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DECOMPOSITION OF SPECIAL PSEUDO PROJECTIVE CURVATURE TENSOR FIELD

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Abstract. The aim of this paper is to study the projective curvature tensor field of the Curvature tensor R^i_{jkh} on a recurrent non Riemannian space admitting recurrent affine motion, which is also decomposable in the form R^i_{jkh}=X^i Y_{jkh}, where X^i and Y_{jkh} are non-null vector and tensor respectively. In this paper we decompose Special Pseudo Projective Curvature Tensor Field. In the sequal of decomposition we established several properties of such decomposed tensor fields. We have considered the curvature tensor field R^i_{jkh} in a Finsler space equipped with non symmetric connection and we study the decomposition of such field. In a special Pseudo recurrent Finsler Space, if the arbitrary tensor field ψ^i_j is assumed to be a covariant constant then, in view of the decomposition rule, ϕ_{kh} behaves as a recurrent tensor field. In the last, we have considered the decomposition of curvature tensor fields in Kaehlerian recurrent spaces and have obtained several related theorems.

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1. Introduction

Curvature tensor R^i_{jkh} on a recurrent non Riemannian space admitting recurrent affine motion is decomposable in the form R^i_{jkh}= X^i Y_{jkh} where X^i and Y_{jkh} are a non null vector and tensor respectively. Saxena, M [11] discussed special structure. Singh, B. B [7] and Ramhit [5] introduced a recurrent Finsler space whose curvature tensor is decomposable in the form H^i_{jkh}= X^i Y_{jkh}. Kowalski, O [6] studied curvature of the diagonal lift and obtained several results.

In present paper, we introduce a Special Pseudo Projective Curvature Tensor Field, and subsequently we decompose it. In the sequel, we established several

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properties of such decomposed tensor fields. In the later section we have considered the curvature tensor field $R^i_{jkh}(x,y)$ in a Finsler space equipped with non symmetric connection and studeied properties of its decomposition. In the last section, we have considered the decomposition of curvature tensor fields in Kaehlerian recurrent spaces and have obtained few related theorems.

2. SPECIAL PSEUDO PROJECTIVE TENSOR FIELDS

The special pseudo projective tensor fields have been defined by Kumar, A [2] as

$$T^i_J(x,y) = PW^i_J + HH^i_J \tag{1}$$

where

$$P(x,y) = \frac{1}{n+1} GG^r_{rs} Y^s \tag{2}$$

is a scalar function positively homogeneous of degree one in its directional arguments, here W and H are (1, 1) tensor field, and G is a tensor field
also we have,

$$H(x,y) = \frac{1}{n-1} H^i_j \tag{3}$$

with the help of (1) two more tensor fields $T^i_{hJ}(x,y)$ and $T^i_{lhj}(x,y)$ have also been defined by Kumar, A [2] as

$$T^i_{hJ}(x,y) = PW^i_{hJ}(x,y) + HH^i_{hJ}(x,y) + \frac{1}{2}\{\partial_l hPW^i_{hJl} + \partial_l hHH^i_{hJl}\} \tag{4}$$

and

$$\begin{aligned} T^i_{lhJ}(x,y) = {} & PW^i_{lhJ}(x,y) + HH^i_{lhJ} + \dot{\partial}_1 PW^i_{Jl} + \dot{\partial}_1 hHH^i_{hJ} \\ & + \frac{2}{3}\{\ddot{\partial}_{1[h} PW^i_{jJ} + \dot{\partial}_{[h} P\dot{\partial}_{<l>} W^i_{jJ} + \ddot{\partial}_{1[h} HH^i_{jJ} + \dot{\partial}_{[h} H\dot{\partial}_{<l>} H^i_{jJ}\}. \end{aligned} \tag{5}$$

The following identities have also been obtained by Kumar, A [2] as

$$\begin{aligned} T^i_{lhJ} = {} & \dot{\partial}_{[l} PW^i_{hjJ} + \dot{\partial}_{[l} HH^i_{jJ} \\ & + \frac{1}{3}\{\ddot{\partial}_{1[h} PW^i_{jJ} - \ddot{\partial}_{[l} HH^i_{hj} + \dot{\partial}_{[h} H\dot{\partial}_{<l>} H^i_{jJ} - \dot{\partial}_{[j} H\dot{\partial}_{<l>} H^i_{hJ}\} \end{aligned} \tag{6}$$

and

$$T^i_{lhJ(k)} + T^i_{lJk(h)} + T^i_{lkh(J)} = E^i_{lhkJ} + Q^i_{lhkJ} \tag{7}$$

where E^i_{lhkJ} and Q^i_{lhkJ} each contain 24 terms including $P(x,y)$, $H^i_J(x,y)$, $W^i_J(x,y)$ and their derivatives. We have the following definitions which shall be used in the later discussions.

Definition 2.1. A Finsler space F_n is said to be special pseudo recurrent of first order if the special pseudo projective curvature tensor field $T^i_{lhJ}(x,y)$ satisfies

$$T^i_{lhJ(k)}(x,y) = \lambda_k T^i_{lhJ}(x,y) \tag{8}$$

3. GENERAL DECOMPOSITION OF SPECIAL PSEUDO PROJECTIVE CURVATURE TENSOR FIELD

We consider the decomposition of a special pseudo projective curvature tensor field $T^i_{Jkh}(x,y)$ as

$$T^i_{Jkh} = \psi^i_j \phi_{kh} \tag{9}$$

together with

$$\psi^i_j \lambda_i = d_j, \tag{10}$$

where ψ^i_j and ϕ_{kh} are arbitrary tensor fields and d_j is a decomposed vector field in the recurrent Finsler space F_n.

In view of (9), the equation (4) takes the following form

$$\psi^i_{[l}\phi_{hj]} = \dot{\partial}_{[l}PW^i_{hj]} + \dot{\partial}_{[1}HH^i_{hJ]} + \frac{1}{3}\Big\{\ddot{\partial}_{1[j}PW^i_{h]} - \ddot{\partial}_{1[j}PW^i_{h]} + \dot{\partial}_{[h}P\dot{\partial}_{<l>}W^i_{j]}$$
$$-\dot{\partial}_{[j}P\dot{\partial}_{<l>}W^i_{h]} + \ddot{\partial}_{1[h}HH^i_{j]} - \ddot{\partial}_{1[j}HH^i_{h]} + \dot{\partial}_{[h}H\dot{\partial}_{<l>}H^i_{j]} - \dot{\partial}_{[j}H\dot{\partial}_{<l>}H^i_{h]}\Big\} \tag{11}$$

Transverting (11) by λ_i and thereafter noting the equation (10) we get

$$d_{[l}\phi_{hj]} = \lambda_i[\dot{\partial}_{[l}PW_{hj]} + \dot{\partial}_{[1}HH^i_{hJ]} + \frac{1}{3}\Big\{\ddot{\partial}_{[1h}PWH^i_{j]} - \ddot{\partial}_{[1j}PW^i_{h]} - \ddot{\partial}_{[1h}HH^i_{j]}$$
$$+\dot{\partial}_{[h}H\dot{\partial}_{<l>}H^i_{j]} - \dot{\partial}_{[j}H\dot{\partial}_{<l>}H^i_{h]}\Big\}, \tag{12}$$

similarly in view of the equation (5) and (10), we get

$$\psi^i_l(\lambda_h\phi_{jk} + \lambda_j\phi_{kh} + \lambda_k\phi_{hj}) = E^i_{lkhj} + H^i_{lkhj} \tag{13}$$

On multiplying (13) by λ_i and then using (10) we get

$$d_i(\lambda_h\phi_{jk} + \lambda_j\phi_{kh} + \lambda_k\phi_{hj}) = \lambda_i(E^i_{lkhj} + HQ^i_{lkhj}) \tag{14}$$

Theorem 3.1. *In view of the decomposition given by (9), the identities for the special pseudo projective curvature tensor fields in a special pseudo recurrent Finsler space are respectively given by (12) and (14).*

Now we multiply (9) by λ_i and use (10) thereafter to get

$$\lambda_i T^i_{jkh} = d_j\phi_{kh}. \tag{15}$$

Transverting (14) by ψ^j_m and thereafter using (10), and the fact that $\psi_{[kh]} = 0$ we get

$$d_l(\phi_{kh}d_m - 2M_{1[k}\lambda_{h]}) = \lambda_i\psi^m_j(E^i_{lhkj} + Q^i_{lkhj}) \tag{16}$$

where

$$M_{lk} = \psi^i_l\psi_{kl} \tag{17}$$

At this stage we use (15) and (17) in (16) and subsequently get

$$d_m\lambda_i T_{lkh} = \lambda_i\psi^j_m(E^i_{lkhj} + Q^i_{lkhj}) + 2d_l M_{m[k}\lambda_{h]} \tag{18}$$

Thus we Have the following theorem

Theorem 3.2. *In a special pseudo recurrent Finsler space under the decomposition rule (9) the special pseudo projective tensor field always satisfies (17).*

Differentiating (9) with respect to X^m we get

$$T^j_{jkh(m)} = \psi^m_{j(m)}\phi_{kh} + \psi^j_j\phi_{kh(m)}. \tag{19}$$

If ψ^j_j be assumed to be a covariant constant then in view of (14), (11) gives

$$\lambda_m T^i_{jkh} = \psi^i_j\phi_{kh(m)}. \tag{20}$$

In view of the decomposition rule (9), (20) gives

$$\phi_{kh(m)-\lambda_m\phi_{kj}\psi^i_j} = 0. \tag{21}$$

Since ψ^i_j is an arbitrary non zero tensor field, from (19) we get,

$$\phi_{kh(m)} = \lambda_m\phi_{kh} \tag{22}$$

Theorem 3.3. *In a special pseudo recurrent Finsler space if the arbitrary tensor field, ψ^i_j given in (9) is assumed to be a covariant constant, then in view of the decomposition rule (9), ϕ_{kh} behaves as recurrent tensor field.*

4. DECOMPOSITION OF CURVATURE TENSOR FIELD $R^i_{jkh}(x,y)$ IN A FINSLER SPACE EQUIPPED WITH NON SYMMETRIC CONNECTION

Definition 4.1. An F_n is said to be an $R - \oplus$ recurrent Finsler space of first/second order (Pandey, H D [4]) when the curvature tensor field $R^i_{jkl}(x,y)$ satisfies.

$$R^i_{jkl}\ {}^+I_s = \lambda_s R^i_{jkl} \tag{23}$$

or

$$R^i_{jkl}\ {}^+I_{sm} = \alpha_{sm} R^i_{jkl} \tag{24}$$

respectively. Hence $\lambda_s(x,y)$ and $\alpha_{sm}(x,y)$ are recurrence vector and recurrence tensor fields respectively. Transverting (23) and (24) by Y^j and noting the facts that $R^i_{jkl}Y^j = R^i_{kl}$, $Y\ {}^+I_s = 0$ as have been given in [1], we get

$$R^i_{kl}\ {}^+I_s = \lambda_s R^i_{kl} \tag{25}$$

$$R^i_{kl}\ {}^+I_{sm} = \alpha_{sm} R^i_{kl} \tag{26}$$

We now consider a Finsler space F^*_m, in which the curvature tensor field R^i_{jkl} is decomposable. Since the curvature tensor R^i_{jkl} is a mixed tensor of rank 4, it may be written either as a tensor product as a vector and tensor of rank 3 or as a tensor product of two tensors each of rank 2. In the first case, the possibilities of its form are as follows.

$$R^i_{jkl} = X^i\psi_{jkl} \tag{27}$$

$$R^i_{jkl} = X_j\psi^i_{kl} \tag{28}$$

$$R^i_{jkl} = X_k\psi^i_{jl} \tag{29}$$

$$R^i_{jkl} = X_l\psi^i_{jk} \tag{30}$$

While in the second case the possibilities are as follows

$$R^i_{jkl} = P^i_j\psi_{kl} \tag{31}$$

$$R^i_{jkl} = P^i_l\psi_{jk} \tag{32}$$

$$R^i_{jkl} = P^i_k\psi_{jl} \tag{33}$$

Out of all these possibilities, we propose to take up the possibility given (27) only. In (27) the vector field X^i satisfies

$$\lambda_i X^i = 1 \tag{34}$$

where as the tensor P^i_j appearing in (31) satisfies

$$P^i_j\lambda_i = d_j \tag{35}$$

where d_j is a decomposed vector field. Transverting (29) by y^j, we get

$$R^i_{jkl}(x,y) = X^i\psi_{kl} \tag{36}$$

where we have taken into account the facts given by

$$\psi^i_{jkl}Y^j = \psi^i_{kl} \tag{37}$$

$$\dot{\partial}_1\psi_{kh} = \psi_{lkh}. \tag{38}$$

Differentiating (29), $\oplus$-covariantly with respect to X^i, we get

$$R^i_{jkl}\ {}^+I_s = X^i\ {}^+I_s\psi_{lkh}\ {}^+I_s \tag{39}$$

using (23) in (29), we get

$$\lambda_s R^i_{jkl} = X^i\ {}^+I_s\psi_{jlk} + X^i\psi_{jlk}\ {}^+I_s \tag{40}$$

again using (29) in (40). we get

$$\lambda_s X^i\psi_{jkl}\ {}^+I_s = X^i\ {}^+I_s\psi_{lkh} + X^i\psi_{jlk}\ {}^+I_s. \tag{41}$$

Now assume the decomposition vector X^i is a covariant constant, then from (41), we immediately get

$$X^i(\lambda_s\psi_{jkl} - \psi_{jkl}\ ^+I_s) = 0. \tag{42}$$

Since X^i is an arbitrary vector, we use (42) to get

$$\psi_{jkl}\ ^+I_s = \lambda_s\psi_{jkl}. \tag{43}$$

Transverting (43) by Y^j and using (37) thereafter, we get

$$\psi_{kl}\ ^+I_s = \lambda_s\psi_{kl}. \tag{44}$$

With the help of (43) and (44), we can therefore state:

Theorem 4.2. *In an R^+-recurrent F_n^* of first order, if the decomposition vector X^i is assumed to be a covariant constant then the decomposition tensor fields $\psi_{jkl}(x,y)$ and $\psi_{kl}(x,y)$ behave like I-recurrent tensor fields.*

Differentiating (29), $\oplus$-covariantly with respect to X^i and X^m, we get

$$R^i_{jkl}\ ^+I_{sm} = X^i\ ^+I_{sm}\psi_{jkl} + X^i\psi_{jkl}\ ^+I_{sm} \tag{45}$$

using (24) and (29) in (45), we get

$$\alpha_m X^i\psi_{jkl} = X^i\psi_{jkl}\ ^+I_{sm} \tag{46}$$

where, we have assumed that X^i is a covariant constant.

$$\psi_{kl}\ ^+I_{sm} = X^i\alpha_{sm}\psi_{kl}. \tag{47}$$

With the help of (46) and (47) we can state the following theorem:

Theorem 4.3. *In an R^+-recurrent F_n^* of second order, the decomposition field $\psi_{jkl}(x,y)$ and $\psi_{kl}(x,y)$ behaves like second order recurrent tensor fields if the decomposition vector X^i be assumed to be a covariant constant.*

In view of communication formula (18), differentiating (47) with respect to Y^j, we get

$$\begin{aligned}(\dot\partial_1\alpha_{sm})\psi_{kh} - \alpha_{sm}(\dot\partial_1\psi_{kl}) &= (\dot\partial_j\psi_{kl}\ ^+I_{sm})\ ^+I_{sm} - \psi\ ^+I_{sm}(\dot\partial_j\Gamma^l_{km}) \\ &-(\dot\partial_p\psi_{kl}\ ^+I_{sm}\dot\partial_p\Gamma_{km})y^i - \psi_{kl}\ ^+I_{sm}\dot\partial_j\Gamma^l_{lm} + \dot\partial_j\psi_{kl}\Gamma^l_{lm}\end{aligned} \tag{48}$$

At this stage, if we assume that the space is affinely connected (in an affinely connected space F_n^*, $\dot\partial_j\Gamma^p_{lm} = 0$), then, from (48), we have

$$(\dot\partial_1\alpha_{sm})\psi_{kl} + \alpha_{sm}(\dot\partial_1\psi_{kl}) = (\dot\partial_j\psi_{kl}\ ^+I_s)\ ^+I_m + (\dot\partial_j\psi_{kl}\ ^+I_s)\Gamma^l_{sm} \tag{49}$$

We now assume that the recurrent tensor $\alpha_{sm}(x,y)$ is homogeneous of degree one in its directional arguments and thereafter making use of (43) in (49), we get

$$(\dot\partial_1\alpha_{sm})\psi_{kl} + \alpha_{sm}\psi_{jkl} = \lambda_s\lambda_m\psi_{jkl} + \lambda_r\psi_{kl}\lambda^r_{sm}. \tag{50}$$

Transverting (50) by Y^j, and then using (37), we get

$$(2\alpha_{sm}\lambda_s\lambda_m)\psi_{kl} = \lambda_r\psi_{kl}\Gamma^r_{sm}Y^j \tag{51}$$

Thus, we can state the theorem:

Theorem 4.4. *In an affinely connected recurrent F_n^* , if the recurrence tensor is assumed to be homogeneous of degree one in its directional arguments then (51) necessarily holds.*

In view of commutation formula (18), commutating (47) with respect to the indices s and m and thereafter using (26), we get

$$(\alpha_{sm} - \alpha_{ms})\psi_{kh} + (\dot{\partial}_j \psi_{kh})R^r_{sm} = \psi_{kh}\ {}^{+}I_s N^r_{ms} - \psi_{lh} R^r_{hm} - \psi_{kl} R^r_{lsm} \qquad (52)$$

Making use of (27) and (37) in (52), we get

$$(\alpha_{sm} - \alpha_{ms})\psi_{kh} + (\psi_{rkh}\psi_{kh} + \psi_{ksm}\psi_{rh} + \psi_{hsm}\psi_{kr})x_i - \lambda_r N^r_{ms}\psi_{kh} = 0 \quad (53)$$

Thus we have the following theorem:

Theorem 4.5. *In the recurrent F_n^* of second order, under the decomposition rule (27) the recurrence tensor field $\alpha_{sm}(x, y)$ behaves like a symmetric tensor field provided*

$$(\psi_{rkh}\psi_{kh} + \psi_{ksm}\psi_{rh} + \psi_{hsm}\psi_{kr})X^i = \lambda_r N^r_{ms}\psi_{kh} \qquad (54)$$

holds, differentiating (47) $\oplus$ - covariantly with respect to X^i, we get

$$(\psi_{kh}\ {}^{+}I_{sm})\ {}^{+}I_j = \alpha_{sm}\ {}^{+}I_j \psi_{kl} + \alpha_{sm}\psi_{kl}\ {}^{+}I_j \qquad (55)$$

Commutating (55) with respect to the indices m and j and thereafter using the commutation formula (18), we get

$$\begin{aligned}&\{(\alpha_{sm}\ {}^{+}I_j - \alpha_{sj}\ {}^{+}I_m)\psi_{kh} + (\psi_{kh}\ {}^{+}I_j\alpha_{sm} - \psi_{kh}\ {}^{+}I_m\alpha_{sj})\}\\ &= \psi_{kh}\ {}^{+}I_s\ {}^{+}I_r N^r_{jm} - (\dot{\partial}_r\psi_{kh}\ {}^{+}I_s)N^r_{mj} - \psi_{kr}\psi_{kh}\ {}^{+}I_s R^r_{hmj}\\ &-\psi_{kh}\ {}^{+}I_r R^r_{smj} - \psi_{kh}\ {}^{+}I_s R^r_{kmj}\end{aligned} \qquad (56)$$

Making use of the provisions of theorem (23) in (56) and the fact that the recurrence vector is independent of direction, we get

$$\begin{aligned}&2\alpha_s[m\ {}^{+}I_{j]}\psi_{kh} + 2\lambda_{[j}\alpha_{<s>}m\psi_{kh} - \lambda_m\psi_{kh}\alpha_{rj}\\ &= \psi_{kh}\alpha_{rs}N^r_{jm} - (\partial\psi_{kh})\lambda_{ij}R^r_{mj} - \lambda_i\psi_{kh}R^r_{smj}\\ &\qquad\qquad -\lambda_h\psi_{ih}R^r_{kmj}\end{aligned} \qquad (57)$$

Now using equation (24), (38) and (37) in (57), we get

$$\begin{aligned}&\psi_{kh}\{2\alpha_{s[m}\ {}^{+}I_{ij}2\lambda_{[j}\alpha_{<s>m]} - \alpha_{rs}N^r_{jm} + \alpha_{smj}\}\\ &= -X^r\lambda_s\{\psi_{rkh}\psi_{mj} + \psi_{hmj}\psi_{kr} + \psi_{rh}\psi_{kmj}\}\end{aligned} \qquad (58)$$

Thus we can state that

Theorem 4.6. *In the recurrent Finsler space F_n^* equipped with non-symmetric connection, in view of the decomposition, rules given by (28) and (37), (57) necessarily hold.*

5. DECOMPOSITION OF CURVATURE TENSOR FIELD IN A KAEHLERIAN RECURRENT SPACE

The Riemann Christoffel curvature tensor, R^h_{ijk}, satisfies the identity

$$R^h_{ijk} + R^h_{jki} + R^h_{kij} = 0 \tag{59}$$

The Bianchi identity for a Riemann Christoffel curvature tensor in $K_n^{(c)}$ is given by

$$R^h_{ijkn} + R^h_{iknj} + R^h_{injk} = 0 \tag{60}$$

It is well known that the Riemann curvature tensor and the Ricci tensor satisfy the identity

$$R_{ijkn} = R_{jki} - R_{ikj} \tag{61}$$

The holomorphically projective curvature tensor is given by Gonul [9]

$$P^h_{ijk} = R^h_{ijk}\frac{1}{n+2}(R_{ik}\delta^h_j - R_{jk}\delta^h_i + S_{ik}F^h_j - S_{jk}F^h_i + 2S_{ij}F^h_k) \tag{62}$$

Where,

$$S_{ij} = F^a_i R_{aj} \tag{63}$$

We can briefly write (62) as

$$P^h_{ijkj} = R^h_{ijk} + E^h_{ijk} \tag{64}$$

Where E^h_{ijk} has been identified as equal to the second member on the right hand side of (62). We now give the following definition.

Definition 5.1. A Kaehler space is said to be the Kaehlerian recurrent if its curvature tensor field satisfies

$$R^h_{ijkn} = \mu_n R^h_{ijk} \tag{65}$$

where μ_n is a non zero recurrence vector field.

We consider the decomposition of the recurrent curvature tensor R^h_{ijk} in the form

$$R^h_{ijk} = P^h_i Q_{jk}. \tag{66}$$

Where P^h_i and Q_{jk} are two non null tensor fields such that

$$\mu_h P^h_i = 0 \tag{67}$$

$$\mu_h P^h = 1 \tag{68}$$

$$Q_{jk} = \mu_{hk} - \mu_{kj} \tag{69}$$

The non null vector field di is called a decomposed vector field. We now use (66) in (59) and get

$$P^h_i Q_{jk} + P^h_j Q_{ki} + P^h_k Q_{ij} = 0. \tag{70}$$

We transverse (70) by μ_h and use (67) thereafter to get

$$d_i Q_{jk} + d_j Q_{ki} + d_k Q_{ij} = 0. \tag{71}$$

At this stage, we make use of (66) and (65) in (60) and get

$$P_j^h[\mu_a Q_{jk} + \mu_j Q_{ka} + \mu_k Q_{aj}] = 0. \tag{72}$$

Multiply (72) by μ_a and using (67) thereafter, we get

$$d_i[\mu_a Q_{jk} + \mu_j Q_{ka} + \mu_k Q_{aj}] = 0. \tag{73}$$

Since d_i is a non null recurrence vector field, we therefore get

$$\mu_a Q_{jk} + \mu_j Q_{ka} + \mu_k Q_{aj} = 0. \tag{74}$$

Thus we can state:

Theorem 5.2. *Under the decomposition rule (67) the Bianchi identities assume the forms as have been given by (71) and (74).*

Using (65), we can have

$$R_{iks} = \mu_s R_{ik} \tag{75}$$

and

$$R_a = \mu_a R. \tag{76}$$

Using the equations (65), (61) and (75), we shall have

$$\mu_a R^a_{ijk} = \mu_i R_{jk} + \mu_j R_{ik}. \tag{77}$$

We now multiply (77) by μ_h and use (68) thereafter to get

$$\mu_h R^h_{ijk} = d_i Q_{jk}. \tag{78}$$

In view of (77) and (78), we get

$$\mu_a R^a_{ijk} = \mu_i R_{jk} + \mu_j R_{ik} = d_i Q_{jk}. \tag{79}$$

Thus, we state:

Theorem 5.3. *In view of the decomposition rule (77) the tensor fields R^h_{ijk}, R_{jk} and Q_{jk} are connected by (79).*

Conflicts of interest : We disclose that we don't have any financial and personal relationships with other people or organizations that could inappropriately influence (bias) our work.

Data availability : The data is available on the request to authors.

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