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SELF-MAPS ON $M(\mathbb{Z}_q, n+2) \vee M(\mathbb{Z}_q, n+1) \vee M(\mathbb{Z}_q, n)$

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Abstract. When G is an abelian group, we use the notation $M(G, n)$ to denote the Moore space. The space X is the wedge product space of Moore spaces, given by $X = M(\mathbb{Z}_q, n+2) \vee M(\mathbb{Z}_q, n+1) \vee M(\mathbb{Z}_q, n)$. We determine the self-homotopy classes group $[X, X]$ and the self-homotopy equivalence group $\mathcal{E}(X)$. We investigate the subgroups of $[M_j, M_k]$ consisting of homotopy classes of maps that induce the trivial homomorphism up to $(n+2)$-homotopy groups for $j \neq k$. Using these results, we calculate the subgroup $\mathcal{E}_\sharp^{dim}(X)$ of $\mathcal{E}(X)$ in which all elements induce the identity homomorphism up to $(n+2)$-homotopy groups of X.

1. Introduction

For a based, finite CW-complex X, we denote by $[X, X]$ the set of homotopy classes of self-maps on X and by $\mathcal{E}(X)$ the group of homotopy classes of self-homotopy equivalences of X. Furthermore, if X is either an H-space or co-H-space then $[X, X]$ has the group structure. For surveys of the known results and applications of $\mathcal{E}(X)$, see [2] and [7]. The subgroup $\mathcal{E}_\sharp^{dim+r}(X)$ of $\mathcal{E}(X)$ consist of self-homotopy equivalences which induce the identity homomorphism on the homotopy groups of X in dimensions $\leq dim\ X + r$. Many authors have studied $\mathcal{E}_\sharp^{dim+r}(X)$ and so see [3], [4] and [6]. When G is an abelian group, we let $M(G, n)$ denote the Moore space. The space X is the wedge product space of Moore-spaces such that $X = M(\mathbb{Z}_q, n+2) \vee M(\mathbb{Z}_q, n+1) \vee M(\mathbb{Z}_q, n)$. In this paper, we study $[X, X]$, $\mathcal{E}(X)$ and $\mathcal{E}_\sharp^{dim}(X)$. We determine $[X, X]$ and $\mathcal{E}(X)$. By Lemma 1, we have

$$[X, X] \equiv \oplus_{j,k=1,2,3}[M_j, M_k].$$

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By Theorem 3, $\mathcal{E}(X)$ is the isomorphic to

$$\begin{aligned}&\mathcal{E}(M_1) \oplus [M_2, M_1] \oplus [M_3, M_1]\\ &\oplus[M_1, M_2] \oplus \mathcal{E}(M_2) \oplus [M_3, M_2]\\ &\oplus\ 0\ \oplus [M_2, M_3] \oplus \mathcal{E}(M_3).\end{aligned}$$

Depending on q, $[X, X]$ and $\mathcal{E}(X)$ may appear differently. By Remark 1 and 3, we calculate special cases. Now, we calculate $\mathcal{E}_\sharp^{dim}(X)$. First of all, we investigate the subgroups $Z_\sharp^{n+2}[M_j, M_k]$ of $[M_j, M_k]$ consisting of homotopy classes of maps that induce the trivial homomorphism up to $(n+2)$-homotopy groups for $j \neq k$. By Remark 4 and Lemma 2, we have

	q is odd	$q \equiv 2\ (mod\ 4)$	$q \equiv 0\ (mod\ 4)$
$Z_\sharp^{n+2}[M_2, M_1]$	$\mathbb{Z}_q$	0	0
$Z_\sharp^{n+2}[M_3, M_2]$	$\mathbb{Z}_q$	0	0
$Z_\sharp^{n+2}[M_1, M_2]$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$Z_\sharp^{n+2}[M_2, M_3]$	0	0	0
$Z_\sharp^{n+2}[M_1, M_3]$	$\mathbb{Z}_{(q,24)}$	$\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2$	$\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2$

Using this result, we have determined $\mathcal{E}_\sharp^{dim}(X)$. By Theorem 4, we see that

	$\mathcal{E}_\sharp^{dim}(X)$
q : odd	$\mathbb{Z}_q \oplus (\mathbb{Z}_{(q,24)}) \oplus \mathbb{Z}_q$
$q \equiv 2\ \ (\text{mod } 4)$	$\mathbb{Z}_2 \oplus (\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2) \oplus \mathbb{Z}_2$
$q \equiv 0\ (\text{mod } 4)$	$\mathbb{Z}_2 \oplus (\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_2)$

2. Preliminaries

In this section, we present some propositions to use.

PROPOSITION 1 ([1]).
(1) $\pi_n(M(\mathbb{Z}_q, n)) \cong \mathbb{Z}_q$ for all q.
(2) $\pi_{n+1}(M(\mathbb{Z}_q, n)) \cong \begin{cases} 0 & \text{if } q \text{ is odd,} \\ \mathbb{Z}_2 & \text{if } q \text{ is even.} \end{cases}$
(3) $\pi_{n+2}(M(\mathbb{Z}_q, n)) \cong \begin{cases} 0 & \text{if } q \text{ is odd,} \\ \mathbb{Z}_4 & \text{if } q \equiv 2 \text{ (mod 4),} \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 & \text{if } q \equiv 0 \text{ (mod 4).} \end{cases}$

(4) $\pi_{n+3}(M(\mathbb{Z}_q, n)) \cong \begin{cases} \mathbb{Z}_{(q,24)} & \text{if } q \text{ is odd,} \\ \mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2 & \text{if } q \equiv 2 \pmod 4, \\ \mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2 & \text{if } q \equiv 0 \pmod 4. \end{cases}$

Proposition 2 ([1]).
(1) $[M(\mathbb{Z}_q, n-1), M(\mathbb{Z}_q, n)] \cong \mathbb{Z}_q$ for all q.
(2) $[M(\mathbb{Z}_q, n), M(\mathbb{Z}_q, n)] \cong \begin{cases} \mathbb{Z}_q & \text{if } q \text{ is odd,} \\ \mathbb{Z}_{2q} & \text{if } q \equiv 2 \pmod 4, \\ \mathbb{Z}_q \oplus \mathbb{Z}_2 & \text{if } q \equiv 0 \pmod 4. \end{cases}$
(3) $[M(\mathbb{Z}_q, n+1), M(\mathbb{Z}_q, n)] \cong \begin{cases} 0 & \text{if } q \text{ is odd,} \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 & \text{if } q \equiv 2 \pmod 4, \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 & \text{if } q \equiv 0 \pmod 4. \end{cases}$
(4) $[M(\mathbb{Z}_q, n+2), M(\mathbb{Z}_q, n)] \cong \begin{cases} \mathbb{Z}_{(q,24)} & \text{if } q \text{ is odd,} \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{(q,24)} & \text{if } q \equiv 2 \pmod 4, \\ \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{(q,24)} & \text{if } q \equiv 0 \pmod 4. \end{cases}$

Proposition 3 ([3]). *If X is $(k-1)$-connected, Y is $(\ell-1)$-connected and, further, if $k, \ell \geq 2$ and $\dim P < k + \ell - 1$, then the projections $X \vee Y \to X$ and $X \vee Y \to Y$ induce a bijection :*

$$[P, X \vee Y] \to [P, X] \oplus [P, Y].$$

Theorem 1 ([3]). *Let $M(G, n)$ be a Moore space. Then*

$$\mathcal{E}_*^{\infty}(M(G, n)) \cong \oplus^{(r+s)s}\mathbb{Z}_2$$

where r is the rank of G and s is the number of 2-torsion sums of G.

Theorem 2 ([3]). *Let $M(G, n)$ be a Moore space. Then*

$$\begin{aligned} \mathcal{E}_\sharp^{n}(M(G, n)) &\cong \mathcal{E}_*^{\infty}(M(G, n)) \\ \mathcal{E}_\sharp^{n+1}(M(G, n)) &\cong 1, \text{ if } n > 3. \end{aligned}$$

For any non-negative integer n, $\mathcal{A}_\sharp^n(X)$ consists of homotopy classes of self-map of X that induce an automorphism from $\pi_i(X)$ to $\pi_i(X)$ for $i = 0, 1, \cdots, n$. $\mathcal{A}_\sharp^k(X)$ is a submonoid of $[X, X]$ and always contains $\mathcal{E}(X)$. If $n = \infty$, we briefly denote $\mathcal{A}_\sharp^\infty(X)$ as $\mathcal{A}_\sharp(X)$. If $k < n$, then $\mathcal{A}_\sharp^n(X) \subseteq \mathcal{A}_\sharp^k(X)$; thus, we have the following chain by inclusion:

$$\mathcal{E}(X) \subseteq \mathcal{A}_\sharp(X) \subseteq ... \subseteq \mathcal{A}_\sharp^1(X) \subseteq \mathcal{A}_\sharp^0(X) = [X, X].$$

Definition 1 ([5]). *The self-closeness number of X is the minimum number n such that $\mathcal{A}_\sharp^n(X) = \mathcal{E}(X)$, and is denoted by $N\mathcal{E}(X)$. If the minimum number n does not exist such that $\mathcal{A}_\sharp^n(X) = \mathcal{E}(X)$, then we write $N\mathcal{E}(X) = \infty$.*

Proposition 4 ([5]). *$N\mathcal{E}(M(G,n)) = n$ for $n \geq 3$.*

Let f be a map from X to Y.

- $\pi_k(f) : \pi_k(X) \to \pi_k(Y)$ is a homomorphism from k-dimensional homotopy group of X to k-dimensional homotopy group of Y.
- $\pi_{\leq k}(f) : \pi_{\leq k}(X) \to \pi_{\leq k}(Y)$ are homomorphisms up to k-dimensional homotopy group.
- $H_k(f) : H_k(X) \to H_k(Y)$ is a homomorphism from k-dimensional homology group of X to k-dimensional homology group of Y.
- $f^\sharp : [Y,Z] \to [X,Z]$ for any Z.

3. Self-maps on $M(\mathbb{Z}_q, n+2) \vee M(\mathbb{Z}_q, n+1) \vee M(\mathbb{Z}_q, n)$

For $n \geq 5$, we let $X = M(\mathbb{Z}_q, n+2) \vee M(\mathbb{Z}_q, n+1) \vee M(\mathbb{Z}_q, n)$. We determine the groups $[X,X]$, $\mathcal{E}(X)$ and $\mathcal{E}_\sharp^{dim}(X)$.
From now on, we set $M_1 = M(\mathbb{Z}_q, n+2)$, $M_2 = M(\mathbb{Z}_q, n+1)$, $M_3 = M(\mathbb{Z}_q, n)$ and $X = M_1 \vee M_2 \vee M_3$.

Lemma 1. $[X,X] \equiv \oplus_{j,k=1,2,3}[M_j, M_k]$.

Proof. By Proposition 3, we have $[X,X] \equiv \oplus_{j,k=1,2,3}[M_j, M_k]$. □

Now, we introduce a notation

$$\begin{aligned}[X,X] \equiv\ & [M_1,M_1] \oplus [M_2,M_1] \oplus [M_3,M_1] \\ & \oplus [M_1,M_2] \oplus [M_2,M_2] \oplus [M_3,M_2] \\ & \oplus [M_1,M_3] \oplus [M_2,M_3] \oplus [M_3,M_3].\end{aligned}$$

Since $[M_3, M_1] = 0$,

$$\begin{aligned}[X,X] \equiv\ & [M_1,M_1] \oplus [M_2,M_1] \oplus 0 \\ & \oplus [M_1,M_2] \oplus [M_2,M_2] \oplus [M_3,M_2] \\ & \oplus [M_1,M_3] \oplus [M_2,M_3] \oplus [M_3,M_3].\end{aligned}$$

Remark 1. *Let q be an odd. By Proposition 2, we have*

$$\begin{aligned}[X,X] \equiv\ & \mathbb{Z}_q \oplus \mathbb{Z}_q \oplus 0 \\ & \oplus 0 \oplus \mathbb{Z}_q \oplus \mathbb{Z}_q \\ & \oplus \mathbb{Z}_{(q,24)} \oplus 0 \oplus \mathbb{Z}_q.\end{aligned}$$

Let $j, k \in \{1, 2, 3\}$ and $f \in [X, X]$.

- $i_j : M_j \to X$ is the inclusion.
- $p_k : X \to M_k$ is the projection.
- $f_{kj} : J \to K$ where $f_{kj} = p_k \circ f \circ i_j$.

Proposition 5. *The function θ which assigns to each $f \in [X, X]$, the 3×3 matrix*

$$\theta(f) = \begin{pmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix},$$

where $f_{kj} \in [M_j, M_k]$ is bijective. In addition,
(1) $\theta(f+g) = \theta(f)+\theta(g)$, so θ is an isomorphism $[X, X] \to \oplus_{j,k=1,2,3}[M_j, M_k]$.
(2) $\theta(f \circ g) = \theta(f)\theta(g)$ where $f \circ g$ denotes composition in $[X, X]$ and $\theta(f)\theta(g)$ denotes matrix multiplication.
(3) If $\alpha_k : \pi_k(M_1) \oplus \pi_k(M_2) \oplus \pi_k(M_3) \to \pi_k(M_1 \vee M_2 \vee M_3)$ and $\beta_k : \pi_k(M_1 \vee M_2 \vee M_3) \to \pi_k(M_1) \oplus \pi_k(M_2) \oplus \pi_k(M_3)$ are the homomorphism induced by the inclusions and projections, respectively. then $\beta_k \circ \pi_k(f) \circ \alpha_k(x, y, z) = (\pi_k(f_{11})(x) + \pi_k(f_{12})(y) + \pi_k(f_{13})(z),\ \pi_k(f_{21})(x) + \pi_k(f_{22})(y) + \pi_k(f_{23})(z),\ \pi_k(f_{31})(x) + \pi_k(f_{32})(y) + \pi_k(f_{33})(z))$ for $x \in \pi_k(M_1)$, $y \in \pi_k(M_2)$ and $z \in \pi_k(M_3)$.

Proof. By Lemma 1, $[X, X] \equiv \oplus_{j,k=1,2,3}[M_j, M_k]$. The rest of proofs are straightforward and hence omitted. □

By Proposition 3, we have the following proposition.

Proposition 6. *$\pi_k(X) \cong \pi_k(M_1) \oplus \pi_k(M_2) \oplus \pi_k(M_3)$ for $k \leq 2n$.*

Remark 2. *By* [4, Remark 3.1], *there is the following table.*

	$k < n+2$	$k = n+2$
$\pi_k(M_1)$	0	$\mathbb{Z}_q\ \{i_1\}$

Theorem 3.

$$\begin{aligned} \mathcal{E}(X) \ \cong\ & \mathcal{E}(M_1) \oplus [M_2, M_1] \oplus 0 \\ & \oplus [M_1, M_2] \oplus \mathcal{E}(M_2) \oplus [M_3, M_2] \\ & \oplus [M_1, M_3] \oplus [M_2, M_3] \oplus \mathcal{E}(M_3). \end{aligned}$$

Proof. For any $f \in [X, X]$, $f \in \mathcal{E}(X)$ if and only if $H_n(f)$, $H_{n+1}(f)$ and $H_{n+2}(f)$ are isomorphism if and only if $H_n(f_{11})$, $H_{n+1}(f_{22})$ and $H_{n+1}(f_{33})$ are isomorphism.
By Proposition 4, $N\mathcal{E}(M(\mathbb{Z}_q, \ell)) = N\mathcal{E}_*(M(\mathbb{Z}_q, \ell)) = \ell$, $f \in \mathcal{E}(X)$ if and only if $f_{11} \in \mathcal{E}(M_1)$, $f_{22} \in \mathcal{E}(M_2)$ and $f_{33} \in \mathcal{E}(M_3)$. □

Remark 3. *By* [7, Theorem 2.1], $\mathcal{E}(M(\mathbb{Z}_q,k)) \cong \mathbb{Z}_{(2,q)} \times \mathbb{Z}_q^*$ *where* $\mathbb{Z}_q^*$ *is the automorphism group of* $\mathbb{Z}_q$ *for* $k \geq 3$. *By Proposition 1 and Theorem 3, let* q *be 2. Then*

$$\begin{aligned}\mathcal{E}(X) \;\cong\; & (\mathbb{Z}_2 \oplus \mathbb{Z}_2^*) \oplus \mathbb{Z}_2 \oplus 0 \\ & \oplus(\mathbb{Z}_2 \oplus \mathbb{Z}_2) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_2^*) \oplus \mathbb{Z}_2 \\ & \oplus(\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_{(q,24)}) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_2) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_2^*).\end{aligned}$$

We define the subgroup $Z_\sharp^k[M_j, M_k] = \{f_{kj} \mid \pi_{\leq k}(f_{kj}) = 0\}$ of $[M_j, M_k]$. From now on, we determine $Z_\sharp^k[M_j, M_k]$ for $j,\ k = 1,2,3$ and $j \neq k$.

Remark 4. *By* [6, Theorems 3.4 and 3.5], *we have*

	q *is odd*	$q \equiv 2\ (mod\ 4)$	$q \equiv 0\ (mod\ 4)$
$Z_\sharp^{n+2}[M_2, M_1]$	$\mathbb{Z}_q$	0	0
$Z_\sharp^{n+2}[M_3, M_2]$	$\mathbb{Z}_q$	0	0
$Z_\sharp^{n+2}[M_1, M_2]$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$
$Z_\sharp^{n+2}[M_2, M_3]$	0	0	0

It sufficiently determines that $Z_\sharp^{n+2}[M_3, M_1]$.

Lemma 2.

	q *is odd*	$q \equiv 2\ (mod\ 4)$	$q \equiv 0\ (mod\ 4)$
$Z_\sharp^{n+2}[M_1, M_3]$	$\mathbb{Z}_{(q,24)}$	$\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2$	$\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2$

Proof. Consider the mapping cone sequence of M_1,

$$S^{n+2} \xrightarrow{q} S^{n+2} \xrightarrow{i_1} M_1 \xrightarrow{\pi_1} S^{n+3} \xrightarrow{q} S^{n+3}.$$

This sequence induces the following exact sequence:

$$[S^{n+3}, M_3] \xrightarrow{q} [S^{n+3}, M_3] \xrightarrow{\pi_1^\sharp} [M_1, M_3] \xrightarrow{i_1^\sharp} [S^{n+2}, M_3] \xrightarrow{q} [S^{n+2}, M_3].$$

By Propositions 1 and 2, we have the split exact sequence

$$0 \longrightarrow [S^{n+3}, M_3] \xrightarrow{\pi_1^\sharp} [M_1, M_3] \xrightarrow{i_1^\sharp} ker(q) \longrightarrow 0.$$

Thus $[M_1, M_3] = \pi_1^\sharp([S^{n+3}, M_3]) \oplus (i_1^\sharp)^{-1}(ker(q))$.

By Remark 2 and properties of split exact sequence, $\pi_1 \circ i_1 = C_*$ and $((i_1^\sharp)^{-1}(ker(q)))(i_1) = i_1^\sharp((i_1^\sharp)^{-1}(ker(q))) = ker(q)$ where C_* is the constant map. We have $Z_\sharp^{n+2}[M_1, M_3] = \pi_1^\sharp([S^{n+3}, M_3])$. Since $\pi_1^\sharp$ is monomorphism, $Z_\sharp^{n+2}[M_1, M_3] \cong [S^{n+3}, M_3]$. □

THEOREM 4.

	$\mathcal{E}_\sharp^{dim}(X)$
q : odd	$\mathbb{Z}_q \oplus (\mathbb{Z}_{(q,24)}) \oplus \mathbb{Z}_q$
$q \equiv 2 \pmod 4$	$\mathbb{Z}_2 \oplus (\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2) \oplus \mathbb{Z}_2$
$q \equiv 0 \pmod 4$	$\mathbb{Z}_2 \oplus (\mathbb{Z}_{(q,24)} \oplus \mathbb{Z}_2) \oplus (\mathbb{Z}_2 \oplus \mathbb{Z}_2)$

Proof. For any $f \in \mathcal{E}_\sharp^{dim}(X)$, by Propositions 5 and 6, we have

$$\theta(\pi_{\leq dim}(f)) = \theta(id_{\pi_{\leq dim}(X)}) = \begin{pmatrix} id_{\pi_{\leq n+2}(M_1)} & 0 & 0 \\ 0 & id_{\pi_{\leq n+2}(M_2)} & 0 \\ 0 & 0 & id_{\pi_{\leq n+2}(M_3)} \end{pmatrix}.$$

Thus $f_{11} \in \mathcal{E}_\sharp^{n+2}(M_1)$, $f_{22} \in \mathcal{E}_\sharp^{n+2}(M_2)$ and $f_{33} \in \mathcal{E}_\sharp^{n+2}(M_3)$. Furthermore, $\pi_{\leq dim}(f_{kj}) = 0$ for $k \neq j$. By Theorems 1 and 2, it is implies that

$$\begin{aligned} \mathcal{E}_\sharp^{dim}(X) \quad \cong \quad & \mathcal{E}_\sharp^{n+2}(M_1) \oplus Z_\sharp^{n+2}[M_2, M_1] \oplus 0 \\ & \oplus Z_\sharp^{n+2}[M_1, M_2] \oplus 1 \oplus Z_\sharp^{n+2}[M_3, M_2] \\ & \oplus Z_\sharp^{n+2}[M_1, M_3] \oplus Z_\sharp^{n+2}[M_2, M_3] \oplus 1. \end{aligned}$$

The proof is completed by Theorem 2, Remark 4 and Lemma 2. □

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