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Identification of indirect effects in the two-condition within-subject mediation model and its implementation using SEM

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Abstract

In the two-condition within-subject mediation design, pairs of variables such as mediator and outcome are observed under two treatment conditions. The main objective of the design is to investigate the indirect effects of the condition difference (sum) on the outcome difference (sum) through the mediator difference (sum) for comparison of two treatment conditions. The natural condition variables mean the original variables, while the rotated condition variables mean the difference and the sum of two natural variables. The outcome difference (sum) is expressed as a linear model regressed on two natural (rotated) mediators as a parallel two-mediator design in two condition approaches: the natural condition approach uses regressors as the natural condition variables, while the rotated condition approach uses regressors as the rotated condition variables. In each condition approach, the total indirect effect on the outcome difference (sum) can be expressed as the sum of two individual indirect effects: within- and cross-condition indirect effects. The total indirect effects on the outcome difference (sum) for both condition approaches are the same. The invariance of the total indirect effect makes it possible to analyze the nature of two pairs of individual indirect effects induced from the natural conditions and the rotated conditions. The two-condition within-subject design is extended to the addition of a between-subject moderator. Probing of the conditional indirect effects given the moderator values is implemented by plotting the bootstrap confidence intervals of indirect effects against the moderator values. The expected indirect effect with respect to the moderator is derived to provide the overall effect of moderator on the indirect effect. The model coefficients are estimated by the structural equation modeling approach and their statistical significance is tested using the bias-corrected bootstrap confidence intervals. All procedures are evaluated using function lavaan() of package {lavaan} in R.

Keywords: within-condition, crossed-condition, orthogonal rotation, natural condition, rotated condition

1. Introduction

A mediation model is composed of three paths: One, from the causal variable to the mediator (path a); two, from the mediator to the outcome (path b); three, from the causal variable to the outcome (path c'). The indirect effect of the causal variable X on the outcome Y through the mediator M implies the effect from the causal steps that X affects M in path a which, according to the effect of M to Y in path b, in turn affects Y. Thus, the indirect effect is the product of the effect of X on M in path a and the

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effect of M on Y in path b. When the indirect effect is statistically different from zero, it is said that the effect of X on Y is mediated by M.

Such mediation analysis is mostly conducted in the "between-subject" data frame. That is, the causal variable, mediator, and outcome are measured once for every subject. This kind of design is called the "between-subject mediation design". When the data for the mediator and the outcome are measured repeatedly for the same subject, it is called the "repeated measures mediation design". We deal with the repeated measures mediation design especially when each subject is measured under two different conditions. This design is called the "two-condition within-subject mediation design". In this design, two different conditions are given, and the mediator and the outcome are measured under two different conditions from the same subject.

Since an approach to testing mediation in designs where each subject is measured on the mediator and the outcome under each of two conditions was studied by Judd *et al*. (2001), subsequent research has been followed by many authors, such as Cole and Maxwell (2003), Morris *et al*. (2007), Cheryan *et al*. (2009), Selig and Preacher (2009), Grant and Gino (2010), Paladino *et al*. (2010), Spiller (2011), Converse and Fishbach (2012), De Kwaadsteniet *et al*. (2013), and Warren and Campbell (2014). Recent works for the repeated measures mediation analysis are Preacher (2015), Josephy *et al*. (2015), Ferguson *et al*. (2017), Vuorre and Bolger (2018), Aung *et al*. (2020), Rijnhart *et al*. (2021), Tofighi (2021), and Montoya (2023).

The inference about the indirect effect is mostly based on the bootstrap confidence interval because the sampling distribution of the product of two estimates (one from the causal variable to the mediator, the other from the mediator to the outcome) is nonnormal. Judd *et al*. (2001) proposed testing the indirect effect as a set of hypothesis tests about individual paths in the model. Montoya and Hayes (2017) developed the mediation analysis in a path-analytic framework rather than a set of discrete hypothesis tests about individual paths in the model. Montoya (2019) developed a moderation analysis method in a two-condition repeated measures design for estimating and conducting inference on an interaction between a repeated measures factor and between-subject moderators using linear regression. Little has been found in the literature for the analysis of the effect of moderators in the repeated measures mediation design. One example is Montoya (2018) who studied the two-condition mediation model analysis where between-subject moderators are added in the model.

There is growing literature on methods for examining the moderation of components of a mediation process, which means that indirect and direct effects can also be tested to see how they depend on other variables. Examples are Muller *et al*. (2005), Edwards and Lambert (2007), Preacher *et al*. (2007), Fairchild and MacKinnon (2009), and Hayes (2018).

Hayes' PROCESS macro for SAS and SPSS is the most widely used program which can handle the mediation models with a variety of model specifications. Also, there are several R packages for mediation models such as {RMediation} by Tofighi and MacKinnon (2011), {mediation} by Tingley *et al*. (2014), and {mma} by Yu and Li (2017). The function lavaan() of package {lavaan} in R also can be implemented to evaluate the effects of the mediation model by defining effects in the model syntax.

Considering the prevalence of the mediation analysis and moderation analysis in within-subject designs independently, the needs for the combination of two analyses in the same design arise for further developments of within-subject designs. Montoya's MEMORE macro for SPSS and SAS (https://www.akmontoya.com/spss-and-sas-macros) estimate and test the mediation and moderation models for two-condition within-subject designs separately, but the macro for the moderated mediation model is underway.

The analysis of the two-condition within-subject mediation model was mainly concerned about the estimation of the indirect effect of the condition difference on the outcome difference through the mediator difference in the literature (Judd *et al*., 2001; Montoya and Hayes, 2017; Montoya, 2018). They used models where the outcome of each condition is affected only by the within-condition mediator which in turn, the outcome difference is regressing on the difference as well as the sum of mediators. Judd *et al*. (2001) defined the effect of the sum of mediators as potential moderator, and Montoya and Hayes (2017) and Montoya (2018) defined as a part of the direct effect.

Park and Park (2023) extended the studies by Judd *et al*. (2001), Montoya and Hayes (2017), and Montoya (2018) by defining models where the outcome of each condition is affected by both within- and cross-condition mediators, and by considering the difference as well as the sum of the two outcomes as responses of interest. Then the outcome difference as well as the outcome sum are regressing on both the mediator difference and the mediator sum. In this article, the sources of indirect effects in Park and Park's (2023) models are identified by defining two separate individual indirect effects on each outcome model, and such identification made it possible to analyze how the indirect effects are resulted from the original sources of data.

Model estimation is typically undertaken with ordinary least squares regression-based path analysis or using a SEM program. Hayes *et al*. (2017) asserted that the choice of which to use is inconsequential, as the results are largely identical. In this article, methods for estimating and testing the indirect and direct effects in the presence of a between-subject moderator are developed under the structural equation modeling (SEM) framework. A hypothetical study for the two-condition within-subject mediation model with a between-subject moderator is illustrated together with the implementation function lavaan() of package {lavaan} in R.

2. Two-condition within-subject mediation design

In a two-condition within-subject design, the two-condition causal variable X is expressed as the condition-specific indicator variable $X_{[j]}$ for condition $j = 1, 2$:

$$X_{[j]} = \begin{cases} 1, & \text{if condition } j, \\ 0, & \text{otherwise.} \end{cases}$$

Similarly, the mediator M and the outcome Y under condition $j\,(= 1, 2)$ are denoted as $M_{[j]}$ and $Y_{[j]}$, respectively.

In analyzing the indirect effect of conditions on the outcome in a two-condition within-subject mediation model, the difference of indirect effects under the two conditions is of the main interest. In addition to the difference of two indirect effects, the sum (or average) of two indirect effects is also of interest. Since the indirect effect is the effect on the outcome through the mediator, the comparison of indirect effects means the research on the difference and the sum of outcomes through possible mediating schemes. The difference and the sum of two outcomes are results by the difference and the sum of mediators, which are direct results by the difference and the sum of two conditions, respectively. The difference and the sum of two measures are the orthogonal transformation using 45° rotation of two measures and called the rotated measures. The original measures are called the natural measures in comparison with the rotated measures.

The difference in two conditions is defined as $X_D = X_{[2]} - X_{[1]}$ and the sum of conditions as $X_S = X_{[2]} + X_{[1]}$. The two rotated conditions, X_D and X_S, are in fact do not exist in reality, while the natural conditions do. Thus, instead of considering their own meaning, the effects of the rotated conditions on the rotated outcomes are considered in practical senses. The effect of the condition difference on the outcome means the difference of two average outcomes when each subject is treated by two conditions or the average changes in the outcome when the condition shifts from condition one to condition two. The effect of the condition sum means twice the average of two outcomes when each subject is treated by two conditions. The effect of the condition sum can be interpreted practically as twice the effect on the outcome when each subject is treated half by condition one and half condition two. Thus, the condition difference is useful when the difference of two conditions is of interest, and the condition sum is useful when the average of two conditions is of interest.

In a two-condition within-subject design, define the natural and rotated measures:

$$\boldsymbol{X} = (X_{[1]}, X_{[2]})' \,; \boldsymbol{M} = (M_{[1]}, M_{[2]})' \,; \boldsymbol{Y} = \left(Y_{[1]}, Y_{[j]}\right)',$$
$$\boldsymbol{X_R} = (X_D, X_S)' \,; \boldsymbol{M_R} = (M_D, M_S)' \,; \boldsymbol{Y_R} = (Y_D, Y_S)' \,,$$

where $M_D = M_{[2]} - M_{[1]}, M_S = M_{[2]} + M_{[1]}, Y_D = Y_{[2]} - Y_{[1]}, Y_S = Y_{[2]} + Y_{[1]}$.

The key difficulty in analyzing the within-subject design is that there is no variable representing the conditions either natural or rotated. To use X_D or X_S as the causal variable, the ability to handle the constant-only variable as the input in regression problems is inevitable, which is not allowed in many statistical programs such SPSS and SAS. In R, however, the function lavaan() in the package {lavaan} can handle the constant-only regression in the model syntax. The difference or the sum in conditions can be implemented as a variable in a constant-only regression by the use of “1” in defining the model syntax in lavaan().

The effect of the condition on the mediator can be formalized as:

$$\boldsymbol{M} = \boldsymbol{a} + \varepsilon_{\mathbf{M}}, \tag{2.1}$$

where a constant vector $\boldsymbol{a} = (a_{[1]}X_{[1]}, a_{[2]}X_{[2]})'$ denotes the mean vector of $\boldsymbol{M}$, $\varepsilon_{\mathbf{M}} = (\varepsilon_{M_{[1]}}, \varepsilon_{M_{[1]}})'$ denotes the error vector with correlated elements.

Suppose that the effect of the condition and the mediator on the outcome is formalized as:

$$\boldsymbol{Y} = \boldsymbol{c}' + \boldsymbol{b} \cdot \boldsymbol{M} + \varepsilon_Y, \tag{2.2}$$

where $\boldsymbol{c}' = (c'_{[1]}\boldsymbol{X}_{[1]}, c'_{[2]}\boldsymbol{X}_{[2]})'$is a constant vector, $\boldsymbol{b} = \begin{pmatrix} b_{[1]1} & b_{[1]2} \\ b_{[2]1} & b_{[2]2} \end{pmatrix}$ is a constant matrix, $\varepsilon_{\mathbf{Y}} = (\varepsilon_{Y_{[1]}}, \varepsilon_{Y_{[2]}})'$ is the error vector with correlated elements.

The indirect effect of the natural conditions on the natural outcomes through the natural mediators are expressed as the product of coefficient matrix, $\boldsymbol{b}$, in equation (2.2) and the mean vector, $\boldsymbol{a}$, in equation (2.1):

$$IE\,(\boldsymbol{X} \rightarrow \boldsymbol{Y}) = \boldsymbol{b} \cdot \boldsymbol{a}.$$

From the above expression, the indirect effect of the natural conditions on the first outcome through the natural mediators is expressed:

$$IE\,(\boldsymbol{X} \rightarrow Y_{[1]}) = b_{[1]1}a_{[1]}X_{[1]} + b_{[1]2}a_{[2]}X_{[2]}. \tag{2.3}$$

Also, the indirect effect of the natural conditions on the first outcome through the natural mediators is expressed:

$$IE\,(\boldsymbol{X} \rightarrow Y_{[2]}) = b_{[2]1}a_{[1]}X_{[1]} + b_{[2]2}a_{[2]}X_{[2]}. \tag{2.4}$$

Taking the difference and the sum of two indirect effects in equations (2.3) and (2.4) produces

$$IE\,(\boldsymbol{X} \rightarrow Y_{[2]}) - IE\,(\boldsymbol{X} \rightarrow Y_{[1]}) = b_{D1}a_{[1]}X_{[1]} + b_{D2}a_{[2]}X_{[2]}, \tag{2.5}$$
$$IE\,(\boldsymbol{X} \rightarrow Y_{[2]}) + IE\,(\boldsymbol{X} \rightarrow Y_{[1]}) = b_{S1}a_{[1]}X_{[1]} + b_{S2}a_{[2]}X_{[2]}, \tag{2.6}$$

where $b_{D1} = b_{[2]1} - b_{[1]1}, b_{D2} = b_{[2]2} - b_{[1]2}, b_{S1} = b_{[2]1} + b_{[1]1}, b_{S2} = b_{[2]2} + b_{[1]2}$.

2.1. Natural condition approach

Let $R = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ and call ‘rotation matrix’ because pre-multiplying R to natural measures vector generates the rotated measures: For two variables, x and y,

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y - x \\ y + x \end{pmatrix}.$$

Let $c'_D = c'_{[2]} - c'_{[1]}$ and $c'_S = c'_{[2]} + c'_{[1]}$, and rotate the natural outcome in equation (2.2) by pre-multiplying the rotation matrix to get:

$$\boldsymbol{Y}_R = \boldsymbol{c}'_R + \boldsymbol{b}_R \cdot \boldsymbol{M} + \varepsilon_{\mathbf{Y_R}}, \tag{2.7}$$

where $\boldsymbol{Y}_R = (Y_D, Y_S)'$, $\boldsymbol{c}'_R = (c'_D X_D, c'_s X_S)'$, $\boldsymbol{b}_R = \begin{pmatrix} b_{D1} & b_{D2} \\ b_{S1} & b_{S2} \end{pmatrix}$ and $\varepsilon_{\mathbf{Y_R}} = (\varepsilon_{\mathbf{Y}_{[2]}} - \varepsilon_{\mathbf{Y}_{[1]}}, \varepsilon_{\mathbf{Y}_{[2]}} + \varepsilon_{\mathbf{Y}_{[1]}})'$.

The indirect effect of the natural conditions to the rotated outcomes through the natural mediators are expressed as the product of coefficient matrix, $\boldsymbol{b}_R$, in equation (2.7) and the mean vector, $\boldsymbol{a}$, in equation (2.1):

$$IE\left(\boldsymbol{X} \to \boldsymbol{Y}_R\right) = \boldsymbol{b}_R \cdot \boldsymbol{a}. \tag{2.8}$$

From the upper row of $IE(\boldsymbol{X} \to \boldsymbol{Y}_R)$ in equation (2.8), it is seen that the total indirect effect on the outcome difference is the sum of two individual indirect effects on the outcome difference through $M_{[1]}$ and $M_{[2]}$:

$$IE\left(\boldsymbol{X} \to Y_D\right) = b_{D1} a_{[1]} X_{[1]} + b_{D2} a_{[2]} X_{[2]}. \tag{2.9}$$

The first element of equation (2.9) implies that the indirect effect of $X_{[1]}$ on Y_D through $M_{[1]}$ ($IE(X_{[1]} \to Y_D) = b_{D1} a_{[1]}$), and the second implies that the indirect effect of $X_{[2]}$ on Y_D through $M_{[2]}$($IE(X_{[2]} \to Y_D) = b_{D2} a_{[2]}$).

From the lower row of $IE(\mathbf{X} \to \mathbf{Y}_R)$ in equation (2.8), it is seen that the total indirect effect on the outcome sum is the sum of two individual indirect effects on the outcome sum through $M_{[1]}$ and $M_{[2]}$:

$$IE\left(\boldsymbol{X} \to Y_S\right) = b_{S1} a_{[1]} X_{[1]} + b_{S2} a_{[2]} X_{[2]}. \tag{2.10}$$

The first element of equation (2.10) implies that the indirect effect of $X_{[1]}$ on Y_S through $M_{[1]}$ ($IE(X_{[1]} \to Y_S) = b_{S1} a_{[1]}$), and the second implies that the indirect effect of $X_{[2]}$ on Y_S through $M_{[2]}$($IE(X_{[2]} \to Y_S) = b_{S2} a_{[2]}$).

Comparing two expressions in equations (2.5) and (2.6) with equations (2.9) and (2.10), the following equalities hold:

$$IE\left(\boldsymbol{X} \to Y_D\right) = IE\left(\boldsymbol{X} \to Y_{[2]}\right) - IE\left(\boldsymbol{X} \to Y_{[1]}\right). \tag{2.11}$$

$$IE\left(\boldsymbol{X} \to Y_S\right) = IE\left(\boldsymbol{X} \to Y_{[2]}\right) + IE\left(\boldsymbol{X} \to Y_{[1]}\right). \tag{2.12}$$

Left part of each equation in (2.11) and (2.12) implies that taking the difference (sum) of $Y_{[1]}$ and $Y_{[2]}$ first and then calculating indirect effect on that difference (sum), while right one implies that calculating indirect effects on $Y_{[1]}$ and $Y_{[2]}$ first and then taking the difference (or sum) of those indirect effects. This is analogous to the expectation as a linear operator, that is taking expectation of difference (sum) of two variables is equivalent to taking difference (sum) of expectations of two variables.

2.2. Rotated condition approach

Let $a_D = a_{[2]} - a_{[1]}$ and $a_S = a_{[2]} + a_{[1]}$, and rotate the natural mediator by pre-multiplying the rotation matrix to get:

$$\boldsymbol{M}_R = \boldsymbol{a}_R + \varepsilon_{\mathbf{M_R}}, \tag{2.13}$$

where $M_R = (M_D, M_S)'$, $\boldsymbol{a}_R = (a_D X_D, a_S X_S)'$, and $\varepsilon_{\mathbf{M_R}} = (\varepsilon_{M_{[2]}} - \varepsilon_{M_{[1]}}, \varepsilon_{M_{[2]}} + \varepsilon_{M_{[1]}})'$.

Note the fact that the product of the "half rotation matrix" $\boldsymbol{R}/2$ and the rotation matrix $\boldsymbol{R}$ is equal to the identity matrix, that is $\mathbf{I} = (\boldsymbol{R}/2) \cdot \boldsymbol{R}$. Then, inserting the identity matrix to a linear function of natural measures makes it a linear function of rotated measures: That is, for coefficients α, β, and measures x, y,

$$(\alpha, \beta) \cdot \begin{pmatrix} x \\ y \end{pmatrix} = (\alpha, \beta) \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \left(\frac{\beta - \alpha}{2}, \frac{\beta + \alpha}{2} \right) \begin{pmatrix} y - x \\ y + x \end{pmatrix}.$$

Note that post-multiplying the half rotation matrix to a row vector produces half of the rotation of the row.

The outcome in equation (2.7) can be rewritten by inserting the identity matrix in $\boldsymbol{b}_R \cdot \boldsymbol{M}$:

$$\boldsymbol{Y}_R = \boldsymbol{c}'_R + \boldsymbol{b}_{RR} \cdot \boldsymbol{M}_R + \varepsilon_{\mathbf{Y_R}}, \tag{2.14}$$

where $\boldsymbol{b}_{RR} = \begin{pmatrix} b_{DD} & b_{DS} \\ b_{SD} & b_{SS} \end{pmatrix}$.

Then the indirect effects of $\boldsymbol{X}_R$ on $\boldsymbol{Y}_R$ through $M_{\mathbf{R}}$ are expressed as the product of the coefficient matrix, $\boldsymbol{b}_{RR}$, in equation (2.14) and the mean vector, $\boldsymbol{a}_R$, in equation (2.13):

$$IE\,(\boldsymbol{X}_R \to \boldsymbol{Y}_R) = \boldsymbol{b}_{RR} \cdot \boldsymbol{a}_R. \tag{2.15}$$

From the upper row of $IE(\boldsymbol{X}_R \to \boldsymbol{Y}_R)$ in equation (2.15), it is seen that the total indirect effect on the outcome difference is the sum of two individual indirect effects on Y_D through M_D and M_S:

$$IE\,(\boldsymbol{X}_R \to \boldsymbol{Y}_D) = b_{DD} a_D X_D + b_{DS} a_S X_S. \tag{2.16}$$

The first element of equation (2.16) implies that the indirect effect of X_D on Y_D through M_D ($IE(X_D \to Y_D) = b_{DD} a_D$), and the second implies that the indirect effect of X_S on Y_D through M_S ($IE(X_S \to Y_D) = b_{DS} a_S$).

From the lower row of $IE(\boldsymbol{X}_R \to \boldsymbol{Y}_R)$ in equation (2.15), it is seen that the total indirect effect on the outcome sum is the sum of two individual indirect effects on Y_S through M_D and M_S:

$$IE\,(\boldsymbol{X}_R \to Y_S) = b_{SD} a_D X_D + b_{SS} a_S X_S. \tag{2.17}$$

The first element of equation (2.17) implies that the indirect effect of X_D on Y_S through M_D ($IE(X_D \to Y_S) = b_{SD} a_D$), and the second implies that the indirect effect of X_S on Y_S through M_S ($IE(X_S \to Y_S) = b_{SS} a_S$).

The statistical path diagram of models in equations (2.13) and (2.14) ignoring error terms is depicted in Figure 1.

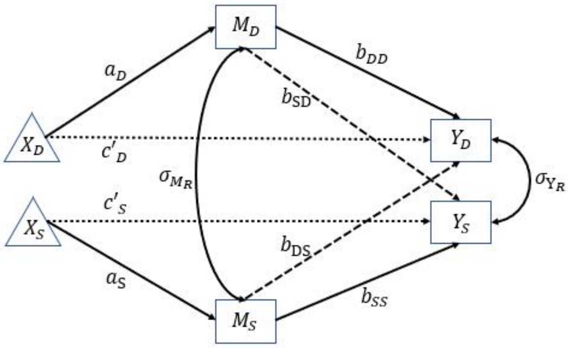

Figure 1: *Statistical path diagram for both the outcome difference and the outcome sum in a two-condition within-subject mediation model based on equations (2.13) and (2.14) ignoring errors. Covariance between two rotated mediators is denoted as σ_{M_R}, and covariance between two rotated outcomes is denoted as σ_{Y_R}.*

3. Identification of the total indirect effect on the rotated outcomes

Insert $\boldsymbol{I} = (\boldsymbol{R}/2) \cdot \boldsymbol{R}$ in $\boldsymbol{b}_R \cdot \boldsymbol{a}$ to get:

$$\boldsymbol{b}_R \cdot \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \boldsymbol{a} = \boldsymbol{b}_{RR} \cdot \boldsymbol{a}_R.$$

Then, from equations (2.8) and (2.15), the total indirect effect on the rotated outcomes in the natural condition approach ($IE(\boldsymbol{X} \to \boldsymbol{Y}_R)$) is the same that in the rotated condition approach ($IE(\boldsymbol{X}_R \to \boldsymbol{Y}_R)$). Thus, from equations (2.9) and (2.16), the following equality holds:

$$b_{D1}a_{[1]} + b_{D2}a_{[2]} = b_{DD}a_D + b_{DS}a_S. \tag{3.1}$$

Also, form equations (2.10) and (2.17), the following equality holds:

$$b_{S1}a_{[1]} + b_{S2}a_{[2]} = b_{SD}a_D + b_{SS}a_S. \tag{3.2}$$

Equation (3.1) (equation (3.2)) implies that the sum of two individual indirect effects on the outcome difference (sum) in the natural condition approach is the same as that in the rotated condition approach. Thus, when the total indirect effect on the outcome difference (sum) is given fixed, the pair of two individual indirect effects in each condition approach may vary under the constraint of the fixed sum.

The direct effect of $\boldsymbol{X}$ on $\boldsymbol{Y}_R$ ($DE(\mathbf{X}_R \to \boldsymbol{Y}_R)$) is the same as the direct effect of $\boldsymbol{X}_R$ on $\boldsymbol{Y}_R$ ($DE(\boldsymbol{X}_R \to Y_R)$), which is $\boldsymbol{c}'_R$ from equations (2.7) and (2.14), and the total effect of $\boldsymbol{X}$ on $\boldsymbol{Y}_R$ ($TE(\boldsymbol{X}_R \to \boldsymbol{Y}_R)$) is also the same as the total effect of $\boldsymbol{X}_R$ on $\boldsymbol{Y}_R$ ($TE(\boldsymbol{X}_R \to \boldsymbol{Y}_R)$), which is sum of the total indirect effect and the direct effect.

When the total indirect effect is calculated, it is possible to analyze how the two individual indirect effects in the rotated condition approach are resulted from the two individual indirect effects in the natural condition approach using the fixed sum constraint of two individual indirect effects in each

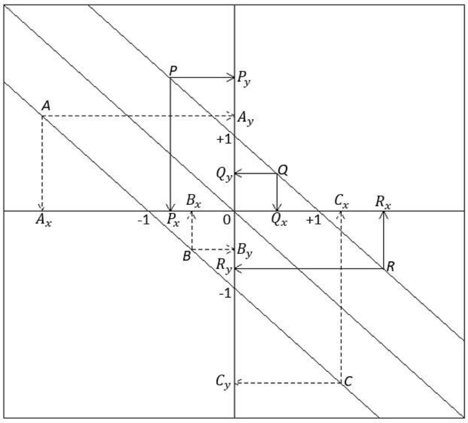

Figure 2: *Geometric positioning of total indirect effects (P, Q, R) and their corresponding two individual indirect effects. Three points P, Q, R (or A, B, C) indicate locations of two individual indirect effects for positive (or negative) total indirect effects. Positive (or negative) total indirect effect is given 1 (or −1) in y-axis, and two individual indirect effects are denoted in x-axis and y-axis.*

condition approach. For illustration, consider constants of models in the natural condition approach are estimated as

$$b_{D1} = -5,\ b_{D2} = 3, \quad a_1 = 2,\ a_2 = 7.$$

Then constants of models in the rotated condition approach are calculated as

$$b_{DD} = 4,\ b_{DS} = -1, \quad a_D = 5,\ a_S = 9.$$

The two pairs of individual indirect effects in both condition approaches are:

$$b_{D1}a_1 = -10,\ b_{D2}a_2 = 21, \quad b_{DD}a_D = 20,\ b_{DS}a_S = -9,$$

which in turn, produces the total indirect effect as 11 in both condition approaches. From such results, it can be inferred: The indirect effects of $X_{[1]}$ and $X_{[2]}$ on the outcome difference are negative (−10) and positive (20), respectively, but that of $X_{[2]}$ is more positive than that of $X_{[1]}$ in magnitude, which in turn produces the indirect effect of X_D and X_S on the outcome difference positive (20) and negative (−9), respectively, but that of X_D is more positive than that of X_S in magnitudes. Thus, the reason why the total indirect effect of the condition difference on the outcome difference is positive (11) is the two individual indirect effects are wide apart in opposite directions.

Figure 2 depicts the geometric locations of two individual indirect effects in a x-y plane when the total indirect effect is given fixed. Typically, the positive value of total indirect effect is given ‘1’ and the negative value is given ‘−1’. Coordinate x-value denotes the indirect effect of the first condition and y-value denotes the indirect effect of the second condition. There are three points (P, Q, R) on the line ‘$x + y = 1$’ corresponding to three possible cases for a positive total indirect effect, and three points (A, B, C) on the line ‘$x + y = -1$’ corresponding to three possible cases for a negative total indirect effect. Coordinates of the three points are denoted by subscript x and y of the point character to denote the x-value and y-value, respectively.

The positive total indirect effect under the natural condition approach can be one of the three points (P, Q, R) and the one from the rotated condition approach can be one of the remaining two points. Point P denotes the case that x-value is negative and y-value is positive but dominates x-value in magnitude to produce the total indirect effect positive. Point Q denotes the case that both x-value and y-value are small positive but are added to produce a positive total indirect effect. Point R denotes the case that y-value is negative and x-value is positive but large enough in magnitude to produce the total indirect effect positive. There are three points (A, B, C) on the line '$x + y = -1$' corresponding to three possible cases for a negative total indirect effect. The geometric interpretation of the three points can be done similarly to the positive case.

4. With a between-subject moderator

A moderator is a concomitant variable which affects the effect of the condition variable or the mediator on the outcome but should not be affected by the variables it is moderating (Kraemer *et al.*, 2001). The moderator in a within-subject model usually refers to a between-subject moderator because a within-subject moderator will be affected by the condition if it exists. A moderator can be specified in any or all of the three paths in the mediation model.

The two-condition within-subject mediation model with a between-subject moderator is considered, and its direct and indirect effects are derived. Let W be the between-subject moderator specified in all three paths. Then the mediator vector under two natural conditions in equation (2.1) is modified to get:

$$\boldsymbol{M} = \boldsymbol{a} + \mathbf{d} \cdot W + \varepsilon_{\mathbf{M}} \tag{4.1}$$

for a constant vector, $\boldsymbol{d} = (d_{[1]}, d_{[2]})'$. Also, the outcome vector under two natural conditions in equation (2.2) is modified to get:

$$\boldsymbol{Y} = \boldsymbol{c}' + \boldsymbol{b} \cdot \boldsymbol{M} + \boldsymbol{f} \cdot W + \boldsymbol{e} \cdot W\boldsymbol{M} + \varepsilon_{\mathbf{Y}} \tag{4.2}$$

for a constant vector, $\boldsymbol{f} = (f_{[1]}, f_{[2]})'$ and a constant matrix, $\boldsymbol{e} = \begin{pmatrix} e_{[1]1} & e_{[1]2} \\ e_{[2]1} & e_{[2]2} \end{pmatrix}$.

4.1. Natural condition approach

Let $f_D = f_{[2]} - f_{[1]}$ and $f_S = f_{[2]} + f_{[1]}$. Then the outcome vector in equation (4.2) is pre-multiplied by the rotation matrix to get the rotated outcome vector:

$$\boldsymbol{Y}_R = \boldsymbol{c}_R + (\boldsymbol{b}_R + \boldsymbol{e}_R \cdot W) \cdot \boldsymbol{M} + \boldsymbol{f}_R \cdot W + \varepsilon_{\mathbf{Y_R}} \tag{4.3}$$

for a constant vector $f_R = (f_D, f_S)'$.

The conditional indirect effect of the natural conditions to the natural outcomes through the natural mediators given W is expressed as the product of coefficient matrix, $\boldsymbol{b}_R + \boldsymbol{e}_R \cdot W$, in equation (4.3) and the mean vector, $\boldsymbol{a} + \boldsymbol{d} \cdot W$, in equation (4.1):

$$IE\,(\boldsymbol{X} \to \boldsymbol{Y}_R \,|\, W) = (\boldsymbol{b}_R + \boldsymbol{e}_R \cdot W)\,(\boldsymbol{a} + \boldsymbol{d} \cdot W)\,. \tag{4.4}$$

From equation (4.4), we get the conditional indirect effect of the natural conditions on Y_D through the natural mediators:

$$IE\,(\boldsymbol{X} \to Y_D \,|\, W) = (b_{D1} + e_{D1}W)\,(a_{[1]} + d_{[1]}W) + (b_{D2} + e_{D2}W)\,(a_{[2]} + d_{[2]}W)\,, \tag{4.5}$$

where the first part in the right-hand side of equation denotes $IE(X_{[\mathbf{1}]} \to Y_D|W)$, and the second $IE(X_{[\mathbf{2}]} \to Y_D|W)$. Also, we get the conditional indirect effect of the natural conditions on Y_S through the natural mediators:

$$IE\left(\boldsymbol{X} \to Y_S \mid W\right) = \left(b_{S1} + e_{S1}W\right)\left(a_{[1]} + d_{[1]}W\right) + \left(b_{S2} + e_{S2}W\right)\left(a_{[2]} + d_{[2]}W\right), \tag{4.6}$$

where the first part in the right-hand side of equation denotes $IE(X_{[\mathbf{1}]} \to Y_S|W)$, and the second $IE(X_{[\mathbf{2}]} \to Y_S|W)$.

Probing the conditional indirect effects against the W-values with confidence intervals is often used to help understanding the regions where the indirect effect is significant and where it is not. Probing effects are done using the flood-light method (Johnson-Neyman approach) for continuous W-values or spot-light method for categorical W-values. The advantage of probing effects is that the regions, where the effect is significant and where it is not, are separated, and it will help understanding the role of the moderator locally in detail. Probing the conditional indirect effects against the W-values with confidence intervals is often used to help understanding the regions where the indirect effect is significant and where it is not. Probing effects are done using the flood-light method (Johnson-Neyman approach) for continuous W-values or spot-light method for categorical W-values. The advantage of probing effects is that the regions, where the effect is significant and where it is not, are separated, and it will help understanding the role of the moderator locally in detail.

Sometimes summarizing the overall magnitude of the moderator's effect on the outcome may be helpful in understanding its global role on the outcome. For such purpose, the expected indirect effect with respect to the moderator value is derived. The expected indirect effect can be derived by replacing terms including the moderator with their means in the expressions of conditional indirect effect given the moderator.

Replace W and W^2 in equation (4.4) by $E(W)$ and $E(W^2)$ to get the expected total indirect effects of the natural conditions on the rotated outcomes through the natural mediators:

$$E\{IE\left(\boldsymbol{X} \to \boldsymbol{Y}_R \mid W\right)\} = \boldsymbol{b}_R \cdot \boldsymbol{a} + \boldsymbol{e}_R \cdot \boldsymbol{a} \cdot \boldsymbol{E}(W) + \boldsymbol{b}_R \cdot \boldsymbol{d} \cdot \boldsymbol{E}(W) + \boldsymbol{e}_R \cdot \boldsymbol{d} \cdot \boldsymbol{E}\left(W^2\right). \tag{4.7}$$

Actually, equation (4.7) implies the conditional indirect effect evaluated at $W = E(W)$ and $W^2 = E(W^2)$.

Similarly, the expected total indirect effects of the natural conditions on Y_D and Y_S through the natural mediators are obtained from equations (4.5) and (4.6) as:

$$\begin{aligned} E\left\{IE\left(\boldsymbol{X} \to Y_D \mid W\right)\right\} = b_{D1}a_{[1]} + e_{D1}a_{[1]}E\left(W\right) + b_{D1}d_{[1]}E\left(W\right) + e_{D1}d_{[1]}E\left(W^2\right) \\ + b_{D2}a_{[2]} + e_{D2}a_{[2]}E\left(W\right) + b_{D2}d_{[2]}E\left(W\right) + e_{D2}d_{[2]}E\left(W^2\right). \end{aligned} \tag{4.8}$$

$$\begin{aligned} E\left\{IE\left(\boldsymbol{X} \to Y_S \mid W\right)\right\} = b_{S1}a_{[1]} + e_{S1}a_{[1]}E\left(W\right) + b_{S1}d_{[1]}E\left(W\right) + e_{S1}d_{[1]}E\left(W^2\right) \\ + b_{S2}a_{[2]} + e_{S2}a_{[2]}E\left(W\right) + b_{S2}d_{[2]}E\left(W\right) + e_{S2}d_{[2]}E\left(W^2\right). \end{aligned} \tag{4.9}$$

4.2. Rotated condition approach

The rotated mediator vector under two rotated conditions is obtained by pre-multiplying the rotation matrix to equation (4.1):

$$\boldsymbol{M}_R = \boldsymbol{a}_R + \boldsymbol{d}_R \cdot W + \varepsilon_{\mathbf{M_R}}. \tag{4.10}$$

The rotated outcome vector under two rotated conditions is obtained by inserting the identity matrix $(\boldsymbol{b}_R + \boldsymbol{e}_R \cdot W) \cdot \boldsymbol{M}$ in equation (4.3):

$$\boldsymbol{Y}_R = \boldsymbol{c}'_R + (\boldsymbol{b}_{RR} + \boldsymbol{e}_{RR} \cdot W) \cdot \boldsymbol{M}_R + \boldsymbol{f}_R \cdot W + \varepsilon_{\mathbf{Y_R}}. \tag{4.11}$$

The indirect effect of the rotated conditions to the rotated outcomes through the rotated mediators are expressed as the product of coefficient matrix, $\boldsymbol{b}_{RR} + \boldsymbol{e}_{RR} \cdot W$, in equation (4.11) and the mean vector, $\boldsymbol{a}_R + \boldsymbol{d}_R \cdot W$, in equation (4.10):

$$IE\,(\boldsymbol{X}_R \to \boldsymbol{Y}_R \mid W) = (\boldsymbol{b}_{RR} + \boldsymbol{e}_{RR} \cdot W)\,(\boldsymbol{a}_R \cdot \boldsymbol{X}_R + \boldsymbol{d}_R \cdot W)\,. \tag{4.12}$$

From the above expression, we get the indirect effect of the rotated conditions on Y_D through the rotated mediators:

$$IE\,(\boldsymbol{X}_R \to Y_D \mid W) = (b_{DD} + e_{DD}W)\,(a_D + d_D W) + (b_{DS} + e_{DS}W)\,(a_S + d_S W)\,, \tag{4.13}$$

where the first part in the right-hand side of equation denotes $IE(X_D \to Y_D|W)$, and the second $IE(X_S \to Y_D|W)$. Also, we get the indirect effect of the rotated conditions on Y_S through the rotated mediators:

$$IE\,(\boldsymbol{X}_R \to Y_S \mid W) = (b_{SD} + e_{SD}W)\,(a_D + d_D W) + (b_{SS} + e_{SS}W)\,(a_S + d_S W)\,, \tag{4.14}$$

where the first part in the right-hand side of equation denotes $IE(X_D \to Y_S|W)$, and the second $IE(X_S \to Y_S|W)$.

Replace W and W^2 in equation (4.12) by $E(W)$ and $E(W^2)$ to get the expected total indirect effects of the rotated conditions on the rotated outcomes through the rotated mediators:

$$E\,\{IE\,(\boldsymbol{X}_R \to \boldsymbol{Y}_R \mid W)\} = \boldsymbol{b}_{RR} \cdot \boldsymbol{a}_R + \boldsymbol{e}_{RR} \cdot \boldsymbol{a}_R \cdot \boldsymbol{E}\,(W) + \boldsymbol{b}_{RR} \cdot \boldsymbol{d}_R \cdot \boldsymbol{E}\,(W) + \boldsymbol{e}_{RR} \cdot \boldsymbol{d}_R \cdot \boldsymbol{E}\left(W^2\right).$$

Similarly, the expected total indirect effects of the rotated conditions on Y_D and the sum through the rotated mediators are obtained from equations (4.13) and (4.14) as:

$$\begin{aligned} E\,\{IE\,(\boldsymbol{X}_R \to Y_D \mid W)\} = {} & b_{DD}a_D + e_{DD}a_D E\,(W) + b_{DD}d_D E\,(W) + e_{DD}d_D E\left(W^2\right) \\ & + b_{DS}a_S + e_{DS}a_S E\,(W) + b_{DS}d_S E\,(W) + e_{DS}d_S E\left(W^2\right). \end{aligned}$$

$$\begin{aligned} E\,\{IE\,(\boldsymbol{X}_R \to Y_S \mid W)\} = {} & b_{SD}a_D + e_{SD}a_D E\,(W) + b_{SD}d_D E\,(W) + e_{SD}d_D E\left(W^2\right) \\ & + b_{SS}a_S + e_{SS}a_S E\,(W) + b_{SS}d_S E\,(W) + e_{SS}d_S E\left(W^2\right). \end{aligned}$$

5. Application to SEM and interpretation using function lavaan( )

The path coefficients involved in the expressions of the conditional and expected effects can be estimated based on the SEM approach. The SEM approach is realized using function lavaan( ) of package {lavaan} in R, wherein all path coefficients, conditional and expected effects are estimated with confidence intervals by defining ‘model’ statement appropriately. The point estimates with bootstrap confidence intervals of total and individual conditional indirect effects are probed against the given moderator values in the same plot space. All path coefficients as well as effects are tested by plotting their 95% bootstrap confidence intervals with 5,000 replications. If the bootstrap confidence interval

Table 1: Expected effects and bootstrap confidence intervals on the outcome difference by natural and rotated condition approaches

Natural condition approach			Rotated condition approach		
Effect	Estimate	Boot CI	Effect	Estimate	Boot CI
$E\{IE(X_{[1]} \to Y_D\|W)\}$	−1.463	[−4.170, 2.199]	$E\{IE(X_D \to Y_D\|W)\}$	0.087	[−0.696, 0.824]
$E\{IE(X_{[2]} \to Y_D\|W)\}$	−0.908	[−5.763, 3.428]	$E\{IE(X_S \to Y_D\|W)\}$	−2.458	[−4.797, −1.298]
$E\{IE(\boldsymbol{X} \to Y_D\|W)\}$	−2.371	[−5.111, −0.777]	$E\{IE(\boldsymbol{X_R} \to Y_D\|W)\}$	−2.371	[−5.120, −0.778]
$E\{DE(\boldsymbol{X} \to Y_D\|W)\}$	0.405	[−1.372, 3.087]	$E\{DE(\boldsymbol{X_R} \to Y_D\|W)\}$	0.405	[−1.369, 3.109]
$E\{TE(\boldsymbol{X} \to Y_D\|W)\}$	−1.966	[−2.230, −1.717]	$E\{TE(\boldsymbol{X_R} \to Y_D\|W)\}$	−1.966	[−2.231, −1.717]

Table 2: Expected effects and bootstrap confidence intervals on the outcome sum by natural and rotated cond ition approaches

Natural condition approach			Rotated condition approach		
Effect	Estimate	Bootstrap CI	Effect	Estimate	Bootstrap CI
$E\{IE(X_{[1]} \to Y_S\|W)\}$	−6.024	[−14.019, −0.877]	$E\{IE(X_D \to Y_S\|W)\}$	1.610	[0.182, 3.106]
$E\{IE(X_{[2]}ßY_S\|W)\}$	9.194	[0.441, 19.033]	$E\{IE(X_S \to Y_S\|W)\}$	1.560	[−1.748, 6.698]
$E\{IE(\boldsymbol{X} \to Y_S\|W)\}$	3.170	[−1.004, 8.965]	$E\{IE(\boldsymbol{X_R} \to Y_S\|W)\}$	3.170	[−1.003, 8.978]
$E\{DE(\boldsymbol{X} \to Y_S\|W)\}$	5.865	[−0.036, 10.254]	$E\{DE(\boldsymbol{X_R} \to Y_S\|W)\}$	5.865	[−0.048, 10.252]
$E\{TE(\boldsymbol{X} \to Y_S\|W)\}$	9.034	[8.440, 9.553]	$E\{TE(\boldsymbol{X_R} \to Y_S\|W)\}$	9.035	[8.437, 9.554]

of an effect does not contain zero, we judge that the effect is positive (or negative) when both upper and lower confidence limits are all positive (or negative).

A hypothetical data with two conditions $(X1, X2)$, two within-subject mediators $(M1, M2)$, and two within-condition outcomes $(Y1, Y2)$ and a between-subject moderator (W) is used to illustrate the estimation and testing of two-condition within-subject mediation model with a between-subject moderator. In this example, the moderator is specified in paths a and c of the mediation model, not in path b. Usually in practice, a moderator is specified in one of two paths, a and b. Otherwise, the model contains the quadratic term W^2, which make it too complicated. The expressions for the corresponding model can be easily modified by simply deleting all terms factored with matrices $\boldsymbol{e}, \boldsymbol{e}_R, \boldsymbol{e}_{RR}$ and their elements. The R code given in Table A.2 of Appendix conducts the analysis using the data given in Table A.1 of Appendix.

Tables 1 and 2 provide estimates of the expected effects and the bootstrap confidence intervals on the outcome difference and the outcome sum, respectively. The expected effects in Table 1 presents the overall effects of natural and rotated conditions on the outcome difference, whose values are evaluated at the sample mean value of the moderator, $W = \bar{W}$. Similarly, the expected effects in Table 2 presents the overall effects of natural and rotated conditions on the outcome sum, whose values are evaluated at $W = \bar{W}$.

In Table 1, both the expected individual indirect effects of $X_{[1]}$ and $X_{[2]}$ on Y_D are estimated non-significant negative ($\text{Estimate} = -1.463, -0.908; \text{Boot CI} = [-4.170, 2.199], [-5.763, 3.428]$), but their sum is significant negative ($\text{Estimate} = -2.371, ; \text{Boot CI} = [-5.111, -0.777]$). The expected indirect effect of X_S on Y_D is significant negative ($\text{Estimate} = -2.458; \text{Boot CI} = [-4.797, -1.298]$), while that of X_D on Y_D is not ($\text{Estimate} = 0.087; \text{Boot CI} = [-0.696, 0.824]$), and thus their sum is still significant negative ($\text{Estimate} = -2.371; \text{Boot CI} = [-5.120, -0.778]$). As a summary, the two individual indirect effects of natural conditions are small negative, but the sum of them becomes large negative.

In Table 2, the individual indirect effect of $X_{[1]}$ on Y_S is significant negative (Estimate = −6.024; Boot CI = $[-14.019, -0.877]$), while that of $X_{[2]}$ on Y_S is significant positive (Estimate = 9.194; Boot CI = $[0.441, 19.033]$). The two individual indirect effects cancels each other to produce the to-

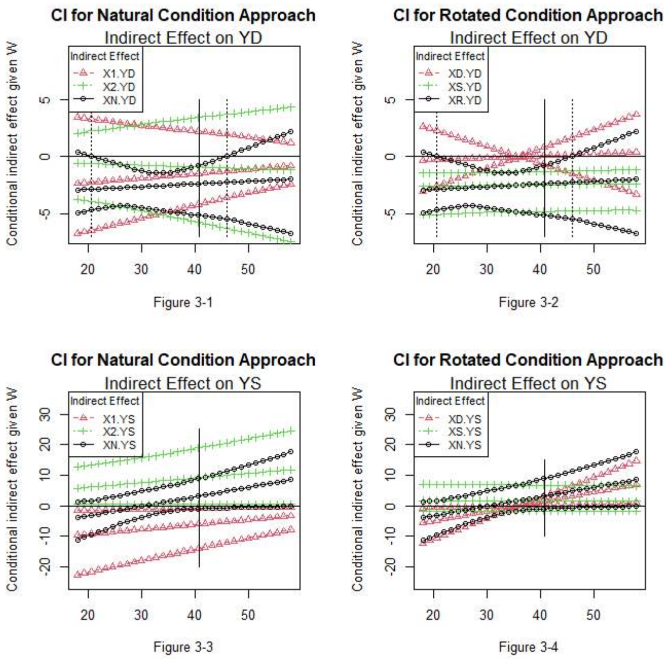

Figure 3: *Bootstrap confidence intervals (95%) of the conditional indirect effects of the natural and rotated conditions on the rotated outcomes.*

tal indirect effect on Y_S non-significant positive (Effect = 3.170; Boot CI = $[-1.004, 8.965]$). Thus, the difference of the two individual indirect effects is significant positive (Effect = 1.610; Boot CI = $[0.182, 3.106]$) while the sum of them is non-significant positive (Effect = 1.560; Boot CI = $[-1.748, 6.698]$), which in turn make the total indirect effect on Y_S non-significant positive.

Note that, in Tables 1 and 2, estimates of the expected total indirect effect (3^{rd} row), the expected direct effect (4^{th} row), and the expected total effect (5^{th} row) are the same for both condition approaches with possible but negligible differences in bootstrap confidence intervals due to different randomizations.

Figure 3 summarizes the probing of the conditional indirect effects against the moderator values. In Figure 3, there are four panels where the two panels in the upper row (Figures 3-1, 3-2) are for Y_D, and lower row (Figures 3-3, 3-4) for Y_S, and the two panels in the left column (Figures 3-1, 3-3) are for natural condition approach and the right column (Figures 3-2, 3-4) for rotated condition approach. In each panel, there are three bunches of curves with three different point characters, "Δ", "+", "$\circ$", denoting the two individual indirect effects and the total indirect effect. Each bunch consists of three curves for the upper CI, the point estimate, and the lower CI of the corresponding indirect effect. In both the natural and rotated condition approach, the total indirect effects are the same as each other, but the individual indirect effects are different.

In the natural condition approach for Y_D (Figure 3-1), values of the conditional indirect effect through $M_{[1]}$ are estimated small negative but not different from zero for all values of W, and those through $M_{[2]}$ are also estimated small negative but not different form zero for all values of W. However, the synergetic effect of the sum of two small negative effects makes the total indirect effect on Y_D significantly negative for middle values of $W \in (20, 46)$ (the two dotted vertical lines). In the rotated condition approach for Y_D (Figure 3-2), the individual indirect effects through M_D are not different from zero for all values of W, and those through M_S are negative for all values of W, which in turn make the total indirect effect negative for middle values of $W \in (20, 46)$ (the two dotted vertical lines). Summarizing results shows that effects of both two natural conditions on the outcome difference are small negative, but their sum is large negative in the middle of W while their difference is not significant.

In the natural condition approach for Y_S (Figure 3-3), the individual indirect effect through $M_{[1]}$ is negative for almost all values of W, while the individual conditional indirect effect through $M_{[2]}$ are positive for almost all values of W. The effects of two individual indirect effects are cancelling out each out, which in turn results in no nonzero total indirect effects for all values of W. In the rotated condition approach for Y_S (Figure 3-4), the two individual indirect effects through M_D and M_S are not different from zero for all values of W, which in turn makes no nonzero total indirect effect for all values of W.

The solid vertical line in each Figure indicates the sample mean value of W(= 40.8).

6. Conclusions and discussions

In this manuscript, the estimation methods for the two-condition within-subject mediation model are developed for comparison of indirect effects between two treatment conditions. The rotated outcomes are expressed as responses regressing on both within- and cross-condition mediators. Natural variables are the original variables, and the rotated variables are the difference and the sum of the natural condition variables. Natural condition approach uses the model where the rotated outcomes are regressing on the natural mediators, while rotated condition approach uses the model where the rotated outcomes are regressing on the rotated mediators. The reason that the rotated outcomes are regressed on both the within- and crossed-condition mediators is to achieve their estimated coefficients match with those of models for the natural outcomes by exact functional relations. The total indirect effects in each of the two condition approaches are developed and shown as sum of two individual indirect effects. The nature between the two pairs of individual indirect effects in the two condition approaches can be explained using the fact that sum of the pair of indirect effects are invariant across the condition approach.

When a between-subject moderator is specified in the simple mediation model, the models for the simple mediation case are extended to adapt the addition of a moderator, and coefficient are estimated simply extending the results of the simple mediation case. Probing of the conditional indirect effects given moderator values is designed to help understanding the regions of significance. Also, the expected indirect effects are derived as the conditional indirect effects evaluated at the sample mean value of the moderator, which helps understanding of the overall effect of the moderator to indirect effects.

A hypothetical study is illustrated for the case where a between-participant moderator is specified in paths a and c'. The estimated effects are tested by the 95% bootstrap confidence interval method for indirect effects. Probing the conditional indirect effect given the moderator is also illustrated by plotting the estimated conditional indirect effects with the 95% bootstrap confidence intervals against the selected moderator values. All these developments are evaluated implementing the function lavaan( )

of package {lavaan} in R.

The main contribution of this article is: First, the outcome difference (sum) is expressed as a response regressing on both within- and cross-condition mediators in the two condition approaches. Second, the two-condition within-subject mediation model can be expressed as a parallel two-mediator mediation model and the total indirect effect of treatment condition on the outcome difference (sum) is identified as sum of two individual indirect effects in the two condition approaches, which in turn makes it possible to understand the consequential relation between two pairs of individual indirect effects from two condition approaches.

For further research, the development of the analysis in the two-condition within-subject mediation design can be extended to the repeated measures mediation design because comparison of a specific condition with the reference condition in the latter design corresponds to a former design.

Appendix A:

Table A.2 contains R syntax for evaluating the indirect effects of the two-condition within-subject mediation model with a between-subject moderator. This program uses the data in Table A.1 which contains the natural measures such as within-subject mediators (M1, M2), within-subject outcomes (Y1, Y2), and between-subject moderator (W).

In function lavaan( ), the options 'se="bootstrap"' and 'bootstrap = 5000' use the bootstrapping method in estimating the confidence intervals with 5,000 replications. In function parameterEstimates( ), the option 'bca.simple' produces the bias-corrected bootstrap result.

For probing the conditional indirect effect given W using the Johnson-Neyman approach, values of the indirect effect for given W-values should be calculated inside the model statement of lavaan( ). For this purpose, R function paste0( ) is used so that values defined outside the model statement can be imported inside. Variable "Wval" contains thirty-one equally spaced values within the sample range of W, and they are used to calculate the conditional indirect effect of the condition difference on the loyalty difference through perception difference.

The mean of W needs to be specified in the 'model' statement, otherwise its mean will be set zero. Covariances between within-subject variables, the mediator and the outcome, are specified.

Table A.1: Data for the two-condition within-subject mediation design

M1	M2	Y1	Y2	W
3	6.33	5.67	5	50
3.67	4	6	3.67	36
5	3.67	6.67	2.67	28
4	5	7	4	43
4.67	4.67	6	3	37
3.33	4.33	5.33	4	40
4	4.67	5	3	41
4.67	4.67	5.67	3	36
4	3.67	5.67	3	34
2.33	6	5.33	5	54
4.67	5	6.33	3.67	39
3.67	4.67	2.67	2.67	45
2.33	5.67	4.33	4.67	54
7	3.67	7.33	2	18
3	6.67	4.33	4	58
3	4.67	5.67	4	39
3.33	4	5	3	39
3	3.67	6	4	39
3	4.67	4.67	3.67	48
5	5.33	5.33	2.67	38

Table A.2: R syntax for the analysis of a two-condition within-subject mediation model where a between-subject moderator is specified in paths, a and b

```
D$YD=D$Y2-D$Y1;D$YS=D$Y2+D$Y1
D$MD=D$M2-D$M1;D$MS=D$M2+D$M1
EW=mean(D$W)
# select 31 values of W across the sample range
Winc=(max(D$W)-min(D$W))/30
Wval=seq(min(D$W),max(D$W),Winc)
```

```
### Natural condition approach
model.natural<-c(
# intercepts
"W ~ w*1",          # mean of W

# regressions
"M1 ~ a1*1+d1*W ",   # (21)
"M2 ~ a2*1+d2*W ",   # (21)
"YD ~ cD*1 + bD1*M1 + bD2*M2 + fD*W",  # (22)
"YS ~ cS*1 + bS1*M1 + bS2*M2 + fS*W",  # (22)

# covariances
"M1 ~~ M2",
"YD ~~ YS",

# indirect effects of natural conditions
# on YD
paste0("X1.YD",1:31," := bD1*(a1+d1*",Wval[1:31],")"), # (24)
paste0("X2.YD",1:31," := bD2*(a2+d2*",Wval[1:31],")"), # (24)
paste0("XN.YD",1:31," := X1.YD",1:31," + X2.YD",1:31),  # (24)
paste0("DIR.YD",1:31," := cD + fD*",Wval[1:31]),
paste0("TOT.YD",1:31," := DIR.YD",1:31," + XN.YD",1:31 ),
paste0("E.X1.YD := bD1*a1 + bD1*d1*",EW),    # (27)
paste0("E.X2.YD := bD2*a2 + bD2*d2*",EW),    # (27)
paste0("E.XN.YD := E.X1.YD + E.X2.YD"),      # (27)
paste0("E.DIR.YD := cD + fD*",EW),
paste0("E.TOT.YD := E.DIR.YD + E.XN.YD "),

# on YS
paste0("X1.YS",1:31," := bS1*(a1+d1*",Wval[1:31],")"), # (25)
paste0("X2.YS",1:31," := bS2*(a2+d2*",Wval[1:31],")"), # (25)
paste0("XN.YS",1:31," := X1.YS",1:31," + X2.YS",1:31), # (25)
paste0("DIR.YS",1:31," := cS +fS*",Wval[1:31]),
paste0("TOT.YS",1:31," := DIR.YS",1:31," + XN.YS",1:31 ),
paste0("E.X1.YS := bS1*a1 + bS1*d1*",EW),    # (28)
paste0("E.X2.YS := bS2*a2 + bS2*d2*",EW),    # (28)
paste0("E.XN.YS := E.X1.YS + E.X2.YS"),      # (28)
paste0("E.DIR.YS := cS + fS*",EW),
paste0("E.TOT.YS := E.DIR.YS + E.XN.YS ")
)

### Rotated condition approach
model.rotated<-c(
# intercepts
"W ~ w*1",          # mean of W

# regressions
"MD ~ aD*1+dD*W ",   # (20)
"MS ~ aS*1+dS*W ",   # (20)
"YD ~ cD*1 + bDD*MD + bDS*MS + fD*W",  # (22)
"YS ~ cS*1 + bSD*MD + bSS*MS + fS*W",  # (22)

# covariances
"MD ~~ MS",
"YD ~~ YS",

# indirect effects of rotated conditions
# on YD
```

```
paste0("XD.YD",1:31," := bDD*(aD+dD*",Wval[1:31],")"), # (24)
paste0("XS.YD",1:31," := bDS*(aS+dS*",Wval[1:31],")"), # (24)
paste0("XR.YD",1:31," := XD.YD",1:31," + XS.YD",1:31), # (24)
paste0("DIR.YD",1:31," := cD + fD*",Wval[1:31]),
paste0("TOT.YD",1:31," := DIR.YD",1:31," + XR.YD",1:31 ),
paste0("E.XD.YD := bDD*aD + bDD*dD*",EW),    # (27)
paste0("E.XS.YD := bDS*aS + bDS*dS*",EW),    # (27)
paste0("E.XR.YD := E.XD.YD + E.XS.YD"),      # (27)
paste0("E.DIR.YD := cD + fD*",EW),
paste0("E.TOT.YD := E.DIR.YD + E.XR.YD "),

# on YS
paste0("XD.YS",1:31," := bSD*(aD+dD*",Wval[1:31],")"), # (25)
paste0("XS.YS",1:31," := bSS*(aS+dS*",Wval[1:31],")"), # (25)
paste0("XR.YS",1:31," := XD.YS",1:31," + XS.YS",1:31), # (25)
paste0("DIR.YS",1:31," := cS + fS*",Wval[1:31]),
paste0("TOT.YS",1:31," := DIR.YS",1:31," + XR.YS",1:31 ),
paste0("E.XD.YS := bSD*aD + bSD*dD*",EW),    # (28)
paste0("E.XS.YS := bSS*aS + bSS*dS*",EW),    # (28)
paste0("E.XR.YS := E.XD.YS + E.XS.YS"),      # (28)
paste0("E.DIR.YS := cS + fS*",EW),
paste0("E.TOT.YS := E.DIR.YS + E.XR.YS ")
)
#####################################################
## boostrap confidence intervals
set.seed(12357)
fit.natural.boot<-lavaan(model.natural,auto.var=TRUE,fixed.x=FALSE,check.gradient
= FALSE,se="bootstrap",bootstrap=5000,data=D)
res.natural.boot=parameterEstimates(fit.natural.boot,boot.ci.type="bca.simple");res.natural.boot

set.seed(12357)
fit.rotated.boot<-lavaan(model.rotated,auto.var=TRUE,fixed.x=FALSE,check.gradient
= FALSE,se="bootstrap",bootstrap=5000,data=D)
res.rotated.boot=parameterEstimates(fit.rotated.boot,boot.ci.type="bca.simple");res.rotated.boot

X1.YD.boot=res.natural.boot[21:51,c("est","ci.lower","ci.upper")]
X2.YD.boot=res.natural.boot[52:82,c("est","ci.lower","ci.upper")]
XN.YD.natural.boot=res.natural.boot[83:113,c("est","ci.lower","ci.upper")]
X1.YS.boot=res.natural.boot[181:211,c("est","ci.lower","ci.upper")]
X2.YS.boot=res.natural.boot[212:242,c("est","ci.lower","ci.upper")]
XN.YS.natural.boot=res.natural.boot[243:273,c("est","ci.lower","ci.upper")]
YD.natural.boot=cbind(X1.YD.boot,X2.YD.boot,XN.YD.natural.boot)
YS.natural.boot=cbind(X1.YS.boot,X2.YS.boot,XN.YS.natural.boot)

XD.YD.boot=res.rotated.boot[21:51,c("est","ci.lower","ci.upper")]
XS.YD.boot=res.rotated.boot[52:82,c("est","ci.lower","ci.upper")]
XR.YD.rotated.boot=res.rotated.boot[83:113,c("est","ci.lower","ci.upper")]
XD.YS.boot=res.rotated.boot[181:211,c("est","ci.lower","ci.upper")]
XS.YS.boot=res.rotated.boot[212:242,c("est","ci.lower","ci.upper")]
XR.YS.rotated.boot=res.rotated.boot[243:273,c("est","ci.lower","ci.upper")]
YD.rotated.boot=cbind(XD.YD.boot,XS.YD.boot,XR.YD.rotated.boot)
YS.rotated.boot=cbind(XD.YS.boot,XS.YS.boot,XR.YS.rotated.boot)
#####################################################
# probing conditional indirect effects
EW # 40.8
par(mfrow=c(2,2))
par(mar=c(5, 4, 4, 3))
matplot(Wval,YD.natural.boot,type="l",lty=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
```

```
rep(1,3)),xlab="Figure 3-1",
ylim=c(-7,9),ylab="Conditional indirect effect given W")
matpoints(Wval,YD.natural.boot,type="p",pch=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),cex=0.8)
title("CI for Natural Condition Approach")
mtext("Indirect Effect on YD",cex=1.1)
legend(title="Indirect Effect","topleft",
c("X1.YD","X2.YD","XN.YD"),
lty=c(2,3,1),pch=c(2,3,1),col=c(2,3,1),cex=0.8)
abline(h=0)
segments(20.5,-7,20.5,4,lty=3)
segments(46,-7,46,5,lty=3)
segments(40.8,-7,40.8,5,lty=1)

par(mar=c(5, 4, 4, 3))
matplot(Wval,YD.rotated.boot,type="l",lty=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),xlab="Figure 3-2",
ylim=c(-7,9),ylab="Conditional indirect effect given W")
matpoints(Wval,YD.rotated.boot,type="p",pch=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),cex=0.8)
title("CI for Rotated Condition Approach")
mtext("Indirect Effect on YD",cex=1.1)
legend(title="Indirect Effect","topleft",
c("XD.YD","XS.YD","XR.YD"),
lty=c(2,3,1),pch=c(2,3,1),col=c(2,3,1),cex=0.8)
abline(h=0)
segments(20.5,-7,20.5,4,lty=3)
segments(46,-7,46,5,lty=3)
segments(40.8,-7,40.8,5,lty=1)

matplot(Wval,YS.natural.boot,type="l",lty=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),xlab="Figure 3-3",
ylim=c(-25,35),ylab="Conditional indirect effect given W")
matpoints(Wval,YS.natural.boot,type="p",pch=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),cex=0.8)
title("CI for Natural Condition Approach")
mtext("Indirect Effect on YS",cex=1.1)
legend(title="Indirect Effect","topleft",
c("X1.YS","X2.YS","XN.YS"),
lty=c(2,3,1),pch=c(2,3,1),col=c(2,3,1),cex=0.8)
abline(h=0)
segments(40.8,-20,40.8,25,lty=1)

matplot(Wval,YS.rotated.boot,type="l",lty=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),xlab="Figure 3-4",
ylim=c(-25,35),ylab="Conditional indirect effect given W")
matpoints(Wval,YS.rotated.boot,type="p",pch=c(rep(2,3),rep(3,3),rep(1,3)),col=c(rep(2,3),rep(3,3),
rep(1,3)),cex=0.8)
title("CI for Rotated Condition Approach")
mtext("Indirect Effect on YS",cex=1.1)
legend(title="Indirect Effect","topleft",
c("XD.YS","XS.YS","XN.YS"),
lty=c(2,3,1),pch=c(2,3,1),col=c(2,3,1),cex=0.8)
abline(h=0)
segments(40.8,-10,40.8,15,lty=1)
```

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