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APPLICATION OF CONTRACTION MAPPING PRINCIPLE IN INTEGRAL EQUATION

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Abstract. In this paper, we establish some common fixed point theorems satisfying contraction mapping principle on partially ordered non-Archimedean fuzzy metric spaces and also derive some coupled fixed point results with the help of established results. We investigate the solution of integral equation and also give an example to show the applicability of our results. These results generalize, improve and fuzzify several well-known results in the recent literature.

1. Introduction

George and Veeramani [14] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [21] with the help of continuous t-norm and defined the Hausdorff topology of fuzzy metric spaces. In [20], Istratescu introduced the concept of non-Archimedean fuzzy metric space.

In [15], Guo and Lakshmikantham introduced the notion of coupled fixed point for single-valued mappings. Using this notion, Gnana-Bhaskar and Lakshmikantham [4] established some coupled fixed point theorems by defining mixed monotone property. After that, Lakshmikantham and Ciric [22] extended the notion of mixed monotone property to mixed $g-$monotone property and established coupled coincidence point results using a pair of commutative mappings, which generalized the results of Gnana-Bhaskar and Lakshmikantham [4]. For more details one can consult [1, $6-13$, 16, 19, 28].

This manuscript is divided into three sections. In first section of this research article, we prove some unique common fixed point theorems satisfying contraction mapping principle on partially ordered non-Archimedean fuzzy metric spaces and

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also give an example to validate our results. In second section of the article, we formulate some coupled fixed point results with the help of the results established in the first section. In the end, we investigate the solution of integral equation to demonstrate the fruitfulness of the established results. We generalize, extend, improve and fuzzify the results of Alotaibi and Alsulami [2], Alsulami [3], Gnana-Bhaskar and Lakshmikantham [4], Harjani et al. [17], Harjani and Sadarangani [18], Lakshmikantham and Ciric [22], Luong and Thuan [23], Nieto and Rodriguez-Lopez [24], Ran and Reurings [25], Razani and Parvaneh [26], Su [28] and many other famous results in the literature.

2. Preliminaries

Definition 2.1 ([27]). A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

(1) $*$ is commutative and associative,
(2) $*$ is continuous,
(3) $a * 1 = a$ for all $a \in [0, 1]$,
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ with $a, b, c, d \in [0, 1]$.

A few examples of continuous t-norm are

$$a * b = ab,\ a * b = \min\{a,\ b\} \text{ and } a * b = \max\{a + b - 1,\ 0\}.$$

Definition 2.2 ([14]). The 3-tuple $(X, M, *)$ is called a *fuzzy metric space* if X is an arbitrary non-empty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

$(FM-1)$ $M(x, y, t) > 0$,
$(FM-2)$ $M(x, y, t) = 1$ iff $x = y$,
$(FM-3)$ $M(x, y, t) = M(y, x, t)$,
$(FM-4)$ $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$,
$(FM-5)$ $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is continuous.

Remark 2.1. If in the above definition (FM-4) is replaced by

$$(NAFM-4)\ M(x,\ z,\ \max\{t,\ s\}) \geq M(x,\ y,\ t) * M(y,\ z,\ s)$$

or equivalently,

$$(NAFM-4)\ M(x,\ z,\ t) \geq M(x,\ y,\ t) * M(y,\ z,\ t),$$

then $(X, M, *)$ is called a *non-Archimedean fuzzy metric space* [20]. It is easy to check that (NAFM-4) implies (FM-4), that is, every non-Archimedean fuzzy metric space is itself a fuzzy metric space.

Example 2.1 ([14]). Let (X, d) be a metric space. Define t-norm by $a * b = ab$ and

$$M(x,\ y,\ t) = \frac{t}{t + d(x,\ y)} \text{ for all } x,\ y \in X \text{ and } t > 0.$$

Then $(X, M, *)$ is a fuzzy metric space. We call this *fuzzy metric* M induced by the metric d the standard fuzzy metric.

Remark 2.2 ([14]). In fuzzy metric space $(X, M, *)$, $M(x, y, \cdot)$ is non-decreasing for all $x, y \in X$.

Definition 2.3 ([14]). Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}_n$ in X is called *Cauchy* if for each $\varepsilon \in (0, 1)$ and each $t > 0$ there is $n_0 \in \mathbb{N}$ such that

$$M(x_n,\ x_m,\ t) > 1 - \varepsilon \text{ whenever } n \geq m \geq n_0.$$

We say that $(X, M, *)$ is *complete* if every Cauchy sequence is convergent, that is, if there exists $y \in X$ such that $\lim_{n\to\infty} M(x_n, y, t) = 1$, for all $t > 0$.

Definition 2.4 ([4]). Let $F : X^2 \to X$ be a given mapping. An element $(x, y) \in X^2$ is called a *coupled fixed point* of F if $F(x, y) = x$ and $F(y, x) = y$.

Definition 2.5 ([4]). Let $(X, \preceq)$ be a partially ordered set and $F : X^2 \to X$ be a given mapping. We say that F has the *mixed monotone property* if for all $x, y \in X$, we have

$$\begin{aligned} x_1,\ x_2 &\in X,\ x_1 \preceq x_2 \implies F(x_1,\ y) \preceq F(x_2,\ y), \\ y_1,\ y_2 &\in X,\ y_1 \preceq y_2 \implies F(x,\ y_1) \succeq F(x,\ y_2). \end{aligned}$$

Definition 2.6 ([22]). Let $F : X^2 \to X$ and $g : X \to X$ be given mappings. An element $(x, y) \in X^2$ is called a *coupled coincidence point* of the mappings F and g if $F(x, y) = gx$ and $F(y, x) = gy$.

Definition 2.7 ([22]). Let $F : X^2 \to X$ and $g : X \to X$ be given mappings. An element $(x, y) \in X^2$ is called a *common coupled fixed point* of the mappings F and g if $x = F(x, y) = gx$ and $y = F(y, x) = gy$.

Definition 2.8 ([22]). Mappings $F : X^2 \to X$ and $g : X \to X$ are said to be *commutative* if $gF(x, y) = F(gx, gy)$, for all $(x, y) \in X^2$.

Definition 2.9 ([22]). Let $(X, \preceq)$ be a partially ordered set. Suppose $F : X^2 \to X$ and $g : X \to X$ are given mappings. We say that F has the *mixed* $g-$*monotone* *property* if for all $x,\ y \in X$, we have

$$\begin{aligned} x_1,\ x_2 &\in X,\ gx_1 \preceq gx_2 \implies F(x_1,\ y) \preceq F(x_2,\ y), \\ y_1,\ y_2 &\in X,\ gy_1 \preceq gy_2 \implies F(x,\ y_1) \succeq F(x,\ y_2). \end{aligned}$$

If g is the identity mapping on X, then F satisfies the mixed monotone property.

Definition 2.10 ([4, 13])**.** A partially ordered metric space $(X,\ d,\ \preceq)$ is a metric space $(X,\ d)$ provided with a partial order $\preceq$. An ordered metric space $(X,\ d,\ \preceq)$ is said to be *non-decreasing-regular* (respectively, *non-increasing-regular*) if for every sequence $\{x_n\} \subseteq X$ such that $\{x_n\} \to x$ and $x_n \preceq x_{n+1}$ (respectively, $x_n \succeq x_{n+1}$) for all $n \geq 0$, we have $x_n \preceq x$ (respectively, $x_n \succeq x$) for all $n \geq 0$. $(X,\ d,\ \preceq)$ is said to be *regular* if it is both non-decreasing-regular and non-increasing-regular.

Definition 2.11 ([13])**.** Let $(X,\ \preceq)$ be a partially ordered set and $\alpha,\ \beta : X \to X$ be two mappings. We say that α is $(\beta,\ \preceq)-$*non-decreasing* if $\alpha x \preceq \alpha y$ for all x, $y \in X$ such that $\beta x \preceq \beta y$. If β is the identity mapping on X, we say that α is $\preceq -$*non-decreasing*.

Definition 2.12 ([5])**.** Let $(X,\ M,\ *)$ be a partially ordered fuzzy metric space. Two mappings $F,\ G : X \to X$ are said to be *compatible* if

$$\lim_{n\to\infty} M(FGx_n,\ GFx_n,\ t) = 1,$$

provided that $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Fx_n = \lim_{n\to\infty} Gx_n \in X.$$

Definition 2.13 ([19])**.** Let $F : X^2 \to X$ and $g : X \to X$ be two mappings. We say that the pair $\{F,\ g\}$ is *compatible* if

$$\begin{aligned} \lim_{n\to\infty} M(F(gx_n,\ gy_n),\ g(F(x_n,\ y_n),\ F(y_n,\ x_n)),\ t) &= 1, \\ \lim_{n\to\infty} M(F(gy_n,\ gx_n),\ g(F(y_n,\ x_n),\ F(x_n,\ y_n)),\ t) &= 1, \end{aligned}$$

whenever (x_n) and (y_n) are sequences in X such that

$$\begin{aligned} \lim_{n\to\infty} gx_n &= \lim_{n\to\infty} F(x_n,\ y_n) = x \in X, \\ \lim_{n\to\infty} gy_n &= \lim_{n\to\infty} F(y_n,\ x_n) = y \in X. \end{aligned}$$

Definition 2.14 ([19])**.** Let X be a non-empty set. Mappings $F : X^2 \to X$ and $g : X \to X$ are called *weakly compatible* if $F(x,\ y) = gx$ and $F(y,\ x) = gy$ imply that $g(F(x,\, y),\, F(y,\, x)) = F(gx,\, gy)$ and $g(F(y,\, x),\, F(x,\, y)) = F(gy,\, gx)$, for all $x,\, y \in X$.

Definition 2.15 ([28])**.** An altering distance function is a function $\psi : [0,\, +\infty) \to [0,\, +\infty)$ which satisfies the following conditions:

(i_ψ) ψ is continuous and non-decreasing,

(ii_ψ) $\psi(t) = 0$ if and only if $t = 0$.

3. Fixed Point Results

In this section we formulate some unique common fixed point theorems for mappings $\alpha,\, \beta : X \to X$ in a partially ordered non-Archimedean fuzzy metric space $(X,\, M,\, \preceq)$, where X is a non-empty set. Let $\beta : X \to X$ be a mapping, we shall denote $\beta(x)$ by βx where $x \in X$.

Theorem 3.1. *Let $(X,\, \preceq)$ be a partially ordered set and $(X,\, M,\, *)$ be a non-Archimedean fuzzy metric space. Suppose $\alpha,\, \beta : X \to X$ are two mappings satisfying*

(i) α is $(\beta,\, \preceq)$-non-decreasing and $\alpha(X) \subseteq \beta(X)$,

(ii) there exists $x_0 \in X$ such that $\beta x_0 \preceq \alpha x_0$,

(iii) there exist an altering distance function ψ and a right upper semi-continuous function $\varphi : [0,\, +\infty) \to [0,\, +\infty)$ such that

$$\psi\left(\frac{1}{M(\alpha x,\ \alpha y,\ t)} - 1\right) \le \varphi\left(\frac{1}{M(\beta x,\ \beta y,\ t)} - 1\right),$$

for all $x,\, y \in X$ with $\beta x \preceq \beta y$, where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. Also assume that, at least, one of the following conditions holds.

(a) $(X,\, M)$ is complete, α and β are continuous and the pair $(\alpha,\, \beta)$ is compatible,

(b) $(\beta(X),\, M)$ is complete and $(X,\, M,\, \preceq)$ is non-decreasing-regular,

(c) $(X,\, M)$ is complete, β is continuous and monotone non-decreasing, the pair $(\alpha,\, \beta)$ is compatible and $(X,\, M,\, \preceq)$ is non-decreasing-regular.

Then α and β have a coincidence point. Moreover, if

(iv) for every $x,\, y \in X$ there exists $z \in X$ such that αz is comparable to αx and αy, and also the pair $(\alpha,\, \beta)$ is weakly compatible.

Then α and β have a unique common fixed point.

Proof. Let $x_0 \in X$ be arbitrary. By (i), we have $\alpha(X) \subseteq \beta(X)$, there exists $x_1 \in X$ such that $\beta x_1 = \alpha x_0$. Then, by (ii), we have $\beta x_0 \preceq \alpha x_0 = \beta x_1$. As α is $(\beta,\, \preceq)$-non-decreasing and so $\alpha x_0 \preceq \alpha x_1$. Now $\alpha x_1 \in \alpha(X) \subseteq \beta(X)$, so there exists $x_2 \in X$ such that $\beta x_2 = \alpha x_1$. Then $\beta x_1 = \alpha x_0 \preceq \alpha x_1 = \beta x_2$. Since α is $(\beta, \preceq)-$non-decreasing, $\alpha x_1 \preceq \alpha x_2$. Continuing in this manner, we get a sequence $\{x_n\}_{n\geq 0}$ such that $\{\beta x_n\}$ is $\preceq$-non-decreasing, $\beta x_{n+1} = \alpha x_n \preceq \alpha x_{n+1} = \beta x_{n+2}$ and

$$\beta x_{n+1} = \alpha x_n \text{ for all } n \geq 0. \tag{3.1}$$

First, we claim that $\{M(\beta x_n, \beta x_{n+1}, t)\} \to 1$. Let

$$\zeta_n = \left(\frac{1}{M(\beta x_n,\ \beta x_{n+1},\ t)} - 1\right), \text{ for all } n \geq 0. \tag{3.2}$$

Now, by using the contractive condition (iii), we have

$$\begin{aligned} \psi\left(\frac{1}{M(\beta x_{n+1},\ \beta x_{n+2},\ t)} - 1\right) &= \psi\left(\frac{1}{M(\alpha x_n,\ \alpha x_{n+1},\ t)} - 1\right) \\ &\leq \varphi\left(\frac{1}{M(\beta x_n,\ \beta x_{n+1},\ t)} - 1\right). \end{aligned}$$

Thus, by (3.2), we have

$$\psi(\zeta_{n+1}) \leq \varphi(\zeta_n). \tag{3.3}$$

It follows, by the fact $\psi(t) > \varphi(t)$ for all $t > 0$, that $\psi(\zeta_{n+1}) < \psi(\zeta_n)$, which, by the monotonicity of ψ, implies $\zeta_{n+1} < \zeta_n$. This indicates that the sequence $\{\zeta_n\}_{n\geq 0}$ is a decreasing sequence of positive numbers. Then there exists $\zeta \geq 0$ such that

$$\lim_{n\to\infty} \zeta_n = \lim_{n\to\infty}\left(\frac{1}{M(\beta x_n,\ \beta x_{n+1},\ t)} - 1\right) = \zeta. \tag{3.4}$$

We shall now prove that $\zeta = 0$. Suppose, to the contrary, that $\zeta > 0$. Taking $n \to \infty$ in (3.3), by using the property of ψ, φ and (3.4), we obtain

$$\psi(\zeta) \leq \lim_{n\to\infty} \psi(\zeta_{n+1}) \leq \lim_{n\to\infty} \varphi(\zeta_n) \leq \varphi(\zeta),$$

which contradicts the fact $\psi(t) > \varphi(t)$ for all $t > 0$ and so $\zeta = 0$. Thus, by (3.4), we get

$$\lim_{n\to\infty} \zeta_n = \lim_{n\to\infty}\left(\frac{1}{M(\beta x_n,\ \beta x_{n+1},\ t)} - 1\right) = 0,$$

that is,

$$M(\beta x_n,\ \beta x_{n+1},\ t) \to 1 \text{ as } n \to \infty. \tag{3.5}$$

We now claim that $\{\beta x_n\}_{n\geq 0}$ is a Cauchy sequence in X. Suppose, to the contrary, that $\{\beta x_n\}$ is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ for which we can find two sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ such that for all positive integers k, and

$$M(\beta x_{n(k)},\ \beta x_{m(k)},\ t) \leq 1 - \varepsilon, \text{ for } n(k) > m(k) > k.$$

Assuming that $n(k)$ is the smallest such positive integer, we have

$$M(\beta x_{n(k)-1},\ \beta x_{m(k)},\ t) > 1-\varepsilon.$$

By (NAFM-4), we have

$$\begin{aligned} 1-\varepsilon &\geq M(\beta x_{n(k)},\ \beta x_{m(k)},\ t) \\ &\geq M(\beta x_{n(k)},\ \beta x_{n(k)-1},\ t) * M(\beta x_{n(k)-1},\ \beta x_{m(k)},\ t) \\ &\geq M(\beta x_{n(k)},\ \beta x_{n(k)-1},\ t) * (1-\varepsilon). \end{aligned}$$

Letting $k \to \infty$ in the above inequality, by using (3.5), we have

$$\lim_{k\to\infty} M(\beta x_{n(k)},\ \beta x_{m(k)},\ t) = 1-\varepsilon. \tag{3.6}$$

By using (NAFM-4), we have

$$\begin{aligned} &M(\beta x_{n(k)+1},\ \beta x_{m(k)+1},\ t) \\ \geq\ &M(\beta x_{n(k)+1},\ \beta x_{n(k)},\ t) * M(\beta x_{n(k)},\ \beta x_{m(k)},\ t) * M(\beta x_{m(k)},\ \beta x_{m(k)+1},\ t). \end{aligned}$$

Letting $k \to \infty$ in the above inequalities, using (3.5) and (3.6), we have

$$\lim_{k\to\infty} M(\beta x_{n(k)+1},\ \beta x_{m(k)+1},\ t) = 1-\varepsilon. \tag{3.7}$$

As $n(k) > m(k)$, $\beta x_{n(k)} \succeq \beta x_{m(k)}$ and so by using contractive condition (iii), we have

$$\begin{aligned} \psi\left(\frac{1}{M(\beta x_{n(k)+1},\ \beta x_{m(k)+1},\ t)} - 1\right) &= \psi\left(\frac{1}{M(\alpha x_{n(k)},\ \alpha x_{m(k)},\ t)} - 1\right) \\ &\leq \varphi\left(\frac{1}{M(\beta x_{n(k)},\ \beta x_{m(k)},\ t)} - 1\right). \end{aligned}$$

Letting $k \to \infty$ in the above inequality, by using the property of ψ, φ and (3.6), (3.7), we have

$$\psi\left(\frac{\varepsilon}{1-\varepsilon}\right) \leq \varphi\left(\frac{\varepsilon}{1-\varepsilon}\right),$$

which is a contradiction due to $\varepsilon > 0$. This shows that $\{\beta x_n\}_{n\geq 0}$ is a Cauchy sequence in X.

Meanwhile, we claim that α and β have a coincidence point distinguishing between cases $(a) - (c)$.

First suppose that (a) holds, that is, $(X,\ M)$ is complete, α and β are continuous and the pair $(\alpha,\ \beta)$ is compatible. Since $(X,\ M)$ is complete, there exists $x \in X$ such that $\{\beta x_n\} \to x$ and (3.1) follows that $\{\alpha x_n\} \to x$. As α and β are continuous and so $\{\alpha\beta x_n\} \to \alpha x$ and $\{\beta\beta x_n\} \to \beta x$. Furthermore the pair $(\alpha,\,\beta)$ is compatible, so

$$\lim_{n\to\infty} M(\alpha\beta x_n,\ \beta\alpha x_n, t) = 1.$$

Thus

$$M(\alpha x,\ \beta x,\ t) = \lim_{n\to\infty} M(\alpha\beta x_n,\ \beta\beta x_{n+1},\ t) = \lim_{n\to\infty} M(\alpha\beta x_n,\ \beta\alpha x_n,\ t) = 1,$$

that is, x is a coincidence point of α and β.

Now suppose that (b) holds, that is, $(\beta(X),\,M)$ is complete and $(X,\,M,\,\preceq)$ is non-decreasing-regular. Since $\{\beta x_n\}$ is a Cauchy sequence in the complete space $(\beta(X),\,M)$, there exists $y \in \beta(X)$ such that $\{\beta x_n\} \to y$. Let $x \in X$ be any point such that $y = \beta x$, then $\{\beta x_n\} \to \beta x$. As $(X,\,M,\,\preceq)$ is non-decreasing-regular, $\{\beta x_n\}$ is $\preceq$ $-$non-decreasing converging to βx and so $\beta x_n \preceq \beta x$ for all $n \geq 0$. Applying the contractive condition (iii), we have

$$\begin{aligned} \psi\left(\frac{1}{M(\beta x_{n+1},\ \alpha x,\ t)} - 1\right) &= \psi\left(\frac{1}{M(\alpha x_n,\ \alpha x,\ t)} - 1\right) \\ &\leq \varphi\left(\frac{1}{M(\beta x_n,\ \beta x,\ t)} - 1\right). \end{aligned}$$

Taking $n \to \infty$ in the above inequality, by using the properties of ψ, φ and the fact $\{\beta x_n\} \to \beta x$, we get $M(\beta x,\, \alpha x,\, t) = 1$, that is, x is a coincidence point of α and β.

In the end, suppose that (c) holds, that is, $(X,\,M)$ is complete, β is continuous and monotone non-decreasing, the pair $(\alpha,\,\beta)$ is compatible and $(X,\,M,\,\preceq)$ is non-decreasing-regular. As $(X,\,d)$ is complete, so there exists $x \in X$ such that $\{\beta x_n\} \to x$ and (3.1) follows that $\{\alpha x_n\} \to x$. Since β is continuous, $\{\beta\beta x_n\} \to \beta x$. Furthermore, since the pair $(\alpha,\,\beta)$ is compatible, we have

$$\lim_{n\to\infty} M(\beta\beta x_{n+1},\ \alpha\beta x_n,\ t) = \lim_{n\to\infty} M(\beta\alpha x_n,\ \alpha\beta x_n,\ t) = 1,$$

and the fact $\{\beta\beta x_n\} \to \beta x$ suggests that $\{\alpha\beta x_n\} \to \beta x$.

Since $(X,\,M,\,\preceq)$ is non-decreasing-regular and $\{\beta x_n\}$ is $\preceq$-non-decreasing and converging to x, $\beta x_n \preceq x$, which, by the monotonicity of β, implies $\beta\beta x_n \preceq \beta x$ for all $n \geq 0$. Applying the contractive condition (iii), we get

$$\psi\left(\frac{1}{M(\alpha\beta x_n,\ \alpha x,\ t)} - 1\right) \leq \varphi\left(\frac{1}{M(\beta\beta x_n,\ \beta x,\ t)} - 1\right).$$

Taking $n \to \infty$ in the above inequality, by using the properties of ψ, φ and the fact $\{\beta\beta x_n\} \to \beta x$ and $\{\alpha\beta x_n\} \to \beta x$, we get $M(\beta x,\, \alpha x,\, t) = 1$, that is, x is a coincidence point of α and β.

Thus the set of coincidence points of α and β is non-empty. Let x and y be two coincidence points of α and β, that is, $\alpha x = \beta x$ and $\alpha y = \beta y$. Now, we claim that $\beta x = \beta y$. By the assumption, there exists $z \in X$ such that αz is comparable with αx and αy. Put $z_0 = z$ and choose $z_1 \in X$ so that $\beta z_0 = \alpha z_1$. Then, we can inductively define the sequence $\{\beta z_n\}$ where $\beta z_{n+1} = \alpha z_n$ for all $n \geq 0$. Hence $\alpha x = \beta x$ and $\alpha z = \alpha z_0 = \beta z_1$ are comparable. Suppose that $\beta z_1 \preceq \beta x$. We claim that $\beta z_n \preceq \beta x$ for each $n \in \mathbb{N}$. In fact, we will use mathematical induction. Since $\beta z_1 \preceq \beta x$, our claim is true for $n = 1$. Now, suppose that $\beta z_n \preceq \beta x$ holds for some $n > 1$. Since α is $\beta-$non-decreasing with respect to $\preceq$, we get $\beta z_{n+1} = \alpha z_n \preceq \alpha x = \beta x$, and this proves our claim.

Let

$$\xi_n = \left(\frac{1}{M(\beta z_n,\ \beta x,\ t)} - 1 \right), \text{ for all } n \geq 0. \tag{3.8}$$

Now, by using the contractive condition (iii), we have

$$\begin{aligned} \psi\left(\frac{1}{M(\beta z_{n+1},\ \beta x,\ t)} - 1 \right) &= \psi\left(\frac{1}{M(\alpha z_n,\ \alpha x,\ t)} - 1 \right) \\ &\leq \varphi\left(\frac{1}{M(\beta z_n,\ \beta x,\ t)} - 1 \right), \end{aligned}$$

Thus, by (3.8), we have

$$\psi(\xi_{n+1}) \leq \varphi(\xi_n). \tag{3.9}$$

It follows, by the fact $\psi(t) > \varphi(t)$ for all $t > 0$, that $\psi(\xi_{n+1}) < \psi(\xi_n)$, which, by the monotonicity of ψ, implies $\xi_{n+1} < \xi_n$. This shows that the sequence $\{\xi_n\}_{n \geq 0}$ is a decreasing sequence of positive numbers. Then there exists $\xi \geq 0$ such that

$$\lim_{n\to\infty} \xi_n = \lim_{n\to\infty} \left(\frac{1}{M(\beta z_n,\ \beta x,\ t)} - 1 \right) = \xi. \tag{3.10}$$

Now, we claim that $\xi = 0$. Suppose, to the contrary, that $\xi > 0$. Taking $n \to \infty$ in (3.9), by using the property of ψ, φ and (3.10), we obtain

$$\psi(\xi) \leq \lim_{n\to\infty} \psi(\xi_{n+1}) \leq \lim_{n\to\infty} \varphi(\xi_n) \leq \varphi(\xi),$$

which contradicts the fact $\psi(t) > \varphi(t)$ for all $t > 0$ and so $\xi = 0$. Thus, by (3.10), we get

$$\lim_{n\to\infty} \xi_n = \lim_{n\to\infty} \left(\frac{1}{M(\beta z_n,\ \beta x,\ t)} - 1 \right) = 0,$$

that is,

$$\lim_{n\to\infty} M(\beta z_n,\ \beta x,\ t) = 1. \tag{3.11}$$

Similarly, one can obtain that

$$\lim_{n\to\infty} M(\beta z_n,\ \beta y,\ t) = 1. \tag{3.12}$$

Hence, by (3.11) and (3.12), we get

$$\beta x = \beta y. \tag{3.13}$$

Since $\alpha x = \beta x$, by weak compatibility of α and β, we have $\alpha\beta x = \beta\alpha x = \beta\beta x$. Let $z = \beta x$, then $\alpha z = \beta z$, that is, z is a coincidence point of α and β. Then from (3.13) with $y = z$, it follows that $\beta x = \beta z$, that is, $z = \alpha z = \beta z$. Hence z is a common fixed point of α and β. To prove the uniqueness, assume that w is another common fixed point of α and β. Then by (3.13) we have $w = \beta w = \beta z = z$, that is, the common fixed point of α and β is unique. □

If we take $\psi(t) = t$ and $\varphi(t) = kt$ with $k < 1$ in Theorem 3.1, we get the following result:

Corollary 3.2. *Let $(X,\ \preceq)$ be a partially ordered set and $(X,\ M,\ *)$ be a non-Archimedean fuzzy metric space. Suppose $\alpha,\ \beta : X \to X$ are two mappings satisfying conditions (i) and (ii) of Theorem 3.1 and*

(i) there exists $k < 1$ such that

$$\frac{1}{M(\alpha x,\ \alpha y,\ t)} - 1 \le k\left(\frac{1}{M(\beta x,\ \beta y,\ t)} - 1\right),$$

for all $x,\ y \in X$ with $\beta x \preceq \beta y$. Also assume that, at least, one of the conditions $(a) - (c)$ of Theorem 3.1 holds. Then α and β have a coincidence point. Moreover, if condition (iv) of Theorem 3.1 holds. Then α and β have a unique common fixed point.

If we put $\beta = I$ (the identity mapping) in Theorem 3.1, we get the following corollary.

Corollary 3.3. *Let $(X,\ \preceq)$ be a partially ordered set and $(X,\ M,\ *)$ be a complete non-Archimedean fuzzy metric space. Suppose $\alpha : X \to X$ is a non-decreasing mapping for which there exist an altering distance function ψ and a right upper semi-continuous function $\varphi : [0,\ +\infty) \to [0,\ +\infty)$ such that*

$$\psi\left(\frac{1}{M(\alpha x,\ \alpha y,\ t)} - 1\right) \le \varphi\left(\frac{1}{M(x,\ y,\ t)} - 1\right),$$

for all $x,\ y \in X$ such that $x \preceq y$ where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. If there exists $x_0 \in X$ such that $x_0 \preceq \alpha x_0$, then α has a fixed point.

If we put $\beta = I$ (the identity mapping) in the Corollary 3.2, we get the following corollary.

Corollary 3.4. *Let* $(X, \preceq)$ *be a partially ordered set and* $(X, M, *)$ *be a complete non-Archimedean fuzzy metric space. Suppose* $\alpha : X \to X$ *is a non-decreasing mapping satisfying*

$$\frac{1}{M(\alpha x,\ \alpha y,\ t)} - 1 \leq k\left(\frac{1}{M(x,\ y,\ t)} - 1\right),$$

for all $x, y \in X$ *such that* $x \preceq y$ *and* $k < 1$*. If there exists* $x_0 \in X$ *such that* $x_0 \preceq \alpha x_0$*, then* α *has a fixed point.*

Example 3.1. Suppose that $X = [0, 1]$, equipped with the usual metric $d : X \times X \to [0, +\infty)$ with the natural ordering of real numbers $\leq$ and $*$ is defined by $a * b = ab$, for all $a, b \in [0, 1]$. Define

$$M(x,\ y,\ t) = \frac{t}{t + d(x,\ y)},\ \text{for all } x,\ y \in X \text{ and } t > 0.$$

Clearly $(X, M, *)$ is a complete non-Archimedean fuzzy metric space. Let $\alpha, \beta : X \to X$ be defined as

$$\alpha x = \frac{x^2}{3} \text{ and } \beta x = x^2,\ \text{for all } x \in X.$$

Define

$$\psi(t) = t,\ \text{for } t \geq 0 \text{ and } \varphi(t) = \begin{cases} t/2, & \text{for } t \neq 1, \\ 3/4, & \text{for } t = 1. \end{cases}$$

Then ψ and φ have all the required properties and the contractive condition of Theorem 3.1 is satisfied for all $x, y \in X$. Furthermore, all the other conditions of Theorem 3.1 are satisfied and $z = 0$ is a unique common fixed point of α and β.

4. Coupled Fixed Point Results

Next, we deduce two dimensional version of Theorem 3.1. Given $n \in \mathbb{N}$ where $n \geq 2$, let X^n be the n^{th} Cartesian product $X \times X \times ... \times X$ (n times). For partially ordered non-Archimedean fuzzy metric space $(X, M, \preceq)$, let us consider the partially ordered non-Archimedean fuzzy metric space $(X^2, M_\delta, \sqsubseteq)$, where $M_\delta : X^2 \times X^2 \times [0, \infty) \to [0, 1]$ is defined by

$$M_\delta(V,\ W,\ t) = \min\{M(x,\ u,\ t),\ M(y,\ v,\ t)\},\ \forall V = (x,\ y),\ W = (u,\ v) \in X^2,$$

and $\sqsubseteq$ is defined by

$$W \sqsubseteq V \Leftrightarrow x \succeq u \text{ and } y \preceq v, \text{ for all } W = (u,\ v),\ V = (x,\ y) \in X^2.$$

It is easy to check that M_δ is a non-Archimedean fuzzy metric on X^2. Let $F : X^2 \to X$ and $G : X \to X$ be two mappings. Define the mapping $\Phi,\ \Psi : X^2 \to X^2$, for all $V = (x,\ y) \in X^2$, as follows:

$$\Phi(V) = (F(x,\ y),\ F(y,\ x)) \text{ and } \Psi(V) = (Gx,\ Gy).$$

Lemma 4.1 ([17]). *Let $(X,\ \preceq)$ be a partially ordered set and $(X,\ M,\ *)$ be a non-Archimedean fuzzy metric space. Let $F,\ G : X^2 \to X$ and $\Phi,\ \Psi : X^2 \to X^2$ be mappings, then the following properties hold.*

(1) $(X,\ M)$ is complete if and only if $(X^2,\ M_\delta)$ is complete.

(2) If $(X,\ M,\ \preceq)$ is regular, then $(X^2,\ M_\delta,\ \sqsubseteq)$ is also regular.

(3) If F is $M-$continuous, then Φ is $M_\delta-$continuous.

(4) F has the mixed monotone property with respect to $\preceq$ if and only if Φ is $\sqsubseteq$-non-decreasing.

(5) F has the mixed $G-$monotone property with respect to $\preceq$ if and only if then Φ is $(\Psi,\ \sqsubseteq)$-non-decreasing.

(6) If there exist two elements $x_0,\ y_0 \in X$ with $Gx_0 \preceq F(x_0,\ y_0)$ and $Gy_0 \succeq F(y_0,\ x_0)$, then there exists a point $X_0 = (x_0,\ y_0) \in X^2$ such that $\Psi(X_0) \sqsubseteq \Phi(X_0)$.

(7) If $F(X^2) \subseteq G(X)$, then $\Phi(X^2) \subseteq \Psi(X^2)$.

(8) If F and G are commuting in $(X,\ M,\ \preceq)$, then Φ and Ψ are also commuting in $(X^2,\ M_\delta,\ \sqsubseteq)$.

(9) If F and G are compatible in $(X,\ M,\ \preceq)$, then Φ and Ψ are also compatible in $(X^2,\ M_\delta,\ \sqsubseteq)$.

(10) If F and G are weak compatible in $(X,\ M,\ \preceq)$, then Φ and Ψ are also weak compatible in $(X^2,\ M_\delta,\ \sqsubseteq)$.

(11) A point $(x,\ y) \in X^2$ is a coupled coincidence point of F and G if and only if it is a coincidence point of Φ and Ψ.

(12) A point $(x,\ y) \in X^2$ is a coupled fixed point of F if and only if it is a fixed point of Φ.

Theorem 4.1. *Let $(X,\ \preceq)$ be a partially ordered set and $(X,\ M,\ *)$ be a complete non-Archimedean fuzzy metric space. Assume $F : X^2 \to X$ and $G : X \to X$ are two mappings such that F has mixed $G-$monotone property with respect to $\preceq$ on X for which there exist an altering distance function ψ and a right upper semi-continuous* *function* $\varphi : [0, +\infty) \to [0, +\infty)$ *such that*

$$\psi\left(\frac{1}{M(F(x,\ y),\ F(u,\ v),\ t)} - 1\right) \tag{4.1}$$

$$\leq \varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}} - 1\right),$$

for all $x, y, u, v \in X$ *with* $Gx \preceq Gu$ *and* $Gy \succeq Gv$, *where* $\psi(t) > \varphi(t)$ *for all* $t > 0$ *and* $\varphi(0) = 0$. *Suppose that* $F(X^2) \subseteq G(X)$, G *is continuous and monotone non-decreasing and the pair* $\{F, G\}$ *is compatible. Also suppose that either*

(a) F *is continuous or*

(b) $(X, d, \preceq)$ *is regular.*

Assume that there exist two elements $x_0, y_0 \in X$ *with*

$$Gx_0 \preceq F(x_0,\ y_0) \text{ and } Gy_0 \succeq F(y_0,\ x_0).$$

Then F *and* G *have a coupled coincidence point. Furthermore, suppose that for every* $(x, y), (x^*, y^*) \in X^2$, *there exists a point* $(u, v) \in X^2$ *such that* $(F(u, v), F(v, u))$ *is comparable to* $(F(x, y), F(y, x))$ *and* $(F(x^*, y^*), F(y^*, x^*))$, *and also the pair* (F, G) *is weakly compatible. Then* F *and* G *have a unique common coupled fixed point.*

Proof. Let $V = (x, y)$ and $W = (u, v) \in X^2$ with $\Psi(V) \sqsubseteq \Psi(W)$. Then $Gx \preceq Gu$ and $Gy \succeq Gv$ and so by using (4.1), we have

$$\psi\left(\frac{1}{M(F(x,\ y),\ F(u,\ v),\ t)} - 1\right) \leq \varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}} - 1\right).$$

Furthermore taking into account that $Gy \succeq Gv$ and $Gx \preceq Gu$, (4.1) also guarantees that

$$\psi\left(\frac{1}{M(F(y,\ x),\ F(v,\ u),\ t)} - 1\right) \leq \varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}} - 1\right).$$

Combining them, we get

$$\max\left\{\begin{array}{c} \psi\left(\dfrac{1}{M(F(x,\ y),\ F(u,\ v),\ t)} - 1\right), \\ \psi\left(\dfrac{1}{M(F(y,\ x),\ F(v,\ u),\ t)} - 1\right) \end{array}\right\}$$

$$\leq \ \varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}} - 1\right).$$

Since ψ is non-decreasing,

$$\psi\left(\max\left\{\begin{array}{c}\left(\dfrac{1}{M(F(x,\ y),\ F(u,\ v),\ t)}-1\right),\\ \left(\dfrac{1}{M(F(y,\ x),\ F(v,\ u),\ t)}-1\right)\end{array}\right\}\right)$$

$$\leq\ \varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}}-1\right). \tag{4.2}$$

Thus, it follows from (4.2) that

$$\begin{aligned}
&\psi\left(\frac{1}{M_\delta(\Phi(V),\ \Phi(W),\ t)}-1\right)\\
=\ &\psi\left(\frac{1}{\min\{M(F(x,\ y),\ F(u,\ v),\ t),\ M(F(y,\ x),\ F(v,\ u),\ t)\}}-1\right)\\
=\ &\psi\left(\max\left\{\left(\frac{1}{M(F(x,\ y),\ F(u,\ v),\ t)}-1\right),\left(\frac{1}{M(F(y,\ x),\ F(v,\ u),\ t)}-1\right)\right\}\right)\\
\leq\ &\varphi\left(\frac{1}{\min\{M(Gx,\ Gu,\ t),\ M(Gy,\ Gv,\ t)\}}-1\right)\\
\leq\ &\varphi\left(\frac{1}{M_\delta(\Psi(V),\ \Psi(W),\ t)}-1\right).
\end{aligned}$$

It is only necessary to apply Theorem 3.1 with $\alpha = \Phi$ and $\beta = \Psi$ in the partially ordered metric space $(X^2,\ M_\delta,\ \sqsubseteq)$ taking into account of all items of Lemma 4.1. □

Corollary 4.2. *Let $(X,\ \preceq)$ be a partially ordered set and $(X,\ M,\ *)$ be a complete non-Archimedean fuzzy metric space. Assume $F : X^2 \to X$ has mixed monotone property with respect to $\preceq$ for which there exist an altering distance function ψ and a right upper semi-continuous function $\varphi : [0,\ +\infty) \to [0,\ +\infty)$ such that*

$$\psi\left(\frac{1}{M(F(x,\ y),\ F(u,\ v),\ t)}-1\right)\leq\varphi\left(\frac{1}{\min\{M(x,\ u,\ t),\ M(y,\ v,\ t)\}}-1\right),$$

for all $x,\ y,\ u,\ v \in X$, with $x \preceq u$ and $y \succeq v$, where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. Also suppose that either

(a) F is continuous or

(b) $(X,\ d,\ \preceq)$ is regular.

Assume that there exist two elements $x_0,\ y_0 \in X$ with

$$x_0 \preceq F(x_0,\ y_0) \text{ and } y_0 \succeq F(y_0,\ x_0).$$

Then F has a coupled fixed point.

In a similar way, we may state the results analogous to Corollary 3.2 for Theorem 4.1 and Corollary 4.2.

5. Applications

In this section, we give an application to integral equation of our results. Consider the integral equation

$$u(t) = \int_0^T K(t,\ s,\ u(s))ds + h(t),\ t \in [0,\ T], \tag{5.1}$$

where $T > 0$. We introduce the following space:

$$C[0,\ T] = \{u : [0,\ T] \to \mathbb{R} : u \text{ is continuous on } [0,\ T]\},$$

equipped with the metric

$$d(x,\ y) = \sup_{t\in[0,\ T]} |x(t) - y(t)|\,,\ \text{for each } x,\ y \in C[0,\ T].$$

It is clear that $(C[0,\ T],\ d)$ is a regular complete metric space. It is easy to check that $(C[0,T],\ M,\ *)$ is a complete non-Archimedean fuzzy metric space with respect to the fuzzy metric

$$M(x,\ y,\ t) = \frac{t}{t + d(x,\ y)},\ \text{for all } x,\ y \in X \text{ and } t > 0,$$

with $*$ is defined by $a * b = ab$, for all $a,\ b \in I$. Furthermore, $C[0,\ T]$ can be equipped with the partial order $\preceq$ as follows: for $x,\ y \in C[0,\ T]$,

$$x \preceq y \Longleftrightarrow x(t) \leq y(t),\ \text{for each } t \in [0,\ T].$$

Now, we state the main result of this section.

Theorem 5.1. *We assume that the following hypotheses hold:*

(i) $K : [0,\ T] \times [0,\ T] \times \mathbb{R} \to \mathbb{R}$ *and* $h : \mathbb{R} \to \mathbb{R}$ *are continuous,*

(ii) for all $s,\ t,\ u,\ v \in C[0,\ T]$ *with* $v \preceq u$, *we have*

$$K(t,\ s,\ v(s)) \leq K(t,\ s,\ u(s)),$$

(iii) there exists a continuous function $g : [0,\ T] \times [0,\ T] \to [0,\ +\infty)$ *such that*

$$|K(t,\ s,\ x) - K(t,\ s,\ y)| \leq g(t,\ s) \cdot \frac{|x - y|}{2},$$

for all $s,\ t \in C[0,\ T]$ *and* $x,\ y \in \mathbb{R}$ *with* $x \succeq y$,

(iv) $\sup_{t\in[0,\ T]} \int_0^T g(t,\ s)^2 ds \leq \frac{1}{T}$.

Then the integral equation (5.1) *has a solution* $u^* \in C[0,\ T]$.

Proof. Define $\alpha : C[0,\ T] \to C[0,\ T]$ by

$$\alpha u(t) = \int_0^T K(t,\ s,\ u(s))ds + h(t),\ \text{for all } t \in [0,\ T] \text{ and } u \in C[0,\ T].$$

Assume that $v \preceq u$. From (ii), for all $s,\ t \in [0,\ T]$, we have $K(t,\ s,\ v(s)) \le K(t,\ s,\ u(s))$. Thus, we get,

$$\alpha v(t) = \int_0^T K(t,\ s,\ v(s))ds + h(t) \le \int_0^T K(t,\ s,\ u(s))ds + h(t) = \alpha u(t).$$

Thus α is non-decreasing. Now, for all $u,\ v \in C[0,\ T]$ with $v \preceq u$, due to (iii) and by using Cauchy-Schwarz inequality, we get

$$\begin{aligned} & |\alpha u(t) - \alpha v(t)| \\ \le & \int_0^T |K(t,\ s,\ u(s)) - K(t,\ s,\ v(s))|\, ds \\ \le & \int_0^T g(t,\ s) \cdot \frac{|u(s) - v(s)|}{2} ds \\ \le & \left(\int_0^T g(t,\ s)^2 ds \right)^{\frac{1}{2}} \left(\int_0^T \left(\frac{|u(s) - v(s)|}{2} \right)^2 ds \right)^{\frac{1}{2}}. \end{aligned}$$

Thus

$$|\alpha u(t) - \alpha v(t)| \le \left(\int_0^T g(t,\ s)^2 ds \right)^{\frac{1}{2}} \left(\int_0^T \left(\frac{|u(s) - v(s)|}{2} \right)^2 ds \right)^{\frac{1}{2}}. \tag{5.2}$$

Taking (iv) into account, we estimate the first integral in (5.2) as follows:

$$\left(\int_0^T g(t,\ s)^2 ds \right)^{\frac{1}{2}} \le \frac{1}{\sqrt{T}}. \tag{5.3}$$

For the second integral in (5.2) we proceed in the following way:

$$\left(\int_0^T \left(\frac{|u(s) - v(s)|}{2} \right)^2 ds \right)^{\frac{1}{2}} \le \sqrt{T} \cdot \frac{d(u,\ v)}{2}. \tag{5.4}$$

Combining (5.2), (5.3) and (5.4), we conclude that

$$d(\alpha u,\ \alpha v) \le \frac{1}{2} d(u,\ v).$$

It yields

$$\frac{1}{M(\alpha u,\ \alpha v,\ t)} - 1 \le \frac{1}{2} \left(\frac{1}{M(u,\ v,\ t)} - 1 \right),$$

for all u, $v \in C[0, T]$ with $v \preceq u$. Thus the contractive condition of Corollary 3.4 is satisfied with $k = 1/2 \in (0, 1)$. Hence, all hypotheses of Corollary 3.4 are satisfied. Thus, α has a fixed point $u^* \in C[0, T]$ which is a solution of (5.1). □

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