

RESEARCH ARTICLE

Elementary Preservice Teachers' Noticing and Evaluation of Digital Mathematical Resources

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Received: June 7, 2023 / Accepted: June 21, 2023 / Published online: June 27, 2023

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Abstract

With the rapid advancement of educational technology, recent studies have connected teachers' professional noticing with the use of digital resources in mathematical instructions. In this study, I examined elementary mathematics preservice teachers' attending and interpreting a mathematical software, ST Math, in the exploring and implementing phases. The findings indicate that preservice teachers paid attention to visual representations and manipulation prior to interactions with children and further took into consideration on task structures and situated context after interactions. They interpreted the events based on connected mathematical knowledge of prior interactions and further reflected on the progression of problem-solving strategies and sequence of tasks. In addition, four distinctive profiles of transitioning of evaluation on ST Math activities were identified with illustrations. Implications for noticing and teacher education were discussed.

Keywords professional noticing, ST Math, evaluating digital resources, elementary mathematics preservice teachers

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- Acknowledgement: This study was extended from a conference proceeding previously presented and I would like to thank Dr. Corey Webel for his guidance, support, and insightful comments on this study.
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I. INTRODUCTION

Teaching is very complicated work due to the multi-layered interaction with students within the learning environment. This makes what and how teachers look at classroom events and phenomena very critical. Researchers examined various professional skills to enhance the quality of instruction. Recently, professional noticing has been paid attention to as an important and essential instructional practice for supporting and extending students' mathematical thinking (Jacobs et al., 2010; Mason, 2002; Sherin & van Es, 2005). Professional noticing is a set of instructional practices for decision-making. Jacobs and her colleagues (2010) suggested a series of practices that enhance students' mathematical thinking: attending, interpreting, and deciding to respond.

Meanwhile, with the rapid advancement of educational technology, recent studies have connected teachers' professional noticing with the use of technology in mathematics instructions, including noticing during evaluating the use of dynamic technology (Smith et al., 2018), technology-mediated teacher noticing (Walkoe et al., 2017), and developing noticing skills through the creation of animated teaching episodes (de Araujo et al., 2015). In teacher education, previous studies have identified a framework for the evaluation of technological tools by having preservice teachers (PSTs) engage with and reflect on the qualities of the tools (Smith et al., 2018). However, little is known about what and how PSTs actually pay attention to and make sense of children's engagement with digital resources.

In this study, "Spatial-Temporal Mathematics" or "ST Math", a game-based instructional software was examined to understand how PSTs notice the use of digital resources. ST Math employed visual representations of quantities and mathematical objects to develop students' construction of mental images ahead in space and time (Peterson et al., 2004; Yeo et al., 2022). The puzzles in ST Math are designed for students to develop mathematical concepts by manipulating the touchscreen with actions and to provide strong connections between concepts (Yeo, 2018). Despite of potential to use ST Math for conceptual development in student mathematical learning, much less is known about how ST Math is appreciated by teachers, especially PSTs. Therefore, in this study, I explore how PSTs notice the mathematical thinking of students as they engage with a particular digital resource, and how this noticing influences their evaluation of the tool.

This study focuses on PSTs' professional noticing related to digital resources, exploring how PSTs attended to and interpreted students' actions and statements when students were engaged with ST Math. As a central focus of the elementary mathematics methods course was for PSTs to develop their noticing skills for student thinking, I would like to understand how PSTs would use noticing skills in the context of the ST Math program. In particular, this study seeks to examine the following questions:

- RQ 1: To what aspects of children's thinking do PSTs attend when the children are engaged in the ST Math activities?
- RQ 2: How do the PSTs interpret evidence about this thinking?
- RQ 3: How do PSTs draw on their noticing of children's thinking when evaluating the ST Math activities?

II. THEORETICAL BACKGROUND

Professional Noticing

Teachers' awareness of aspects of students' work in the classrooms is crucial to make effective teaching. Teachers use this awareness to make pedagogical decisions (Goodwin, 1994) and the awareness of student's work and classroom events have been described as intentional noticing (Mason, 2002). *Noticing* has been implied with various meanings in mathematics education. In a recent review, König et al. (2022) categorized noticing into four categories in terms of their theoretical underpinnings: a cognitive-psychological perspective, a socio-cultural perspective, a discipline-specific perspective, and an expertise-related perspective of teacher noticing.

A cognitive-psychological perspective implies noticing as what teachers pay attention to and make sense of during mental decision-making. For example, van Es and Sherin (2002) defined noticing with the following components: "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions" (p. 573). A socio-cultural perspective focuses on social nature of teacher noticing. This notion is based on the professional vision which refers "the ability to see a meaningful event is not a transparent, psychological process, but instead a socially situated activity" (Goodwin, 1994, p. 606). That is, a certain event would be interpreted differently depending on their professions by constructing 'objects of knowledge'. A discipline-specific perspective of teacher noticing is conceptualized as practices for raising teacher awareness of one's own practices: "At the heart of all practices lies noticing: noticing an opportunity to act appropriately. To notice an opportunity to act requires three things: being present and sensitive at the moment, having a reason to act, and having a different act come to mind" (Mason, 2002, p. 1). Mason (2002) identified four interrelated actions for professional noticing: (1) systematic reflection, which involves keeping accounts by noticing and recording important moments and retrospectively identifying threads; (2) recognizing, which is based on the interrelated processes of identifying and labeling typical situations, distinguishing choices, and accumulating alternatives; (3) preparing and noticing, which consists of the sensitizing processes of imagining possibilities and enhancing the opportunities for noticing; and (4) validating with others, which is based on describing moments and refining task exercises to highlight important issues or sensitivities (p. 95). Lastly, an expertise-related perspective concentrates on the difference between novices and experts. From this perspective, teachers have different stages of skill development (Berliner, 2004). For example, novices might have difficulty interpreting appropriately classroom events, while experts would do better explanations of the events relatively. Although professional noticing might not directly connect with the expertise-related perspective, there is a clear correlation between noticing and teacher expertise (Lachner et al., 2016). In this study, I draw upon the cognitive-psychological perspective to examine PSTs' noticing and evaluation of digital mathematical resources.

From empirical studies, some focus on what teachers pay attention to in the

mathematical classrooms (e.g., Star et al., 2011), while others view noticing as consisting of multiple practices by attending to specific events and making sense of the events (Sherin et al., 2011). For example, Jacobs et al. (2010) suggested a triad of practices as professional noticing that consists of three components: attending to students' mathematical ideas, interpreting their understanding, and deciding how to respond to their understanding. *Attending* refers to focusing on "noteworthy aspects of complex situations" (Jacobs et al., 2010, p. 172). For example, during a lesson on fraction addition ($1/2+1/4$), a teacher found a student who added two denominators as 6. *Interpreting* includes reasoning about children's strategies and comprehending their understanding based on details (Jacobs et al., 2010). The teacher thought this student had a common misconception about whole number addition in fraction operations. At last, Jacobs and her colleague (2010) described *deciding* as the degree to which teachers' responses were related to the children's thinking and to research on children's understanding of the mathematical concept. For example, the teacher decided to ask the student to draw a circular representation to express a half and a fourth. In this study, professional noticing is regarded as a collection of instructional practices with an emphasis on *attending* and *interpreting* from the cognitive-psychological perspective (van Es & Sherin, 2022). Note here, *deciding to respond* was considered as a part of the evaluation of a digital tool.

Professional Noticing to Digital Resources

Professional noticing has been widely expanded to other research topics: group coordination (Campbell & Yeo, 2022; Campbell et al., 2022), curriculum design (Dietiker et al., 2018), and equity (van Es et al., 2022). Recently, researchers also have explored the possibility to use noticing framework for the use of educational technology (Amador et al., 2021; Chao et al., 2016; Kosko et al., 2021). For example, Smith et al. (2018) found professional noticing for the evaluation of interactive dynamic geometry activities as educational technology in mathematics education. They reported that secondary mathematics PSTs attended mostly to mathematical features of the dynamic geometry activities and considered the activities' ability to focus on student engagement and student mathematical thinking. PSTs also valued the ease of implementation of technological activities. Their evaluation was associated with how the activities worked by attending to their appearance, rather than focusing on the mathematical features or student thinking. Previous studies have shown that teachers tend to evaluate online resources and activities positively with little consideration of mathematical or pedagogical features. That is, teachers attended to surface-level characteristics, such as whether students would be familiar with the problem types or if the activities had a game-like interface (e.g., Webel et al., 2015). In this study, I focus on how PSTs use noticing practices on student thinking in the use of digital resources.

III. METHODS

Participants

The participants were 21 elementary PSTs (2 males, 19 females) enrolled in a

methods course for teaching elementary students at a Midwest university in the U.S. The participants were in their third year of the four-year program for elementary teacher preparation. The instructor of the course emphasized an understanding of how children’s mathematical thinking develops in the domain of numbers and operations (Carpenter et al., 2014) and developing the core teaching practices of eliciting and extending to children’s thinking (Jacobs & Empson, 2016). To provide authentic opportunities for PSTs to develop their pedagogical skills with students, the PSTs were engaged in daily one-on-one interactions with an assigned student in 3rd, 4th, or 5th grade (a “Math Buddy”). PSTs and their buddy had a long-term relationship during a single semester in a local elementary school. This embedded course design between the university and the local school allowed PSTs to have a rich experience by exploring children’s mathematical thinking with traditional instructional materials as well as digital resources.

ST Math Activities

PSTs were engaged in two-phase of ST Math activities: exploring and implementing phases. In the exploring phase, they were asked to explore specific puzzles in ST Math as a small group and to reflect on the activity individually (Table 1).

Table 1. Groups of PSTs

Group	PSTs
Group A	Olivia, Emma, Charlotte, Amelia
Group B	Harper, Chloe, Aria, Willow*, Zoe
Group C	Sophia, Isabella, Beth, Naomi
Group D	Ava, Mia, Evelyn, Hannah
Group E	Ellie, Madison, Ivy, Christopher*

* denotes male PSTs.

Then, in the implementing phase, PSTs implemented one of the puzzles to their Math Buddies. During this implementation, PSTs were asked to elicit the child’s thinking about the mathematics they had learned during the course and to write a reflection paper about the experience with Math Buddies. ST Math was a digital tool that had been adopted by the school district in which PSTs were placed for their field experience. Therefore, their Math Buddies were already familiar with the use of ST Math.

In this study, three tasks in ST Math were chosen (Figure 1): *Pie Monster* (subtraction), *How Many Petals?* (place value), and *Building Expressions* (multiplication and division). These tasks were chosen to ensure that children could engage in mathematical puzzles with the appropriate use of the digital game. In addition, the tasks represented a range of opportunities for children to develop a conceptual understanding of the domain of numbers and operations. Specifically, the *Pie Monster* task involves whole number subtraction with various structures, such as start-unknown, change-unknown, and result-unknown (Carpenter et al., 2014) including three types of direct modeled representations. The screen (see Figure 1- left) uses two red circles to represent the change (subtrahend), seven orange circles to represent the start (minuend), and the white circles in the Monster’s belly to represent the result (difference). When choosing the number of white

circles, *JiJi* (penguin character) attempts to cross the screen. If the provided answer is correct, the boxes are burnt by the Monster's fire and *JiJi* can cross the screen. If not, *JiJi* would go back to the starting place and one trial would be lost. The *How many petals?* task involves two-digit and three-digit place value concepts with the representations of petals (ones), flowers (tens), and a bunch of flowers (hundreds). Each tap on the 'ten' section on the screen (Figure 1-middle) collects ten petals, and so on for each place value. If the 'ones' section has more individual petals than ten, a flower would be automatically made of the ten petals. Ten tens will automatically transform into a bunch of flowers (hundreds). The *Building Expressions* task involves the relationship between multiplication and division (e.g., $24 \div 4 = 6$, $4 \times 6 = 24$). A number of green dots must be selected according to the first number of a given number expression and the user decides how to drag the slider to partition the set of dots into the number of pink segments as designated by the second number. The quotient is the number of dots corresponding to each segment.



Figure 1. Pie Monster (left), How Many Petals? (middle), and Building Expression (right)

Data Sources

The data consisted of three parts: 1) PSTs' group solution for each ST Math task and individual reflections on the problem-solving in the exploring phase (e.g., what mathematical ideas they believed the task was targeting, whether the task provided a good opportunity to learn those ideas, what questions they would ask children to better understand their thinking while engaging in the task; 2) screencasting videos while working with their individual Math Buddy, which captured manipulations on a tablet device and verbal explanations at the same time; and 3) individual reflection papers in which PSTs described the children's strategies, compared the strategies to how the children solved story problems, and gave an evaluation of each activity.

Data Analysis

The constant comparative method was administrated to establish a coding scheme from the collected data (Strauss & Corbin, 1998). This method allows me to identify categories and to find the relationships between categories for establishing the whole structure by comparing similarities and differences. To address RQs 1 and 2, PSTs' responses regarding the ST Math activities were coded into attending and interpreting from both exploring and implementing phases (see Table 2 for specific codes). In this analysis, I did not include the codes for deciding how to respond. Instead, PSTs were expected to evaluate the digital tool, ST Math, based on attending and interpreting (RQ 3). The screencasting data were reviewed to redefine and modify the initial coding scheme.

After establishing the coding scheme, a mathematics teacher educator and I reviewed the data to validate and revise the coding scheme. Resolving all inconsistencies with the teacher educator, we finalized the coding scheme of noticing in terms of attending and interpreting in the exploring and implementing phases (Table 2). Compared to the exploring phase, there were additional codes in the implementing phase. Therefore, Table 2 includes additional codes in the row of the implementing phase.

Table 2. Professional Noticing Coding Scheme in Exploring and Implementing Phases

Noticing	Phases	Codes	Examples
Attending	Exploring Phase	-Instructions	- “It doesn’t tell you what to do.”
		-Visual representations	- “Students get to see a visual representation of every step of the multiplication and division process, furthering their understanding.”
		-Manipulation	- “Students are physically seeing the number how it is broken down into the place value parts and their individual values.”
		-Mathematics concepts	- “Because the petals are in groups of 10s, students will be likely to count by 10s.”
	Implementing Phase	-Task structures	- “The result is always located inside the green monster and the unknowns are always located in the metal machine and underneath of the pie monster.”
		-Strategy	- “The strategies my buddy uses for ST Math is the same that he uses for solving word problems.”
		-Verbal explanations	- “She responded and said, ‘Because I clicked it just to see what it was going to do’.”
		-Semiotic actions	- “When first starting the problem my math buddy counts by 1s to get the original number, such as 12.”
		-Gamified features	- “Yes, ST Math is fun because it is in the form of a game.”
		-Situated context	- “I think the pie monster portrays math in a fun way, but not necessarily a real or relatable one.”

Table 2. Professional Noticing Coding Scheme in Exploring and Implementing Phases (continued)

Noticing	Phases	Codes	Examples
	Exploring Phase	-Using sense-making to solve tasks	- “Some kids might know how to visually play the game but not understand that they are actually doing subtraction.”
		-Potential mathematical concepts	- “Requires understanding of place value and number sense.”
		-Making a connection between representation and concept	- “Some students may not connect the images with mathematical concepts.”
		-Requirement of prior knowledge	- “This requires previous knowledge of math concepts that the game develops.”
Interpreting	Implementing Phase	-Progression of problem-solving strategies	- “However at the beginning he struggled with knowing how many groups there are, he would test out different numbers and then was able to make the connection that it was the number given in the problem.”
		-Solving tasks with given representations	- “If she clicked too many times and added too many petals all she had to do was break the flowers down and move them to the ones place.”
		- Sequence of the tasks	- “What I mean is that when the game changed from a result unknown problem to a change unknown problem, he recognized that shift in mathematical concepts.”

Next, to address RQ 3, I analyzed PSTs’ responses and reflections from the exploring activity to characterize individualized initial evaluation of the ST Math activities. The baseline data in the exploring phase was compared to their final reflection paper which was submitted after one-to-one interactions with their Math Buddies in the implementing phase. I examined whether the PSTs’ evaluation of the ST Math activities changed, and how their noticing of student thinking appeared to influence their evaluations. This provides distinctive profiles of transitioning PSTs’ evaluations of the ST Math.

IV. FINDINGS

In this section, PSTs' attending to and interpreting children's mathematical thinking were addressed when children engaged in the puzzles of ST Math. In addition, PST's noticing was categorized into positive and/or negative features of ST Math in relation to potential learning opportunities.

Attending to Children's Thinking

PSTs showed evidence of noticing when reflecting on their own exploration of the ST Math tasks. They understandably tended to pay attention to student thinking when reflecting on the interactive experience with their Math Buddies in the implementing phase. That is, they anticipated how students would think about and solve the tasks based on the visual representations and strategies in the ST Math environment. Math buddies manipulated the representations strategically to understand the targeted mathematical concepts in the tasks. For example, in the exploring phase, PSTs in Group A attended to the instructions of the Pie Monster task (Figure 2), which represents whole number subtraction with various number choices.

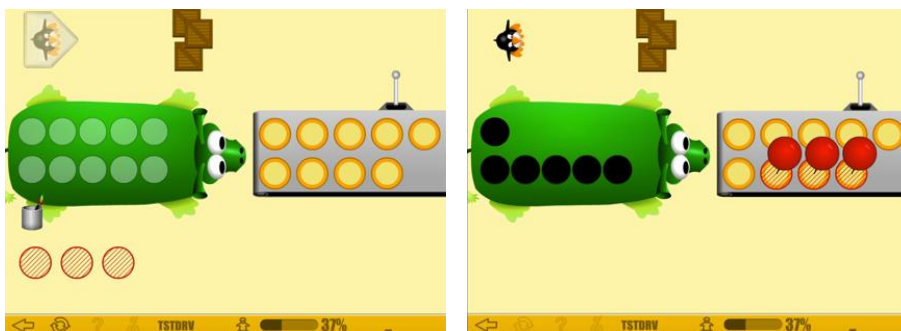


Figure 2. Problem-Solving of the Pie Monster Task

This group noted that the activity did not provide any instructions about how to play the game, although such lack of instruction is a central design feature of ST Math. This unique feature of ST Math could lead to confusion about how to start and what they are supposed to do: “I feel like the game is simple, but the instructions are not there and it can take them a while to figure out what numbers they need to subtract.”

The initial attending pattern was expanded through one-to-one interaction with their buddies. PSTs attended to task structures, learning goals, verbal explanations, semiotic actions, gamified features, and situated context. For example, Emma reflected on her student's understanding of task structures:

Something I noticed while asking him about the game is that...when the game changed from a result unknown problem to a change unknown problem, he recognized that shift in mathematical concepts.

Since there are various levels of difficulty in a single task, students have further opportunities to engage in multiple problem structures with a similar context. Emma

attended to this transitioning of tasks and the mathematical structure of the Pie Monster task.

Interpreting of Children's Thinking

In the exploring phase, PSTs' initial interpretations focused on how children might make sense of the tasks, what mathematical concepts they might engage with, how they might make a connection between a concept and the ST Math representation, and what prior knowledge might be required. For example, PSTs in Group C anticipated a possible way to use interpreting when students solve the Pie Monster task: "Some kids might know how to visually play the game but not understand that they are actually doing subtraction." This group of PSTs anticipated that students might use the visual representations to solve the task without understanding the embedded mathematical concepts (e.g., subtraction).

On the other hand, in the implementing phase, PSTs' interpretations of student thinking included descriptions of how students solved the tasks, how they engaged with different representations, and how the sequence of the tasks impacted students' approaches. For example, Harper noticed the progression of her Math Buddy's strategies.

She was beginning to use other strategies that weren't simply guess-and-check, such as counting on. She counted the red circles, then found that amount in the yellow circles. She then counted the yellow circles that were left to find the answer.

Harper articulated the way how her buddy solved ST Math tasks, which revealed evidence of her pedagogical knowledge. The student used a guess-and-check strategy at first, but this strategy changed to a counting strategy over time. This progression of problem-solving strategies was interpreted by PSTs.

PSTs' Evaluations of ST Math

When comparing PSTs' evaluations of ST Math activities, four distinct profiles of evaluation types were identified. While PSTs in Profile 1 show they become more positive about ST Math activities after engaging with their Math buddy, PSTs in Profile 2 increase their negative disposition after the interaction. On the other hand, PSTs in Profiles 3 and 4 appeared sustained in their evaluations.

Profile 1: Increased positive evaluations. Eight PSTs had the opposite shift from a negative evaluation of ST Math to a somewhat more positive evaluation, though these were sometimes the result of relatively sophisticated reasoning. For example, Sophia initially criticized the How Many Petals? task, anticipating that students might not use mathematical thinking: "The kids do not really have to do much thinking; they just need to memorize the different flowers." However, after she saw her Math Buddy demonstrated strong understanding in her explanation for why certain ones go in the tens column and others in the one's column, her evaluation was more positive:

This leads me to believe that she was thinking mathematically rather than just playing the game without thought... I think that she does better at ST math because

it is easier for her to visualize, as she uses direct modeling as her primary strategy for solving problems... I do not think ST Math should be discounted, it seems to be a big help for students to refresh on previously learned material.

Sophia modified her evaluation of ST Math, focusing on its potential to “refresh on previously learned material.” Indeed, the child appeared to be bringing her understanding of place value concepts to the task through the activity. In particular, Sophia attended to the visual representation of ST Math (*Attending-Visual representation*). Since her buddy employed the direct modeling strategy through other mathematics problems, she affirmed ST Math was helpful to support mathematical learning.

Profile 2: Increased negative evaluations. Six PSTs had a positive evaluation based on their initial explorations, but during their interactions with children they began to question whether some features were likely to foster mathematical thinking relevant to targeted mathematical concepts. For example, Christopher was initially positive about the potential of the How Many Petals? task to develop place value concepts (Figure 3). The major mathematical idea in this task is to recognize a group of 10 petals is the same as one flower and 10 groups of flowers are the same as a bunch of flowers. He believed that the task could provide an opportunity to learn place value concepts (e.g., hundreds, tens, ones): “I think that the students can learn that they need 10 petals to make a flower and that they need to know how many flowers they have”.



Figure 3. Problem-Solving of the How Many Petals? Task

However, he noticed that his Math Buddy was able to get the right answers by just tapping the columns repeatedly and did not demonstrate an understanding of the relationship between different places.

Christopher: [Before solving the second problem] So on this one you explain all your thinking out loud and how you do it. So, what’s the first thing you do?

Math Buddy [MB]: So, like if there is a big pile you press tens. These are all tens. And then or if you have not enough tens press ones.

...

Christopher: [At the third problem] Basically you just keep pressing tens until you run out.

MB: Yeah.

...
 Christopher: [At the final problem] What are you learning on this game you play?
 So, what do you learn when you do this?
 MB: I don't really know.

Even though the student completed all problems successfully, Christopher was not sure what his buddy was learning from the ST Math activity. He noticed this lack of understanding of the mathematical concept:

My understanding is that my buddy just counts the petals and that's it. You can even hear him clicking on the tablet screen rapidly to get rid of as many petals as you can. To me, there isn't much learning going on during this game, other than being able to identify where the hundreds, tens, and one's value is.

Initially, Christopher believed that tapping the counting button could help develop an understanding of the relationship between ones, tens, and hundreds. However, when working with his Math Buddy, he noticed that the student was able to mindlessly tap the button until the solution was represented as a number of bunches, individual flowers, and petals (*Interpreting-Solving tasks with given representations*). This interpretation in solving tasks with given representations made him evaluate the ST Math activity negatively ("isn't much learning going on").

Profile 3: Consistently positive. Four PSTs were included in this category. For example, Beth kept evaluating ST Math positively. She expected her Math Buddy to understand and use the relationship between multiplication and division embedded in the given pictorial model (e.g., dots, boxes) in the Building Expressions task (Figure 4). She initially appreciated the potential of the tasks: "Students get to see a visual representation of every step of the multiplication and division process, furthering their understanding".

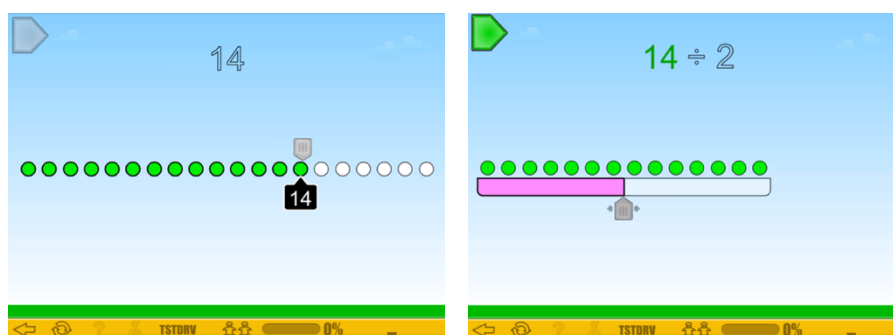


Figure 4. Problem-Solving of the Building Expression Task

During the one-to-one interaction, Beth noticed her Math Buddy expressing more of his understanding within ST Math tasks, and eventually concluded that ST Math could provide good opportunities for children to engage in mathematical concepts and thinking:

When working with my Math Buddy I realized that ST Math reveals more about his thinking and understanding than story problems... This allows him to focus on showing his understanding of multiplication and division, or any other concept he is working on, instead of focusing on the words in a word problem.

Perhaps since her Math Buddy was an English Language Learner, Beth might have some challenges for her buddy to understand the meaning of words in story problems. However, she interpreted his work in ST Math as evidence that the visual representations could support his mathematical understanding better than story problems by concentrating on the visual medium solely (*Interpreting-Making a connection between representation and concept*).

Profile 4: Consistently negative. The analysis showed three PSTs remaining consistent in their evaluations of ST Math. For example, Hannah initially expressed the concern that her Math Buddy might use only an unsophisticated “counting by ones” strategy, “Because they can always count single units within the 10 petals, so as long as they can count by 1’s they can finish the levels.”

When working with her Math Buddy, Hannah noticed that the student focused on getting the right answer only without considering other strategies: “She seemed to just pick up patterns of how to pick out the correct answers and numbers to move to the next problem... I do not believe that there is any strategy for solving the problem besides counting.” She was also worried about her student’s misconceptions since it was possible to get the correct answers without understanding the base-10 structure of the petals representation: “It also may give students the impression that they understand the content just because they are able to find the pattern of the game and fill in the rest of the answers.” She noticed that her buddy did not make a connection between representations and mathematical concepts within the ST Math environment (*Interpreting-Making a connection between representation and concept*).

V. DISCUSSION

In this study, I analyzed how PSTs’ attend to and interpret student thinking during the use of ST Math tasks. I evidenced how PSTs drew on their noticing of student mathematical thinking in their evaluations of the ST Math activities. In some cases, PSTs came to different conclusions from their original evaluations, prior to interacting with students (Profiles 1 and 2). Often, these conclusions were based not just on whether children were able to complete the tasks and answer with correct answers, but how PSTs were thinking about the mathematical ideas embedded in the digital resources (Dick & Hollebrands, 2011). This shed light on the potential that developing noticing skills, in general, might help PSTs be better consumers of technology, especially if they are asked to evaluate tools while simultaneously attending to student thinking (Yeo, 2020).

One possible implication is that interactions with real children, with whom PSTs have relationships, could stimulate critical reflection on the value of learning experiences.

Although digital resources have a dynamic feature or a rich environment, those do not guarantee the quality of mathematics learning for students. For example, many PSTs were positive about the How many petals? task after the exploring activity, explaining that the task seemed accessible and helpful in developing the place-value concept with broken-down visual models. However, most changed their evaluation after engaging with students and seeing them struggle to make sense of the connection between the symbolic and quantitative representations. In particular, the tapping is considered as grouping with dynamic representations. However, this feature might be used by children with less focus on the original function that transfers 10 one to 1 ten.

On the other hand, others often did not change their evaluation of ST Math activities across exploring and implementing phases. This result might not be surprising since PSTs' acceptance of technology is associated with multiple factors in the educational environment. Yeo et al. (2022) found various factors that influence teachers' use of ST Math in the context of mathematics instructions: perceived ease of use, attitude towards ST Math use, perceived usefulness for mathematical learning, environmental support, and learning outcomes. In other words, while the evaluation of ST Math activities is a very simple decision for PSTs, the process of such decision-making is never simple. This brings attention to further studies of how PSTs accept the use of ST Math activities with multi-layered factors that surround PSTs.

I also realize the limitation of this single case would not represent all digital resources and need further studies to investigate the impact of different types of tools. I selected ST Math for this study. However, it is possible to have different attending and interpreting components with different digital resources. In addition, Math Buddies are not a common course design for general teacher education programs. Therefore, further studies are needed to investigate PSTs' noticing with different forms of teacher education.

In this study, I illustrated how PSTs' attending and interpreting interplay with the evolution of digital mathematical recourses by using examples of ST Math tasks. These data extended the analytic framework of teachers' technological noticing from earlier studies (e.g., Smith et al., 2018). The developed framework could be used in the embedded field experiences to enhance PSTs' noticing skills. PSTs might take a critical stance on the use of digital mathematical resources based on their noticing of children's mathematical thinking or vice versa. I believe that a digital resource that PSTs have more accessibility in their placement increases the quality of noticing in terms of scope and depth. Such opportunities help PSTs better understand children's mathematical thinking through their evaluations.

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