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THE H-INFINITY ESTIMATOR DESIGN FOR IPMSM WITH DISTURBANCE OBSERVER†

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Abstract. To achieve precise control performance of an electric motor, it is necessary to compensate for load torque, various parameter errors, and model errors. To this end, the load torque acting on the motor rotation shaft is designed and estimated using a disturbance estimator. The estimated load torque was used as input, and the estimated error in the load torque was assumed to be state noise, with current and speed states estimated using the H^∞ filter. As a result of applying the disturbance and state values estimated from the 1-horsepower IPMSM to speed control, a speed tracking error within 0.1[%] was obtained in steady state.

AMS Mathematics Subject Classification : 65H05, 65F10.
Key words and phrases : Interior permanent magnet synchronous motor, disturbance observer, load torque, state estimation.

1. Introduction

Electric motors with various performance structures have been developed, and with the development of battery technology, the production and research of driving devices using electric motors are actively underway [1-5]. Particularly, to control autonomous driving devices that include the concept of autonomous driving, power and drive control of the electric motor are fundamentally required. Additionally, controlling the speed and direction of the moving object's movement requires driving control as a higher-level control. To control the operation of an electric motor, observation of the motor's state is required, and a sensor is configured for this purpose. By designing a feedback control system using the observed state, the entire system is configured to satisfy various control performances. Sensors for measuring conditions can generate significant noise depending on the environment and may also include system uncertainty.

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Additionally, certain control conditions may be difficult or require great effort to measure. The Kalman Filter theory, a probabilistic estimation theory developed in the 1950s, studies how to remove noise included in signals measured by sensors. It is still applied, developed, and widely used today [1-8]. There are several considerations when estimating signals using the Kalman filter. First, we must know the average and interdependence of system uncertainty and sensor noise. "Knowing" here means being able to express something numerically or mathematically. Second, the covariance of system uncertainty and sensor noise must be known. The Kalman filter is an estimation algorithm that minimizes the standard deviation of the state error by matching the state estimated through a mathematical system with the state signal measured through an observer, thereby minimizing covariance. Lastly, we must know the system and observer matrices. However, because these matrices are derived from dynamic modeling and undergo a linearization process, parameter errors between the system and observer exist. Excellent state estimation performance is urgently needed in many fields where control is currently used. The Kalman filter, which minimizes the variance of estimation error, can achieve excellent estimation performance if the above considerations are met. However, even in situations where this is not the case, state estimation performance is required. Therefore, an algorithm that minimizes estimation error in the worst-case scenario, which includes errors in the system model, is proposed [1, 6]. This study assumes a systematic error and investigates the H$^\infty$ filter algorithm that minimizes the estimation error even when the systematic error is large. It designs and estimates a disturbance observer with load torque acting on the motor's rotating shaft [9-12] and models the estimated load torque as an input to the motor mechanical system. The difference between the actual load torque and the estimated load torque is assumed to be noise and applied to the H$^\infty$ filter algorithm. Even if there is a large difference between the actual and estimated load torque, the design aims to achieve performance that minimizes the state estimation error.

2. The Discrete time H^∞ filter [1]

The H^∞ estimation algorithm defines the cost function as in Eq. (1) based on game theory.

$$J = \frac{\sum_{k=0}^{N-1} ||z_k - \hat{z}_k||^2_{S_k}}{||x_0 - \hat{x}_0||^2_{P_0^{-1}} + \sum_{k=0}^{N-1}(||w_k||^2_{Q_k^{-1}} + ||v_k||^2_{R_k^{-1}})} \tag{1}$$

The discrete time system associated with the algorithm for finding the optimal estimate $\hat{z}_k$ that minimizes the cost function J is as follows:

$$x_{k+1} = F_k x_k + w_k \tag{2}$$

$$y_k = H_k x_k + v_k \tag{3}$$

$$z_k = L_k x_k,\ \hat{z}_k = L_k \hat{x}_k \tag{4}$$

Eq. (2) is a discrete system, and x_k,u_k are the state vector and input vector. w_k is noise added to the state vector. In Eq. (3), y_k is the output vector, and v_k is the noise included in the output vector. x_k in Eq. (4) is an estimate. And S_k, P_0, Q_k, R_k in Eq. (1) are the weight matrices multiplied by each error, which are symmetric matrices and positive definite matrices. Since Eq. (1) is a fractional expression, the equation can be converted to allow calculation as follows.

$$J = -\frac{1}{\theta}||x_0 - \hat{x}_0||^2_{P_0^{-1}} + \sum_{k=0}^{N-1}(||z_k - \hat{z}_k||^2_{S_k} - \frac{1}{\theta}(||w_k||^2_{Q_k^{-1}} + ||v_k||^2_{R_k^{-1}})) < 1 \quad (5)$$

$$J^* = \min_{\hat{z}_k} \max_{x_0,w_k,v_k} J \quad (6)$$

The solution to obtain the optimal cost function J^* is to find a stationary point such that J is maximized for the noise w_k, v_k and initial values x_0, and J is minimized for the state $\hat{z}_k$. θ is a parameter used to set the maximum of the cost function. To make Eq. (6) easier to process, the following equation is used.

$$||z_k - \hat{z}_k||^2_{S_k} = ||x_k - \hat{x}_k||^2_{\bar{S}_k},\ \bar{S}_k = L_k^T S_k L_k \quad (7)$$

$$|||v_k||^2_{R_k^{-1}} = ||y_k - H_k x_k||^2_{R_k^{-1}} \quad (8)$$

Eq. (7) is an expression converted using Eq. (4) and $\hat{z}_k = L_k\hat{x}_k$, and Eq. (8) is an equation created from Eq. (3). Converting the cost function from Eq. (5) to Eq. (6) using Eq. (7) to Eq. (8) is as follows:

$$J = -\frac{1}{\theta}||x_0 - \hat{x}_0||^2_{P_0^{-1}} + \sum_{k=0}^{N-1}(||x_k - \hat{x}_k||^2_{\bar{S}_k} - \frac{1}{\theta}(||w_k||^2_{Q_k^{-1}} + ||y_k - H_k x_k||^2_{R_k^{-1}})) \quad (9)$$

$$J^* = \min_{\hat{x}_k} \max_{x_0,w_k,y_k} J \quad (10)$$

The optimal state $\hat{x}_k$ that leads to the optimal cost function J^* is finding the stationary point where J is maximized for noise w_k, observed value y_k, and initial value x_0, and J is minimized for state $\hat{x}_k$.

3. The H^∞ filter design of the IPMSM

The IPMSM model is as follows Eq. (11) to Eq. (14). Eq. (11) to Eq. (12) is the electrical system expressed on the virtual DQ axis, and Eq. (13) is the mechanical system of the motor rotation shaft. And Eq. (14) shows how to generate torque with the currents in Eq. (11) to Eq. (12). In this way, the electromagnetically generated torque Eq. (14) is input into Eq. (13) and operates to generate rotation of the electric motor.

$$\frac{d}{dt}i_d = -\frac{R_s}{L_d}i_d + \frac{pL_q}{L_d}\omega_r i_q + \frac{1}{L_d}V_d \quad (11)$$

$$\frac{d}{dt}i_q = -\frac{R_s}{L_q}i_q - \frac{pL_d}{L_q}\omega_r i_d - \frac{p\psi_f}{L_q}\omega_r + \frac{1}{L_d}V_d \quad (12)$$

$$\frac{d}{dt}\omega_r = -\frac{B_m}{J_m}\omega_r - \frac{1}{J_m}T_l + \frac{1}{J_m}T_e \quad (13)$$

$$T_e = \frac{3}{2}p(\psi_f + (L_d - L_q)i_d)i_q \tag{14}$$

V_d, V_q are the input voltage of d-axis and q-axis respectively, i_d, i_q are the currents of d-axis and q-axis respectively, ω_r is mechanical velocity of rotor, T_e is the torque caused by electromagnetic forces, T_l is the load torque, R_s is the phase resistance of the stator, L_d and L_q are the inductances of the d-axis and q-axis respectively, ψ_f is the flux of the rotor permanent magnet, p is the pole pair of the permanent magnet, J_m is the moment of inertia of the motor rotor, and B_m is viscous friction coefficient of the motor rotor.

$$\frac{d}{dt}x_e = \frac{\partial f(x,u)}{\partial x}|_{\hat{x}}\, x_e + \frac{\partial f(x,u)}{\partial u}|_{\hat{u}}\, u_e + w \tag{15}$$

$$y_e = (y - \hat{y})^T = (i_d - \hat{i}_d\ i_q - \hat{i}_q\ \omega_r - \hat{\omega}_r)^T \tag{16}$$

$$x_e = (x - \hat{x})^T = (i_d - \hat{i}_d\ i_q - \hat{i}_q\ \omega_r - \hat{\omega}_r)^T \tag{17}$$

$$u_e = (u - \hat{u})^T = (V_d - \hat{V}_d\ V_q - \hat{V}_q\ T_l - \hat{T}_l)^T \tag{18}$$

$$\frac{\partial f(x,u)}{\partial x}|_{\hat{x}} = \begin{bmatrix} -\frac{R_s}{L_d} & p\frac{L_q}{L_d}\omega_r & p\frac{L_q}{L_d}i_q \\ -p\frac{L_d}{L_q}\omega_r & -\frac{R_s}{L_q} & -p\frac{L_d i_d + \psi_f}{L_q} \\ \frac{3p}{2J_m}(L_d - L_q)i_q & \frac{3p}{2J_m}((L_d - L_q)i_d + \psi_f) & -\frac{B_m}{J_m} \end{bmatrix}$$

$$\frac{\partial f(x,u)}{\partial u}|_{\hat{u}} = (\frac{1}{L_d} \quad \frac{1}{L_q} \quad -\frac{1}{J_m})^T \tag{19}$$

Eq. (15) to Eq. (19) perform Taylor series expansion of the nonlinear function $f(x,u)$ from Eq. (11) to Eq. (14) for the states $\hat{x}$ and $\hat{u}$ and omit the quadratic or higher order terms. u_e is the difference between the actual control input value and the calculated control input value, and this value is assumed to be zero. $\hat{T}_l$ is the estimated disturbance value, and the difference between the actual disturbance T_l and this is considered as the system noise w. Assuming that the discrete sampling period T_s is small, the motor model discretized using Euler's approximation is as follows.

$$x_{e,k+1} = (I + T_s\frac{\partial f(x,u)}{\partial x}|_{\hat{x}})_k x_{e,k} + w = F_k x_{e,k} + w_k \tag{20}$$

Eq. (20) reconstructs Eq. (9) using the Lagrange multiplier λ so that it is included as a constraint in the cost function Eq. (9).

$$J = -\frac{1}{\theta}||x_{e,0} - \hat{x}_{e,0}||^2_{P_0^{-1}} + \sum_{k=0}^{N-1}(||x_{e,k} - \hat{x}_{e,k}||^2_{\bar{S}_k^{-1}}$$
$$- \frac{1}{\theta}(||w_k||^2_{Q_k^{-1}} + ||y_{e,k} - H_k x_{e,k}||^2_{R_k^{-1}}) + \frac{2\lambda^T_{k+1}}{\theta}(F_k x_{e,k} + w_k - x_{e,k+1})) \tag{21}$$

There are two strategies for finding the value that optimizes the cost function in Eq. (21). First, we find the stationary point of the cost function for system noise w_k, state $x_{e,k}$, and initial value $x_{e,0}$. Next, by finding the observation value $y_{e,k}$ where the cost function is maximum and $\hat{x}_{e,k}$ where the cost function is minimum among the stationary points, the estimation algorithm is obtained as follows.

$$\bar{S}_k = L_k^T S_k L_k \tag{22}$$

$$K_k = P_k[I - \theta\bar{S}_k P_k + H_k^T R_{-1}^k H_k P_k]^{-1} H_k^T R_k^{-1} \tag{23}$$

$$\hat{x}_{e,k+1} = F_k \hat{x}_{e,k} + F_k K_k (y_{e,k} - H_k \hat{x}_{e,k}) \tag{24}$$

$$P_{k+1} = F_k P_k [I - \theta\bar{S}_k P_k + H_k^T R_{-1}^k H_k P_k]^{-1} F_k^T + Q_k \tag{25}$$

$$P_k^{-1} - \theta\bar{S}_k + H_k^T R_{-1}^k H_k > 0 \tag{26}$$

Eq. (24) is an estimated value that minimizes the cost function for state error. The equation for estimating the state value is as follows.

$$\hat{x}_{k+1} = \hat{x}_k + \hat{x}_{e,k+1} \tag{27}$$

4. The Disturbance Observer

The load torque T_l, which acts on the motor rotation shaft in real time, is an essential factor for achieving good performance in controlling the motor and estimating state variables. However, since load torque is not easy to measure, a disturbance observer is designed to reflect the estimate in the control input and is used as an input to the state estimator to improve state estimation performance. The disturbance observer is illustrated in the mechanical system of the motor as shown in Eq. (13).

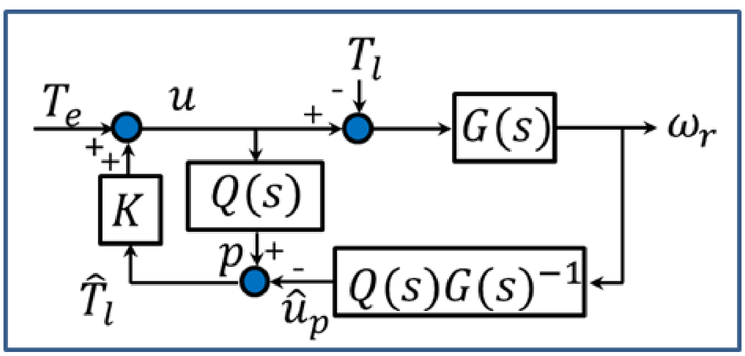

FIGURE 1. IPMSM mechanical system with the Disturbance Observer.

The transfer function $G(s)$, which uses ω_r as the input and T_e as the output by Laplace transforming Eq. (13), is as follows:

$$\omega_r = \frac{1}{J_m s + B_m}(T_e(s) - T_l(s)) = G(s)(T_e(s) - T_l(s)) \tag{28}$$

$\hat{G}(s)^{-1}$ in Figure 1 is the inverse function of $G(s)$, with ω_r as input and u as output. The load torque $\hat{T}_l$ is as follows: $G(s)^{-1}$ in Figure 1 is the inverse function of $G(s)$, with ω_r as input and u as output. The load torque $\hat{T}_l$ is as follows:

$$p = Q(s)u \tag{29}$$

$$\hat{u}_p = Q(s)G(s)^{-1}G(s)(u - T_l(s)) \tag{30}$$

$$\hat{T}_l(s) = p - \hat{u}_p \tag{31}$$

$Q(s)$ and $G(s)^{-1}$ are the filters to be designed, and by substituting Eq. (29) and Eq. (30) into Eq. (31), the result is as follows:

$$\hat{T}_l(s) = Q(s)((1 - G(s)^{-1}G(s))u + G(s)^{-1}G(s)T_l(s)) \tag{32}$$

In Eq. (32), $G(s)^{-1}$ is designed so that $G(s)^{-1}G(s) \approx 1$. If Q(s) is designed as a low-pass filter with a gain of 1 in the steady state, $\hat{T}_l(s)$ can estimate $T_l(s)$.

$$u = T_e(s) + K\hat{T}_l \tag{33}$$

$$\begin{aligned}(1 - Q(s)(1 - G(s)^{-1}G(s))K)\hat{T}_l \\ = Q(s)((1 - G(s)^{-1}G(s))T_e(s) + G(s)^{-1}G(s)T_l(s))\end{aligned} \tag{34}$$

Eq. (33) forms the input u by adding $\hat{T}_l$ (s) to the input $T_e(s)$, and the result of substituting Eq. (33) into Eq. (32) is Eq. (34).

$$G(s)^{-1}G(s) \approx 1 \tag{35}$$

$$Q(s) = \frac{1}{\tau s + 1} \tag{36}$$

$$\frac{\hat{u}_p}{\omega_r} = Q(s)G(s)^{-1} = \frac{\bar{J}_m + \bar{B}_m}{\tau s + 1} \tag{37}$$

Therefore, $G(s)^{-1}$ is designed according to Eq. (35), and Eq. (36) is designed as a system with the first delay element as the transfer function. $\bar{J}_m$ and $\bar{B}_m$ are parameters set for mathematical calculation processing. Eq. (37) is formed based on the conditions outlined in Eq. (35) and Eq. (36). To convert Eq. (37) to the discrete time domain, the parameters are configured as follows:

$$\frac{\hat{u}_p}{q}\frac{p}{\omega_r} = \frac{\bar{J}_m + \bar{B}_m}{\tau s + 1} \tag{38}$$

$$\hat{u}_p = \bar{J}_m sq + \bar{B}_m q \tag{39}$$

$$\omega_r = \tau sq + q \tag{40}$$

The inverse Laplace transformation of Eq. (39) and Eq. (40) into the time domain is as follows:

$$\hat{u}_p(t) = \bar{J}_m\dot{q}(t) + \bar{B}_m q(t) \tag{41}$$

$$\omega_r(t) = \tau\dot{q}(t) + q(t) \tag{42}$$

This result is obtained by Euler's approximation of the equations of the state equations in Eq. (41) and Eq. (42) with a sampling period T_s.

$$\hat{u}_p(k) = (\bar{B}_m - \frac{\bar{J}_m}{\tau})q(k) + \frac{\bar{J}_m}{\tau}\omega_r(k) \tag{43}$$

If Eq. (29) is designed using Eq. (36), it is as follows, where k represents discrete time steps.

$$p(k) = (1 - \frac{T_s}{\tau})p(k-1) + \frac{T_s}{\tau}u(k-1) \tag{44}$$

Using the estimation results from Eq. (43) and Eq. (44), the load torque is estimated as follows:

$$\hat{T}_l(k) = p(k) - \hat{u}_p(k) \tag{45}$$

5. Simulation

PSIM is used for simulation program. The IPMSM model provided in PSIM uses the motor parameters shown in Table 1.

Table 1. IPMSM parameter.

Motor Rated Power	3-phase 1hp
Motor Rated Speed	1200 RPM
P (Pole Number)	4
R_s (Stator Resistance)	0.048 Ω
L_d (D-axis Inductance)	0.42 mH
L_q (Q-axis Inductance)	1.2 mH
J_m (Moment of Inertia)	0.0008 Kgm^2
B_m (Friction coefficient)	0.001 $Nm/rad/s$
ψ_f (Magnetic Flux Constant)	0.04135 $volt/rad/s$

To evaluate the performance of the designed disturbance observer and H^∞ filter, the IPMSM was operated at a reference angular velocity of 125.6 $[rad/s]$. It operated with no load from 0 to 2 seconds, after which a constant load of 0.5 $[Nm]$ was applied to the rotation axis at 2 seconds. An error of 10 [%] in the system model was also applied. R_s was 0.0432 $[\Omega]$, L_d was 0.378 $[mH]$, L_q was 1.08 $[mH]$, and J_m was 0.00072 $[Kgm^2]$.

The first picture in Figure 2 shows the applied constant load and estimated load torque. As a result of the disturbance estimation, the average is about 0.02 $[Nm]$ from 1 second to 2 seconds, with an amplitude of about 0.1 $[Nm]$, and noise of 0.1 $[A]$ and ± 0.5 $[rad/sec]$ is added to the phase current and angular velocity, respectively. After 2 seconds, the estimation of a constant load of 0.5 $[Nm]$ reached steady state after about 2.3 seconds. The second and third figures show the results of applying the H^∞ filter with the estimated load torque as input.

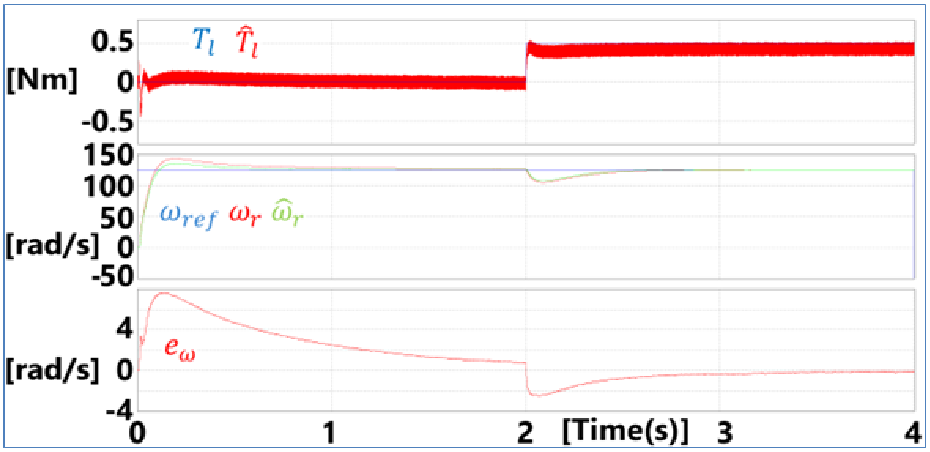

FIGURE 2. IPMSM driving result at angle velocity 125.6 rad/s.

The angular velocity in the second picture is about 143 $[rad/sec]$ at 0.19 seconds, resulting in an overshoot of 13.9 [%], and about 105 $[rad/sec]$ at 2.1 seconds, resulting in an undershoot of 16.4 [%]. The parameter θ used in the H^∞ filter was set to 10. The third figure shows the estimation error of angular velocity. At 0.14 seconds, the maximum value is 7.7 $[rad/sec]$, and at 2.1 seconds, the minimum value is -2.4 $[rad/sec]$.

6. Conclusion

In this paper, for precise tracking control of IPMSM, load torque was estimated using a disturbance observer. Noise included in the state was removed, and an H^∞ filter was designed for state estimation. The smaller the error of the system model and the estimation error of the disturbance, the better the control performance; however, this is not the case in reality, and the control performance deteriorates. The H^∞ filter has the advantage of minimizing estimation error even in the worst-case scenario where model error is included. By setting the system model error to 10 [%], the state estimation results achieved robust performance with a steady-state error within 1 [%]. Future research will focus on developing an algorithm that can operate robustly despite parameter changes by designing a disturbance observer for the electrical system and estimating and updating system parameters.

Conflicts of interest : The author declares no conflict of interest.

Data availability : Not applicable

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