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IDEAL THEORY OF SUBTRACTION SEMIGROUPS BASED ON $(3, 2)$-FUZZY SETS

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Abstract. In this paper, we define the notions of $(3, 2)$-fuzzy ideal of subtraction semigroup and near subtraction semigroup. Also, we discuss some of its properties with examples.

1. Introduction

The concept of fuzzy sets was proposed by Zadeh [21]. The theory of fuzzy sets has several applications in real-life situations, and many scholars have researched fuzzy set theory. After the introduction of the concept of fuzzy sets, several research studies were conducted on the generalizations of fuzzy sets. The integration between fuzzy sets and some uncertainty approaches such as soft sets and rough sets has been discussed in [2, 5, 6]. The idea of intuitionistic fuzzy sets suggested by Atanassov [3] is one of the extensions of fuzzy sets with better applicability. Applications of intuitionistic fuzzy sets appear in various fields, including medical diagnosis, optimization problems, and multicriteria decision making [7, 8, 9]. Yager [20] offered a new fuzzy set called a Pythagorean fuzzy set, which is the generalization of intuitionistic fuzzy sets. Fermatean fuzzy sets were introduced by Senapati and Yager [18], and they also defined basic operations over the Fermatean fuzzy sets. The concept of $(3, 2)$-fuzzy sets are introduced and studied in [11]. There are several generalizations of fuzzy sets such as Intuitionistic fuzzy sets, Pythagorean fuzzy sets and Fermatean fuzzy sets. The concept of (3,2)-fuzzy sets is a generalization of intuitionistic fuzzy sets. This type produces membership grades larger than

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Intutionistic and Pythagorean fuzzy sets. In this paper, we introduce the concept of $(3,2)$-fuzzy ideals of subtraction algebras and obtain several characterizations of $(3,2)$-fuzzy ideals in subtraction algebras.

2. PRELIMINARIES

In this section, we shall recall some basic definitions and results that are required in the sequel.

A subtraction algebra is defined as an algebra $(X, -)$ with a single binary operation that satisfies the following identities:

$$(\forall x, y, z \in X)(x - (y - x) = x), \tag{2.1}$$

$$(\forall x, y, z \in X)(x - (x - y) = y - (y - x)), \tag{2.2}$$

$$(\forall x, y, z \in X)((x - y) - z = (x - z) - y). \tag{2.3}$$

The last identity permits us to omit parentheses in expressions of the form $(x-y)-z$. The subtraction determines an order relation on X $a \leq b \Leftrightarrow a-b = 0$, where $0 = a-a$ is an element that does not depend on the choice of $a \in X$. The ordered set $(X, \leq)$ is a semi-Boolean algebra in the sense of [1], that is, it is a meet semilattice with zero 0 in which every interval $[0, a]$ is a Boolean algebra with respect to the induced order. Here $a \wedge b = a - (a-b)$, the complement of an element $b \in [0, a]$ is $a-b$; and if $b, c \in [0, b]$, then $b \vee c = (b' \wedge c')' = a-((a-b)\wedge(a-c)) = a-((a-b)-((a-b)-(a-c)))$. In a subtraction algebra X, the following are true: [13]

$$(\forall x, y \in X)((x - y) - y = x - y), \tag{2.4}$$

$$(\forall x \in X)(x - 0 = x, 0 - x = 0), \tag{2.5}$$

$$(\forall x, y \in X)((x - y) - x = 0), \tag{2.6}$$

$$(\forall x, y \in X)(x - (x - y) \leq y), \tag{2.7}$$

$$(\forall x, y, z \in X)((x - y) - (y - x) = x - y), \tag{2.8}$$

$$(\forall x, y \in X)(x - (x - (x - y)) = x - y), \tag{2.9}$$

$$(\forall x, y, z \in X)((x - y) - (z - y) \leq x - z), \tag{2.10}$$

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x = y - w \ for\ some\ w \in X), \tag{2.11}$$

$$(\forall x, y, z \in X)(x \leq y \Rightarrow x - z \leq y - z, z - y \leq z - x), \tag{2.12}$$

$$(\forall x, y, z \in X)(x, y \leq z \Rightarrow x - y = x \wedge (z - y)). \tag{2.13}$$

Definition 2.1 ([13])**.** A non-empty subset A of a subtraction algebra X is called an *ideal* of X if it satisfies

(1) $a - x \in A$ for all $a \in A$ and $x \in X$,
(2) for all $a, b \in A$, whenever $a \vee b$ exists in X, then $a \vee b \in A$.

Proposition 2.2 ([13])**.** *Let X be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y, then the element $x \vee y = w - ((w - y) - x)$ is a least upper bound for x and y.*

Definition 2.3. For any $t \in [0,1]$, and a fuzzy set f in a non-empty set S, the set $U(f,t) = \{x \in S : f^3(x) \geq t\}$ is called an upper t-level cut of f and the set $L(f,t) = \{x \in S : f^3(x) \leq t\}$ is called a *lower t-level cut* of f.

Definition 2.4 ([15])**.** A fuzzy set f in a subtraction algebra X is called a *fuzzy ideal* of X if it satisfies:

$$(\forall x, y \in X)(f(x - y) \geq f(x)), \tag{2.14}$$

$$(\forall\, x, y \in X)\left(\ x \vee y\ \Rightarrow f(x \vee y) \geq \min\{f(x), f(y)\}\ \right) \tag{2.15}$$

Definition 2.5 ([11])**.** Let X be a nonempty set. The $(3,2)$-fuzzy set on X is defined to be a structure

$$\mathcal{C}_X := \{\langle x, f(x), g(x)\rangle \mid x \in X\} \tag{2.16}$$

where $f : X \to [0,1]$ is the degree of membership of x to $\mathcal{C}$ and $g : X \to [0,1]$ is the degree of non-membership of x to $\mathcal{C}$ such that $0 \leq (f(x))^3 + (g(x))^2 \leq 1$.

In what follows, we use the notations $f^3(x)$ and $g^2(x)$ instead of $(f(x))^3$ and $(g(x))^2$, respectively, and the $(3,2)$-fuzzy set in (2.16) is simply denoted by $\mathcal{C}_X := (X, f, g)$.

3. Ideals in Subtraction Semigroups

In this section X denotes subtraction semigroup.

Definition 3.1. A $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ of X is called $(3,2)$-*fuzzy subtraction sub-semigroup* of X if

$$(\forall\, x, y \in X)\begin{pmatrix} f^3(x - y) \geq \min\{f^3(x), f^3(y)\} \\ g^2(x - y) \leq \max\{g^2(x), g^2(y)\} \end{pmatrix} \tag{3.1}$$

$-$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

Example 1. Let $X = \{0, a, b, c\}$ be a subtraction sub-semigroup with two binary operations $-$ and $\cdot$ is defined as follows.
We define a $(3, 2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ as follows:

X	0	1	2	3
f	0.8	0.1	0.3	0.2
g	0.4	0.9	0.6	0.8

Then $\mathcal{C}$ is a $(3, 2)$-fuzzy subtraction semi-group of X. Hence $\mathcal{C}_X := (X, f, g)$ is a subtraction sub-semigroup of X.

Definition 3.2. A $(3, 2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ of X is called $(3, 2)$-*fuzzy left ideal* (resp. $(3, 2)$-*fuzzy right ideal*) of X, if $\forall\ x, y \in X$

(1) $f^3(x) \geq \min\{f^3(x - y), f^3(y)\}$
(2) $g^2(x) \leq \max\{g^2(x - y), g^2(y)\}$
(3) $f^3(xy) \geq \min\{f^3(x), f^3(y)\}$
(4) $g^2(xy) \leq \max\{g^2(x), g^2(y)\}$
(5) $f^3(xy) \geq f^3(y)$ (resp. $f^3(xy) \geq f^3(x)$)
(6) $g^2(xy) \leq g^2(y)$ (resp. $g^2(xy) \leq g^2(x)$).

If f and g are both $(3, 2)$-fuzzy left ideal and $(3, 2)$-fuzzy right ideal of X, then f and g are both $(3, 2)$-fuzzy ideal of X.

Example 2. Let $X = \{0, a, b, c\}$ be a subtraction sub-semigroup with two binary operations $-$ and $\cdot$ is defined as follows.

$-$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	a	b	0

We define a $(3, 2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ as follows:

X	0	a	b	c
$f(x)$	0.71	0.32	0.53	0.14
$g(x)$	0.31	0.72	0.53	0.84

Then $\mathcal{C}$ is a $(3,2)$-fuzzy left (resp.$(3,2)$-fuzzy right) ideal of X. Hence $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal ($(3,2)$-fuzzy right ideal) of X.

Definition 3.3. Let $\mathcal{C}_1 = (f_1, g_1)$ and $\mathcal{C}_2 = (f_2, g_2)$ be any two $(3,2)$-fuzzy sets of X. Then the following $(3,2)$-fuzzy sets of X are defined as follows.

$$(\mathcal{C}_1 \star \mathcal{C}_2)(z) = \begin{cases} (f_1^3 \star f_2^3)(z) = \begin{cases} \bigvee\limits_{z \leq xy} \min\{f_1^3(x), f_1^3(y)\} & \text{if } z \leq xy \\ [0,0] \ \ otherwise \end{cases} \\ (g_1^2 \star g_2^2)(z) = \begin{cases} \bigwedge\limits_{z \leq xy} \min\{g_2^2(x), g_2^2(y)\} & \text{if } z \leq xy \\ [1,1] \ \ otherwise \end{cases} \end{cases}$$

$$(\mathcal{C}_1 \cap \mathcal{C}_2)(x) = \begin{cases} (f_1^3 \cap f_2^3)(x) \\ .(g_1^2 \cup g_2^2)(x) \ \forall x \in X \end{cases}$$

$$(\mathcal{C}_1 - \mathcal{C}_2)(z) = \begin{cases} (f_1^3 - f_2^3)(z) = \begin{cases} \bigvee\limits_{z = x-y} \min\{f^3(x), f^3(y)\} & \text{if } z = x - y \ \forall \ x, y, z \in X \\ [0,0] \ otherwise \end{cases} \\ (g_1^2 - g_2^2)(z) = \begin{cases} \bigwedge\limits_{z = x-y} \min\{g^2(x), g^2(y)\} & \text{if } z = x - y \ \forall \ x, y, z \in X \\ [1,1] \ otherwise \end{cases} \end{cases}$$

Theorem 3.4. *Every $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X is a $(3,2)$-fuzzy subtraction semi-group of X.*

Proof. Let f and g be an $(3,2)$-fuzzy ideal of X. Then $\forall x, y, z \in X$, we have

$$\begin{aligned} f^3(x-y) &\geq \min\{f^3((x-y)-z), f^3(z)\} \ \forall z \in X \\ &\geq \min\{f^3((x-y)-z), f^3(x)\} \ for \ z = x \\ &= \min\{f^3(0), f^3(x)\} \\ &= f^3(x), \end{aligned}$$

$$\begin{aligned} g^2(x-y) &\leq \max\{g^2((x-y)-z), g^2(r)\} \ \forall z \in X \\ &\leq \max\{g^2((x-y)-z), g^2(x)\} \ for \ z = x \\ &= \max\{g^2(0), g^2(x)\} \\ &= g^2(x). \end{aligned}$$

Again consider

$$\begin{aligned} f^3(x-y) &\geq \min\{f^3((x-y)-z), f^3(l)\} \ \ \forall z \in X \\ &\geq \min\{f^3((x-y)-y), f^3(y)\} \ \ for \ z = y \\ &= \min\{f^3(x-y), f^3(y)\} \ \ since \ (x-y)-y = x-y \\ &= \min\{f^3(x), f^3(y)\}, \end{aligned}$$

$$\begin{array}{rcl} g^2(x-y) & \leq & \max\{g^2((x-y)-z), g^2(l)\} \ \forall z \in X \\ & \leq & \max\{g^2((x-y)-y), g^2(y)\} \ \ for\ z = y \\ & = & \max\{g^2(x-y), g^2(y)\} \ \ since\ (x-y)-y = x-y \\ & = & \max\{g^2(x), g^2(y)\}. \end{array}$$

Then f and g are $(3,2)$-fuzzy subtraction semi-group of X. The converse is not true. $\square$

Theorem 3.5. *If $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy set of a sub-semigroup X, then the following conditions are equivalent:*

$$(\forall x, y \in X)\left(\ f^3 * f^3 \leq f^3, g^2 * g^2 \geq g^2\ \right) \tag{3.2}$$

$$(\forall x, y \in X)\left(\begin{array}{c} f^3(xy) \geq \min\{f^3(x), f^3(y)\}, \\ g^2(xy) \leq \max\{g^2(x), g^2(y)\}\} \end{array}\right). \tag{3.3}$$

Proof. (3.2) $\Rightarrow$ (3.3): Let $x, y \in X$. Then $(f^3 * f^3)(jk) = \bigvee_{xy \leq jk} \{\min\{f^3(j), f^3(k)\}\} \geq \min\{f^3(x), f^3(y)\}$. By (3.2), $f^3 * f^3 \leq f^3$. Then we have $f^3(xy) \geq (f^3 * f^3)(xy) \geq \min\{f^3(x), f^3(y)\}$. Hence $f^3(xy) \geq \min\{f^3(x), f^3(y)\}$. It is clear that $(g^2 * g^2)(jk) = \bigwedge_{xy \leq jk} \{\max\{g^2(j), g^2(k)\}\} \leq \max\{g^2(x), g^2(y)\}$. By (3.2), $g^2 * g^2 \geq g^2$. $g^2(xy) \leq (g^2 * g^2)(xy) \leq \max\{g^2(x), g^2(y)\}$. Hence $g^2(xy) \leq \max\{g^2(x), g^2(y)\}$.
(3.3) $\Rightarrow$ (3.2): Let $j \in X$. Consider $(f^3 * f^3)(j) = \bigvee_{j \leq xy} \min\{f^3(x), f^3(y)\} \leq \bigvee_{j \leq xy} \{f^3(xy)\} \leq \bigvee_{j \leq xy} \{f^3(j)\} = f^3(j)$. Thus $f^3 * f^3 \leq f^3$. If j cannot be expressed as $j \leq xy$, then $(f^3 * f^3)(x) = 0 \leq f^3(j)$. Thus $(f^3 * f^3)(j) \leq f^3(j)\ \forall j \in X$. Let $j \in X$. Consider $(g^2 * g^2)(j) = \bigvee_{j \leq xy} \max\{g^2(x), g^2(y)\} \geq \bigvee_{j \leq xy} \{g^2(xy)\} \geq \bigvee_{j \leq xy} \{g^2(j)\} = g^2(j)$. Thus $g^2 * g^2 \geq g^2$. If j cannot be expressed as $j \leq ab$, then $(g^2 * g^2)(x) = 0 \geq g^2(j)$. Thus $(g^2 * g^2)(j) \geq g^2(j)\ \forall j \in X$. This implies that $f^3 * f^3 \leq f$ and $g^2 * g^2 \geq g^2$. $\square$

Theorem 3.6. *Let $\mathcal{C}_X := (X, f, g)$ be a $(3,2)$-fuzzy set of X. If $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy sub-semigroup ($(3,2)$-fuzzy left ideal, $(3,2)$-fuzzy right ideal) of X. Then $f - f = f$ and $g - g = g$.*

Proof. Let $\mathcal{C}_X := (X, f, g)$ be a $(3,2)$-fuzzy sub-semigroup of X. Let $x, y, x \in X$. Then

$$\begin{array}{rcl} (f-f)^3(z) & = & \bigvee_{z = x-y} \{\min\{f^3(x), f^3(y)\}\} \\ & \geq & \min\{f^3(z), f^3(0)\} \ since\ z = z - 0 \\ & = & f^3(z), \end{array}$$

$$\begin{aligned}(g-g)^2(z) &= \bigvee_{z=x-y}\{\max\{g^2(x), g^2(y)\}\} \\ &\leq \max\{g^2(z), g^2(0)\} \text{ since } z = z - 0 \\ &= g^2(z).\end{aligned}$$

On the other hand if $z = x - y, x, y \in X$, then

$$\begin{aligned}f^3(z) &= f^3(x-y) \\ &\geq \min\{f^3(x), f^3(y)\} \\ &\geq \bigvee_{z=x-y}\{\min\{f^3(x), f^3(y)\}\} \\ &= (f-f)^3(z),\end{aligned}$$

$$\begin{aligned}g^2(z) &= g^2(x-y) \\ &\leq \max\{g^2(x), g^2(y)\} \\ &\leq \bigvee_{j=x-y}\{\max\{g^2(x), g^2(y)\}\} \\ &= (g-g)^2(z).\end{aligned}$$

Hence $f^3(z) = (f-f)^3(z)$ and $g^2(z) = (g-g)^2(z)$ $\forall z \in X$. Thus $f^3 = f^3 - f^3$ and $g^2 = g^2 - g^2$. □

Theorem 3.7. *Let $\mathcal{C}_1 = (f_1, g_1)$ and $\mathcal{C}_2 = (f_2, g_2)$ be any two $(3,2)$-fuzzy sets of X. If $\mathcal{C}_1$ and $\mathcal{C}_2$ are $(3,2)$-fuzzy left ideals (resp. $(3,2)$-fuzzy right ideals) of X. Then $\mathcal{C}_1$ and $\mathcal{C}_2$ is also $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X.*

Proof. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be any two (3,2)-fuzzy left ideals of X. Let $x, y \in X$. Consider

$$\begin{aligned}(f_1^3 \cap f_2^3)(xy) &= \min\{f_1^3(xy), f_2^3(xy)\} \\ &\geq \min\{\min\{f_1^3(x), f_1^3(y)\}, \min\{f_2^3(x), f_2^3(y)\}\} \\ &\geq \min\{\min\{f_1^3(x), f_2^3(x)\}, \min\{f_1^3(y), f_2^3(y)\}\} \\ &= \min\{(f_1^3 \cap f_2^3)(x), (f_1^3 \cap f_2^3)(y)\},\end{aligned}$$

$$\begin{aligned}(f_1^3 \cap f_2^3)(xy) &= \min\{f_1^3(xy), f_2^3(xy)\} \\ &\geq \min\{f_1^3(y), f_2^3(y)\} \\ &= (f_1^3 \cap f_2^3)(y).\end{aligned}$$

$$\begin{aligned}(g_1^2 \cup g_2^2)(xy) &= \max\{g_1^2(xy), g_2^2(xy)\} \\ &\leq \max\{\max\{g_1^2(x), g_1^2(y)\}, \max\{g_2^2(x), g_2^2(y)\}\} \\ &\leq \max\{\max\{g_1^2(x), g_2^2(x)\}, \max\{g_1^2(y), g_2^2(y)\} \\ &= \max f^3(g_1^2 \cup g_2^2)(x), (g_1^2 \cup g_2^2)(y)\},\end{aligned}$$

$$\begin{aligned}(g_1^2 \cup g_2^2)(xy) &= \max\{g_1^2(xy), g_2^2(xy)\} \\ &\leq \max\{g_1^2(y), g_2^2(y)\} \\ &= (g_1^2 \cup g_2^2)(y).\end{aligned}$$

And

$$\begin{aligned}(f_1^3 \cap f_2^3)(x) &= \min\{f_1^3(x), f_2^3(x)\} \\ &\geq \min\{\min\{f_1^3(x-y), f_1^3(y)\}, \min\{f_2^3(x-y), f_2^3(y)\}\} \\ &\geq \min\{\min\{f_1^3(x-y), f_2^3(x-y)\}, \min\{f_1^3(y), f_2^3(y)\}\} \\ &= \min\{(f_1^3 \cap f_2^3)(x-y), (f_1^3 \cap f_2^3)(y)\}, \\ (g_1^2 \cup g_2^2)(x) &= \max\{g_1^2(x), g_2^2(x)\} \\ &\leq \max\{\max\{g_1^2(x-y), g_1^2(y)\}, \max\{g_2^2(x-y), g_2^2(y)\}\} \\ &\leq \max\{\max\{g_1^2(x-y), g_2^2(x-y)\}, \max\{g_1^2(y), g_2^2(y)\}\} \\ &= \max\{(g_1^2 \cup g_2^2)(x-y), (g_1^2 \cup g_2^2)(y)\}.\end{aligned}$$

Hence the intersection of two $(3,2)$-fuzzy left ideals of X is also $(3,2)$-fuzzy left ideal of X. □

Theorem 3.8. *If $\mathcal{C}_i = (f_i, g_i)_{i\in\Delta}$ is a family of $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of a sub-semigroup X. Then $\bigcap\limits_{i\in\Delta} \mathcal{C}_i = (\bigcap\limits_{i\in\Delta} f_i^3, \bigcup\limits_{i\in\Delta} g_i^2)$ is also a $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X, where Δ is any index set.*

Proof. Let $\mathcal{C}_i = (f_i, g_i)_{i\in\Delta}$ be a family of $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X. Let $x, y \in X$ and $f^3(x) = \bigcap\limits_{i\in\Delta} f_i^3(x) = \bigwedge f_i^3(x)$, $g^2(x) = \bigcup\limits_{i\in\Delta} g_i^2(x) = \bigvee g_i^2(x)$.

$$\begin{aligned}f^3(x) &= \bigwedge f_i^3(x) \\ &\geq \bigwedge \min\{f_i^3(x-y), f_i^3(y)\} \\ &= \min\{\bigwedge f_i^3(x-y), \bigwedge f_i^3(y)\} \\ &= \min\{f_i^3(x-y), f_i^3(y)\} \\ &= \min\{f^3(x-y), f^3(y)\}, \\ g^2(x) &= \bigvee g_i^2(x) \\ &\leq \bigvee \max\{g_i^2(x-y), g_i^2(y)\} \\ &= \max \bigvee\{g_i^2(x-y), \bigvee g_i^2(y)\} \\ &= \max\{\cup g_i^2(x-y), \cup g_i^2(y)\} \\ &= \max\{g^2(x-y), g^2(y)\}, \\ f^3(xy) &= \bigwedge f_i^3(xy) \\ &\geq \bigwedge \min\{f_i^3(x), f_i^3(y)\} \\ &= \min\{\bigwedge f_i^3(x), \bigwedge f_i^3(y)\} \\ &= \min\{f_i^3(x), f_i^3(y)\} \\ &= \min\{f^3(x), f^3(y)\}, \\ g^2(xy) &= \bigvee g_i^2(xy) \\ &\leq \bigvee \max\{g_i^2(x), g_i^2(y)\} \\ &= \max\{\bigvee g_i^2(x), \bigvee g_i^2(y)\} \\ &= \max\{\cup g_i^2(x), \cup g_i^2(y)\} \\ &= \max\{g^2(x), g^2(y)\},\end{aligned}$$

$f^3(xy) = \bigwedge f_i^3(xy) \geq \bigwedge f_i^3(y) \geq f^3(y)$ and $g^2(xy) = \bigvee g_i^2(xy) \leq \bigvee g_i^2(y) \leq g^2(y)$. Hence, $\bigcap_{i\in\Delta} \mathcal{C}_i = (\bigcap_{i\in\Delta} f_i, \bigcup_{i\in\Delta} g_i)$ is also a $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X. $\square$

Theorem 3.9. *If $\mathcal{C}_X := (X, f, g)$ is any $(3,2)$-fuzzy set of a sub-semigroup X, then $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X if and only if every $(3,2)$-fuzzy level set $\bigcup(\mathcal{C}_X, t, n)$ is a left ideal (resp. right ideal) of X when it is non-empty.*

Proof. Suppose that $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X. Let $x, y, x - y \in \bigcup(\mathcal{C}_X, t, n)$ $\forall t \in [0,1]$ and $n \in [0,1]$. Then $f^3(x) \geq t$, $f^3(x-y) \geq t$, $f^3(y) \geq t$ and $g^2(x) \leq n$, $g^2(x-y) \leq n, g^2(y) \leq n$. Suppose $y, x-y \in \bigcup(\mathcal{C}_X, t, n)$. Then $f^3(x) \geq \min\{f^3(x-y), f^3(y)\} \geq \min\{t,t\} = t$ and $g^2(x) \leq \max\{g^2(x-y), g^2(y)\} \leq \max\{n,n\} = n$. Hence $xy \in \bigcup(\mathcal{C}_X, t, n)$. Suppose $x, y \in \bigcup(\mathcal{C}_X, t, n)$. Then $f^3(xy) \geq \min\{f^3(x), f^3(y)\} \geq \min\{t,t\} = t$ and $g^2(xy) \leq \max\{g^2(x), g^2(y)\} \leq \max\{n,n\} = n$. Hence $xy \in \bigcup(\mathcal{C}_X, t, n)$. Let $x \in X$ and $y \in \bigcup(\mathcal{C}_X, t, n)$. Then $f^3(xy) \geq f^3(y) \geq t$ and $g^2(xy) \leq g^2(y) \leq n$. This implies that $xy \in \bigcup(\mathcal{C}_X, t, n)$. Hence $\bigcup(\mathcal{C}_X, t, n)$ is a left ideal of X. Conversely, let $t \in [0,1]$ and $n \in [0,1]$ be such that $\bigcup(\mathcal{C}_X, t, n) \neq 0$ and $\bigcup(\mathcal{C}_X, t, n)$ is a left ideal (right ideal) of X. Suppose $f^3(x) \ngeq \min\{f^3(x-y), f^3(y)\}$ or $g^2(x) \nleq \max\{g^2(x-y), g^2(y)\}$. If $f^3(x) \ngeq \min\{f^3(x-y), f^3(y)\}$, then there exists $t \in [0,1]$ such that $f^3(x) < t < \min\{f^3(x-y), f^3(y)\}$; hence $x - y, y \in (\mathcal{C}_X, t, \max\{g^2(x-y), g^2(y)\})$ but $x \notin \bigcup(\mathcal{C}_X, t, \max\{g^2(x-y), g^2(y)\})$, a contradiction. If $g^2(x) \nleq \max\{g^2(x-y), g^2(y)\}$, then there exists $n \in [0,1]$ such that $g^2(x) > n > \max\{g^2(x-y), g^2(y)\}$, hence $x - y, y \in \bigcup(\mathcal{C}_X, \min\{f^3(x-y), f^3(y)\}, n)$ but $x \notin \bigcup(\mathcal{C}_X, \min\{f^3(x-y), f^3(y)\})$, a contradiction. Hence $f^3(x) \geq \min\{f^3(x-y), f^3(y)\}$ and $g^2(x) \leq \max\{g^2(x-y), g^2(y)\}$. Suppose $f^3(xy) \ngeq \min\{f^3(x), f^3(y)\}$ or $g^2(xy) \nleq \max\{g^2(x), g^2(y)\}$. If $f^3(xy) \ngeq \min\{f^3(x), f^3(y)\}$, then there exists $t \in [0,1]$ such that $f^3(xy) < t < \min\{f^3(x), f^3(y)\}$, hence we have $x, y \in \bigcup(\mathcal{C}_X, t, \max\{g^2(x), g^2(y)\})$ but $xy \notin \bigcup(\mathcal{C}_X, t, \max\{g^2(x), g^2(y)\})$, a contradiction. If $g^2(xy) \nleq \max\{g^2(x), g^2(y)\}$, then there exists $n \in [0,1]$ such that $g^2(xy) > n > \max\{g^2(x), g^2(y)\}$, hence $x, y \in \bigcup(\mathcal{C}_X, \min\{f^3(x), f^3(y)\}, n)$ but $xy \notin \bigcup(\mathcal{C}_X, \min\{f^3(x), f^3(y)\})$, which is contradiction. Hence $f^3(xy) \geq \min\{f^3(x), f^3(y)\}$ and $g^2(xy) \leq \max\{g^2(x), g^2(y)\}$. Suppose $f^3(xy) \ngeq f^3(y)$ or $g^2(xy) \nleq g^2(y)$. If $f^3(xy) \ngeq f^3(y)$, then there exists $t \in [0,1]$ such that $f^3(xy) < t < f^3(y)$, hence $y \in \bigcup(\mathcal{C}_X, t, g^2(y))$ but $xy \notin \bigcup(\mathcal{C}_X, t, g^2(y))$, a contradiction. If $g^2(xy) \nleq g^2(y)$, then there exists $n \in [0,1]$ such that $g^2(xy) > n >$

$g^2(y)$, hence $y \in \bigcup(\mathcal{C}_X, f^3(y), n)$ but $xy \notin \bigcup(\mathcal{C}_X, f^3(y), n)$, a contradiction. Hence, $f^3(xy) \geq f^3(y)$ and $g^2(xy) \leq g^2(y)$. Therefore, $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal (resp. $(3,2)$-fuzzy right ideal) of X. $\square$

4. $(3,2)$-Fuzzy Ideal of Near-subtraction Semigroup

Definition 4.1. Let X be a near-subtraction semigroup. A $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ is called a $(3,2)$-*fuzzy ideal* of X, if

(1) $f^3(x-y) \geq \min\{f^3(x), f^3(y)\}$ and $g^2(x-y) \leq \max\{g^2(x), g^2(y)\}$,
(2) $f^3(xj - x(y-j)) \geq f^3(j)$ and $g^2(xj - x(y-j)) \leq g^2(j)$,
(3) $f^3(xy) \geq f^3(x)$ and $g^2(xy) \leq g^2(x)$, $\forall\ j, x, y \in X$.

If $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal of X if it satisfies (1) and (2) and if $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy right ideal of X if it satisfies (1) and (3).

Theorem 4.2. *If $\mathcal{C}_i = (f_i, g_i)$, $i \in \Delta$ is a family of $(3,2)$-fuzzy ideal of a near-subtraction semigroup X, then $\bigcap\limits_{i\in\Delta} \mathcal{C}_i = (\bigcap\limits_{i\in\Delta} f_i, \bigcup\limits_{i\in\Delta} g_i)$ is also a $(3,2)$-fuzzy ideal of X.*

Proof. If $\{\mathcal{C}_i\}_{i\in\Delta}$ is a family of $(3,2)$-fuzzy ideal of a near-subtraction semigroup X. Let $\bigcap f_i(x) = (\bigwedge f_i)(x) = \bigwedge f_i(x)$ and $\bigcup g_i(x) = (\bigvee g_i)(x) = \bigvee g_i(x)$ for all $x, y \in X$.
Let $x, y \in X$. Then

$$\begin{aligned}
(\bigcap_{i\in\Delta} f_i)^3(x-y) &= \bigwedge_{i\in\Delta}\{f_i^3(x-y)\} \\
&\geq \bigwedge_{i\in\Delta} \min\{f_i^3(x), f^3(y)\} \\
&= \min\{\bigwedge_{i\in\Delta}\{f_i^3(x)\}, \bigwedge_{i\in\Delta}\{f_i^3(y)\}\} \\
&= \min\{(\bigcap_{i\in\Delta} f_i)^3(x), (\bigcap_{i\in\Delta} f_i)^3(y)\},
\end{aligned}$$

$$\begin{aligned}
(\bigcup_{i\in\Delta} g_i)^2(x-y) &= \bigvee_{i\in\Delta}\{g_i^2(x-y)\} \\
&\leq \bigvee_{i\in\Delta} \max\{g_i^2(x), g^2(y)\} \\
&= \max\{\bigvee_{i\in\Delta}\{g_i^2(x)\}, \bigvee_{i\in\Delta}\{g_i^2(y)\}\} \\
&= \max\{(\bigcup_{i\in\Delta} g_i)^2(x), (\bigcup_{i\in\Delta} g_i)^2(y)\}.
\end{aligned}$$

For all $j, x, y \in X$, we have

$$\begin{aligned}
(\bigcap_{i\in\Delta} f_i)^3(xj - x(y-j)) &= \bigwedge_{i\in\Delta}\{f_i^3(xj - x(y-j))\}\\
&\geq \bigwedge_{i\in\Delta}\{f_i^3(j)\}\\
&= (\bigcap_{i\in\Delta} f_i)^3(j),\\
(\bigcup_{i\in\Delta} g_i)^2(xj - x(y-j)) &= \bigvee_{i\in\Delta}\{g_i^2(xj - x(y-j))\}\\
&\leq \bigvee_{i\in\Delta}\{g_i^2(j)\}\\
&= (\bigcup_{i\in\Delta} g_i)^2(j).
\end{aligned}$$

For all $x, y \in X$, we have

$$\begin{aligned}
(\bigcap_{i\in\Delta} f_i)^3(xy) &= \bigwedge_{i\in\Delta}\{f_i^3(xy)\}\\
&\geq \bigwedge_{i\in\Delta}\{f_i^3(x)\}\\
&= (\bigcap_{i\in\Delta} f_i)^3(x),\\
(\bigcup_{i\in\Delta} g_i)^2(xy) &= \bigvee_{i\in\Delta}\{g_i^2(xy)\}\\
&\leq \bigvee_{i\in\Delta}\{g_i^2(x)\}\\
&= (\bigcup_{i\in\Delta} g_i)^2(x).
\end{aligned}$$

Hence $\bigcup_{i\in\Delta} \mathcal{C}_i = (\bigcup_{i\in\Delta} f_i, \bigcap_{i\in\Delta} g_i)$ is a $(3,2)$-fuzzy ideal of X. □

Definition 4.3. A $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ of X is said to be a $(3,2)$-fuzzy bi-ideal of X if $\forall x, y \in X$

(1) $f^3(x-y) \geq \min\{f^3(x), f^3(y)\}$
(2) $g^2(x-y) \leq \max\{g^2(x), g^2(y)\}$
(3) $(f^3 \cdot X \cdot f^3) \cap (f^3 \cdot X) * f^3 \subset f^3$
(4) $(g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) * g^2 \supset g^2$.

Example 3. Let $X = \{0, a, b, c\}$ be a subtraction sub-semigroup with two binary operations $-$ and $\cdot$ is defined as follows.

$-$	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

$*$	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	0	0	0	c

X	0	a	b	c
f	0.8	0.6	0.3	0.2
g	0.3	0.4	0.6	0.7

We define a $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ as follows: It is clear that

X	0	a	b	c
$f^3 \cdot X \cdot f^3$	0.8	0.7	0.5	0.2
$(f^3 \cdot X) \cdot f^3$	0.7	0.6	0.3	0.1
$g^2 \cdot X \cdot g^2$	0.4	0.5	0.6	0.8
$(g^2 \cdot X) \cdot g^2$	0.3	0.6	0.7	0.8

Proposition 4.4. *If a $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy left ideal of X, then $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy bi-ideal of X.*

Proof. Let $j' \in X$ be such that $j' = xyz = jl - j(k-l)$, where $x, y, z, j, k, l \in X$. Then $((f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3))(j')$

$$\begin{aligned}
&= \min\{(f^3 \cdot X \cdot f^3)(j'), ((f^3 \cdot X) \cdot f^3)(j')\} \\
&= \min\{\bigvee_{j'=xyz} \min\{f^3(x), f^3(y), f^3(z)\}\}, \bigvee_{j'=jl-j(k-l)} \min\{(f^3 \cdot X)(j), f^3(l)\} \\
&= \min\{\bigvee\{f^3(x), f^3(z)\}, \bigvee\{(f^3 \cdot X)(j), f^3(l)\}\} \\
&\leq \min\{X(x), X(z), X(j), f^3(jl - j(k-l))\} \\
&= \min\{1, 1, 1, f^3(jl - j(k-l))\} \\
&= f^3(jl - j(k-l)) \\
&= f^3(j').
\end{aligned}$$

If j' is not expressible as $j' = xyz = jl - j(k-l)$, then $(f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3)(j') = 0 \leq f^3(j')$. Then $(f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3) \subset f^3$. Hence $\mathcal{C}$ is a $(3,2)$-fuzzy bi-ideal of X. And

$((g^2 \cdot X \cdot g^2) \cup ((g^2 \cdot X) \cdot g^2))(j')$

$$\begin{aligned}
&= \max\{(g^2 \cdot X \cdot g^2)(j'), (g^2 \cdot X) \cdot g^2)(j')\} \\
&= \max\{\bigvee_{j'=xyz} \max\{g^2(x), g^2(y), g^2(z)\}\}, \bigvee_{j'=jl-j(k-l)} \max\{(g^2 \cdot X)(j), g^2(l)\} \\
&= \max\{\max\{g^2(x), g^2(z)\}, \max\{(g^2 \cdot X)(j), g^2(l)\}\} \\
&\geq \max\{X(x), X(z), X(j), g^2(jl - j(k-l))\} \\
&= \max\{0, 0, 0, g^2(jl - j(k-l))\} \\
&= g^2(jl - j(k-l)) \\
&= g^2(j').
\end{aligned}$$

If j' is not expressible as $j' = xyz = jl - j(k-l)$ then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2)(j') = 0 \geq g^2(j')$. Then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2) \supset g^2$. Hence $\mathcal{C}$ is a $(3,2)$-fuzzy bi-ideal of X. $\square$

Proposition 4.5. *If a $(3,2)$-fuzzy set $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy right ideal of X, then $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy bi-ideal of X.*

Proof. Let $j' \in X$ be such that $j' = xy = jl - j(k-l)$, $x = x_1x_2$, where x, x_1, x_2, y, j, k and l are in X. Consider,

$$
\begin{aligned}
&((f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3))(j') \\
&= \min\{(f^3 \cdot X \cdot f^3)(j'), ((f^3 \cdot X) \cdot f^3)(j')\} \\
&= \min\{\bigvee_{j'=xyz} \min\{(f^3 \cdot X)(x), f^3(y)\}, ((f^3 \cdot X) \cdot f^3)(jl - j(k-l))\} \\
&= \min\{\bigvee_{j'=xyz} \min\{\bigvee_{j'=xyz} \min\{f^3(x_1), X(x_2)\}, f^3(y)\}, ((f^3 \cdot X) \cdot f^3)(jl - j(k-l))\} \\
&= \min\{\bigvee_{j'=xyz} \min\{\bigvee_{j'=xyz} \{f^3(a_1)\}, f^3(b)\}, ((f^3 \cdot X) \cdot f^3)(jl - j(k-l))\} \\
&= \min\{f^3(x_1), f^3(y), ((f^3 \cdot X) \cdot f^3)(jl - j(k-l))\} \\
&\leq \min\{f^3(xy), 1, 1\} \\
&= f^3(xy) \\
&= f^3(j').
\end{aligned}
$$

If j' is not expressible as $j' = xyz = jl - j(k-l)$ then $((f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3)(j') = 0 \leq f^3(j')$. Then $(f^3 \cdot X \cdot f^3) \cap ((f^3 \cdot X) \cdot f^3) \subset f^3$. Hence $\mathcal{C}$ is a $(3,2)$-fuzzy bi-ideal of X. And

$$
\begin{aligned}
&((g^2 \cdot X \cdot g^2) \cup ((g^2 \cdot X) \cdot g^2))(j') \\
&= \max\{(g^2 \cdot X \cdot g^2)(j'), ((g^2 \cdot X) \cdot g^2)(j')\} \\
&= \max\{\bigvee_{j'=xyz} \max\{(g^2 \cdot X)(x), g^2(y)\}, ((g^2 \cdot X) \cdot g^2)(jl - j(k-l))\} \\
&= \max\{\bigvee_{j'=xyz} \max\{\bigvee_{j'=xyz} \max\{g^2(x_1), X(x_2)\}, g^2(y)\}, ((g^2 \cdot X) \cdot g^2)(jl - j(k-l))\} \\
&= \max\{\bigvee_{j'=xyz} \max\{\bigvee_{j'=xyz} \{g^2(x_1)\}, g^2(y)\}, ((g^2 \cdot X) \cdot g^2)(jl - j(k-l))\} \\
&= \max\{g^2(x_1), g^2(y), ((g^2 \cdot X) \cdot g^2)(jl - j(k-l))\} \\
&\geq \max\{g^2(xy), 1, 1\} \\
&= g^2(xy) \\
&= g^2(j').
\end{aligned}
$$

If j' is not expressible as $j' = xyz = jl - j(k-l)$ then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2)(j') = 0 \leq g^2(j')$. Then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2) \subset g^2$. Hence D is a $(3,2)$-fuzzy bi-ideal of X. □

Theorem 4.6. *Let $\mathcal{C}_X := (X, f, g)$ be a $(3,2)$-fuzzy subalgebra of X. If $\mathcal{C}X\mathcal{C} \subset \mathcal{C}$, then $\mathcal{C}$ is a $(3,2)$-fuzzy bi-ideal of X.*

Proof. Assume that f is a $(3,2)$-fuzzy subalgebra of X and $f^3 \cdot X \cdot f^3 \subset f^3$. Let $j \in X$. Then $((f^3 \cdot X \cdot f^3) \cap (f^3 \cdot X) \cdot f^3)(j) = \min\{(f^3 \cdot X \cdot f^3)(j), ((f^3 \cdot X) \cdot f^3)(j)\} \leq$

$(f^3 \cdot X \cdot f^3)(j) \leq f^3(j)$. Thus $((f^3 \cdot X \cdot f^3) \cap (f^3 \cdot X) \cdot f^3) \subset f^3$ and f is a $(3,2)$-fuzzy bi-ideal of X and assume that g is a $(3,2)$-fuzzy subalgebra of X and $g^2 \cdot X \cdot g^2 \supset g^2$. Let $j \in X$. Then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2)(j) = \max\{(g^2 \cdot X \cdot g^2)(j), ((g^2 \cdot X) \cdot g^2)(j)\} \geq (g^2 \cdot X \cdot g^2)(j) \geq g^2(j)$. Thus $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2) \supset g^2$ and g is a $(3,2)$-fuzzy bi-ideal of X. □

Theorem 4.7. *If X is a zero symmetric near-subtraction semigroup and $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy bi-ideal of X, then $f^3 \cdot X \cdot f^3 \subset f^3$ and $g^2 \cdot X \cdot g^2 \supset g^2$.*

Proof. Let f be a $(3,2)$-fuzzy bi-ideal of X. Then $((f^3 \cdot X \cdot f^3) \cap (f^3 \cdot X) \cdot f^3) \subset f^3$. Clearly $f^3(0) \geq f^3(j)$. Thus $(f^3 \cdot X)(0) \geq (f^3 \cdot X)(j)$ $\forall j \in X$. Since X is a zero symmetric near-subtraction semigroup, $f^3 \cdot X \cdot f^3 \subset (f^3 \cdot X) \cdot f^3$. So $((f^3 \cdot X \cdot f^3) \cap (f^3 \cdot X) \cdot f^3) = f^3 \cdot X \cdot f^3 \subset f^3$, and let g be a $(3,2)$-fuzzy bi-ideal of X. Then $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2) \supset g^2$. Clearly $g^2(0) \leq g^2(j)$. Thus $(g^2 \cdot X)(0) \leq (g^2 \cdot X)(j)$ $\forall j \in X$. Since X is a zero symmetric near-subtraction semigroup, $g^2 \cdot X \cdot g^2 \supset (g^2 \cdot X) \cdot g^2$. So $((g^2 \cdot X \cdot g^2) \cup (g^2 \cdot X) \cdot g^2) = g^2 \cdot X \cdot g^2 \supset g^2$. □

Theorem 4.8. *If $\mathcal{C}_X := (X, f, g)$ is a $(3,2)$-fuzzy bi-ideal of a zero symmetric near-subtraction semigroup X, then $f^3(jkl) \geq \min\{f^3(j), f^3(l)\}$ and $g^2(jkl) \leq \max\{g^2(j), g^2(l)\}$.*

Proof. Let f be a $(3,2)$-fuzzy bi-ideal of zero symmetric near-subtraction semigroup X. It follows that $f^3 \cdot X \cdot f^3 \subset f^3$ and $g^2 \cdot X \cdot g^2 \supset g^2$.
Let $j, k, l \in X$. Then

$$\begin{array}{rcl}
f^3(jkl) & \geq & (f^3 \cdot X \cdot f^3)(jkl) \\
& = & \bigvee\limits_{jkl=xy} \min\{(f^3 \cdot X)(x), f^3(y)\} \\
& \geq & \min\{(f^3 \cdot X)(jk), f^3(l)\} \\
& \geq & \min\{(f^3 \cdot X)(j), X(k), f^3(l)\} \\
& = & \min\{(f^3 \cdot X)(j), 1, f^3(l)\} \\
& = & \min\{(f^3 \cdot X)(j), f^3(l)\} \\
& \geq & \min\{f^3(j), f^3(l)\}, \\
g^2(jkl) & \geq & (g^2 \cdot X \cdot g^2)(jkl) \\
& = & \bigwedge\limits_{jkl=xy} \max\{(g^2 \cdot X)(x), g^2(y)\} \\
& \leq & \max\{(g^2 \cdot X)(jk), g^2(l)\} \\
& \leq & \max\{(g^2 \cdot X)(j), X(k), g^2(l)\} \\
& = & \max\{(g^2 \cdot X)(j), 1, g^2(l)\} \\
& = & \max\{(g^2 \cdot X)(j), g^2(l)\}. \\
& \leq & \max\{g^2(j), g^2(l)\}.
\end{array}$$

□

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References

1. J.C. Abbott: Sets, Lattices and Boolean Algebras. Allyn and Bacon, Boston, 1969.
2. B. Ahmad & A. Kharal: On fuzzy soft sets. Advances in Fuzzy Systems **2009**, Article ID 586507, 6 pages, 2009. `https://doi.org/10.1155/2009/586507`
3. K.T. Atanassov: Intuitionistic fuzzy sets. Fuzzy Sets and Systems **20** (1986), no. 1, 87-96. `https://doi.org/10.1016/S0165-0114(86)80034-3`
4. K.T. Atanassov: New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets and Systems **61** (1994), 137-142. `https://doi.org/10.1016/0165-0114(94)90229-1`
5. M. Atef, M.I. Ali & T.M. Al-shami: Fuzzy soft covering based multi-granulation fuzzy rough sets and their applications. Computational and Applied Mathematics **40** (2021), no. 4, 115. `https://doi.org/10.1007/s40314-021-01501-x`
6. N. Cağman, S. Enginoğlu & F. Citak: Fuzzy soft set theory and its application. Iranian Journal of Fuzzy Systems **8** (2011), no. 3, 137-147. `10.22111/IJFS.2011.292`
7. H. Garg & S. Singh: A novel triangular interval type-2 intuitionistic fuzzy set and their aggregation operators. Iranian Journal of Fuzzy Systems **15** (2018), 69-93. `10.22111/IJFS.2018.4159`
8. H. Garg & K. Kumar: An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making. Soft Computing **22** (2018), no. 15, 4959-4970. `https://doi.org/10.1007/s00500-018-3202-1`
9. H. Garg & K. Kumar: Distance measures for connection number sets based on set pair analysis and its applications to decision-making process. Applied Intelligence **48** (2018), no. 10, 3346-3359.
10. A. Iampan: A new branch of the logical algebra: UP-algebras. J. Algebra Relat. Top. **5** (2017), no. 1, 35-54. `10.22124/JART.2017.2403`
11. H.Z. Ibrahim, T.M. Al-shami & O.G. Elbarbary: (3, 2)-fuzzy sets and their applications to topology and optimal choice. Computational Intelligence and Neuroscience **2021**, Article ID 1272266, 14 pages. `https://doi.org/10.1155/2021/1272266`
12. Y.B. Jun, G. Muhiuddin & S.A. Romano: On filters in UP-algebras, A review and some new reflections. J. Int. Math. Virtual Inst. **11** (2021), no. 1, 35-52. `DOI: 10.7251/JIMVI2101035J`
13. Y.B. Jun, H.S. Kim & E.H. Roh: Ideal theory of subtraction algebras. Scientiae Mathematicae Japonicae **61** (2005), no. 3, 459-464.

14. Y.B. Jun & H.S. Kim: On ideals in subtraction algebras. Scientiae Mathematicae Japonicae **65** (2007), no. 1, 129-134.
15. K.J. Lee & C.H. Park: Some questions on fuzzifications of ideals in subtraction algebras. Communications of the Korean Mathematical Society **22** (2007), no. 3, 359-363. `10.4134/CKMS.2007.22.3.359`
16. S.M. Mostafa, M.A.A. Naby & M.M.M. Yousef: Fuzzy ideals of KU-algebras. Int. Math. Forum **63** (2011), 3139-3149.
17. B.M. Schein: Difference Semigroups. Communications in Algebra **20** (1992), 2153-2169. `https://doi.org/10.1080/00927879208824453`
18. T. Senapati & R.R. Yager: Fermatean fuzzy sets. Journal of Ambient Intelligence and Humanized Computing **11** (2020), no. 2, 663-674.
19. J. Somjanta, N. Thuekaew, P. Kumpeangkeaw & A. Iampan: Fuzzy sets in UP-algebras. Ann. Fuzzy Math. Inform. **12** (2016), 739-756.
20. R.R. Yager: Pythagorean fuzzy subsets. in Proceedings of the 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS), 57-61, IEEE, Edmonton, Canada, 2013. `10.1109/IFSA-NAFIPS.2013.6608375`
21. L.A. Zadeh: Fuzzy sets. Information and Control **8** (1965), no. 3, 338-353. `https://doi.org/10.1016/S0019-9958(65)90241-X`

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