

KF AND H-INFINITY FILTER DESIGN FOR IPMSM WITH D.O.B[†]

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ABSTRACT. In order to control the speed and torque of IPMSM, the noise included in the state values such as the measured angular velocity and current must be removed. In addition, the designed controller must be able to operate well even when the mathematical model error of the motor is included and there is a disturbance. To this end, the KF and H^∞ filters is designed to estimate the current and speed states in the situation where the model error exists. In addition, the estimated current and angular velocity are used in the designed disturbance observer to estimate the disturbance. Using the estimated disturbance and state values, robust control performance can be obtained in the angular velocity and current control of a 1-horsepower IPMSM.

AMS Mathematics Subject Classification : 65H05, 65F10.

Key words and phrases : Interior permanent magnet synchronous motor, disturbance observer, Kalman filter, H infinity filter.

1. Introduction

There are several advantages to using electric motors to make driving devices in the current industrial field. It is compact and lightweight because it uses electricity as an energy source to generate kinetic energy. It is environmentally friendly because it is quiet and has no exhaust gas. It has fewer breakdowns, is easy to maintain, and is easy to operate and control [1-5]. In order to control the motor, an inverter, which is a power converter, is required. The inverter changes the power from the power supply to the form that the motor needs and supplies it. The IPMSM(Interior Permanent Magnet Synchronous Motor) motor is based on the principle that the rotor is a permanent magnet and supplies current to the stator winding to form a rotating magnetic field and generate rotational power.

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The basic switch control is configured by supplying three-phase current to the stator winding according to the position of the magnet rotor in the inverter. The basic control concept of the motor is current control, which controls the amount of current flowing through the winding to control the size of the generated rotational force. Voltage control is a method of controlling the rotational speed of the rotating shaft by controlling the voltage supplied from the inverter. Therefore, in order to control the motor, state values such as the speed of the rotating shaft, voltage and current, and motor parameters are essential. When measuring the actual state values of the motor, a lot of noise is included, which becomes a factor of error in the state measurement, and the motor parameters increase the uncertainty in calculating the target amount of voltage and current required for motor control. Therefore, in order to precisely control the motor, efforts are needed to eliminate noise in the state measurement [1-8]. In addition, disturbances such as load fluctuations occurring in the rotating shaft cannot be measured, but they should be considered for precise control [9-12]. In this study, we study a state estimation algorithm that combines Kalman filter and H^∞ filter to estimate the state of the motor. It is an algorithm that minimizes the optimal RMS state estimation error based on the estimation error boundary in a situation where the system model error of the motor exists. IPMSM is divided into an electrical system that generates electrical torque with current and a mechanical system. This system considers the electrical system as linear and designs a state estimation algorithm, and assumes the remaining nonlinear terms as disturbances. Using the estimated state, a disturbance observer is designed to estimate the disturbance and applied to the current controller. The mechanical system designs a state estimation algorithm considering load variation as a disturbance and estimates the load torque using the estimated state of the disturbance observer. And this system considers an estimation algorithm that can robustly estimate the state against the uncertainty of the system model, including the model error of the system.

2. Kalman & H^∞ filtering

The system for estimating state variables is as follows.

$$x_{k+1} = F_k x_k + G_k u_k + w_k \quad (1)$$

$$y_k = H_k x_k + v_k \quad (2)$$

Eq. (1) is a discrete system, x_k , u_k are the state vector and input vector, respectively, and the input vector is assumed to be a known control input. w_k is noise added to the state vector. y_k in Eq. (2) is the output vector, and v_k is noise included in the output vector. w_k and v_k are uncorrelated and are white noise with zero mean. And F_k , G_k in Eq. (1) are the discretized system matrix and input matrix, respectively. The steady-state Kalman filter is a state estimator

structured to minimize the variance value of the following state.

$$J_2 = \lim_{N \rightarrow \infty} \sum_{k=0}^N E(\|x_k - \hat{x}_k\|_2) \quad (3)$$

The H^∞ estimation algorithm has the following cost function based on game theory.

$$J_\infty = \lim_{N \rightarrow \infty} \max_{x_0, w_k, v_k} \frac{\sum_{k=0}^N \|x_k - \hat{x}_k\|_{P_k}^2}{\|x_0 - \hat{x}_0\|_{P_0^{-1}}^2 + \sum_{k=0}^N (\|w_k\|_{Q_k^{-1}}^2 + \|v_k\|_{R_k^{-1}}^2)} < \frac{1}{\theta} \quad (4)$$

It is a state estimator that minimizes the estimation error in the worst-case situation with system noise and observer noise.

P_0, Q_k, R_k in Eq. (4) are weight matrices that are multiplied by each error, and are symmetric matrices and positive definite matrices. $\frac{1}{\theta}$ is the boundary of the cost function J_∞ and has a positive value.

$$P_{k+1} = F_k [P_k^{-1} - \theta I + H_k^T R_k^{-1} H_k]^{-1} F_k^T + Q_k \quad (5)$$

$$\hat{x}_{k+1} = F_k \hat{x}_k + G_k u_k + F_k K_k (y_k - H_k \hat{x}_k) \quad (6)$$

$$K_k = [P_k^{-1} - \theta I + H_k^T R_k^{-1} H_k]^{-1} H_k^T R_k^{-1} \quad (7)$$

$$P_k^{-1} - \theta I + H_k^T R_k^{-1} H_k > 0 \quad (8)$$

The Riccati equation for the cost function of Eq. (4) is the same as Eq. (5), and the optimal solution is Eq. (6). At this time, the gain is obtained as in Eq. (7). And P_k is a positive definite matrix and satisfies the condition of Eq. (8) [1]. Eq. (7) becomes the solution that minimizes the cost function of Eq. (3) when θ is set to 0 in Eq. (5).

In order to reduce the computational burden of Eq. (5), the following relationship is used.

$$\begin{aligned} & [P_k^{-1} - \theta I + H_k^T R_k^{-1} H_k]^{-1} = \\ & (P_k^{-1} - \theta I)^{-1} - (P_k^{-1} - \theta I)^{-1} H_k^T (R_k + H_k (P_k^{-1} - \theta I)^{-1} H_k^T)^{-1} H_k (P_k^{-1} - \theta I)^{-1} \\ & = P_a - P_a H_k^T (R_k + H_k P_a H_k^T)^{-1} H_k P_a \end{aligned} \quad (9)$$

$$P_a = (P_k^{-1} - \theta I)^{-1} = P_k + P_k (\theta^{-1} I - P_k)^{-1} P_k = \theta^{-1} P_k (\theta^{-1} I - P_k)^{-1} \quad (10)$$

Using Eq. (9) and Eq. (10), Eq. (5) is transformed as follows.

$$P_{k+1} = F_k P_a F_k^T + Q_k - F_k P_a H_k^T V^{-1} H_k P_a F_k^T \quad (11)$$

$$V = (R_k + H_k P_a H_k^T) \quad (12)$$

Using Eq. (9) and Eq. (10), Eq. (7) is transformed as follows.

$$\begin{aligned} K_k &= [P_a - P_a H_k^T (R_k + H_k P_a H_k^T)^{-1} H_k P_a] H_k^T R_k^{-1} \\ &= [P_a H_k^T - P_a H_k^T (R_k + H_k P_a H_k^T)^{-1} H_k P_a H_k^T] R_k^{-1} \end{aligned}$$

$$\begin{aligned}
&= P_a H_k^T \left[I - (R_k + H_k P_a H_k^T)^{-1} H_k P_a H_k^T \right] R_k^{-1} \\
&= P_a H_k^T \left[(R_k + H_k P_a H_k^T)^{-1} R_k \right] R_k^{-1} \\
&= P_a H_k^T (R_k + H_k P_a H_k^T)^{-1} \\
&= P_a H_k^T V^{-1}
\end{aligned} \tag{13}$$

Eq. (11) and Eq. (13) show the same results as the equations derived using the transfer function of the discrete system [1][7].

The estimator of Eq. (6) is reconstructed as follows.

$$\hat{x}_{k+1} = \hat{F}_k \hat{x}_k + G_k u_k + F_k K_k y_k \tag{14}$$

$$\hat{F}_k = F_k (I - K_k H_k) \tag{15}$$

Equation (14) is an estimator of system parameters, and the system parameters are updated as in Eq. (15).

2.1. Mechanical parameter estimation and Kalman & H^∞ filter design of the IPMSM. The discrete-time model of the mechanical part of the IPMSM is as follows.

$$\omega_{k+1} = a_m \omega_k + b_m (T_{e,k} - T_l) \tag{16}$$

$$a_m = 1 - \frac{B_m}{J_m} T_m, \quad b_m = \frac{T_m}{J_m} \tag{17}$$

Eq. (11) is the mechanical system of the motor shaft. ω_k is mechanical velocity of rotor, $T_{e,k}$ is the torque caused by electromagnetic, T_l is load torque, J_m is moment of inertia of motor rotor, and B_m is viscous friction coefficient of motor rotor. T_m is the discrete-time sampling period.

When the load torque T_l is estimated using an observer and input, the estimation error of the disturbance is assumed as noise added to the system. When the output variable is ω_k , the equation related to the estimation is as follows.

$$P_{m,k+1} = a_m^2 R_{m,k} K_{m,k} + Q_{m,k} \tag{18}$$

$$K_{m,k} = P_{m,a} (R_{m,k} + P_{m,a})^{-1} \tag{19}$$

$$P_{m,a} = [\theta_m^{-1} P_{m,k} (\theta_m^{-1} - P_{m,k})]^{-1} \tag{20}$$

And the equations for estimating the machine parameter time constant a_m and speed $\hat{\omega}_k$ are as follows.

$$\hat{\omega}_{k+1} = a_m \hat{\omega}_k + b_m T_{e,k} + a_m K_{m,k} (\omega_k - \hat{\omega}_k) \tag{21}$$

$$\hat{a}_m = a_m (1 - K_{m,k}) \tag{22}$$

Eq. (21) estimates the optimal state $\hat{\omega}_k$ using Eq. (19).

2.2. Current parameter estimation and Kalman & H^∞ filter design of the IPMSM. The discrete-time model for the d-q axis current of the motor is as follows.

$$i_{d,k+1} = a_d i_{d,k} + b_d (V_{d,k} + d_{id,k}) \quad (23)$$

$$a_d = 1 - \frac{R_s}{L_d} T_i, \quad b_d = \frac{T_i}{L_d}, \quad d_{id,k} = p L_q \omega i_q \quad (24)$$

$$i_{q,k+1} = a_q i_{q,k} + b_q (V_{q,k} + d_{iq,k}) \quad (25)$$

$$a_q = 1 - \frac{R_s}{L_q} T_i, \quad b_q = \frac{T_i}{L_q}, \quad d_{iq,k} = -p L_d \omega i_d - p \psi_f \omega \quad (26)$$

$V_{d,k}$, $V_{q,k}$ is input voltage of d-axis and q-axis respectively, $i_{d,k}$, $i_{q,k}$ is current of d-axis and q-axis respectively, R_s is phase resistance of stator, L_d , L_q is inductance of d-axis and q-axis respectively, ψ_f is flux of rotor permanent magnet, p is pole pair of permanent magnet. T_i is the discrete-time sampling period. $d_{id,k}$ is assumed to be a disturbance acting on the d-axis current, and $d_{iq,k}$ is assumed to be a disturbance acting on the q-axis current. That is, the disturbance estimated by the disturbance estimator is used as input. Therefore, the difference between the estimated disturbance and the actual disturbance value is assumed as state noise.

When the output variable is i_d , the equation related to the estimation is as follows.

$$P_{d,k+1} = a_d^2 R_{d,k} K_{d,k} + Q_{d,k} \quad (27)$$

$$K_{d,k} = P_{d,a} (R_{d,k} + P_{d,a})^{-1} \quad (28)$$

$$P_{d,a} = \theta_d^{-1} P_{d,k} (\theta_d^{-1} - P_{d,k})^{-1} \quad (29)$$

And the equations for estimating the d-axis current parameter time constant a_d and state $i_{d,k}$ are as follows.

$$\hat{i}_{d,k+1} = a_d \hat{i}_{d,k} + b_d V_{d,k} + a_d K_{d,k} (i_{d,k} - \hat{i}_{d,k}) \quad (30)$$

$$\hat{a}_{d,k} = a_d (1 - K_{d,k}) \quad (31)$$

Eq. (30) estimates the optimal state $\hat{i}_{d,k}$ using Eq. (28).

When the output variable is i_q , the equation related to the estimation is as follows.

$$P_{q,k+1} = a_q^2 R_{q,k} K_{q,k} + Q_{q,k} \quad (32)$$

$$K_{q,k} = P_{q,a} (R_{q,k} + P_{q,a})^{-1} \quad (33)$$

$$P_{q,a} = \theta_q^{-1} P_{q,k} (\theta_q^{-1} - P_{q,k})^{-1} \quad (34)$$

And the equation for estimating the q-axis current parameter time constant a_q is as follows.

$$\hat{i}_{q,k+1} = a_q \hat{i}_{q,k} + b_q V_{q,k} + a_q K_{q,k} (i_{q,k} - \hat{i}_{q,k}) \quad (35)$$

$$\hat{a}_{q,k} = a_q (1 - K_{q,k}) \quad (36)$$

Eq. (35) estimates the optimal state $\hat{i}_{q,k}$ using Eq. (33).

3. The Disturbance Observer

The load torque T_l of Eq. (16) and $d_{id,k}$ of Eq. (23) and $d_{iq,k}$ of Eq. (25) that act on the motor shaft in real time are disturbances acting on each system, and are observed in real time using a disturbance observer.

The design of the observer for the system of Eq. (16) is as follows.

$$p_{m,k} = (1 - a_\omega T_s) p_{m,k-1} + a_\omega T_s u_{m,k-1} \quad (37)$$

$$q_{m,k} = (1 - a_\omega T_s) q_{m,k-1} + a_\omega T_s y_{m,k-1} \quad (38)$$

$$\hat{u}_{m,k} = J_m \left(-a_\omega q_{m,k-1} + a_\omega y_{m,k-1} + \frac{B_m}{J_m} q_{m,k-1} \right) \quad (39)$$

$$\hat{d}_{m,k} = p_{m,k} - \hat{u}_{m,k} \quad (40)$$

Eq. (37) is a nominal difference equation that converts the nominal system of the continuous-time mechanical equation Eq. (16) into the discrete-time sampling period T_s , and Eq. (38) is a difference equation that implements the inverse function of the transfer function of Eq. (16). Here, $p_{m,k}$ and $q_{m,k}$ are the discrete-time state variables of the filter, a_ω is the gain of the filter, and $y_{m,k}$ is the measured angular velocity, and $u_{m,k}$ corresponds to the real-time control input.

By substituting the angular velocity $\hat{\omega}_k$ estimated in Eq. (21) into $y_{m,k}$ of Eq. (39), the disturbance $\hat{d}_{m,k}$ is estimated as in Eq. (40).

The following is the design of the observer for the nominal system of Eq. (23).

$$p_{id,k} = (1 - a_{id} T_s) p_{id,k-1} + a_{id} T_s u_{id,k-1} \quad (41)$$

$$q_{id,k} = (1 - a_{id} T_s) q_{id,k-1} + a_{id} T_s y_{id,k-1} \quad (42)$$

$$\hat{u}_{id,k} = L_d \left(-a_{id} q_{id,k-1} + a_{id} y_{id,k-1} + \frac{R_s}{L_d} q_{id,k-1} \right) \quad (43)$$

$$\hat{d}_{id,k} = p_{id,k} - \hat{u}_{id,k} \quad (44)$$

Eq. (41) is a nominal difference equation that converts the nominal system of the current equation of the continuous-time d-axis Eq. (23) into the discrete-time sampling period T_s , and Eq. (42) is a difference equation that implements the inverse function of the transfer function of Eq. (23). Here, $p_{id,k}$ and $q_{id,k}$ are the discrete-time state variables of the filter, and a_{id} is the gain of the filter. And $y_{id,k}$ is the measured current $i_{id,k}$, and $u_{id,k}$ corresponds to the real-time control input. The current $i_{id,k}$ estimated in Eq. (30) is used for $y_{id,k}$ of Eq. (43), and the disturbance $\hat{d}_{id,k}$ is estimated as in Eq. (44).

Finally, the observer for the nominal system of Eq. (25) is designed as follows. The following is the design of the observer for the nominal system of Eq. (23).

$$p_{iq,k} = (1 - a_{iq} T_s) p_{iq,k-1} + a_{iq} T_s u_{iq,k-1} \quad (45)$$

$$q_{iq,k} = (1 - a_{iq}T_s) q_{iq,k-1} + a_{iq}T_s y_{iq,k-1} \quad (46)$$

$$\hat{u}_{iq,k} = L_q \left(-a_{iq}q_{iq,k-1} + a_{iq}y_{iq,k-1} + \frac{R_s}{L_q}q_{iq,k-1} \right) \quad (47)$$

$$\hat{d}_{iq,k} = p_{iq,k} - \hat{u}_{iq,k} \quad (48)$$

Eq. (45) is a nominal difference equation that converts the nominal system of the continuous-time q-axis current equation Eq. (25) into the discrete-time sampling period T_s , and Eq. (46) is a difference equation that implements the inverse function of the transfer function of Eq. (25). Here, $p_{iq,k}$ and $q_{iq,k}$ are the discrete-time state variables of the filter, a_{iq} is the gain of the filter, and $y_{iq,k}$ is the measured current $i_{iq,k}$, and $u_{iq,k}$ corresponds to the real-time q-axis control input. In Eq. (47), the current $\hat{i}_{q,k}$ estimated in Eq. (35) is used for $y_{iq,k}$, and the disturbance $\hat{d}_{iq,k}$ is estimated as in Eq. (48).

4. Simulation

PSIM is used for simulation program. The model of IPMSM uses the model provided in PSIM, the motor parameters in Table 1 was used.

TABLE 1. IPMSM parameter.

Motor Rated Power	3-phase 1hp
Motor Rated Speed	1200 RPM
P (Pole Number)	4
R_s (Stator Resistance)	0.048 Ω
L_d (D-axis Inductance)	0.42 mH
L_q (Q-axis Inductance)	1.2 mH
J_m (Moment of Inertia)	0.0008 Kgm^2
B_m (Friction coefficient)	0.001 $Nm/rad/s$
ψ_f (Magnetic Flux Constant)	0.04135 $volt/rad/s$

Using the IPMSM model, the reference angular velocity was set to 62.8 [rad/s] and 600 [RPM]. Then, when the motor was rotating at the steady state speed, a constant load of 1 [Nm] was applied at time 1 [s] to observe the status and disturbance estimation results. Also, at time 2 [s], the motor rotation direction was set to the reference angular velocity of -62.8 [rad/s] so that it rotated in reverse, and the response performance and estimation performance were judged. At this time, the error of the system model was applied as 10 [%]. R_s was 0.0432 [Ω], L_q was 1.08 [mH], and B_m was 0.0009 [$Nm/rad/s$].

The results for the current and speed status are as shown in Figure 1.

The first figure in Figure 1 shows the d-axis current i_d and the estimated current \hat{i}_d . The second figure in Figure 1 shows the q-axis current i_q and the estimated current \hat{i}_q , and the third figure shows the angular velocity ω of the

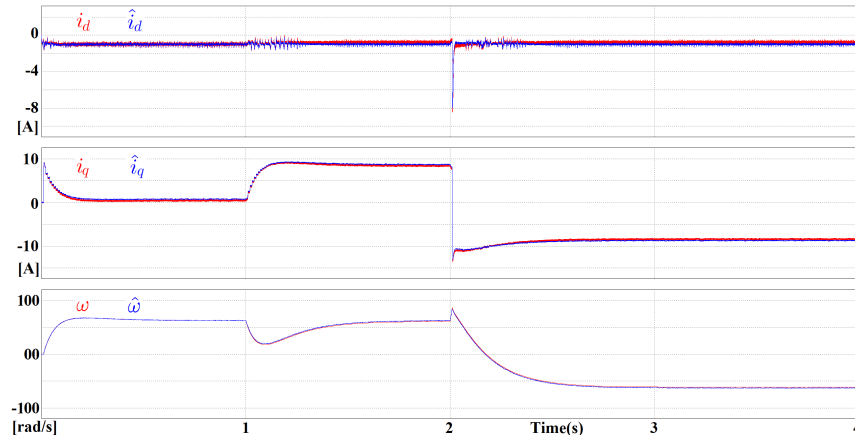


FIGURE 1. The state estimation result at angle velocity 62.8 rad/s

motor rotation shaft and the estimated angular velocity $\hat{\omega}$. The estimation error of the current i_d is approximately 0.2 [A, rms], the estimation error of the current i_q is approximately 0.3 [A, rms], and the estimation error for the rotational angular velocity is 1.5 [rad/s]. The controlled appearance and the results of the state estimation can be confirmed when a constant load is applied at 1 [s]. These are the state control and estimation results in the state with model errors.

The estimation results of the disturbance are as follows in Figure 2.

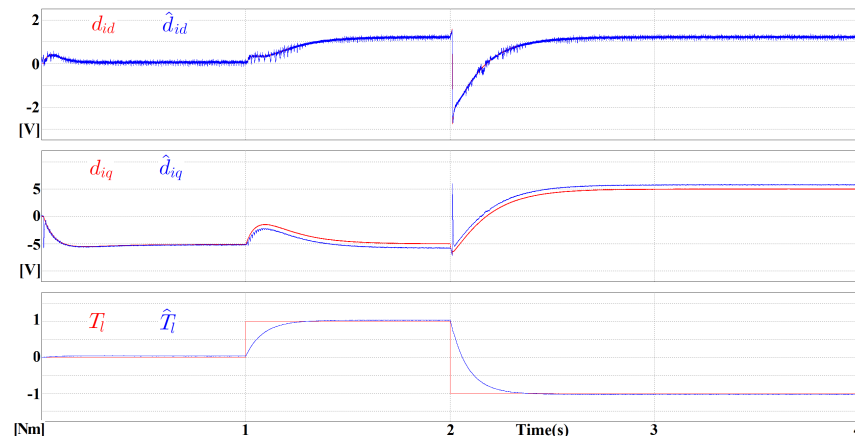


FIGURE 2. The Disturbance estimation result at angle velocity 125.6 rad/s

The first figure in Figure 2 is the result of estimation assuming the back electromotive force generated in the current d-axis as a disturbance. The second figure is the result of estimation assuming the back electromotive force generated in the current q-axis as a disturbance, and the third figure is the result of estimating the constant load acting on the rotation axis. The estimation result of the back electromotive force generated in the d-axis has an error of about 2 [%], and the estimation result of the back electromotive force on the q-axis has an error of about 15 [%]. And the result of estimating the constant load has an error of about 4 [%]. This is the result of estimation by setting the resistance R_s value of the back electromotive force estimator to 10 [%] smaller than the actual value, and the constant load estimator is the result of estimation by setting the value of parameter B_m to 10[%] smaller.

5. Result

This paper designed an observer structured with a Kalman Filter and H^∞ combined to estimate the current and angular velocity of the motor required for the control of IPMSM. As a result, even with parameter uncertainty, the speed estimation error was within 2.5[%] of accuracy. It was also confirmed that the state estimation was well performed for load fluctuations. The disturbance observer designed using the estimated state value can estimate the constant load fluctuation applied externally with an error of within 4[%]. It was also confirmed that the disturbance observer helped improve the response performance of the controller as a result of estimation with parameter errors.

The observer designed for the real system, which includes errors in the system model and noise in the state meter, was able to obtain results with robust performance that minimizes the estimation error even in bad situations that include model errors. In the future, we plan to study an algorithm that reduces energy consumption by designing an optimal controller that finds a solution to control the IPMSM motor with the minimum power based on the H^∞ state observer.

Conflicts of interest : The author declares no conflict of interest.

Data availability : Not applicable

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