields parallel light rays after collimation. The random phase screen of atmospheric turbulence generated in accordance with the sparse spectral model distorts the phase of the beam, simulating wave-front aberrations caused by atmospheric turbulence. Parallel light is incident perpendicular to the target surface of the liquid-crystal spatial light modulator and drives the LC-SLM, after its reflection can change the wave-front phase information of the incident parallel light. The light with turbulence information is reflected back to the beam splitter to change the transmission direction, and then after passing the convergence system and collimated optical system the collimated parallel light is used to simulate the infinitely distant star point.

The accuracy and stability of the parallel light output of the collimating optical system directly affects the performance of the star simulator. The function of the collimating optical system is to convert the light passing through the star holes on the star-point reticle into parallel light, and then to project it to the entrance pupil of the star sensor to simulate the luminous characteristics of stars in space at infinity. To avoid the situation in which the exit pupil of the star simulator does not match the entrance pupil of the star sensor, the two need to form a coaxial optical system and satisfy the pupil-connection principle. Therefore, star simulators usually adopt a structure with an external exit pupil, and the exit pupil’s diameter is generally designed to be slightly larger than the entrance pupil’s diameter.

III. OPTICAL SYSTEM DESIGN

3.1. Basic Concepts of Sparse Spectral Methods

The sparse spectral approach used in this paper is similar to the traditional Fourier-transform simulation of atmo-
spheric turbulence modeling, which essentially involves each individual phase sample having a random discrete support in the wave-vector plane. The wave vector $\mathbf{k}_s$ is a random vector with probability distribution $p_s(\mathbf{k})$ and a random spectral amplitude.

$$P\{\mathbf{K}_s = \mathbf{k} + d\mathbf{K}\} = p_s(\mathbf{k})d\mathbf{K}. \tag{1}$$

Under phase-screen structure function obeying the turbulent power-law form, we can express the sparse-spectral-phase structure function as:

$$D(\mathbf{r}) = 2\int d^2\Phi(\mathbf{K})[1 - \cos(\mathbf{K}\cdot\mathbf{r})]. \tag{2}$$

The spectral density with respect to the wave vector is expressed as:

$$\phi(\mathbf{k}) = \frac{C(\alpha)p_s(\mathbf{k})}{(k^2 + k_0^2)^{\alpha/2}} \exp\left(\frac{k^2}{k_0^2}\right), 1 < \alpha < 2, \tag{3}$$

where the spectral density prefactor is:

$$C(\alpha) = \frac{\alpha^2}{\pi^{\alpha/2}} \Gamma\left(\frac{1 + \alpha}{2}\right). \tag{4}$$

Included among these, $k_0 = 2\pi/L_0$ and $k_n = 2\pi/l_n$. $L_0$ and $l_0$ are the outer and inner scales of atmospheric turbulence respectively, for which $\alpha = 5/3$.

### 3.2. Combining Sparse Spectral Methods to Simulate Phase Screens

Use the triangular series to represent the two-dimensional random phase field as the sum of harmonics with random normally distributed complex amplitudes $a_n$,

$$S(\mathbf{r}) = \text{Re} \sum_{n=1}^{N} a_n \exp(\mathbf{i} \mathbf{k}_n \cdot \mathbf{r}) n = 1, 2, \ldots, N, \tag{5}$$

where $\mathbf{k}_n$ is a random wave vector with uniform distribution. From Eq. (5), the structure function of the sparse spectral model can be expressed as:

$$D(\mathbf{r}) = \int d^2 K \sum_{n=1}^{N} s_n p_s(\mathbf{k}) \left[1 - \cos(\mathbf{k}_n \cdot \mathbf{r})\right]. \tag{6}$$

The Eq. (5) is to be satisfied by a structure function of the power-law form of Eq. (2):

$$\sum_{n=1}^{N} s_n p_s(\mathbf{k}) = 2\Phi(\mathbf{k}). \tag{7}$$

Assuming that the direction of the wave vector $\mathbf{k}_s$ obeys a uniform distribution on $[-\pi, \pi]$ and conforms to the statistical isotropy of the phase, it is required that:

$$p_s(\mathbf{k}) d^2 K = p_s(K) dk d\phi / 2\pi. \tag{8}$$

Then Eq. (7) can be expressed as:

$$s_n p_s = 4\pi C(\alpha) r_n^{-\alpha} k^{1-\alpha}. \tag{9}$$

We divide the interval into $N$ subintervals using a non-overlapping partition of the desired wave-number range. When $K_{n-1} \leq k \leq K_n$,

$$p_s(\mathbf{k}) = k \Phi(k) \left[\int_{k_{n-1}}^{k} kd\Phi(k)\right]. \tag{10}$$

Thus the spectrum of the $n^{th}$ interval can be expressed in logarithmic form as:

$$K_n = K_n \exp \left[\frac{n}{N} \ln \left(\frac{k_n}{K_n}\right)\right]. \tag{11}$$

Weighted on the $n^{th}$ subinterval:

$$s_n = 4\pi C(\alpha)(k_n^{\alpha} - k_n^{\alpha}), 1 \leq n \leq N. \tag{12}$$

Thus the wave number of the subinterval is obtained as follows:

$$K = \left[K_{n-1}^{\alpha} + (K_{n-1}^{\alpha} - K_n^{\alpha})\xi\right]^{-1/\alpha}, \tag{13}$$

where $\xi$ is a $[0, 1]$ uniformly distributed random number. Because atmospheric turbulence is unsteady and random, the random numbers cause a change in phase, making the simulated phase screen more realistic.

### 3.3. Phase-screen Simulation Results

To simulate a more realistic atmospheric environment, continuous dynamic atmospheric turbulence is needed. In the case where only one spot is received at the receiving end, a large phase screen is simulated, and it is then subjected to the interception of multiple small phase screens for dynamic effects, where LC-SLM 512 × 512 pixels in size. The LC-SLM cannot receive a color image, so the generated phase screen needs to be normalized first.

Figure 2 shows the atmospheric turbulence phase screen generated based on the sparse-spectrum theory and the normalization process. The turbulence outer scale is $L_0 = 10$ m, and the turbulence inner scale is $l_0 = 0.01$ m. The expression for atmospheric coherence length is given as:

$$r_n = \left[0.423k^2 C_0(z)dx\right]^{1/3}, \tag{14}$$

where $\lambda = 785$ nm and $L = 1$ km. Three different values for the atmospheric refractive-index structure constant are $C_n^2 = 1.4 \times 10^{-15}$, $C_2^2 = 3 \times 10^{-15}$, and $C_4^2 = 8.2 \times 10^{-14}$, respectively.

The relative error of the phase structure function can be used to determine the effectiveness of the atmospheric
turbulence phase screen generated by the simulation. In this paper the classical value of is \( \alpha = 1.67 \) chosen, and other simulation conditions remain unchanged to simulate the atmospheric turbulence phase screen at the atmospheric coherence length of \( r_0 = 0.1 \) m. 300 frames of phase screens are obtained separately, and the relative-error curves for their phase structures are calculated. The theoretical structure function of the turbulence model is given by:

\[
D_{th}(r) = 45.25 r^{1.67}, \tag{15}
\]

where \( r \) is the distance between any two points in space and \( D_{th}(r) \) is the value of the theoretical phase structure function.

The classical fast-Fourier-transform method can have an error of up to 23% at low frequencies, and the subharmonic method can have an error of up to 17% [18]; The mean value of the phase difference of the structure function of the simulated atmospheric turbulence phase screen derived from the sparse spectral method is 6.8%, which makes up for the insufficiency of traditional Zernike polynomials in the high-frequency component and the insufficiency of the power-spectral method in the low-frequency component.

The simulations in Fig. 3 illustrate that the simulated atmospheric turbulence matches the real atmospheric turbulence.

IV. MODELING SIMULATION ANALYSIS

4.1. Star-point Simulation

In this section, the Gaussian spot after passing through the atmospheric turbulence phase screen is simulated using Matlab software. The flow chart for the star-point simulation is shown in Fig. 4.

In this study the star-chart images are simulated with night as the background, so all of the grayscale values of the image background are low. The size of the star-point spot is related to the brightness (magnitude) of the star and the point-spread function of the optical system. The grayscale distribution of the star-point spot is determined by the point-spread function of the optical system, which can be approximated by a two-dimensional Gaussian distribution function. Therefore, the Gaussian model of the star point can be expressed as:

\[
I(x, y) = A_0 \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-x_0}{\sigma_x} \right)^2 + 2\rho \frac{x-x_0}{\sigma_x} \frac{y-y_0}{\sigma_y} + \left( \frac{y-y_0}{\sigma_y} \right)^2 \right] \right\}, \tag{16}
\]

where \( A_0 \) is the amplitude of the spot, indicating the maximum grayscale value for the pixels of the star point, which is closely related to the magnitude of the star point. \( \sigma_x \) and \( \sigma_y \) are the standard deviations in the \( x \) and \( y \) directions respectively, and \( \rho \) is the correlation coefficient. Since the star-map image is divided into pixels along the \( x \) and \( y \) directions, each star point consists of a number of neighbor-
ing pixels; the diameter of the star-point spot is generally 3 to 5 pixels. From the symmetry of the Gaussian distribution, we can see that $\sigma_x = \sigma_y = \sigma$. From the Gaussian model of the star point, we can see that when the star point spreads to $3 \times 3$ pixels the corresponding $\sigma$ is 0.671, and when it is $5 \times 5$ pixels the corresponding $\sigma$ is 1.058. Therefore, the range for the values of $\sigma$ is $0.671 \leq \sigma \leq 1.058$. Because this paper simulates a single star point, the value of $\sigma$ does not affect the simulation results, and for convenience in the calculations it is taken as $\sigma_x = \sigma_y = 1$, $\rho = 0$.

Figure 5 shows the simulated star-spot images for the original Gaussian spot and when adding $r_0 = 1.0165$ m for weak turbulence, $r_0 = 0.1616$ m for medium turbulence, and $r_0 = 0.0222$ m for strong turbulence respectively. It can be seen that weak turbulence has little effect on the spot, and as the intensity of atmospheric turbulence gradually increases, the spot distribution is uneven and the beam energy becomes smaller and smaller.

**FIG. 4.** Flow chart for star-point simulation with a turbulent phase screen.

**FIG. 5.** Simulated star-point images: (a) Original Gaussian spot, (b) $r_0 = 1.0165$ m, (c) $r_0 = 0.0222$ m, (d) $r_0 = 0.1616$ m.

**FIG. 6.** Some simulation results (a) shattered spot 1, (b) coordinates of the center of mass of spot 1, (c) shattered spot 2, (d) coordinates of the center of mass of spot 2, (e) shattered spot 3, (f) coordinates of the center of mass of spot 3.
4.2. Star Point Center of Mass Positioning Offset

Due to the sparse spectral model, the turbulence phase screen with transverse axis much larger than longitudinal axis can be generated. In this study we take the size of the mother phase screen as 1 m, so we only need to intercept the subphase screen according to the transverse wind speed from left to right and load it into the LC-SLM, which can yield a dynamic turbulence simulation effect.

The current image resolution is 512 × 512 pixels, and the localization effect is calculated based on the proportion and size of the broken spots occupying the entire image at a transmission distance of 1 km. Selecting $C_2^n = 3 \times 10^{-15}$, the change in the center-of-mass position of some of the star points is shown in Fig. 6.

The position of a star point is shifted after passing through atmospheric turbulence, and the shift in the position of the spot’s center of mass is generally used to describe the overall spot shift. The main methods are the weighted average center of mass (WAM) method and the center of mass method, which weights the center of mass of a star point based on the gray value of the star point pixel. The weighted value of the former is the square of the gray value of the pixel in the image of the star point, and this method is more accurate than the common center of mass method. Therefore, the plus-all average center of mass method is used in this paper. The star point’s center of mass $(x_0, y_0)$ can be expressed as in Eq. (17):

$$
\begin{align*}
    x_0 &= \frac{\sum_{x=1}^{m} \sum_{y=1}^{n} f^2(x, y) x}{\sum_{x=1}^{m} \sum_{y=1}^{n} f^2(x, y)} \\
    y_0 &= \frac{\sum_{x=1}^{m} \sum_{y=1}^{n} f^2(x, y) y}{\sum_{x=1}^{m} \sum_{y=1}^{n} f^2(x, y)}
\end{align*}
$$

(17)

Localization of the spot’s center of mass is performed by taking 100 consecutive images for each value of the atmospheric coherence length; The amount of center-of-mass drift is shown in Fig. 7. It can be seen that the atmospheric turbulence is randomly perturbed. When $r_0 = 1.0165$ m, the mean drift variance is $7.62 \times 10^{-8}$ m; When $r_0 = 0.1616$ m, the mean drift variance is $1.745 \times 10^{-7}$ m; And when $r_0 = 0.0222$ m, the mean drift variance is $2.188 \times 10^{-7}$ m. It can be seen from the figure that as the intensity of atmospheric turbulence increases, the disturbance becomes more intense.

V. CONCLUSION

To analyze the effects of atmospheric turbulence on star points, this paper presents the system design for an atmospheric turbulence static star simulator. First the sparse-spectrum method is used to simulate the atmospheric turbulence phase screen, which avoids the situation in which the traditional method produces a periodic phase screen.

**FIG. 7.** Centroid offsets of stars for different atmospheric coherence lengths: (a) $r_0 = 1.0165$ m, (b) $r_0 = 0.1616$ m, (c) $r_0 = 0.0222$ m.
that leads to discontinuity of the image when it is loaded to the star points. The sparse-spectrum method yields a mean value of the phase difference of the simulated atmospheric turbulence phase screen’s structural function of 6.8%. Next the simulated atmospheric turbulence is added to a static star-simulator device for star-point simulation and analysis, and finally the center-of-mass drift of the star point is obtained for different coherence lengths. As the atmospheric coherence length decreases, the turbulence intensity increases and the perturbation becomes more intense. When \( r_0 = 1.0165 \text{ m}, 0.1616 \text{ m}, \text{ and } 0.0222 \text{ m} \), the mean drift variance was respectively \( 7.62 \times 10^{-8} \text{ m}, 1.745 \times 10^{-7} \text{ m}, \) and \( 2.188 \times 10^{-7} \text{ m} \). In summary, the research in this paper can provide a basis for the subsequent improvement of the ground-calibration accuracy of a star sensitizer.

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**DISCLOSURES**

The authors declare no conflicts of interest.

**DATA AVAILABILITY**

The data underlying the results presented in this paper are not publicly available at the time of publication, but can be obtained from the authors upon reasonable request.

**REFERENCES**