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ANALYTIC FUNCTIONS RELATED WITH q-CONIC DOMAIN AND ASSOCIATED WITH A CONVOLUTION OPERATOR

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Abstract. In this paper, we defined some new classes of analytic functions in conic domains. We investigate some important properties such as necessary and sufficient conditions, coefficient estimates, convolution results, linear combination, weighted mean, arithmetic mean, radii of starlikeness and distortion for functions in these classes. It is important to mentioned that our results are generalization of number of existing results in the literature.

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1. Introduction

The theory of the q-calculus operators are used in many areas of science such as fractional q-calculus, optimal control, q-difference, and q-integral equations. This theory is many applications in geometric function theory of complex analysis as is discuss by Srivastava [41] in his recent survey-cum-expository review article.

In 1908, Jackson [19] defined the q-analogs of the ordinary derivative and integral operators, and presented some of their applications. More recently, Ismail et al. [21] gave the idea of a q-extension of the familiar class of starlike functions in $\mathbb{U}$. Many researchers have since studied the q-calculus in the context of Geometric Functions Theory. For example, Kanas and Răducanu [24] introduced the q-analogue of the Ruscheweyh derivative operator and Zang et al. in [47] studied q-starlike functions related with a generalized conic domain $\Omega_{k,\alpha}$. By using the concept of convolution, Srivastava et al. [45] introduced the q-Noor integral operator and studied some of its applications. Ahmad et al. [2]

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introduced a family of meromorphic multivalent functions associated with the domain of lemniscate of Bernoulli in q-analogue. Some of latest innovations in the field can be seen in the work of Arif et al. [14] in which they investigated the q-generalization of Harmonic starlike functions. While Srivastava in [39, 42, 46] investigated some general families in q-analogue related to Janowski functions and obtained some interesting results. Ramachandran et al.[40] obtained coefficient bounds for some subclasses of fractional q-derivative multivalent functions together with generalized Ruscheweyh derivative. Frasin et al. [15] derived a subordination result and integral mean for certain class of analytic functions defined by means of a fractional q-differintegral operator. Furthermore, Shamsan et al. [36] derived some convolution conditions for q-Sakaguchi-Janowski type functions, (see also, [3, 4, 5, 6, 7, 8, 9, 10, 12, 16, 18, 37, 43]).

In most of the cases it is much harder to use a random domain, so Riemann mapping theorem allows us to replace it with open unit disk defined as:

$$\mathbb{U}=\{\xi \in \mathbb{C} : |\xi| < 1\}.$$

A function $\hat{g}$ is analytic at a point ξ_0 if $\hat{g}^{'}(\xi)$ exists at ξ_0 as well as in some neighborhood of ξ_0. An analytic function $\hat{g}$ is univalent in $\mathbb{U}$, if $\hat{g}(\xi_1) = \hat{g}(\xi_2)$ then $\xi_1 = \xi_2$, for all $\xi_1, \xi_2 \in \mathbb{U}$. A function $\hat{g}(\xi)$ is said to be the class $\mathfrak{A}$ if it has a Taylor series of the form

$$\hat{g}(\xi) = \xi + \sum_{t=2}^{\infty} a_t \xi^t, \quad \xi \in \mathbb{U}, \tag{1.1}$$

A collection of functions of the form (1.1), which are analytic and univalent in $\mathbb{U}$ are placed in the class $\mathfrak{S}$. An analytic function $p(\xi)$ having positive real part that is, $\operatorname{Re}\{p(\xi)\} > 0$ and $p(0) = 1$ is placed in class $\mathfrak{P}$, or equivalently

$$p \in \mathfrak{P} : p(\xi) = 1 + \sum_{t=1}^{\infty} a_t \xi^t \iff \operatorname{Re}\{p(\xi)\} > 0, \quad \xi \in \mathbb{U}. \tag{1.2}$$

The class of normalized convex functions is given by

$$C = \left\{ \hat{g} : \hat{g} \in \mathfrak{S}; \operatorname{Re}\left(\frac{\left(\xi \hat{g}^{'}(\xi)\right)^{'}}{\hat{g}(\xi)} \right) > 0, \quad \xi \in \mathbb{U} \right\}.$$

Similarly, the class of normalized starlike functions is defined as:

$$\mathbb{S}^* = \left\{ \hat{g} : \hat{g} \in \mathfrak{S}; \ \operatorname{Re}\left(\frac{\xi \hat{g}^{'}(\xi)}{\hat{g}^{'}(\xi)} \right) > 0, \quad \xi \in \mathbb{U} \right\},$$

for details, see [11]. If a function $\hat{g}(\xi) \in QC$, the class of quasi-convex function if and only if there exist $\hat{h}(\xi) \in C$ such that $\operatorname{Re}\left(\frac{(\xi \hat{g}^{'}(\xi))^{'}}{\hat{h}^{'}(\xi)} \right) > 0$. In 1952, Kaplan [26] introduced the class KC of close-to-convex function. A function is of the form (1.2) is in KC if and only if there exists $\hat{h}(\xi) \in \mathbb{S}^*$ such that $\mathrm{Re}\left(\frac{\xi \hat{g}^{'}(\xi)}{\hat{h}(\xi)}\right) > 0$. Let $\hat{g}(\xi)$ is of the form (1.1) and $\hat{h}(\xi)$ is of the form

$$\hat{h}(\xi) = \xi + \sum_{t=2}^{\infty} b_t \xi^t, \ \xi \in \mathbb{U}. \tag{1.3}$$

Then the Hadamard product(convolution) of $\hat{g}$ and $\hat{h}$ is defined as:

$$\left(\hat{g} * \hat{h}\right)(\xi) = \xi + \sum_{t=2}^{\infty} a_t b_t \xi^t = \left(\hat{h} * \hat{g}\right)(\xi). \tag{1.4}$$

The q-derivative of a function $\hat{g}$ belonging to $\mathfrak{A}$ defined as:

$$D_q \hat{g}(\xi) = \frac{\hat{g}(q\xi) - \hat{g}(\xi)}{\xi(q-1)} \qquad \text{for } \xi \neq 0, \tag{1.5}$$

for details, see [19], where $q \in (0,1)$ and $\xi \in \mathbb{U}$. For $\xi = 0$, (1.5) can be written as $\hat{g}^{'}(0)$ provided that the derivative exists. By using (1.1) and (1.5) the Maclaurin's series representation of $D_q \hat{g}$ is given by

$$D_q \hat{g}(\xi) = 1 + \sum_{t=0}^{\infty} [t,q]\, a_t \xi^{t-1}.\mathbb{N} \tag{1.6}$$

It can be noted from (1.5) that

$$\lim_{q \to 1^-} \left(D_q \hat{g}(\xi)\right) = \lim_{q \to 1^-} \left(\frac{\hat{g}(q\xi) - \hat{g}(\xi)}{\xi(q-1)}\right) = \hat{g}^{'}(\xi), \text{ where } [t,q] = \frac{1-q^t}{1-q}.$$

For any non negative integer t see [13] the q-number shift factorial is given by

$$[t,q]! = \begin{cases} 1, & t = 0 \\ [1,q]\,[2,q] \cdots [t,q]\,, & t \in \mathbb{N} \end{cases}, \tag{1.7}$$

For $y > 0$, the q-generalized Pochammar symbol is defined as:

$$[y,q]_t = \begin{cases} 1, & t = 0 \\ [y,q]\,[y+1,q] \cdots [y+t-1,q]\,, & t \in \mathbb{N} \end{cases}. \tag{1.8}$$

The study of operators plays an important role in the geometric function theory. Many differential and integral operators can be written in terms of convolution of certain analytic functions. For $\mu > -1$, we defined a function $\mathfrak{F}_{q,1+\mu}^{-1}(\xi)$ such that

$$\mathfrak{F}_{q,1+\mu}(\xi) * \mathfrak{F}_{q,1+\mu}^{-1}(\xi) = \xi D_q \hat{g}(\xi), \tag{1.9}$$

where

$$\mathfrak{F}_{q,1+\mu}(\xi) = \xi + \sum_{t=2}^{\infty} \left(\frac{[1+\mu,q]_{t-1}}{[t-1,q]!} \xi^t\right), \qquad \text{for } \xi \in \mathbb{U}. \tag{1.10}$$

In [13] q-analogue of Noor integral operator $\Im_q^\mu : \mathfrak{A} \to \mathfrak{A}$ is define as:

$$\Im_q^\mu \hat{g}(\xi) = \hat{g}(\xi) * \mathfrak{F}_{q,1+\mu}^{-1}(\xi) = \xi + \sum_{t=2}^{\infty} \psi_{t-1} a_t \xi^t, \tag{1.11}$$

where

$$\psi_{t-1} = \frac{[t,q]!}{[1+\mu,q]_{t-1}}. \tag{1.12}$$

From (1.9) It can be easily seen that

$$[1+\mu,q]\,\Im_q^{\mu}\hat{g}(\xi) = [\mu,q]\,\Im_q^{\mu+1}\hat{g}(\xi) + q^{\mu}\xi D_q\left(\Im_q^{\mu+1}\hat{g}(\xi)\right). \tag{1.13}$$

It is worth mentioned that $\Im_q^0\hat{g}(\xi) = \xi D_q\hat{g}(\xi)$, $\Im_q^1\hat{g}(\xi) = \hat{g}(\xi)$ and

$$\lim_{q\to 1^-}\left(\Im_q^{\mu}\hat{g}(\xi)\right) = \xi + \sum_{t=2}^{\infty}\frac{t!}{(1+\mu)_{t-1}}a_t\xi^t. \tag{1.14}$$

From (1.14), we can observe that by applying limit $q \to 1^-$, the operator defined in (1.11) reduces to well known Noor integral operators see ([31, 32]).

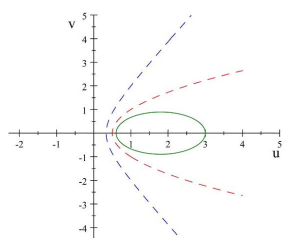

FIGURE 1. $u = l\sqrt{(u-1)^2 + v^2}$

In [22, 23], Kanas and Waniowska introduced the concept of a conic domain Ξ_l for $l \geq 0$ as:

$$\Xi_l = \left\{u + iv : u > l\sqrt{v^2 + (u-1)^2}\right\}. \tag{1.15}$$

This domain merely represent the right half plane for $l = 0$, a hyperbola for $0 < l < 1$, parabola for $l = 1$ and ellipse for $l > 1$ as shown in Figure 1. The extremal functions ϖ_l for this conic region Ξ_l is given by

$$\varpi_l(\xi) = \begin{cases} \frac{1+\xi}{1-\xi} & l = 0, \\ 1 + \left\{\frac{2}{\pi^2}\left(\log\frac{\sqrt{\xi}+1}{1-\sqrt{\xi}}\right)^2\right\} & l = 1, \\ 1 + \frac{2}{1-l^2}\sinh^2\left[\left(\frac{2}{\pi}\arccos l\right)\left(\arctan h\sqrt{\xi}\right)\right] & 0 < l < 1, \\ 1 + \frac{1}{l^2-1}\sin\left[\frac{\pi}{2R(n)}\int_0^{\frac{U(\xi)}{\sqrt{n}}}\left(\frac{1}{\sqrt{1-n^2y^2}\sqrt{1-x^2}}\right)dx\right] + \frac{1}{l^2-1} & l > 1, \end{cases} \tag{1.16}$$

where $U(\xi)=\frac{\xi-\sqrt{n}}{1-\sqrt{n}\xi}$, for all $\xi\in\mathbb{U}$, $0<l<1$ and $l=\cosh\left[\frac{\pi R^{'}(n)}{4R(n)}\right]$ where $R(n)$ is Legendre's complete elliptic integral of first kind and $R^{'}(n)$ is complementary integral of $R(n)$ for more details, see [22, 23, 1]. If we take $\varpi_l(\xi)=1+\delta(l)\xi+\delta_1(l)\xi^2+\cdots$, then

$$\delta(l)=\begin{cases}\frac{8(\arccos l)^2}{\pi^2(1-l^2)} & 0\leq l<1,\\ \frac{8}{\pi^2} & l=1,\\ \frac{\pi^2}{4\sqrt{n}(l^2-1)(1+n)R^2(n)} & l>1.\end{cases}\tag{1.17}$$

Let $\delta_1(l)=\delta_2(l)\,\delta(l)$, where

$$\delta_2(l)=\begin{cases}\frac{2+\left(\frac{2}{\pi}\arccos l\right)^2}{3} & 0\leq l<1,\\ \frac{2}{3} & l=1,\\ \frac{4R^2(n)\left(1+n^2+6n\right)-\pi^2}{24(1+n)\sqrt{n}R^2(n)} & l>1.\end{cases}\tag{1.18}$$

Motivated by the above cited work, we now define the following more general class of analytic functions associated with conic domain with a convolution operator.

Definition 1.1. [20] Let p be a analytic function with $p(0)=1$. Then $p\in\mathfrak{P}(\lambda,R)$ if and only if

$$p(\xi)\prec\frac{\lambda\xi+1}{R\xi+1},\qquad\text{where } -1\leq R<\lambda\leq 1.\tag{1.19}$$

In [20] it was shown that $p\in\mathfrak{P}(\lambda,R)$ if and only if there exists a function $p\in\mathfrak{P}$ such that

$$\frac{(1+\lambda)\,p(\xi)-(\lambda-1)}{(1+R)\,p(\xi)-(R-1)}\prec\frac{\lambda\xi+1}{R\xi+1}.$$

Definition 1.2. [29] A function $\hat{g}\in\mathfrak{A}$ considered in the class $k-ST_q(N,M)$ if and only if

$$\begin{aligned}&\Re\left[\frac{(ML_1-L_2)\left(\frac{\xi D_q(\hat{g}(\xi))}{\hat{g}(\xi)}\right)-(NL_1-L_2)}{(ML_1+L_2)\left(\frac{\xi D_q(\hat{g}(\xi))}{\hat{g}(\xi)}\right)-(NL_1+L_2)}\right]\\ >\quad & k\left|\frac{(ML_1-L_2)\left(\frac{\xi D_q(\hat{g}(\xi))}{\hat{g}(\xi)}\right)-(NL_1-L_2)}{(ML_1+L_2)\left(\frac{\xi D_q(\hat{g}(\xi))}{\hat{g}(\xi)}\right)-(NL_1+L_2)}-1\right|,\end{aligned}\tag{1.20}$$

where $k\geq 0$, $-1\leq M<N\leq 1$, $L_1=q+1$ and $L_2=3-q$. One can observe that, for $q\to 1^-$, the class $k-ST_q(N,M)$ reduced to well known class defined in [33].

Definition 1.3. Let $\hat{g}(\xi) \in \mathfrak{A}$. Then $k - ST_q(\mu, N, M)$ if and only if

$$\Re\left[\frac{(ML_1 - L_2)\left(\frac{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}{\Im_q^\mu \hat{g}(\xi)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}{\Im_q^\mu \hat{g}(\xi)}\right) - (NL_1 + L_2)}\right]$$

$$> \quad k\left|\frac{(ML_1 - L_2)\left(\frac{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}{\Im_q^\mu \hat{g}(\xi)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}{\Im_q^\mu \hat{g}(\xi)}\right) - (NL_1 + L_2)} - 1\right|,$$

where $k \geq 0$, $-1 \leq M < N \leq 1$, $\mu > -1$, $L_1 = 1 + q$ and $L_2 = 3 - q$.

Definition 1.4. Let $\hat{g}(\xi) \in \mathfrak{A}$. Then $k - UCV_q(\mu, N, M)$ if and only if

$$\Re\left[\frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)}\right]$$

$$> \quad k\left|\begin{array}{c}\frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} \\ -1\end{array}\right|,$$

where $k \geq 0$, $-1 \leq O < N \leq 1$, $\mu > -1$, $L_1 = 1 + q$ and $L_2 = 3 - q$. One can easily verify this

$$\hat{g} \in k - UCV_q(\mu, N, M) \Longleftrightarrow \xi D_q\left(\Im_q^\mu \hat{g}\right) \in k - ST_q(\mu, N, M). \tag{1.21}$$

It is noted that, for $\mu = 1$, the function class $k - UCV_q(\mu, N, M)$ reduced to well known class $k - UCV_q(N, M)$ introduced by Naeem et al. in [30] and for $\mu = 1$ along with $q \to 1^-$, the class $k - UCV_q(\mu, N, M)$ bring to well-known class interpreted in [33].

In this paper, we will presume that $\mu > -1$, $k \geq 0$, $-1 \leq M < N \leq 1$, $L_1 = 1 + q$ and $L_2 = 3 - q$, if not mentioned.

To establish our main results, we need the following lemma.

Lemma 1.5. [29] *A function $\hat{g} \in \mathfrak{A}$ will be in the class $k - ST_q(N, M)$, if*

$$\sum_{t=2}^{\infty}\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1 + L_2)\,[t,q] - (NL_1 + L_2)\right|\right\}|a_t| < L_1\,|M - N|\,. \tag{1.22}$$

Motivated by the work of Mahmood et al. [29], Noor and Malik [33] and Arif et al. [13], in this paper we find some properties such as necessary and sufficient conditions, coefficient estimates, convolution results, linear combination, weighted mean, arithmetic mean, radii of starlikeness and distortion for functions in the class $k - UCV_q(\mu, N, M)$.

2. Main Results

In this section of the paper, we will prove some major results.

2.1. Necessary and Sufficient Conditions.

Theorem 2.1. *Let $\hat{g}(\xi) \in \mathfrak{A}$ is of the form (1.1). Then $\hat{g}(\xi) \in k - UCV_q(\mu, N, M)$, if it fulfill the following inequality*

$$\sum_{t=2}^{\infty} \{2(k+1)qL_2 [t-1,q] + |(ML_1+L_2)[t,q] - (NL_1+L_2)|\} [t,q]\, \psi_{t-1} |a_t| < L_1 |M-N| . \tag{2.1}$$

Proof. Presume that (2.1) be valid, then it is suffices to prove that

$$k \left| \frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} - 1 \right| - \Re \left[\frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} - 1 \right] < 1.$$

We consider

$$\begin{aligned}
&k \left| \frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} - 1 \right| \\
&- \Re \left[\frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} - 1 \right] \\
&\leq (k+1) \left| \frac{(ML_1 - L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 - L_2)}{(ML_1 + L_2)\left(\frac{D_q\left\{\xi D_q\left(\Im_q^\mu \hat{g}(\xi)\right)\right\}}{D_q\left(\Im_q^\mu \hat{g}(\xi)\right)}\right) - (NL_1 + L_2)} - 1 \right| \\
&= \frac{2L_2 (k+1) \sum_{t=2}^{\infty} |1 - [t\ \ ,q]|\, \psi_{t-1} [t,q] |a_t|}{L_1 |M-N| - \sum_{t=2}^{\infty} |\{(ML_1+L_2)[t,q] - (NL_1+L_2)\} \psi_{t-1} [t,q]| |a_t|}.
\end{aligned}$$

The finale declaration is restricted over by 1 if

$$2L_2 (1+k) \sum_{t=2}^{\infty} |[t-1,q]\, q\, [t,q]\, \psi_{t-1}| |a_t|$$

$$< \quad L_1 \left|M-N\right| - \sum_{t=2}^{\infty} \left| \left\{ \begin{array}{c} (ML_1+L_2)\,[t,q] \\ -\,(NL_1+L_2)\,b_t \end{array} \right\} \psi_{t-1}\,[t,q] \right| \left|a_t\right|,$$

which reduced to

$$\sum_{t=2}^{\infty} \left\{2L_2(k+1)q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,\psi_{t-1}\left|a_t\right|$$

$$< L_1 \left|M-N\right|.$$

□

By taking $\mu = 1$ and $q \to 1^-$, we have the following results proved by Noor and Malik [33].

Corollary 2.2. *Let $\hat{g}(\xi) \in \mathfrak{A}$; then, $\hat{g} \in k - UCV(N,M)$, if the following inequality satisfies*

$$\sum_{t=2}^{\infty} \left\{2(1+k)\,(t-1) + \left|t\,(M+1) - (1+N)\right|\right\} t\left|a_t\right| < \left|M-N\right|.$$

Theorem 2.3. *If $\hat{g} \in k - UCV_q(\mu, N, M)$, then*

$$\left|a_t\right| \leq \frac{L_1\left|M-N\right|}{\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,\psi_{t-1}}. \tag{2.2}$$

Equality attains for the function

$$\hat{g}(\xi) = \xi + \frac{L_1\left|M-N\right|}{\left\{2(1+k)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,\psi_{t-1}}\xi^t. \tag{2.3}$$

Proof. Since $\hat{g} \in k - UCV_q(\mu, N, M)$, (1.20) holds. Since

$$\sum_{t=2}^{\infty} \left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,\psi_{t-1}\left|a_t\right|$$

$$< L_1\left|M-N\right|,$$

we have

$$\left|a_t\right| \leq \frac{L_1\left|M-N\right|}{\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,\psi_{t-1}}.$$

Clearly the function given by (2.3) satisfies (2.2) and therefore $\hat{g}(\xi)$ given by (2.3) is in $k - UCV_q(\mu, N, M)$. For above function, the result is clearly sharp. □

2.2. Coefficient Bound for the class $k-UCV_q(\mu,N,M)$.

Theorem 2.4. *Let $\hat{g}\in k-UCV_q(\mu,N,M)$, is of the form (1.1), then*

$$|a_t|\leq\frac{1}{[t,\ q]}\prod_{j=0}^{t-2}\frac{|(N-M)\,(q+1)\,\delta_l\psi_{j-1}-4qM\psi_j\,[j,\ q]|}{4\psi_{j+1}\,[j+1,q]\,q},\ \ (t\in\mathbb{N}\setminus\{1\})\,.$$

Proof. By using Lemma 1.5 and relation (1.21), this proof is straightforward. □

By taking $\mu=1$, in Theorem 2.4 we obtained the result due to proved by Naeem et al. [30].

2.3. Linear Combination. Linear combination for our defined classes are defined as following.

Theorem 2.5. *Let $\hat{g}_i\in k-UCV_q(\mu,N,M)$ and have the form $\hat{g}_i\,(\xi)=\xi+\sum_{t=1}^{\infty}a_{t,\ i}\xi^t$, for $i=1,2,3,\cdots,n$. Further, let $\sum_{i=1}^{n}c_i=1$ and $F\,(\xi)=\sum_{i=1}^{n}c_i\hat{g}_i\,(\xi)$. Then $F\in k-UCV_q(\mu,N,M)$*

Proof. As $\hat{g}_i\in k-UCV_q(\mu,N,M)$, by the virtue of (2.1), we have

$$\sum_{t=2}^{\infty}\left[\frac{\{2(k+1)L_2q\,[t-1,q]+|(ML_1+L_2)\,[t,q]-(NL_1+L_2)|\}\,[t,q]\,\psi_{t-1}}{L_1\,|M-N|}\right]|a_{t,\ i}|$$
$$<1.$$

Therefore

$$F\,(\xi)=\sum_{i=2}^{n}c_i\left(\xi+\sum_{t=2}^{\infty}a_{t,\ i}\cdot\xi^t\right)=\xi+\sum_{t=2}^{\infty}\left(\sum_{i=2}^{n}c_i\cdot a_{t,\ i}\right)\xi^t.$$

Consider

$$\sum_{t=2}^{\infty}\left[\frac{\left\{2(k+1)L_2q\,[t-1,q]+|(ML_1+L_2)\,[t,q]-(NL_1+L_2)|\right\}[t,q]\,\psi_{t-1}}{L_1\,|M-N|}\right]\left(\sum_{i=2}^{n}c_ia_{t,\ i}\right)$$

$$=\sum_{i=2}^{n}\left(\sum_{t=2}^{\infty}\left[\frac{\left\{2(k+1)L_2q\,[t-1,q]+|(ML_1+L_2)\,[t,q]-(NL_1+L_2)|\right\}\psi_{t-1}}{L_1\,|M-N|}\right]a_{t,\ i}\right)c_i$$
$$\leq 1.$$

Then $F\in k-UCV_q(\mu,N,M)$. □

2.4. Weighted Mean. Let $\hat{g}$ and $\hat{h}$ be two analytic functions and $W\geq 0$, then their weighted mean is defined as:

Theorem 2.6. *Let $h_W\in k-UCV_q(\mu,N,M)$, Then*

$$h_W\,(\xi)=\left\{\frac{(1-W)\,\hat{g}\,(\xi)+(1+W)\,\hat{h}\,(\xi)}{2}\right\}.$$

Proof. If $\hat{g}$ and $\hat{h}$ belongs to $k-UCV_q(\mu, N, M)$, then their weighted mean h_W is also in $k-UCV_q(\mu, N, M)$, where h_W is defined by $h_W(\xi) = \left\{ \frac{(1-W)\hat{g}(\xi)+(1+W)\hat{h}(\xi)}{2} \right\}$. As

$$\begin{aligned} h_W(\xi) &= \left\{ \frac{(1-W)\,\hat{g}(\xi) + (1+W)\,\hat{h}(\xi)}{2} \right\}, \\ &= \left\{ \frac{2\xi + \sum_{t=2}^{\infty}(1-W)\,a_t\xi^t + \sum_{t=2}^{\infty}(1+W)\,b_t\xi^t}{2} \right\}, \\ &= \xi + \sum_{t=2}^{\infty} \left\{ \frac{(1-W)\,a_t + \sum_{t=2}^{\infty}(1+W)\,b_t}{2} \right\} \xi^t. \end{aligned}$$

To prove that $h_W(\xi) \in k-UCV_q(\mu, N, M)$, we need to show

$$\begin{aligned} &\sum_{t=2}^{\infty} \left[\frac{\{2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)|\}\,[t,q]\,\psi_{t-1}}{L_1\,|M-N|} \right] \\ &\times \left\{ \frac{(1-W)\,a_t + (1+W)\,b_t}{2} \right\} \\ &< 1. \end{aligned}$$

For this, consider

$$\begin{aligned} &\sum_{t=2}^{\infty} \left\{ \frac{\{2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)|\}\,[t,q]}{L_1\,|M-N|} \right\} \\ &\times \left\{ \frac{(1-W)\,a_t + (1+W)\,b_t}{2} \right\} \psi_{t-1} \\ &= \frac{(1-W)}{2} \\ &\times \sum_{t=2}^{\infty} \left\{ \frac{\{2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)|\}\,[t,q]}{L_1\,|M-N|} \right\} \psi_{t-1} a_t \\ &+ \frac{(1+W)}{2} \\ &\times \sum_{t=2}^{\infty} \left\{ \frac{\{2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)|\}\,[t,q]}{L_1\,|M-N|} \right\} \psi_{t-1} b_t. \end{aligned}$$

Since $\hat{g}$, $\hat{h} \in k-UCV_q(\mu, N, M)$, so by using (2.1), we have

$$\begin{aligned} &\sum_{t=2}^{\infty} \left\{ \frac{\{2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)|\}\,[t,q]}{L_1\,|M-N|} \right\} \\ &\times \left\{ \frac{(1-W)\,a_t + (1+W)\,b_t}{2} \right\} \psi_{t-1} < \frac{(1-W)}{2}\,(1) + \frac{(1+W)}{2}\,(1) = 1. \end{aligned}$$

Hence the result follows. $\square$

Corollary 2.7. *For $\mu = 1$, if $\hat{g}$ and $\hat{h}$ belongs to $k - UCV_q(\mu, N, M) = k - UCV_q(N, M)$, then their weighted mean h_W is also in $k - UCV_q(N, M)$.*

Corollary 2.8. *For $\mu = 1$ and $q \to 1$, if $\hat{g}$ and $\hat{h}$ belongs to $k-UCV_{q\to 1}(1, N, M) = k-UCV(N, M)$, then their weighted mean h_W is also in $k-UCV(N, M)$. Where $h_W(\xi)$ is defined by*

$$h_W(\xi) = \left\{ \frac{(1-W)\,\hat{g}(\xi) + (1+W)\,\hat{h}(\xi)}{2} \right\}.$$

2.5. Arithmetic Mean.

Theorem 2.9. *Let $\hat{g}_i \in k-UCV_q(\mu, N, M)$ where $i = 1, 2, \cdots, \nu$ then the "arithmetic mean" $A_M(\xi)$ of the function $\hat{g}_i$ is defined by $A_M(\xi) = \frac{1}{\nu}\sum_{i=1}^{\nu} \hat{g}_i(\xi)$, and this also belongs to the class $k - UCV_q(\mu, N, M)$.*

Proof. As

$$A_M(\xi) = \frac{1}{\nu}\sum_{i=1}^{\nu}\left(\xi + \sum_{t=2}^{\infty} a_{t,\,i}\xi^t\right) = \xi + \sum_{t=2}^{\infty}\left(\frac{1}{\nu}\sum_{i=1}^{\nu} a_{t,\,i}\right)\xi^t. \tag{2.4}$$

Since $\hat{g}_i \in k - UCV_q(\mu, N, M)$, for every $i = 1, 2, \cdots, \nu$, by using (2.1) and (2.4), we have

$$\sum_{t=2}^{\infty}\psi_{t-1}\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\left(\frac{1}{\nu}\sum_{i=1}^{\nu} a_{t,\,i}\right)$$
$$= \frac{1}{\nu}\sum_{i=1}^{\nu}\left(\sum_{t=2}^{\infty}\psi_{t-1}\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\right) a_{t,\,i}$$
$$\le \frac{1}{\nu}\sum_{i=1}^{\nu}\left(L_1\left|M-N\right|\right) = L_1\left|M-N\right|.$$

So,

$$\sum_{t=2}^{\infty}\psi_{t-1}\left\{2(k+1)L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\left(\frac{1}{\nu}\sum_{i=1}^{\nu} a_{t,\,i}\right)$$
$$\le L_1\left|M-N\right|.$$

$\square$

2.6. Radii of Starlikeness. A function $\hat{g} \in k - UCV_q(\mu, N, M)$, is said to be starlike of order $\alpha\,(0 \le \alpha < 1)$ if $\operatorname{Re} S^*(\alpha) > \alpha$.

Theorem 2.10. *Let $\hat{g} \in k - UCV_q(\mu, N, M)$. Then $\hat{g} \in S^*(\alpha)$ for $|\xi| < \mathfrak{s}_1$, where*

$$\mathfrak{s}_1 = \left[\frac{(1-\alpha)\left\{2\,(1+k)\,L_2 q\,[t-1,q] + \left|(ML_1+L_2)\,[t,q] - (NL_1+L_2)\right|\right\}[t,q]\,[t,q]!}{L_1\left|M-N\right|(t-\alpha)\,[\mu+1,q]_{t-1}}\right]^{\left(\frac{1}{t-1}\right)}.$$

Proof. To prove $\hat{g} \in S^*(\alpha)$, it is enough to show that

$$\left| \frac{\xi \hat{g}^{'}(\xi)/\hat{g}(\xi) - 1}{\xi \hat{g}^{'}(\xi)/\hat{g}(\xi) + 1 - 2\alpha} \right| < 1.$$

By using (1.1) we have

$$\left| \frac{\xi \hat{g}^{'}(\xi)/\hat{g}(\xi) - 1}{\xi \hat{g}^{'}(\xi)/\hat{g}(\xi) + 1 - 2\alpha} \right| = \sum_{t=2}^{\infty} \left(\frac{t-\alpha}{1-\alpha} \right) |a_t| \, |\xi|^{t-1}. \tag{2.5}$$

Since $\hat{g} \in k - ST_q(\mu, N, M)$, so from (2.1), we can easily obtain

$$\sum_{t=2}^{\infty} \frac{[t,q]!}{[\mu+1,q]_{t-1}} \left(\frac{\left\{ 2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)| \right\}[t,q]}{L_1\,|M-N|} \right) |a_t| < 1.$$

Now inequality (2.5), holds true, if

$$\sum_{t=2}^{\infty} \left[\frac{t-\alpha}{1-\alpha} \right] |a_t| \, |\xi|^{t-1}$$
$$< \sum_{t=2}^{\infty} \frac{[t,q]!}{[\mu+1,q]_{t-1}} \left[\frac{\left\{ 2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)| \right\}[t,q]}{L_1\,|M-N|} \right] |a_t|,$$

which implies that

$$|\xi| < \left(\frac{(1-\alpha)\left\{ 2(k+1)L_2 q\,[t-1,q] + |(ML_1+L_2)\,[t,q] - (NL_1+L_2)| \right\}[t,q]\,[t,q]!}{L_1\,|M-N|\,(t-\alpha)\,[\mu+1,q]_{t-1}} \right)^{\left(\frac{1}{t-1}\right)}.$$

Which completes the proof. □

2.7. Growth and Distortion Theorems.

Theorem 2.11. *If $\hat{g} \in k - UCV_q(\mu, N, M)$ has the form (1.1), then $r(1-\Upsilon) \leq |\hat{g}(\xi)| \leq r(1+\Upsilon)$, where,*

$$\Upsilon = \frac{L_1\,|M-N|}{\{2(k+1)L_2 q + |(ML_1+L_2)(1+q) - (NL_1+L_2)|\}\,\psi_1(1+q)},$$

with $|\xi| = r < 1$, $0 < r < 1$.

Proof. Consider

$$|\hat{g}(\xi)| = \left| \xi + \sum_{t=2}^{\infty} a_t \xi^t \right| = r + \sum_{t=2}^{\infty} |a_t|\, r^t,$$

since $0 < r < 1$, so $r^t < r \Rightarrow r^2 < r$

$$|\hat{g}(\xi)| \leq r + r \sum_{t=2}^{\infty} |a_t| = r \left(1 + \sum_{t=2}^{\infty} |a_t| \right). \tag{2.6}$$

Similarly, by using triangular inequalities, we have

$$|\hat{g}(\xi)| \geq r\left(1-\sum_{t=2}^{\infty}|a_t|\right). \tag{2.7}$$

Has $\hat{g} \in k-UCV_q(\mu, N, M)$, so we have

$$\{2(k+1)L_2q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1\sum_{t=2}^{\infty}a_t$$

$$\leq \sum_{t=2}^{\infty}\{2(k+1)L_2q\,[t-1,q]+|(ML_1+L_2)\,[t,q]-(NL_1+L_2)|\}\,[t,q]\,\psi_{t-1}\,|a_t|\,.$$

By using (2.1), we obtain

$$\{2(k+1)L_2q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1\sum_{t=2}^{\infty}|a_t|$$
$$\leq L_1\,|M-N|\,,$$

which gives

$$\sum_{t=2}^{\infty}|a_t| \leq \frac{L_1\,|M-N|}{\{2(k+1)L_2q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1}$$
$$= \frac{L_1\,|M-N|}{\{2(k+1)L_2q+|(ML_1+L_2)\,(1+q)-(NL_1+L_2)|\}\,(1+q)\,\psi_1}.$$

Using above relativity in (2.6), (2.7), we get required results. □

Theorem 2.12. *If $\hat{g} \in k-UCV_q(\mu, N, M)$ has the form (1.1), then $(1-rt\varrho) \leq \left|\hat{g}'(\xi)\right| \leq (1+rt\varrho)$, where,*

$$\varrho = \frac{L_1\,|M-N|}{\{2(k+1)L_2q+|(ML_1+L_2)\,(1+q)-(NL_1+L_2)|\}\,\psi_1\,(1+q)},$$

with

$$|\xi| = r < 1, 0 < r < 1.$$

Proof. Consider

$$\left|\hat{g}'(\xi)\right| = \left|1+\sum_{t=2}^{\infty}ta_t\xi^{t-1}\right| = 1+r^{t-1}\sum_{t=2}^{\infty}t\,|a_t|\,,$$

since $0 < r < 1$, so $r^t < r$,

$$\left|\hat{g}'(\xi)\right| \leq r+r\sum_{t=2}^{\infty}|a_t| = 1+\sum_{t=2}^{\infty}t\,|a_t|\,. \tag{2.8}$$

Similarly

$$\left|\hat{g}'(\xi)\right| \geq 1-\sum_{t=2}^{\infty}|a_t|\,. \tag{2.9}$$

It can easily be observed that

$$\{2(1+k)L_2 q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1\sum_{t=2}^{\infty}a_t$$

$$\leq\sum_{t=2}^{\infty}\{2(k+1)L_2 q\,[t-1,q]+|(ML_1+L_2)\,[t,q]-(NL_1+L_2)|\}\,[t,q]\,\psi_{t-1}\,|a_t|\,.$$

By using (2.1), we obtain

$$\{2(k+1)L_2 q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1\sum_{t=2}^{\infty}|a_t|$$

$$\leq L_1\,|M-N|\,,$$

which gives

$$\begin{aligned}\sum_{t=2}^{\infty}|a_t| &\leq \frac{L_1\,|M-N|}{\{2(k+1)L_2 q\,[1,q]+|(ML_1+L_2)\,[2,q]-(NL_1+L_2)|\}\,[2,q]\,\psi_1}\\ &= \frac{L_1\,|M-N|}{\{2(k+1)L_2 q+|(ML_1+L_2)\,(1+q)-(NL_1+L_2)|\}\,(1+q)\,\psi_1}.\end{aligned}$$

Using above relation in (2.8), (2.9), we get required results. □

Remark 2.13. (i)For $\mu=1$, and $\mu=1$, $q\to 1^-$, in Theorem 2.11, were discuss in Noor and Malik [33]. (ii)For $\mu=1$, and $\mu=1$, $q\to 1^-$, in Theorem 2.12, were discuss in Noor and Malik [33].

3. Conclusions

In this article, we defined a new class $k-UCV_q(\mu,N,M)$ of analytic functions in conic domains, by using q-analogue of Noor integral operator. We studied various properties such as necessary and sufficient conditions, coefficient bounds, convolution properties, linear combinations, weighted means, arithmetic means, distortion and covering theorems and radii of starlikenss for function belonging to this class. We also pointed out many special cases in the form of corollaries by specializing the parameters.

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