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ON LIFTING C_k^f-STRUCTURES

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Abstract. In this paper, we study some properties about C_k^f-structures and obtain a sufficient condition to be lifting C_k^f-structure, and using the above result, we can obtain a Stasheff's result about to be lifting H-structure as a corollary.

1. Introduction

J. Stasheff [16] obtained a result about a sufficient condition to be lifting an H-structure. In [1], Aguade introduced a T-space as a space X having the property that the evaluation fibration $\Omega X \to X^{S^1} \to X$ is fibre homotopically trivial. It is easy to show that any H-space is a T-space. We [7] introduced the concept of C_k-spaces for any integer $k \geq 1$ and $k = \infty$ which is located at an intermediate stage between H-spaces and T-spaces, where C_∞-space is an H-space and C_1-space is a T-space. We [7] also generalized the concept of C_k-spaces to the concepts of C_k^f-spaces for a map $f : A \to X$. In this paper, we obtain a result if X is a C_k^f-space with a C_k^f-structure $F : P^k(\Omega X) \times A \to X$ and X' is a $C_k^{f'}$-space with a $C_k^{f'}$-structure $F' : P^k(\Omega X') \times A' \to X'$ and a map $(m, l) : f \to f'$ is a C_k-map from F to F', then there is a $C_k^{\bar{f}}$-structure $\bar{F} : P^k(\Omega E_m) \times E_l \to E_m$ for E_m such that a map $(p_m, p_l) : \bar{f} \to f$ is a C_k-map from $\bar{F}$ to F. Taking $k = \infty$, we know, from the fact $P^k(\Omega X) \simeq X$ and $e_\infty^X \sim 1 : X \to X$, that any C_∞ map from F to F' is just an H-map from F to F'. Thus we have a Stasheff's result which is if X is an H^f-space with an H^f-structure $F : X \times A \to X$ and X' is an $H^{f'}$-space with an $H^{f'}$-structure $F' : X' \times A' \to X'$ and a map $(m, l) : f \to f'$ is an H-map from F to F', then there is an $H^{\bar{f}}$-structure

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$\bar{F} : E_m \times E_l \to E_m$ on E_m such that a map $(p_m, p_l) : \bar{f} \to f$ is an H-map from $\bar{F}$ to F as a corollary. Throughout this paper, space means a space of the homotopy type of 1-connected locally finite CW complex. We assume also that spaces have non-degenerate base points. All maps shall mean continuous functions. All homotopies and maps are to respect base points. The base point as well as the constant map will be denoted by $*$. For simplicity, we use the same symbol for a map and its homotopy class. Also, we denote by $[X, Y]$ the set of homotopy classes of pointed maps $X \to Y$. The identity map of space will be denoted by 1 when it is clear from the context. The diagonal map $\Delta\colon X \to X \times X$ is given by $\Delta(x) = (x, x)$ for each $x \in X$ and the folding map $\nabla\colon X \vee X \to X$ is given by $\nabla(x, *) = \nabla(*, x) = x$ for each $x \in X$. ΣX denote the reduced suspension of X and ΩX denote the based loop space of X. The adjoint functor from the group $[\Sigma X, Y]$ to the group $[X, \Omega Y]$ will be denoted by τ. The symbols e and e' denote $\tau^{-1}(1_{\Omega X})$and $\tau(1_{\Sigma X})$ respectively.

2. C_k^f-spaces

In [1], Aguade introduced a T-space as a space X having the property that the evaluation fibration $\Omega X \to X^{S^1} \to X$ is fibre homotopically trivial. It is easy to show that any H-space is a T-space. We [7] introduced the concept of C_k-spaces for any integer $k \geq 1$ and $k = \infty$ which is located at an intermediate stage between H-spaces and T-spaces, where C_∞-space is an H-space and C_1-space is a T-space. We [7] also generalized the concept of C_k-spaces to the concepts of C_k^f-spaces for a map $f : A \to X$. Let $f : A \to X$ be a map. A based map $g : B \to X$ is said to be *f-cyclic* [14] if there exists a map $\phi : B \times A \to X$ such that the diagram

$$\begin{array}{ccc} B \times A & \xrightarrow{\phi} & X \\ {\scriptstyle j}\uparrow & & {\scriptstyle \nabla}\uparrow \\ B \vee A & \xrightarrow{g \vee f} & X \vee X \end{array}$$

is homotopy commutative, where $j : B \vee A \to B \times A$ is the inclusion and $\nabla : X \vee X \to X$ is the folding map. We call such a map ϕ an *associated map* of an f-cyclic map g. Clearly, g is f-cyclic if and only if f is g-cyclic. A based map $g\colon B \to X$ is said to be *cyclic* [17] if g is 1_X-cyclic. *The Gottlieb set* denoted by $G(B, X)$ is the set of all homotopy classes of cyclic maps from B to X. We denote the set of all homotopy classes of f-cyclic maps from B to X by

$$G(B; A, f, X) = G^f(B, X) \subset [B, X]$$

which is called *the Gottlieb set for a map* $f : A \to X$ [21]. If $f = 1_X : X \to X$, then we recover the set

$$G(B, X) = G(B; X, 1_X, X) = G^{1_X}(B, X).$$

In general, $G(B; X) \subset G^f(B, X) \subset [B, X]$ for any spaces A, B, X and any map $f : A \to X$. It is known [21] that $G(S^5, S^5 \times S^5) \cong 2\mathbb{Z} \oplus 2\mathbb{Z} \neq G^{i_1}(S^5, S^5 \times S^5) \cong 2\mathbb{Z} \oplus \mathbb{Z} \neq \pi_5(S^5 \times S^5) \cong \mathbb{Z} \oplus \mathbb{Z}$. In 1934, Lusternik and Schnirelmann [11] proposed the concept of category of a space and proved that *cat* M is a lower bound for the number of critical points of a smooth function on a smooth manifold M. For a space X, *cat* $X = n$ [11] if $n = min\{m \mid X \subset \bigcup_{k=0}^m U_k,\ U_k : open,\ i_k \sim * : U_k \hookrightarrow X\}$, where $i_k : U_k \hookrightarrow X$ is the inclusion. It is clear that *cat* $X = 0$ if and only if X is itself contractible, and *cat* $S^n = 1$. A space X is called a T-*space* [1] if the fibration $\Omega X \to X^{S^1} \to X$ is fibre homotopically trivial. There is a close connection between the concept of a T-space and the Gottlieb set as follows. It is known [1] that X is a T-space if and only if $e : \Sigma\Omega X \to X$ is cyclic. Also, we showed [18] that X is a T-space if and only if $G(\Sigma B, X) = [\Sigma B, X]$ for any space B.

It is well known [8] that *cat* $X \leq 1$ if and only if X is a co-H-space. Thus we know that for any suspension space ΣB, *cat* $\Sigma B \leq 1$. For any space X, the space ΩX has the homotopy type of an associative H-space and so by Milnor's [12] and Stasheff's results [16], we can consider X as filtered by the projective spaces of ΩX. $\Sigma\Omega X = P^1(\Omega X) \hookrightarrow P^2(\Omega X) \hookrightarrow \cdots \hookrightarrow P^\infty(\Omega X) \simeq X$. For each k, let $e_k : P^k(\Omega X) \hookrightarrow P^\infty(\Omega X) \simeq X$ be the natural inclusion. The following Ganea's result is recovered by Iwase [6].

Definition 2.1. [7] *A space X is called a C_k-space if the inclusion $e_k : P^k(\Omega X) \to X$ is cyclic.*

The property of the T-spaces is extended to the C_k-spaces using LS category in the sense that the category of any suspension space ΣA satisfies *cat* $\Sigma A \leq 1$.

Proposition 2.2. [7] *X is a C_k-space if and only if $G(B, X) = [B, X]$ for any space B with cat $B \leq k$.*

It is clear, from the fact [3] that *cat* $B \leq 1$ if and only if B has the homotopy type of a suspension, that X is a C_1-space if and only if X is a T-space.

Definition 2.3. [20] *A space X is called an H^f-space for a map $f : A \to X$ if 1_X is f-cyclic, that is, there is a map $F : X \times A \to X$ such that $Fj \sim \nabla(1 \vee f)$ which is called an H^f-structure.*

Definition 2.4. [21] *A space X is called a T^f-space for a map $f : A \to X$ if $e^X : \Sigma\Omega X \to X$ is f-cyclic, that is, there is a map $F : \Sigma\Omega X \times A \to X$ such that $Fj \sim \nabla(e \vee f)$ which is called a T^f-structure.*

Any H-space X is an H^f-space and any H^f-space X is a T^f-space for any map $f : A \to X$. We showed [21] that the 2-dimensional sphere S^2 is not an H-space nor a T-space, but it is an H^{η_2}-space and a T^{η_2}-space for the Hopf map $\eta_2 : S^3 \to S^2$. We introduced the concepts of C_k^f-spaces as intermediate stages between H^f-spaces and T^f-spaces for a map $f : A \to X$ and showed that they were closely related with Gottlieb sets for maps and LS-categories. In fact,H^1-space is an H-space, and T^1-space is a T-space.

Definition 2.5. [7] *Let $f : A \to X$ be any map. A space X is called a C_k^f-space if the inclusion $e_k : P^k(\Omega X) \to X$ is f-cyclic, that is, there is a map $F : P^k(\Omega X) \times A \to X$ such that $Fj \sim \nabla(e_k \vee f)$, where $j : P^k(\Omega X) \vee A \to P^k(\Omega X) \times A$ is the inclusion and $\nabla : X \vee X \to X$ is the folding map. If X is a C_k^f-space, then we call such a map F a C_k^f-structure for a space X.*

We see that a $C_k^{1_X}$-space X is a C_k-space.

Proposition 2.6. [7] *Let $f : A \to X$ be any map.*
(1) A space X is a C_1^f-space if and only if X is a T^f-space.
(2) Any C_m^f-space is a C_n^f-space for $\infty \geq m \geq n \geq 1$.
(3) A space X is a C_∞^f-space if and only if X is an H^f-space.
(4) For $p \neq 2$, the lens space $L^3(p)$ is a C_1-space that means T-space, but not a C_2-space.

Proposition 2.7. [7] *Let $f : A \to X$ be any map. A space X is a C_k^f-space if and only if $G^f(B, X) = [B, X]$ for any space B with cat $B \leq k$.*

3. Lifting C_k^f-structures

Let $f : A \to X$, $f' : A' \to X'$, $l : A \to A'$, $m : X \to X'$ be maps. Then a pair of maps $(m, l) : (X, A) \to (X', A')$ is called *a map from f*

to f' if the following diagram is homotopy commutative;

$$\begin{array}{ccc} A & \xrightarrow{f} & X \\ {\scriptstyle l}\big\downarrow & & {\scriptstyle m}\big\downarrow \\ A' & \xrightarrow{f'} & X'. \end{array}$$

It will be denoted by $(m,l) : f \to f'$. Given maps $f : A \to X$, $f' : A' \to X'$, let $(m,l) : f \to f'$ be a map from f to f'. Let PX' and PA' be the spaces of paths in X' and A' which begin at $*$ respectively. Let $\epsilon_{X'} : PX' \to X'$ and $\epsilon_{A'} : PA' \to A'$ be the fibrations given by evaluating a path at its end point. Let $p_m : E_m \to X$ be the fibration induced by $m : X \to X'$ from $\epsilon_{X'}$. Let $p_l : E_l \to A$ induced by $l : A \to A'$ from $\epsilon_{A'}$. Then there is a map $\bar{f} : E_l \to E_m$ such that the following diagram is commutative

$$\begin{array}{ccc} E_l & \xrightarrow{\bar{f}} & E_m \\ {\scriptstyle p_l}\big\downarrow & & {\scriptstyle p_m}\big\downarrow \\ A & \xrightarrow{f} & X, \end{array}$$

where $E_l = \{(a,\xi) \in A \times PA' | l(a) = \epsilon(\xi)\}$, $E_m = \{(x,\eta) \in X \times PX' | m(x) = \epsilon(\eta)\}$, $\bar{f}(a,\xi) = (f(a), f' \circ \xi)$, $p_m(x,\eta) = x$, $p_l(a,\xi) = a$.

Definition 3.1. *Given maps $f : A \to X$, $f' : A' \to X'$, let $(m,l) : f \to f'$ be a map from f to f'. Assume that X is a C_k^f-space with a C_k^f-structure $F : P^k(\Omega X) \times A \to X$ and X' is a $C_k^{f'}$-space with a $C_k^{f'}$-structure $F' : P^k(\Omega X') \times A' \to X'$. Then we say that $(m,l) : f \to f'$ is a C_k-map from F to F' if the following diagram is homotopy commutative;*

$$\begin{array}{ccc} P^k(\Omega X) \times A & \xrightarrow{F} & X \\ {\scriptstyle P^k(\Omega m)\times l}\big\downarrow & & {\scriptstyle m}\big\downarrow \\ P^k(\Omega X') \times A' & \xrightarrow{F'} & X'. \end{array}$$

The following lemmas are well known results.

Lemma 3.2. *A map $g : B \to X$ can be lifted to a map $B \to E_m$ if and only if $mg \sim *$.*

Lemma 3.3. [5] *Given maps $g_i : A_i \to E_m$, $i = 1,\ 2$ and $g : A_1 \times A_2 \to E_m$ satisfying $p_m g|_{A_i} \sim p_m g_i$, $i = 1,\ 2$, then there is a map $h : A_1 \times A_2 \to E_m$ such that $p_m h = p_m g$ and $h|_{A_i} \sim g_i, i = 1,\ 2$.*

Theorem 3.4. *If X is a C_k^f-space with a C_k^f-structure $F : P^k(\Omega X) \times A \to X$ and X' is a $C_k^{f'}$-space with a $C_k^{f'}$-structure $F' : P^k(\Omega X') \times A' \to X'$ and a map $(m,l) : f \to f'$ is a C_k-map from F to F', then there is a $C_k^{\bar{f}}$-structure $\bar{F} : P^k(\Omega E_m) \times E_l \to E_m$ for E_m such that a map $(p_m, p_l) : \bar{f} \to f$ is a C_k-map from $\bar{F}$ to F.*

Proof. Since a map $(m,l) : f \to f'$ is a C_k-map from F to F', we have that $mF \sim F'(P^k(\Omega m) \times l)$. Thus we know that $mF(P^k(\Omega p_m) \times p_l) \sim F'(P^k(\Omega(mp_m)) \times (lp_l)) \sim F'(* \times *) \sim * : P^k(\Omega E_m) \times E_l \to X'$. From Lemma 3.2, there is a lifting $\tilde{F} : P^k(\Omega E_m) \times E_l \to E_m$ of $F(P^k(\Omega p_m) \times p_l)$, that is, $p_m \tilde{F} = F(P^k(\Omega p_m) \times p_l) : P^k(\Omega E_m) \times E_l \to X$. We see $e_k^X \circ P^k(\Omega p_m) \sim p_m \circ e_k^{E_m}$ by the naturality of the construction of $P^k(\Omega E_m)$ as is shown in the following homotopy commutative diagram:

$$\begin{array}{ccc} P^k(\Omega E_m) & \xrightarrow{P^k(\Omega p_m)} & P^k(\Omega X) \\ {\scriptstyle e_k^{E_m}}\big\downarrow & & {\scriptstyle e_k^X}\big\downarrow \\ E_m & \xrightarrow{p_m} & X. \end{array}$$

Then $p_m \circ \tilde{F}|_{P^k(\Omega E_m)} \sim F|_{P^k(\Omega X)} \circ P^k(\Omega p_m) \sim e_k^X \circ P^k(\Omega p_m) \sim p_m \circ e_k^{E_m}$ and $p_m \circ \tilde{F}|_{E_l} \sim F|_A \circ p_l \sim f \circ p_l = p_m \circ \bar{f}$. Thus we have, from Lemma 3.3, that there is a map $\bar{F} : P^k(\Omega E_m) \times E_l \to E_m$ such that $p_m \bar{F} = p_m \tilde{F} = F(P^k(\Omega p_m) \times p_l)$ and $\bar{F}|_{P^k(\Omega E_m)} \sim e_k^{E_m}$, $\bar{F}|_{E_l} \sim \bar{f}$. Thus $\bar{F} : P^k(\Omega E_m) \times E_l \to E_m$ is a $C_k^{\bar{f}}$-structure for E_m such that a map $(p_m, p_l) : \bar{f} \to f$ is a C_k-map from $\bar{F}$ to F. This proves the theorem. □

Let X be an H^f-space with an H^f-structure $F : X \times A \to X$ and X' an $H^{f'}$-space with an $H^{f'}$-structure $F' : X' \times A' \to X'$. Then a map $(m,l) : f \to f'$ is called a *H-map from F to F'* [21] if the following diagram is homotopy commutative;

$$\begin{array}{ccc} X \times A & \xrightarrow{F} & X \\ {\scriptstyle m\times l}\big\downarrow & & {\scriptstyle m}\big\downarrow \\ X' \times A' & \xrightarrow{F'} & X'. \end{array}$$

Taking $k = \infty$, we know, from the fact $P^\infty(\Omega X) \simeq X$ and $e_\infty^X \sim 1 : X \to X$, a C_∞-map from F to F' is just an H-map from F to F'. Thus we have the following corollary.

COROLLARY 3.5. [20] *If X is an H^f-space with an H^f-structure $F : X \times A \to X$ and X' is an $H^{f'}$-space with an $H^{f'}$-structure $F' : X' \times A' \to X'$ and a map $(m, l) : f \to f'$ is an H-map from F to F', then there is an $H^{\bar{f}}$-structure $\bar{F} : E_m \times E_l \to E_m$ on E_m such that a map $(p_m, p_l) : \bar{f} \to f$ is an H-map from $\bar{F}$ to F.*

Taking $f = 1_X$, $f' = 1_{X'}$ and $l = m$, we can obtain the following Stasheff's result as a corollary.

COROLLARY 3.6. [16] *If X is an H-space with an H-structure $F : X \times X \to X$ and X' is an H-space with an H-structure $F' : X' \times X' \to X'$ and $m : X \to X'$ is an H-map, then there is an H-structure $\bar{F} : E_m \times E_m \to E_m$ on E_m such that a map $p_m : E_m \to X$ is an H-map.*

In 1951, Postnikov [15] introduced the notion of the Postnikov system as follows; *A Postnikov system for X(or homotopy decomposition of X)* $\{X_n, i_n, p_n\}$ consists of a sequence of spaces and maps satisfying (1) $i_n : X \to X_n$ induces an isomorphism $(i_n)_\# : \pi_i(X) \to \pi_i(X_n)$ for $i \le n$. (2) $p_n : X_n \to X_{n-1}$ is a fibration with fiber $K(\pi_n(X), n)$. (3) $p_n i_n \sim i_{n-1}$. It is well known fact [13] that if X is a 1-connected space having a homotopy type of CW-complex, then there is a Postnikov system $\{X_n, i_n, p_n\}$ for X such that $p_{n+1} : X_{n+1} \to X_n$ is the fibration induced from the path space fibration over $K(\pi_{n+1}(X), n+2)$ by a map $k^{n+2} : X_n \to K(\pi_{n+1}(X), n+2)$.

THEOREM 3.7. *Let A and X be spaces having the homotopy type of 1-connected countable CW-complexes, and $\{A_n, i'_n, p'_n\}$ and $\{X_n, i_n, p_n\}$ be Postnikov systems for A and X respectively. If X is a C_k^f-space with C_k^f-structure $F : P^k(\Omega X) \times A \to X$, then for each $n \ge 2$, there exists a $C_k^{f_n}$-structure $F_n : P^k(\Omega X_n) \times A_n \to X_n$ for X_n such that a map $(p_n, p'_n) : f_n \to f_{n-1}$ is a C_k-map from F_n to F_{n-1}, where f_n and F_n are induced maps from f and F respectively.*

Proof. It is known [2, Proposition 27.9] that if X is simply connected so is each $P^k(\Omega X)$. Clearly $\{P^k(\Omega X_n) \times A_n, P^k(\Omega i_n) \times i'_n, P^k(\Omega p_n) \times p'_n\}$ is a Postnikov system for $P^k(\Omega X) \times A$. Then we have, by Kahn's result [9,Theorem 2.2], that there are families of maps $f_n : A_n \to X_n$ and $F_n : P^k(\Omega X_n) \times A_n \to X_n$ such that $p_n f_n = f_{n-1} p'_n$ and $i_n f \sim f_n i'_n$, and $p_n F_n = F_{n-1}(P^k(\Omega p_n) \times p'_n)$ and $i_n F \sim F_n(P^k(\Omega i_n) \times i'_n)$ for $n = 2, 3, \cdots$ respectively. Since $F|_{P^k(\Omega X)} \sim e_k^X$ and $F|_A \sim f$, we know, from Kahn's another result [10, Theorem 1.2], that $F_{n|P^k(\Omega X_n)} = (F|_{P^k(\Omega X)})_n \sim e_k^{X_n}$ and $F_{n|A_n} = (F|_A)_n \sim f_n$. Thus there exists a $C_k^{f_n}$-structure F_n :

$P^k(\Omega X_n) \times A_n \to X_n$ for each stage X_n such that

$$\begin{array}{ccc} P^k(\Omega X_n) \times A_n & \xrightarrow{F_n} & X_n \\ {\scriptstyle P^k(\Omega p_n)\times p'_n}\Big\downarrow & & {\scriptstyle p_n}\Big\downarrow \\ P^k(\Omega X_{n-1}) \times A_{n-1} & \xrightarrow{F_{n-1}} & X_{n-1}, \end{array}$$

where f_n and F_n are induced maps from f and F respectively. □

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