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ERRATUM TO "PSEUDO-RIEMANNIAN SASAKI SOLVMANIFOLDS" [J. KOREAN MATH. SOC. 60 (2023), NO. 1, PP. 115–141]

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Abstract. In this erratum, we offer a correction to [J. Korean Math. Soc. 60 (2023), No. 1, pp. 115–141]. We rectify Theorem 5.7 and Table 1 of the original paper.

We discovered after publication that the Lie algebras appearing in our classification of $\mathfrak{z}$-standard Sasaki structures in dimension 5 and 7 contain some errors. The errors occur in the computation of

$$db^\flat = \check{D}\omega;$$

this means that for some Lie algebras in the classification, exactly one entry in their representation in the form $(de^1, \dots, de^n)$ must be corrected.

The statement of Theorem 5.7 should be amended as follows:

Theorem 5.7. *Let $\tilde{\mathfrak{g}}$ be a Lie algebra of dimension 5 with a $\mathfrak{z}$-standard Sasaki structure. Then, up to isometry and $\mathcal{D}$-homothety, $\tilde{\mathfrak{g}}$ is one of*

$$(0,0,0,-2e^{12}-2\tau e^{35},0),$$
$$(0,0,2e^{35},-2e^{12}-2\tau e^{35},0),$$
$$(e^{15},e^{25},2\tau e^{12}+2e^{35},-2e^{12}-2\tau e^{35},0),$$

and the Sasaki structure is given by

$$\tilde{g}=\pm(e^1\otimes e^1+e^2\otimes e^2)+\tau e^3\otimes e^3+e^4\otimes e^4+\tau e^5\otimes e^5,\quad \xi=e_4,\quad \Phi=-e^{12}-\tau e^{35}.$$

Similarly, Table 1 in [1] should be amended as below.

We also note that each Lie algebra in rows 1–4 of Table 1 is isomorphic to a Lie algebra in rows 5–8 under the transformation $e_3\mapsto -e_3$. This isomorphism is not an isometry; indeed, there is no isometry preserving the standard decomposition $\operatorname{Span}\{e_1,\dots,e_6\}\rtimes\operatorname{Span}\{e_7\}$, because the nilpotent factors $\operatorname{Span}\{e_1,\dots,e_6\}$ have different signatures.

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TABLE 1. 7-dimensional Lie algebras with a $\mathfrak{z}$-standard Sasaki structure that reduces to an abelian pseudo-Kähler Lie algebra $\check{\mathfrak{g}}$ up to isometry and $\mathcal{D}$-homothety

n.	$\tilde{\mathfrak{g}}$	Metric $\tilde{g}$
1.	$0,0,0,0,0,-2e^{12}-2e^{34}-2\tau e^{57},0$	$(\pm[1]_4,\tau,+1,\tau)$
2.	$0,0,0,0,2e^{57},-2e^{12}-2e^{34}-2\tau e^{57},0$	$(\pm[1]_4,\tau,+1,\tau)$
3.	$0,0,e^{37},e^{47},2\tau e^{34}+2e^{57},-2e^{12}-2e^{34}-2\tau e^{57},0$	$(\pm[1]_4,\tau,+1,\tau)$
4.	$e^{17},e^{27},e^{37},e^{47},2\tau e^{12}+2\tau e^{34}+2e^{57},-2e^{12}-2e^{34}-2\tau e^{57},0$	$(\pm[1]_4,\tau,+1,\tau)$
5.	$0,0,0,0,0,-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
6.	$0,0,0,0,2e^{57},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
7.	$0,0,e^{37},e^{47},-2\tau e^{34}+2e^{57},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
8.	$e^{17},e^{27},e^{37},e^{47},2\tau e^{12}-2\tau e^{34}+2e^{57},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
9.	$\frac{1}{2}e^{17}+2\lambda e^{27}-\frac{1}{2}e^{37}-\lambda e^{47},-2\lambda e^{17}+\frac{1}{2}e^{27}+\lambda e^{37}-\frac{1}{2}e^{47},$ $\frac{1}{2}e^{17}+\lambda e^{27}-\frac{1}{2}e^{37},-\lambda e^{17}+\frac{1}{2}e^{27}-\frac{1}{2}e^{47},$ $\tau e^{12}-\tau e^{14}+\tau e^{23}+\tau e^{34},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
10.	$\frac{1}{2}e^{17}+2\lambda e^{27}-\frac{3}{2}e^{37}-\lambda e^{47},-2\lambda e^{17}+\frac{1}{2}e^{27}+\lambda e^{37}-\frac{3}{2}e^{47},$ $-\frac{1}{2}e^{17}+\lambda e^{27}-\frac{1}{2}e^{37},-\lambda e^{17}-\frac{1}{2}e^{27}-\frac{1}{2}e^{47},$ $\tau e^{12}-\tau e^{14}+\tau e^{23}+\tau e^{34}+2e^{57},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$
11.	$\frac{3}{2}e^{17}+2\lambda e^{27}+\frac{1}{2}e^{37}-\lambda e^{47},-2\lambda e^{17}+\frac{3}{2}e^{27}+\lambda e^{37}+\frac{1}{2}e^{47},$ $\frac{3}{2}e^{17}+\lambda e^{27}+\frac{1}{2}e^{37},-\lambda e^{17}+\frac{3}{2}e^{27}+\frac{1}{2}e^{47},$ $3\tau e^{12}-\tau e^{14}+\tau e^{23}-\tau e^{34}+2e^{57},-2e^{12}+2e^{34}-2\tau e^{57},0$	$(\pm[1]_2,\mp[1]_2,\tau,+1,\tau)$

References

[1] D. Conti, F. A. Rossi, and R. Segnan Dalmasso, *Pseudo-Riemannian Sasaki solvmanifolds*, J. Korean Math. Soc. **60** (2023), no. 1, 115–141. `https://doi.org/10.4134/JKMS.j220232`

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