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α-F-CONVEX CONTRACTION MAPPINGS IN b-METRIC SPACES AND UNIQUENESS IN FIXED POINT THEOREMS

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Abstract. We introduce α-F-convex contraction mappings in b-metric space in this study and establish the uniqueness and existence of fixed points for these mappings. It offers α-F-Convex outcomes in several instances that bolster our primary findings, which are also provided.

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1. Introduction

Fixed point theory plays a pivotal role in functional and nonlinear analysis. The Banach contraction principle is an important result of the fixed point theory. Fixed point theory is a valuable resource for learning about nonlinear analysis. It is thought to be the primary link between applied and pure mathematics. It is also extensively used in many other academic disciplines, including practically all engineering disciplines, chemistry, physics, and economics. There are numerous uses for Banach's contraction mapping principle in fixed point theory. Banach proved the following well-known fixed point theorem in 1922. Given a complete metric space (X,d) and a contraction P:X $\to$ X, it can be shown that $x_0 \in$ X of T Banach has a single fixed point. The Banach contraction principle, as this theorem is known, is a powerful tool in nonlinear analysis.

In recent years [1] studied some fixed point theorems and generalized two sided cone convex contraction mapping of the Banach contraction Principle defined on non-normal cone metric spaces. The attention to the concept of b-metric spaces has been given with examples to illustrate the multivalued mapping by [8]. [5] introduced α-F convex contraction mapping in F Metric spaces

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and discussed some fixed points results with helpful hypothetical results. [2] generalized F Contraction with sufficient conditions for existence and uniqueness of fixed points theorems with samples. [3] discussed various results of fixed points for partial metric spaces, G-metrics, GB-metrics, extended B-metric spaces and many others convex contraction mappings.

The results of Chatterjea two sided convex contractive and Hardy and Rogers convex contraction mappings are defined and the generalized result are studied in [04]. F- Contractive mapping is proved as a Picard operator on complete b-metric space and illustrated with examples by [6]. New contractive mapping defined in b metric space and the uniqueness of fixed point was proved for new contractive mapping to solve simultaneous linear equations with examples of b metric space by [7]. [9] defined the orthogonal cone metric space and orthogonal complete cone metric space as an extended version and the uniqueness of fixed point is proved with an application of periodic boundary value problem.

Fixed point theorems for $\alpha - \psi$ contractive mapping are discussed by [10] and initiated the mappings in fuzzy metric spaces with some ordinary differential equations applications. [11] obtained F Convex contractive by non-linear Fredholm integral equation with examples and convex contractive is defined, some results and properties discussed.

2. Preliminaries

If ϕ is the set of all functions $f : R^+ \to R$, then ϕ is a mapping that satisfies the following conditions:
(1) f is strictly increasing, that is, for all $x_1, x_2 \in R^+$
if $x_1 < x_2$ then $f(x_1) < f(x_2)$;
(2) For each sequence α_n of positive numbers,

$$\lim_{n\to\infty} \alpha_n = 0$$

if and only if

$$\lim_{n\to\infty} f(\alpha_n) = -\infty$$

(3) There exists k $\in$ (0,1) such that

$$\lim_{\alpha\to 0^+} (\alpha)^k f(\alpha) = 0$$

Definition 2.1 (12)**.** Let r $\geq$ 1 be a given real integer and let X be a nonempty set. A function d: X $\times$ X $\to R^+$ is deemed a b-metric if and only if each of the subsequent requirements holds true for every $x_1, x_2, x_3 \in^X$
(a) $d(x_1, x_2) = 0$ if and only if $x_1 = x_2$;
(b) $d(x_1, x_2) = 0$=$d(x_2, x_1) = 0$;
(c) $d(x_1, x_2) \leq r[d(x_1, x_3) + d(x_3, x_2)]$.
The pair (X, d) is called a b-metric space.

It should be noted that, the class of b-metric spaces is functionally bigger than that of metric spaces, since a b-metric is a metric when $r = 1$. But, in general, the converse is not true.

Definition 2.2 (12)**.** Given a b-metric space (X,d) with coefficient r ≥ 1, consider the following: P: X $\to X$ is a given mapping. If and only if, for any sequence $x_n \in X$, we have $x_n \to x_0$ then $Px_n \to Px_0$ as $n \to \infty$ then P is continuous at $x_0 \in X$. We state that P is continuous on X if P is continuous at every point of $x_0 \in X$.

In general, a b-metric is not necessarily continuous.

Definition 2.3 (7)**.** Let X be a b-metric space and x_n be a sequence in X we say that (a) x_n is b-converges to $x_n \in X$ if $d(x_n, x_m) \to 0$ as n $\to \infty$;
(b) x_n is a b-Cauchy sequence if $d(x_n, x) \to 0$ as n, m $\to \infty$;
(c) (X,d) is b-complete if every b-Cauchy sequence in X is b-convergent.

Definition 2.4 (7)**.** A continuous mapping P: X $\to X$ defined on a metric space X is called two-sided convex contraction mapping if there exist positive numbers $a_1, a_2, b_1, b_2 \in (0, 1)$ such that the following inequality holds:
$d(P^2x_1, P^2x_2) \leq a_1d(x_1, Px_1)+a_2d(Px_1, P^2x_1)+b_1d(x_2, Px_2)+b_2d(Px_2, P^2x_2)$ for $x_1, x_2 \in X$ and $a_1 + a_2 + b_1 + b_2 < 1$.

Theorem 2.5 (3)**.** *Let P: $X \to X$ be any two-sided convex contraction mapping and let (X,d) be a full metric space. Then P has a unique fixed point.*

Theorem 2.6 (4)**.** *Let P be a self-mapping that satisfies the Chatterjea two sided convex contraction criteria, and let (X, d) be a full metric space. Assume that P has an orbital continuity. In X, P then has a single fixed point. The Picard iteration x_n, defined as $x_{n=1} = P^n x, n \geq 0$ converges to the fixed point of P for any $x_0 \in X$.*

Definition 2.7 (4)**.** Hardy and Rogers convex contraction mapping of type 2 is a continuous mapping P: X $\to X$defined on a metric space X if there exist positive values $l_1, l_2, k_1, k_2, s_1, s_2, m_1, m_2, f_1, f_2 \in (0, 1)$ such that the following inequality holds:
$d(P^2x_1, P^2x_2) \leq l_1d(x_1, x_2) + l_2d(Px_1, Px_2) + k_1d(x_1, Px_1) + k_2d(Px_1, P^2x_1) + s_1d(Px_2, Px_2) + s_2d(Px_2, P^2x_2) + m_1d(Px_1, Px_2) + m_2d(Px_2, P^2x_2)$
$+ f_1d(x_1, Px_1) + f_2d(Px_1, P^2x_1)$
for $x_1, x_2 \in X$ and $l_1 + l_2 + k_1 + k_2 + s_1 + s_2 + m_1 + m_2 + f_1 + f_2 < 1$.

3. Main results

In the context of b-metric spaces, we established fixed point solutions for α-F convex contraction mappings.

Definition 3.1. A b-metric space (X,d) with parameters $r \geq 1, P : X \to X, \alpha : X \times X \to R^+$ and $F \in \phi$ is defined. If $a_i, b_i \in [0, 1)$ with $\sum_{i=1,2}(a_i + b_i) < \frac{1}{r}$ exists and meets the following criteria, then P is referred to as a Chatterjea

two-sided $\alpha - F$-convex contraction mapping.
$d(P^2x_1, P^2x_2) > 0$

$$\Rightarrow \tau + F(\alpha(x_1, x_2)d(P^2x_1, P^2x_2)) \leq F(a_1d(x_1, Px_2) + a_2d(Px_2, P^2x_2) + b_1d(x_2, Px_1) + b_2d(Px_1, P^2x_1)) \quad (1)$$

for all $x_1, x_2 \in X$ and $\tau > 0$.

Theorem 3.2. *Let $r \geq 1$ and let (X, d) be a complete b-metric space. $P : X \to X$ is a two-sided $\alpha - F$-convex contraction mapping by Chatterjea that satisfies the subsequent requirements:*
(a) P is α-admissible;
(b) There exists $x_0 \in X$ such that $\alpha(x_0, Px_0) \geq 1$;
(c) P is continuous or arbitrarily continuous on X.
Then, P has a fixed point in X. Further, if P is α^*- admissible, then P has a unique fixed point $z \in X$ iff $x_{n+1} = P^{n+1} \neq Px_n$ for all $n \geq 0$, then

$$\lim_{n\to\infty}(P^nx_0) = z$$

Proof. By Theorem 3.1(b),there exists a point $x \in X$ such that $\alpha(x_0, Px_0) \geq 1$ and define a sequence x_n by
$x_1 = Px_0, x_2 = Px_1, x_3 = Px_2, x_(n + 1) = Px_n$ for all $n = 0, 1, 2,$
If $x_n = x_(n + 1)$ for some n, $x_n = x_(n + 1) = Px_n, x_n$ is fixed point of P.
Assume $x_n \neq x_{n+1}$ for all $n = 0, 1, 2,$. Then $d(x_n, x_{n+1}) > 0$ for all $n = 0, 1, 2,$.
Since, P is α admissible, $\alpha(x_0, Px_0) \geq 1 \Rightarrow \alpha(x_1, x_2) = \alpha(Px_0, P^2x_0) \geq 1$.
Therefore, one can obtain inductively that
$\alpha(x_n, x_{n+1}) = \alpha(P^nx_0, P^{(n+1)}x_0) \geq 1$ for all $n = 0, 1, 2,$
By definition 3.1, $x_1 = Px_0$ and $x_2 = P^2x_0$. we have
$d(P^2x_0, P^3x_0) > 0 \Rightarrow \tau + F(\alpha(x_0, Px_0)d(P^2x_0, P^3x_0))$
$\leq F[a_1d(x_0, P^2x_0) + a_2d(P^2x_0, P^3x_0) + b_1d(Px_0, Px_0) + b_2d(Px_0, P^2x_0)]$
$= F[a_1d(x_0, P^2x_0) + a_2d(P^2x_0, P^3x_0) + b_2d(Px_0, P^2x_0)]$
$\leq F[a_1(sd(x_0, Px_0) + rd(Px_0, P^2x_0)) + a_2d(P^2x_0, P^3x_0) + b_2d(Px_0, P^2x_0)]$
$= F[a_1sd(x_0, Px_0) + a_1rd(Px_0, P^2x_0) + a_2d(P^2x_0, P^3x_0) + b_2d(Px_0, P^2x_0)]$
$= F[a_1rd(x_0, Px_0) + (a_1r + b_2)d(Px_0, P^2x_0) + a_2d(P^2x_0, P^3x_0)]$
$\leq F[(2a_1r + b_2)\max(d(x_0, Px_0), d(Px_0, P^2x_0)) + a_2d(P^2x_0, P^3x_0)]$
$= F[(2a_1r + b_2)\upsilon + a_2d(P^2x_0, P^3x_0)]$,
where $\gamma = \max(d(x_0, Px_0), d(Px_0, P^2x_0))$.
Since F is strictly increasing,and $\tau > 0$,
$d(P^2x_0, P^3x_0) < (2a_1r + b_2)\upsilon + a_2d(P^2x_0, P^3x_0)$
$(1 - a_2)d(P^2x_0, P^3x_0) < (2a_1s + b_2)\upsilon$
$d(P^2x_0, P^3x_0) < \frac{2a_1r+b_2}{1-a_2}\gamma, (1 - a_2) > 0$
$d(P^2x_0, P^3x_0) < \lambda\gamma$, where $\lambda = \frac{2a_1r+b_2}{1-a_2}$.
Similarly, we get $d(P^4x_0, P^5x_0) < \lambda^2\gamma$.
Continuing this process inductively, we get
$d(P^mx_0, P^{m+1}x_0) < \lambda^t\gamma$.
when m is even and odd, $l \geq l$.

Now we get x_n is a b-metric Cauchy sequence in X.

Case 1: If m value even, $l \geq 1$.
$d(P^m x_0, P^n x_0) = d(P^{2l} x_0, P^n x_0)$
$d(P^m x_0, P^n x_0) \leq (r + r^2)(\lambda^l)(1 - r^2\lambda)^{-1}\gamma \to 0$ as $l \to \infty$.

Case 2: If m value odd, $l \geq 1$.
$(P^m x_0, P^n x_0) = d(P^{2l+1} x_0, P^n x_0)$
$d(P^m x_0, P^n x_0) \leq (r + r^2)(\lambda^l)(1 - r^2\lambda)^{-1}\gamma \to 0$ as $l \to \infty$.
We have x_n is a b-metric Cauchy sequence in X.
There exists $z \in X$ such that

$$\lim_{n\to\infty} (x_n) = \lim_{n\to\infty} (P^n x_0) \to z$$

.
Next, we prove that z is a fixed point of P.
By the continuity of P, we obtain $z =$

$$\lim_{n\to\infty} P(P^n x_0) = Pz$$

Therefore, z is a fixed point of P.
Uniqueness: We suppose that P is α^*-admissible. Since Fix$(P) \neq \emptyset$,
let $z, z^* \in$ Fix(P), by α^*-admissible of P, we have $\alpha(z, z^*) \geq 1$. From Eq. (2)
$F(d(z, z^*)) = F(d(P^2 z, P^2 z^*)) = F(\alpha(z, z^*) d(P^2 z, P^2 z^*))$
$\leq F(a_1 d(z, Pz^*) + a_2 d(Pz^*, T^2 z^*) + b_1 d(z^*, Pz) + b_2 d(Pz, P^2 z)) - \tau$
$\leq F(a_1 d(z, Tz^*) + b_1 d(z^*, Pz)) - \tau$
Since $\tau > 0$ and F is strictly increasing, we obtain
$d(z, z^*) < a_1 d(z, Pz^*) + b_1 d(z^*, Tz) < (a_1 + b_1) d(z, z^*)$
$d(z, z^*) < d(z, z^*)$,a contradiction, which in turn gives $z^* = z$.
Therefore, P has a unique fixed point in X. □

Example 3.3. Let $X = [0, 1]$ and $d : X \times X \to R^+$ be given by $d(x_1, x_2) = |x_1 - x_2|^2$ for $x_1, x_2 \in X$. Then (X, d) a complete b-metric space with $r = 2$. We define a mapping $P : X \to X$ by
$P(x_1) = \begin{cases} 0 & \text{if } x_1 \in [0, \frac{1}{2}) \\ \frac{{x_1}^2}{5} + \frac{1}{10} & \text{if } x_1 \in [\frac{1}{2}, 1] \end{cases}$
and
$\alpha(x_1, x_2) = \begin{cases} 1 & \text{for all } x_1, x_2 \in X \\ 0 & \text{otherwise} \end{cases}$
Then P is $\alpha-$ admissible. Setting $F \in \phi$ such that $F : R^+ \to R$ given by $F(\vartheta) = \ln(\vartheta)$. Then for $x_1, x_2 \in X, x_1 \neq x_2$.

Definition 3.4. Let (X, d) be a metric space with parameters $r \geq 1, P : X \to X, \alpha : X \times X \to R^+$ and $F \in \phi$. Then P is called Hardy and Rogers $\alpha - F$-convex contraction mapping if there exists $a_i, b_i, c_i, e_i, f_i \in [0, 1)$ with $\sum_{i=1,2}(a_i + b_i +$

$c_i + e_i + f_i) < \frac{1}{r}$ and satisfies the following condition:
$d(P^2x_1, P^2x_2) > 0$

$$\Rightarrow \tau + F(\alpha(x_1,x_2)d(P^2x_1,P^2x_1)) \leq F[a_1 d(x_1,Px_2) + a_2 d(Tx_1,T^2x_2) + b_1 d(x_2,Px_1) + b_2 d(Px_1,P^2x_1)) + c_1 d(x_2,Px_2) + c_2 d(Px_2,P^2x_2)) + e_1 d(x_1,Px_2) + e_2 d(Px_2,P^2x_2)) + f_1 d(x_2,Px_1) + f_2 d(Tx_1,T^2x_1))] \quad (2)$$

for all $x_1, x_2 \in X$ and $\tau > 0$.

Theorem 3.5. *Let (X,d) be a complete b-metric space with $r \geq 1$, $P : X \to X$ be Hardy and Rogers $\alpha - F$-convex contraction mapping satisfying the satisfy the following conditions:*
(a) P is α-admissible;
(b) There exists $x_0 \in X$ such that $\alpha(x_0, Px_0) \geq 1$;
(c) P is continuous or arbitrarily continuous on X.
Then, P has a fixed point in X. Further, if P is α^*-admissible, then P has a unique fixed point $z \in X$. Moreover, for any $x_0 \in X$ if $x_{n+1} = P^{n+1} \neq Px_n$ for all $n \geq 0$, then

$$\lim_{n\to\infty}(P^n x_0) = z$$

Proof. By Theorem 3.2(b),there exists a point $x_0 \in X$ such that $\alpha(x_0, Px_0) \geq 1$ and define a sequence x_n by
$x_1 = Px_0, x_2 = Px_1, x_3 = Px_2, x_(n+1) = Tx_n$ for all $n = 0, 1, 2,$
If $x_n = x_(n+1)$ for some n, $x_n = x_{n+1} = Px_n, x_n$ is fixed point of P.
Assume $x_n \neq x_{n+1}$ for all $n = 0, 1, 2,$. Then $d(x_n, x_{n+1} > 0$ for all $n = 0, 1, 2,$.
Since, P is α admissible, $\alpha(x_0, Px_0) \geq 1 \Rightarrow \alpha(x_1, x_2) = \alpha(Px_0, P^2x_0) \geq 1$.
Therefore, one can obtain inductively that
$\alpha(x_n, x_{n+1}) = \alpha(P^n x_0, P^{n+1}x_0) \geq 1$ for all $n = 0, 1, 2,$
By definition 3.2, $x_1 = Px_0$ and $x_2 = P^2x_0$. we have
$d(P^2x_0, P^3x_0) > 0 \Rightarrow \tau + F(\alpha(x_0, Px_0)d(P^2x_0, P^3x_0))$
$d(P^2x_0, P^3x_0) \leq \lambda\gamma$,where $\lambda = \frac{a_1+b_1+c_1+2e_1r+a_2+b_2+f_2}{1-c_1-e_2}$
By definition 3.2 $x_1 = Px_0$ and $x_2 = P^2x_0$. we get
$d(P^3x_0, P^4x_0) > 0 \Rightarrow \tau + F(\alpha(Px_0, P^2x_0)d(P^3x_0, P^4x_0)) < \lambda\gamma$
Similarly the process inductively, we get
$d(P^m x_0, P^{m+1}x_0) < \lambda^l\gamma$.
when m is even and odd, or for $l \geq l$.
Now show that x_n is a b-Cauchy sequence in X.
Case 1: If m value even, $l \geq 1$.
$d(P^m x_0, P^n x_0) = d(P^{2l}x_0, P^n x_0)$
$\leq r(d(P^{2l}x_0, P^{2l+1}x_0) + d(P^{2l+1}x_0, P^n x_0))$
$d(P^m x_0, P^n x_0) \leq (r + r^2)(\lambda^l)(1 - r^2\lambda)^{-1}\gamma \to 0$ as $l \to \infty$.

Case 2: If m value odd, $l \geq 1$.
$d(P^m x_0, P^n x_0) = d(P^{2l+1}x_0, P^n x_0)$
$\leq r(d(P^{2l+1}x_0, P^{2l+2}x_0) + d(P^{2l+2}x_0, P^n x_0))d(P^m x_0, P^n x_0)$

$\leq (r + r^2)(\lambda^l)(1 - r^2\lambda)^{-1}\gamma \to 0$ as $l \to \infty$.
We have x_n is a b-metric Cauchy sequence in X.
There exists $z \in X$ such that

$$\lim_{n\to\infty}(x_n) = \lim_{n\to\infty}(P^n x_0) \to z$$

.
Next, we prove that z is a fixed point of P.
By the continuity of P, we obtain z=

$$\lim_{n\to\infty} P(P^n x_0) = Pz$$

Therefore, z is a fixed point of P.

Uniqueness: We suppose that P is α^*-admissible. Since Fix $(P) \neq \emptyset$, let $z, z^* \in$ Fix(P), by α^*-admissible of P, we have $\alpha(z, z^*) \geq 1$.
By definition 3.2,
$F(d(z, z^*)) = F(d(P^2z, P^2z^*)) = F(\alpha(z, z^*)d(PP^2z, P^2z^*))$
$\leq F(a_1d(z, z^*) + a_2d(Pz^*, Pz^*) + b_1d(z, Pz) + b_2d(Pz, P^2z)) + c_1d(z^*, Pz^*) + c_2d(z, P^2z^*)) + e_1d(z, Pz^*) + e_2d(Pz, Pz^*)) + f_1d(z^*, Pz) + f_2d(Pz, P^2z)) - \tau$
$\leq F(a_1d(z, z^*) + a_2d(z, z^*) + e_1d(z, z^*) + f_1d(z, z^*)) - \tau$
$d(z, z^*) < (a_1d(z, z^*) + a_2d(z, z^*) + e_1d(z, z^*) + f_1d(z, z^*)$
$< (a_1 + a_2 + e_1 + f_1)d(z, z^*)$
$d(z, z^*) < d(z, z^*) + b_1d(z^*, Tz) < (a_1 + b_1)d(z, z^*)$
$d(z, z^*) < d(z, z^*)$, a contradiction, which in turn gives $z^* = z$.

Hence, P has a unique fixed point in X. Now we give an example in support of Theorem 3.2. □

Example 3.6. Assume that $X = [0, 1]$ and $d : X \times X \to R^+$ is provided by $d(x_1, x_2) = |x_1 - x_2|^2$ for all $x_1, x_2 \in X$. Then, with $r = 2, (X, d)$ a complete b-metric space .
$P(x_1) = \frac{x_1^2}{2} + \frac{1}{4}$ defines a mapping $P : X \to X$ for all $x_1 \in X$. For every $x_1, x_2 \in X, \alpha(x_1, x_2) = 1$. Then P is α-admissible. Assume that $F : R^+ \to R$ is given by $F(\vartheta) = ln\vartheta > 0, \vartheta$.

4. Conclusion

In the context of complete b-metric spaces, we established and demonstrated the uniqueness of fixed points and the existence of a $\alpha-$F-convex contraction mappings. Our findings build upon and broaden the similar findings in the literature. We have also provided evidence for the primary findings of this study by utilizing a few relevant cases.

Conflicts of interest : The authors declare that they have no conflicts of interest.

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