

A Simulation Study on Queueing Delay Performance of Slotted ALOHA under Time-Correlated Channels

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Abstract

Slotted ALOHA (S-ALOHA) is a classical medium access control protocol widely used in multiple access communication networks, supporting distributed random access without the need for a central controller. Although stability and delay have been extensively studied in existing works, most of these studies have assumed ideal channel conditions or independent fading, and the impact of time-correlated wireless channels has been less addressed. In this paper, we investigate the queueing delay performance in S-ALOHA networks under time-correlated channel conditions by utilizing a Gilbert-Elliott model. Through simulation studies, we demonstrate how temporal correlation in the wireless channel affects the queueing delay performance. We find that stronger temporal correlation leads to increased variability in queue length, a larger probability of having queue overflows, and higher congestion levels in the S-ALOHA network. Consequently, there is an increase in the average queueing delay, even under a light traffic load. With these findings, we provide valuable insights into the queueing delay performance of S-ALOHA networks, supplementing the existing understanding of delay in S-ALOHA networks.

Keywords: Slotted ALOHA, Random Access, Queueing Performance, Delay, Gilbert-Elliott Channel

1. Introduction

Slotted ALOHA (S-ALOHA) is a medium access control (MAC) protocol for multiple access in communication networks. It is distinguished from existing MAC protocols in that it can support *distributed* random access for nodes without requiring a central controller. This decentralized feature greatly simplifies implementation and facilitates massive access in large-scale networks. Leveraging these advantages, S-ALOHA has been used in a wide range of communication networks, including 5G and Wi-Fi networks, with diverse applications such as Internet of Things (IoT) [1-3].

Stability and delay are critical concerns in the design of S-ALOHA and its variations. However, the exact characterization of stability and delay has been a challenging problem due to the interdependencies among the queues. In this paper, we focus on the queueing delay performance in S-ALOHA networks. There have been

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extensive works on the delay performance of S-ALOHA networks, e.g., [4-6]. However, most of these works either assumed ideal channel conditions without fading or considered a more realistic channel with fading, but under the strong assumption of independence without accounting for temporal correlation. Nevertheless, the wireless channel is inherently subject to time-correlated, random fluctuations caused by fading. In this paper, we address this issue by adopting a Gilbert-Elliott channel model, as it can effectively capture this phenomenon with a two-state Markov chain [7].

It is worth noting that there are few studies on S-ALOHA under time-correlated fading channels [8-10]. In [8], the stability region and average delay are presented for S-ALOHA under a Gilbert-Elliott channel, with the assumption that the network consists of two nodes. In [9], a delay-optimal control policy is proposed for S-ALOHA under finite state Markov fading channels, but the impact of channel correlation is less addressed. In [10], S-ALOHA under Gilbert-Elliott channels is considered with a focus on information freshness measured by the Age of Information (AoI).

In comparison with the existing works, the main contributions of this paper are summarized as follows:

- (i) We investigate the queueing delay performance of a node in S-ALOHA networks under time-correlated channel conditions. Unlike previous works that assumed ideal channel conditions or independent fading, this work specifically addresses the impact of temporal channel correlation on queueing delay.
- (ii) We assume that the S-ALOHA network consists of n nodes. Based on the availability of channel state information (CSI) at these nodes, the S-ALOHA network can support two types of access schemes: transmission attempt with and without CSI [11]. We compare the queueing delay performance for these schemes under various channel parameters and traffic rates.
- (iii) Our findings from simulation studies indicate that temporal channel correlation has a negative impact on the queueing performance. This leads to increased variability in queue length, a higher probability of having large queues, and higher congestion in the overall network. These factors collectively result in an increase in the average queueing delay, even under a light traffic load.
- (iv) Our study supplements the existing understanding of delay performance in S-ALOHA networks. This understanding can help design a more efficient MAC protocol capable of supporting real-time applications with stringent delay-based guarantees.

The remainder of the paper is organized as follows. In Section 2, we describe the S-ALOHA network, along with a detailed queueing model. In Section 3, we present simulation results demonstrating the impact of temporal channel correlation on the queueing delay performance. In Section 4, we give conclusions.

2. System Description

We consider an S-ALOHA network consisting of n nodes and an Access Point (AP), where nodes transmit packets to the AP through a shared wireless medium. The time axis of this system is divided into slots of fixed duration T , indexed by $t = 1, 2, 3, \dots$. Without loss of generality, we assume $T = 1$, making the t -th slot correspond to the time interval $[t - 1, t)$ for $t = 1, 2, 3, \dots$. All nodes are synchronized to this slot timing.

2.1 Queueing Model

The network is equipped with n queues, where each queue is allocated to each node for storing packets to be transmitted to the AP. Throughout this paper, we use the notation “node i ” to refer to the i -th node and denote its allocated queue as “queue i ” for $i = 1, 2, \dots, n$. The behavior of each queue is governed by two

main processes, namely the arrival process and the service process, as described below.

We first describe the arrival process. Whenever node i generates a packet, it arrives at queue i . We assume that queue i has an infinite capacity, ensuring that the arriving packets can always enter the queue without any blocking. Let $A_i(t)$ be a random variable denoting the number of packets that arrive at queue i during the t -th slot. A common arrival model used in many scenarios is the Bernoulli process, in which $P(A_i(t) = 1) = \lambda_i$ and $P(A_i(t) = 0) = 1 - \lambda_i$, where λ_i represents the arrival rate at queue i . In practical situations, nodes independently generate their packets without relying on other nodes. Hence, the arrival process $\{A_i(t)\}_{t=1}^{\infty}$ is considered independent across i .

We next describe the service process. The service discipline of queue i follows a first-in-first-out (FIFO) policy, and hence packets in queue i are transmitted to the AP in the order of their arrivals. A packet departs from queue i when it is *successfully* transmitted to the AP; otherwise, it remains in queue i until successful transmission. Let $D_i(t)$ be a random variable denoting the number of packets that depart from queue i during the t -th slot. The value of $D_i(t)$ is determined by (i) the current wireless channel state, (ii) the specific S-ALOHA protocol employed, and (iii) the receiver model at the AP. Further details on these factors will be provided in Sections 2.2, 2.3, and 2.4, respectively.

We are ready to describe the queue length process. Let $Q_i(t)$ be the length of queue i (i.e., the number of packets in queue i) at the beginning of the t -th slot. Assuming that packet departure occurs before packet arrival within each slot, the queue length $Q_i(t)$ evolves over time according to the following recursion:

$$Q_i(t + 1) = Q_i(t) - D_i(t) + A_i(t)$$

2.2 Wireless Channel Model

In wireless networks, the wireless medium is prone to time-correlated, random fluctuations caused by fading. In this paper, we adopt a Gilbert-Elliott channel model to capture this phenomenon with a two-state Markov chain. Specifically, we let $C_i(t)$ be a random variable denoting the state of link i during the t -th slot, where link i refers to the wireless channel between node i and the AP. Then, we have $C_i(t) \in \{G, B\}$ with the following interpretation:

$$C_i(t) = \begin{cases} G, & \text{if link } i \text{ is in a good state during the } t\text{th slot,} \\ B, & \text{if link } i \text{ is in a bad state during the } t\text{th slot.} \end{cases}$$

We assume that the transmission of one packet consumes a single slot duration, as is typical in many S-ALOHA networks. Consequently, if a packet in queue i is transmitted by node i at the beginning of the t -th slot, then it will arrive at the AP at the end of the t -th slot. The state of link i during the t -th slot affects whether this packet is successfully transmitted to the AP or not. In the case of $C_i(t) = G$, the AP can decode the packet successfully, provided that no collisions occur over the t -th slot, resulting in $D_i(t) = 1$. On the other hand, in the case of $C_i(t) = B$, the AP cannot decode the packet successfully, regardless of the occurrence of a collision over the t -th slot, resulting in $D_i(t) = 0$. This assumption is in line with the signal-to-noise ratio (SNR) threshold model for reception, where a packet is successfully decoded at the receiver only when the SNR exceeds some threshold value.

In the Gilbert-Elliott model, the channel process $\{C_i(t)\}_{t=1}^{\infty}$ forms a discrete-time Markov chain, which is considered independent across i . The transition diagram of $\{C_i(t)\}_{t=1}^{\infty}$ is shown in Figure 1. In the figure, β_i is the one-step transition probability from the good state to the bad state, and γ_i is the one-step transition probability from the bad state to the good state. Notably, the stationary probability for link i to be in the bad state is given by $\pi_i^B = \beta_i / (\beta_i + \gamma_i)$. Furthermore, the number of consecutive slots over which link i persists in the bad state before transitioning back to the good state is geometrically distributed with a mean of $1/\gamma_i$.

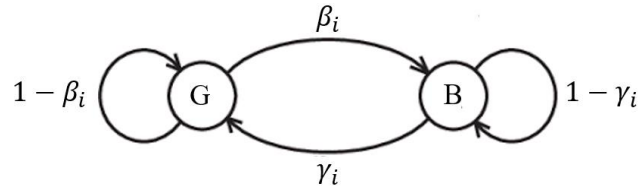


Figure 1. Transition diagram of the channel process for link i

2.3 S-ALOHA with and without CSI

Depending on the availability of CSI at nodes, the S-ALOHA network can support different random access schemes. In this paper, we consider two types of random access schemes, namely transmission attempts with and without CSI, as described below.

2.3.1 Transmission Attempt without CSI

We first consider the scenario where nodes have no knowledge of their CSI. In this case, transmission attempt without CSI is used to support random access for nodes in the system. This scheme follows the classical S-ALOHA protocol, where each backlogged node attempts to transmit a packet at the beginning of each slot with a probability τ , known as the transmission probability. To be precise, we define

$$\bar{N}(t) = \{i \in N \mid Q_i(t) \geq 1\}$$

where $N = \{1, 2, \dots, n\}$ is the set of all nodes in the network. The set $\bar{N}(t)$ distinguishes backlogged nodes from the other nodes in each slot t . Node $i \notin \bar{N}(t)$ stays silent during the t -th slot without packet transmission, since it has no packets to transmit. On the contrary, node $i \in \bar{N}(t)$ makes a transmission decision in a probabilistic manner: with probability τ_i , node i attempts to transmit a packet in queue i to the AP at the beginning of the t -th slot; with probability $1 - \tau_i$, it stays silent during the t -th slot without transmission. This decision is made independently of the other nodes as well as the decisions made in the previous slots. Let $I_i(t) \in \{0, 1\}$ be an indicator random variable that takes 1 when node i attempts to transmit its packet at the beginning of the t -th slot and takes 0 otherwise. Then, for transmission attempt without CSI, we have

$$P(I_i(t) = 1) = \begin{cases} \tau_i, & \text{for node } i \in \bar{N}(t), \\ 0, & \text{for node } i \notin \bar{N}(t). \end{cases} \quad (1)$$

Notably, due to the lack of CSI, a node may attempt to transmit even when its channel is in the bad state, resulting in the waste of resources. The use of CSI can prevent wasting resources as follows.

2.3.2 Transmission Attempt with CSI

Now, we consider the scenario where each node has knowledge of its CSI, i.e., node i is aware of $C_i(t)$. In this case, transmission attempt with CSI is used to support random access for nodes in the network. To prevent the waste of resources, this scheme follows a variant of the classical S-ALOHA protocol, where each backlogged node in good channel state attempts to transmit a packet at the beginning of each slot with a probability τ . To be precise, we define

$$\hat{N}(t) = \{i \in N \mid Q_i(t) \geq 1, C_i(t) = G\}$$

The set $\hat{N}(t)$ distinguishes backlogged nodes in good channel state from the other nodes in each slot t . Node $i \notin \hat{N}(t)$ stays silent during the t -th slot without packet transmission, since it has no packets to transmit or its channel cannot transmit packets successfully. On the contrary, node $i \in \hat{N}(t)$ makes a transmission decision in a probabilistic manner: with probability τ_i , node i attempts to transmit a packet in queue i to

the AP at the beginning of the t -th slot; with probability $1 - \tau_i$, it stays silent during the t -th slot without transmission. Let $I_i(t) \in \{0, 1\}$ be the indicator random variable as defined above. Then, for transmission attempt with CSI, we have

$$P(I_i(t) = 1) = \begin{cases} \tau_i, & \text{for node } i \in \hat{N}(t), \\ 0, & \text{for node } i \notin \hat{N}(t). \end{cases} \quad (2)$$

2.4 Receiver Model at the AP

Let $I_{tot}(t) = I_1(t) + I_2(t) + \dots + I_n(t)$ be a random variable denoting the total number of nodes that attempt to transmit at the beginning of the t -th slot. Since transmission decisions are independently made by each node without central coordination, it is possible to have $0 \leq I_{tot}(t) \leq n$. In the case of $I_{tot}(t) = 0$, no nodes attempt transmission, making the t -th slot idle. In the case of $I_{tot}(t) \geq 2$, there are concurrent transmissions during the t -th slot, and a collision occurs. In this paper, we adopt the collision model, where no transmissions are successful when $I_{tot}(t) \geq 2$ [12]. In the case of $I_{tot}(t) = 1$, only one node transmits a packet during the t -th slot. This transmission is successful when the channel is in the good state and unsuccessful otherwise.

3. Simulation

In this section, we investigate the queueing delay performance of a node in the S-ALOHA network. To this end, we have developed a simulator using MATLAB that accurately emulates the system model described in Section 2. The simulator conducts Monte Carlo simulations with different random seeds in each run. To ensure statistical significance, we execute 10^7 simulations for a given set of simulation parameters and take the averaged value.

3.1 Parameter Setup

In our simulation, we consider an S-ALOHA network consisting of $n = 20$ nodes. In order to focus on the impact of temporal channel correlation on queueing delay performance, we make a simplifying assumption that all nodes in the network are homogeneous in their parameters. This assumption entails the following equalities: $\tau_1 = \tau_2 = \dots = \tau_n = \tau$ for the transmission probability of each node; $\lambda_1 = \lambda_2 = \dots = \lambda_n = \lambda$ for the arrival rate at each queue; and $\gamma_1 = \gamma_2 = \dots = \gamma_n = \gamma$ and $\beta_1 = \beta_2 = \dots = \beta_n = \beta$ for the channel parameters of each link. We set the values of τ, λ, γ , and β as follows.

We set $\tau = 1/n = 0.05$ for both transmission attempts with and without CSI (see Equations (1) and (2)). For the arrival process, we assume a Bernoulli process with rate $\lambda \in \{0.0045, 0.0090, 0.0135\}$ [packets/slot], resulting in aggregate traffic rate of $n\lambda \in \{0.09, 0.18, 0.27\}$ [packets/slot]. Considering that the maximum network throughput of S-ALOHA with the collision model is $e^{-1} \approx 0.3679$ [packets/slot], we can classify the arrival rates as follows: $\lambda = 0.0045$ represents a light traffic scenario, $\lambda = 0.0090$ represents a moderate traffic scenario, and $\lambda = 0.0135$ represents a heavy traffic scenario.

For the channel process, we set $\pi_1^B = \pi_2^B = \dots = \pi_i^B = \pi^B = 0.2$, indicating that 20% of slots are in the bad state in the long-term. With a fixed $\pi^B = 0.2$, we vary the transition probability γ in the range from 0.001 to 0.1, and then compute β using the relation $\pi^B = \beta/(\gamma + \beta)$. It is important to note that once link i transitions to a bad state, it will remain in that state for an average duration of γ^{-1} slots. Hence, γ governs the short-term dynamics of the channel state: a smaller γ induces a higher degree of temporal channel correlation, while a larger γ induces a lower degree of temporal channel correlation. We utilize the simulation parameters as summarized in Table 1, unless otherwise mentioned.

Table 1. Simulation parameters

Parameter	Notation	Value
Number of nodes	n	20
Transmission probability	τ	0.05
Arrival rate	λ	{0.0045, 0.0090, 0.0135} [packets/slot]
Long-term channel statistics in the bad state	π^B	20%
Short-term channel statistics in the bad state	γ^{-1}	{10, 100, 1000} [slots]

3.2 Scenario I: Impact of Temporal Channel Correlation on the Queue Length Process

In Scenario I, we investigate how temporal channel correlation affects the evolution of the queue length over time. To this end, we vary γ to have $\gamma^{-1} \in \{10, 100, 1000\}$ [slots], while keeping the arrival rate fixed at $\lambda = 0.0135$ [packets/slot]. The other parameters are set according to the values presented in Table 1.

Figure 2(a), Figure 2(b), and Figure 2(c) show the sample paths of the queue length process $\{Q_i(t)\}_{t=1}^{\infty}$ for $\gamma^{-1} \in \{10, 100, 1000\}$ [slots], respectively, in the cases of transmission attempts with and without CSI. From the figures, we observe that as the value of γ^{-1} increases, the queue length $Q_i(t)$ exhibits a wider range of fluctuations over time. This result indicates that higher temporal channel correlation leads to increased variability in the queue length process. The variability is more pronounced in the case of transmission attempt without CSI compared to the one with CSI.

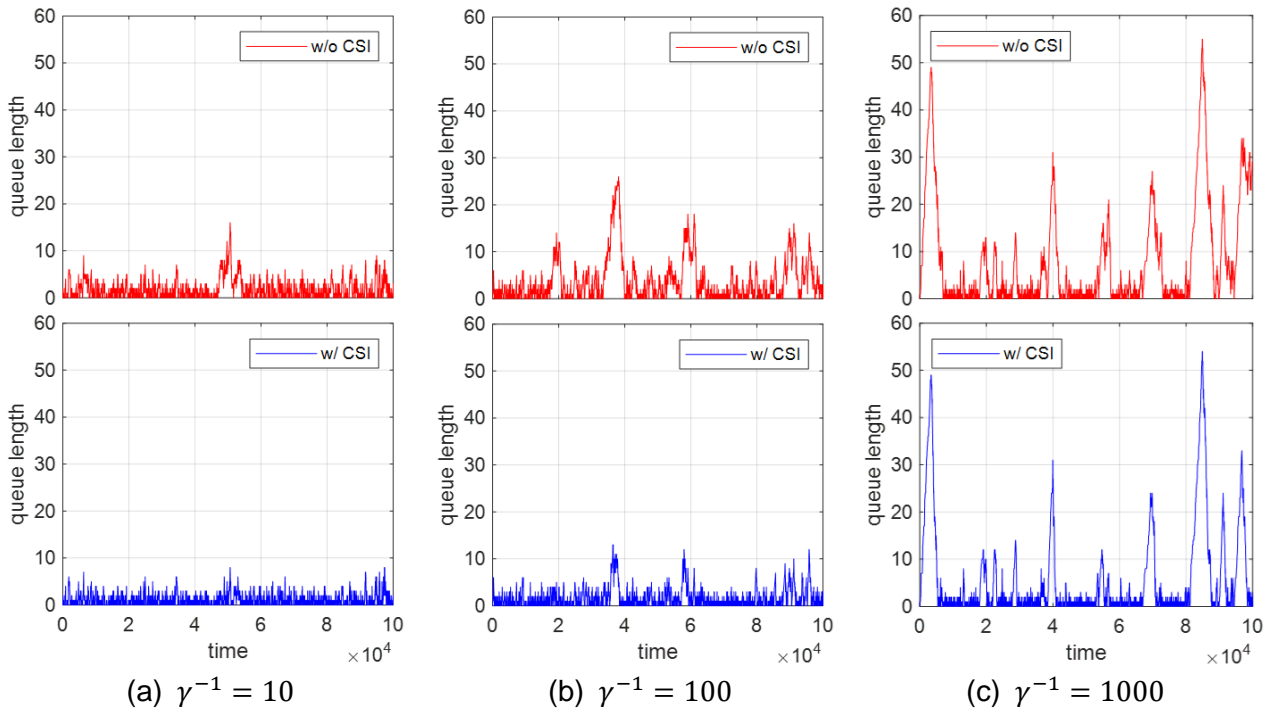
**Figure 2. Sample path of queue length**

Figure 3(a), Figure 3(b), and Figure 3(c) compare the probability mass functions (PMFs) of the queue length $Q_i(t)$ for $\gamma^{-1} \in \{10, 100, 1000\}$ [slots], respectively, in the cases of transmission attempts with and without CSI. Notably, the PMFs in both cases show an exponentially decaying behavior. Furthermore, as the value of γ^{-1} increases, the decay rates in both cases decrease. This result indicates that higher temporal channel correlation can lead to larger queue lengths.

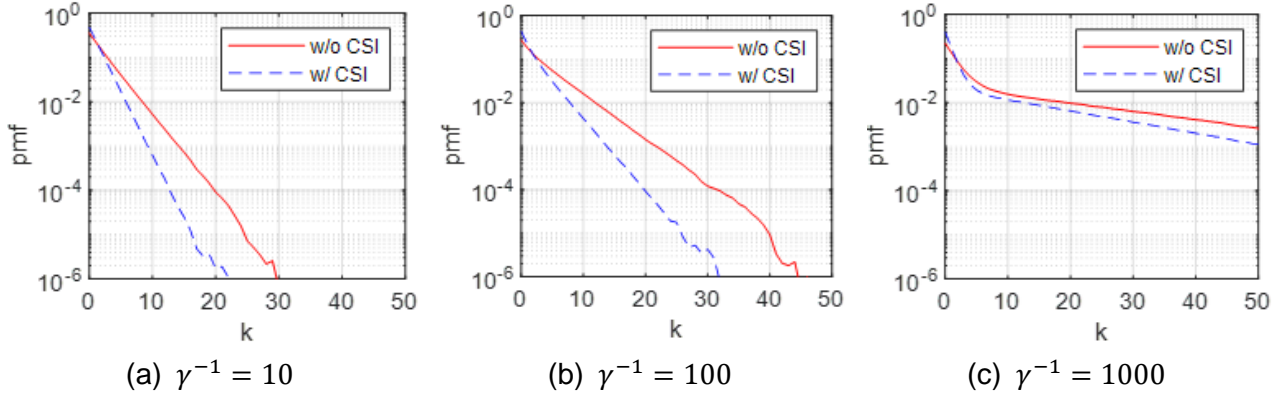


Figure 3. PMF of queue length

In summary, our simulation study in Scenario I illustrates the impact of temporal channel correlation on the queue length process: for both transmission attempts with and without CSI, the presence of temporal correlation in the wireless medium has a negative effect on the queueing performance. It results in the queue length experiencing higher variability, increasing the probability of having a large queue.

3.3 Scenario II: Impact of Temporal Channel Correlation on the Average Queueing Delay

In Scenario II, we investigate the impact of temporal channel correlation on the average queueing delay. To this end, we vary the channel parameter γ to have γ^{-1} in the range from 10 to 1000 [slots], for each of the arrival rates $\lambda \in \{0.0045, 0.0090, 0.0135\}$ [packets/slot]. The other parameters are set according to the values presented in Table 1.

The average queueing delay for $\lambda \in \{0.0045, 0.0090, 0.0135\}$ [packets/slot] is shown in Figure 4(a), Figure 4(b), and Figure 4(c), respectively, in the cases of transmission attempts with and without CSI. From the figures, we observe that as the value of γ^{-1} or λ increases, the average queueing delay also increases. This observation aligns with the findings in Scenario I that shows that higher temporal channel correlation negatively impacts the queueing performance. Furthermore, each of Figure 4(a), Figure 4(b), and Figure 4(c) also illustrates that the transmission attempt with CSI can reduce the average queueing delay compared to the case without CSI. This reduction becomes more significant as the arrival rate λ increases.

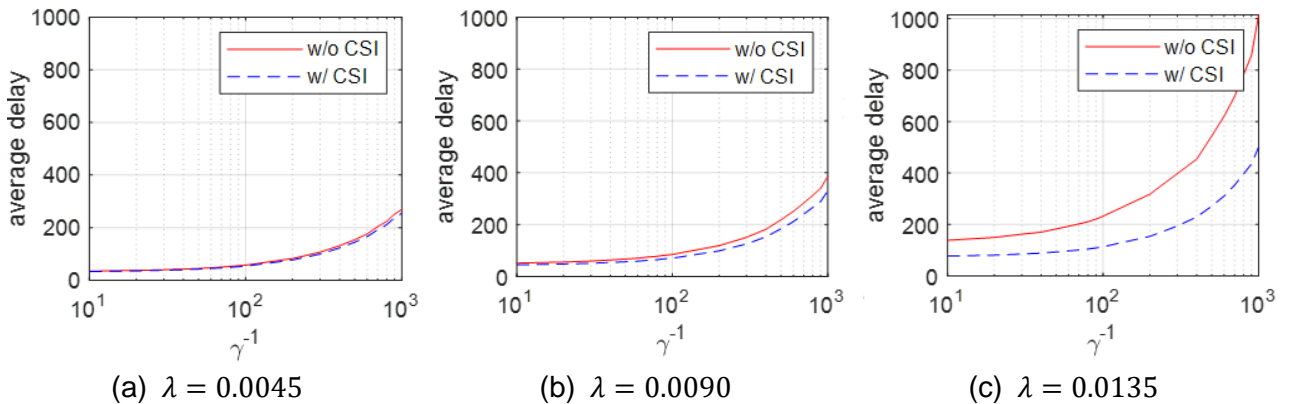


Figure 4. Average queueing delay

In the S-ALOHA network, each node operates in an unsaturated condition, i.e., a node's queue can be empty. A node initiates a transmission attempt only when its queue is not empty. As a result, the number of nodes with non-empty queue varies with time, directly affecting the queueing delay performance. To gain further insights into the impact of temporal channel correlation on the queueing delay performance, we investigate the congestion level by measuring the average number of nodes with non-empty queues. The result is shown in Figure 5(a), Figure 5(b), and Figure 5(c) for $\lambda \in \{0.0045, 0.0090, 0.0135\}$ [packets/slot], respectively, in the cases of transmission attempts with and without CSI. It is evident that as the value of γ^{-1} or λ increases, the network becomes more congested, which in turn increases the average queueing delay.

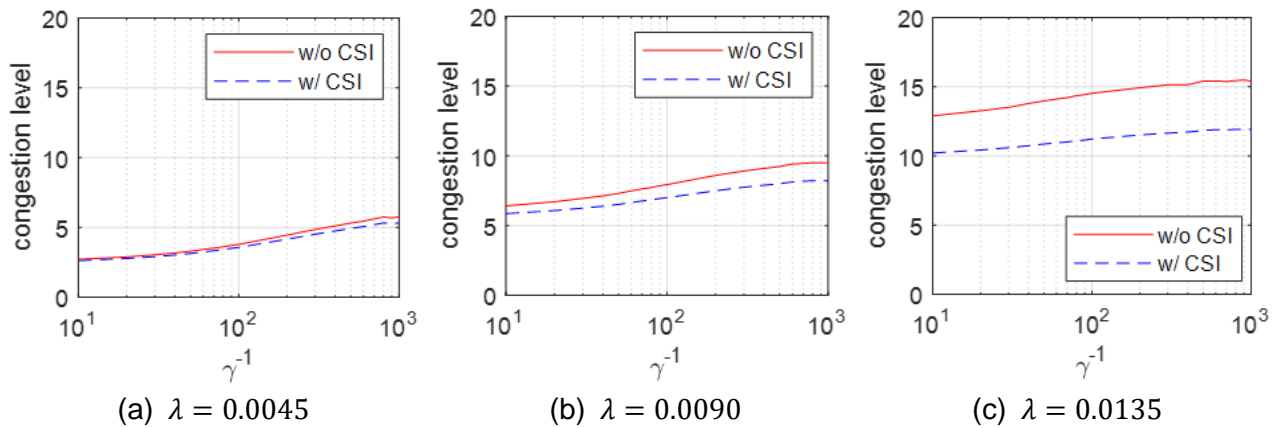


Figure 5. Average congestion level

In summary, our simulation study in Scenario II illustrates the impact of temporal channel correlation on the average queueing delay: for both transmission attempts with and without CSI, the presence of temporal correlation in the wireless medium increases the congestion level of the S-ALOHA network, resulting in an increase in the average queueing delay, even under light traffic load.

4. Conclusion

In this paper, we analyze the queueing performance of a node in the S-ALOHA network under time-correlated channels. Specifically, we investigate through simulation studies how temporal channel correlation affects the queueing delay performance in the cases of transmission attempts with and without CSI. We find that the presence of temporal channel correlation results in an increase in the average queueing delay for both cases, even under light traffic conditions.

Our future work aims to provide a theoretical analysis to complement our findings in this paper. Additionally, we can consider a more generalized version of the Gilbert-Elliott channel, where the successful decoding probability takes a value between 0 and 1 based on its state. Another interesting problem is to consider a more sophisticated receiver model at the AP, such as the capture model, which enables the successful decoding of multiple packets in the case of concurrent transmissions.

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