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Modeling The Dynamics of Grit; Goal, Status, Effort & Stress (GSES)

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Abstract

Grit or perseverance as a factor for student success and life has gained increasing attention. Statistical methods have been the norm in analyzing various aspects of grit, but they do not address the transient and dynamic behavior well. We, for the first time, developed two linear dynamical models that specifically address the feedback structure of a child's desire to achieve a high grade point average (GPA) and the necessary effort that will increase stress between parents and a child. We call the dynamical model as GSES (Goal, Status, Effort & Stress). The two dynamical models incorporate the positive (i.e., achieving a high GPA) and the negative sides (i.e., effort and elevated stress and thus unhappiness) for being gritty or perseverant. Different types of parenting style and a child's characteristics were simulated whether parents and a child are empathetic or stubborn to their expectations and stress (i.e., willing or unwilling to change). Simulations show that when both parents and a child are empathetic to each other's expectation and stress, the most stable situations with minimal stress and effort occur. When a stubborn parent's and a stubborn child were studied together, this resulted in the highest elevation of stress and effort. Stubborn parents and a complying or empathetic child resulted in considerably high stress to a child. Interference from parents may unexpectedly result in a situation in which a child's stress is seriously elevated. The GSES model shows the U-shaped happiness curve (i.e., reciprocal of stress) caused by the increasing and then decreasing goal

Keywords: Mathematical Modeling, Grit, Grade Point Average(GPA), U-shaped Happiness, System Dynamics

1. INTRODUCTION

As we progress in our increasingly competitive job force, grit as a factor for student success has gained increasing attention. Angela Duckworth (1970~) and others defined grit as *perseverance* and *passion* for long-term goals as a personality characteristic [1]. These researchers claimed that students with high levels of grit are more successful in both academic and non-academic pursuits. They also stated that individuals with high grit maintain their determination and motivation over long periods of time despite the presence of failures and difficulty. A child's academic success is influenced by

several factors such as personal characteristics and social/economic environments: (1) individual differences in intelligence (i.e., higher mental ability) and personality (i.e., perseverance, conscientiousness linked to effort, and achievement motivation), (2) parenting styles (i.e., empathetic or stubborn) and parent's socio-economic status, (3) socio-economic environments such as economic inequality, racism, diversity, etc [2].

To this point, research has mainly focused on the positive impacts of grit (e.g., a child's successful achievement in academic excellence) yet have lacked to provide evidence on the adverse effects of grit (e.g., stress or depression). Since grit is a concept that has been introduced recently, it can be seen that the factors affecting grit and the factors affected by grit are in the process of being identified. Therefore, it is of utmost importance to develop the constitutional variables of grit more clearly, through various empirical studies such as longitudinal analysis [3]. Not all children may achieve academic excellence, especially in an environment in which academic performance is graded on a curve such as in a relative evaluation system, such as pass or fail. Students who excel in an absolute evaluation system may end up being graded as average in the relative evaluation system due to severe competition. Parents who put emphasis on their children achieving academic excellence in order to successfully compete in a severely competitive environment often demand objectivity in grading and evaluation systems. When the demands from parents make it difficult to apply an absolute evaluation system, this can lead to the use of multiple-choice exams, and distribution in grading. In addition to the demands of parents, severe competition can make exams and curriculum more difficult and less flexible in allowing for differentiation in a child's grade. This, in turn, could make children fail despite their increasing effort to study and gain knowledge.

Some parents who wish success in life for their children will push and pressure them to achieve perfection and to be the best in all subjects. Tiger parenting is a term which refers to strict or authoritarian parenting methods that push and/or pressure a child for perfection [4]. Similar terms include authoritarian or helicopter parenting. A child raised in a strict, controlling, and punitive environment might suffer chronic social and psychological problems. Authoritarian parenting and academic stress could lead to self-harm and has been found to be associated with high rates of suicide in East Asia [5]. In Asia, academic performance and parental expectations are the top two factors leading to stress in children [6][7].

2. OBJECTIVES

The goal of this paper is to formulate two linear dynamical models to better understand the various factors that affect academic achievement, focusing primarily on the characteristics of a child and their parent/s. We used the analogy of a child's academic achievement and applied System Dynamics to develop these two dynamical models. The first model analyzes the dynamics of a child and the second model examines the dynamics between a child and their parent/s.

In our review of the literature regarding grit, GPA achievement and stress, we found no other developed dynamic models that measure academic achievement and stress [8]. Statistical methods including various regression approaches, factor analysis, and especially structural equation modeling are frequently applied to identify the relationship among variables [9]. Factor analysis, and structural

equation modeling are useful in identifying correlated variables. Structural equation modeling has an advantage compared to various regression methods where the paths of influence among variables are specified and tested even though predictors are well correlated. One weakness of statistical methods is that they usually analyze unidirectional cause and effect relationships; input variables affect output variables, but output variables do not affect input variables [10]. Many studies on grit are cross-sectional and thus analyze a snapshot of transient behavior because observing the same children over many years would be very difficult. It is likely that children's characteristics such as grit, self-control of stress, may change overtime. More importantly, variables studied in statistical analysis do not distinguish whether they are a stock (or state variables), a flow (or rate variables), or a parameter, which is a key concept in economics and System Dynamics. The distinction between stock and flow is very important because causality exists but correlation does not and vice versa. For example, pouring water (e.g., constant inflow) into a cup increase the water level inside (e.g., stock) but the constant inflow and the increasing water level will show no correlation.

The contributions of this paper, for the first time, are the formulation of dynamical models to explore the feedback structure among goal, status, effort and stress between a child and parents and consistent classification of variables (i.e., stock, flow, parameter). Focus is on the derivation of models that will allows us to simulate the effects of various factors despite lack of longitudinal data. All the four variables (i.e., goal, status, effort and stress) are modeled as a stock and the rest are modeled as a parameter (i.e., grit or perseverance not to lower desired GPA, self-awareness of GPA gap, intellectual ability to improve GPA, self-control of stress, and maintaining effort).

3. GSES, A GENERIC MODEL OF GOAL, STATUS, EFFORT & STRESS IN LIFE

The two linear dynamical model analyze the four state variables and named as GSES_C (Goal, Status, Effort, and Stress for a Child) and GSES_PC (Goal, Status, Effort, and Stress for Parent and a Child). The first model, GSES_C, analyzes the dynamics among goal, status, effort and stress of a child. A child has a goal to achieve a high GPA. They perceive the difference between desired and actual GPA and makes effort in order to reduce the GPA gap. The effort increases a child's stress which will then dampen the effort and make a child to adjust his or her desired GPA and sustained effort. The GSES_C include seven parameters of a child; grit or perseverance not to lower the desired GPA, intellectual ability to improve one's actual GPA, self-conscious to the GPA gap, and self-control to cope with the stress and effort. The GSES_C also includes parental influence as a sinusoidal forcing function that reveals a resonance frequency, which indicates that a parent's influence could dangerously increase a child's stress. A simulation from the GSES models can produce the U-shaped happiness curve [11]. Researchers claimed that happiness follows a U-shaped trajectory, high at 20s and followed by low in 40s before starting to rise again after 60s. The U-shaped happiness curve is the results of the increasing and then decreasing goals.

The second model, GSES_PC, explores the detailed relationship between parents and a child. The stocks, flows, and parameters for parents and children are identical except in the fact that parents aren't the ones who have to improve the actual GPA, (i.e., parents require no effort to improve actual GPA). Parents' GPA expectations for their children might be different from the GPA goal a child sets

for themselves. For example, parents may want their children to be successful and achieve only *A*s while their child may think that *A*s and *B*s are good enough. Some parents may be willing, or unwilling, to lower their expectations in response to the stress created for the child and themselves. The two models do not consider the socio-economic factors that are beyond personal ability levels such as poverty, racism, economic inequality, socio-economic status of parents, political system and other obstacles.

The GSES models are generic in that they represent generalized situations including elements such as a person's ambition in life and the stress from the discrepancy between one's goal and their current status. If a person's ambition is not achieved, even after prolonged effort, this discrepancy will increase stress. Some people can handle this increased stress well while others experience negative effects such as depression. People's ambition and stress are also influenced by family, society or cultural norms that are represented as a simple sinusoidal function in this paper.

4. MATHEMATICAL MODELING

4.1 Dynamics of Goal, Status, Effort, and Stress of a Child (GSES_C)

The causal loop diagram of the GSES_C is shown in Figure 1. When a child wants to achieve the highest GPA (e.g., all A's), they remain conscious of the discrepancy between the desired and actual GPA (i.e., self-awareness of the GPA gap). Some children are more cognizant of this gap and make more effort to improve the actual GPA. Some children may improve the actual GPA with ease, while others may have to work much harder (i.e., intellectual ability) due to various reasons such as family influence, school environment, or economy in general. The increased effort will improve the actual GPA, thus will reduce the GPA gap, but also increase the stress that will make him or her lower the desired GPA. A child's effort may lead to an increase a child's stress. If a child is not able to reach their desired GPA after prolonged effort, for whatever reason, they may lower their desired GPA to avoid additional stress. If not, the individual's stress level could elevate critically and be detrimental to their well-being, possibly leading to anxiety and depression. The stock and flow diagram of the causal loop is shown in Figure 2, and identifies four stocks, four flows, and seven parameters (i.e., the *b*s in Figure 1).

Stress, anxiety, and depression are three progressing states or stocks of emotional symptoms. Some children may handle stress well and exert more self-control to manage their stress level, some children will dampen their effort and other children remain workaholics, spending a huge number of hours to obtain their goal GPA. The self-control of effort (β_5) represents whether a person is workaholic or not. The value of zero indicates a person is a workaholic and a positive value represents a person that will dampen their effort if a lot of effort is spent.
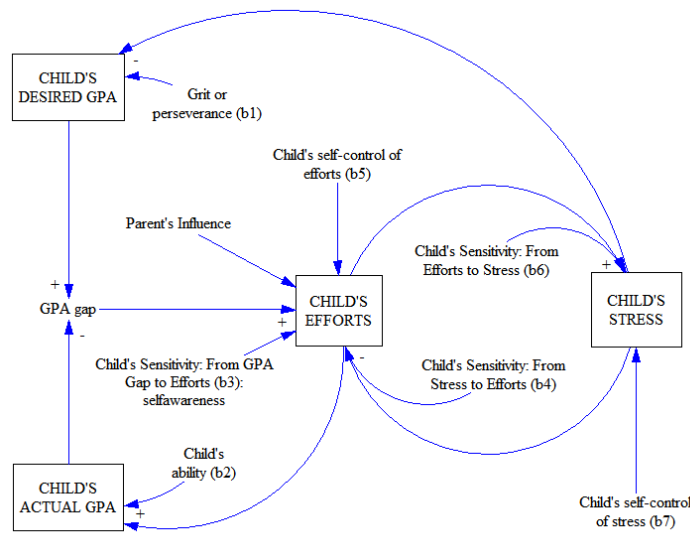
Figure 1. Causal loop diagram for goal, status, effort, and stress. Rectangular box represents a stock.
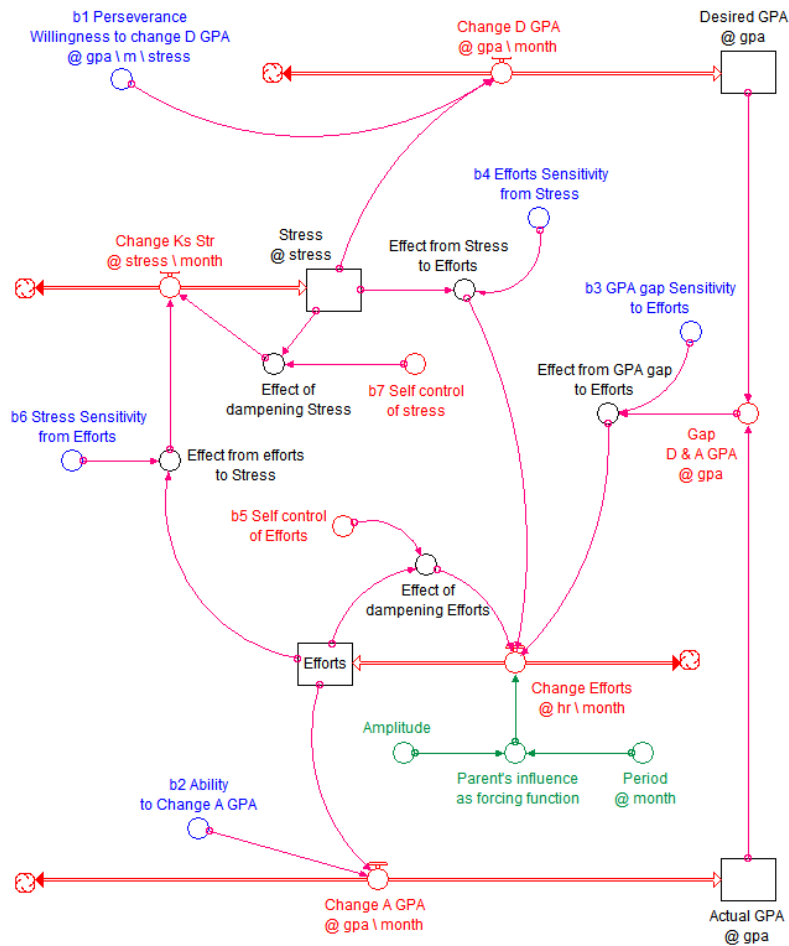

Figure 2. Stock and flow diagram of goal, status, effort, and stress (GSES) for a child. Parents' influence is included as a sinusoidal forcing function.

4.2 Equations for GSES_C for a Child

Equations for GSES_C are as follows. This is a linear, fourth-order, constant coefficient ordinary differential equation.

$$\frac{d\,DGPA_t}{dt} = -\beta_1 * STRESS_t = -perseverance * STRESS_t \tag{1}$$

$$\frac{d\,AGPA_t}{dt} = \beta_2 * EFFORT_t = ability * EFFORT_t \tag{2}$$

$$\frac{d\,EFFORT_t}{dt} = \beta_3 * (DGPA_t - AGPA_t) - \beta_4 * STRESS_t - \beta_5 * EFFORT_t \\ + Parent's\ influence \tag{3}$$

$$\frac{d\,STRESS_t}{dt} = \beta_6 * EFFORT_t - \beta_7 * STRESS_t \tag{4}$$

(1) Desired GPA (i.e., Goal):

$$\frac{d\,DGPA_t}{dt} = -\beta_1 * STRESS_t = -perseverance * STRESS_t$$

- Grit, perseverance and passion for long-term goals, can be modeled as unwillingness to lower the goal (i.e., high GPA) against raised stress. A child wants to achieve the highest GPA but will lower his desired GPA if they are under a lot of stress. β_1 represent a child's perseverance or unwillingness to lower the desired GPA in spite of high stress. A value close to zero indicates that a child is unwilling to lower the desired GPA. Positive value of β_1 indicates a child is willing to adjust the desired GPA in proportion to stress.
- The initial value of the desired GPA = 100.

(2) Actual GPA (i.e., Status):

$$\frac{d\,AGPA_t}{dt} = \beta_2 * EFFORT_t = ability * EFFORT_t$$

- Actual GPA is influenced by the amount of effort exerted and a child's ability (β_2). A value of β_2 close to zero indicates that a child takes a very long time to improve the actual GPA. Positive value of β_2 indicates a child is able to improve the actual GPA in proportion to effort.
- The initial value of the actual GPA = 80

(3) Effort:

$$\frac{d\,EFFORT_t}{dt} = \beta_3 * (DGPA_t - AGPA_t) - \beta_4 * STRESS_t - \beta_5 * EFFORT_t + Parent's\ influence \\ = +child's\ sensitivity\ to\ adjust\ efforts\ against\ GPA\ gap \\ - child's\ sensitivity\ to\ adjust\ efforts\ against\ stress$$

$$- child's\ sensitivity\ to\ lower\ efforts + Parent's\ influence$$

- A child will adjust their effort based on four factors;
- A child will increase his effort as they perceive the difference between desired and actual GPA. β_3 represents a child's sensitivity of the GPA gap to adjust his or her effort. Value of zero indicates a child is insensitive to the gap and makes no effort. Positive value of β_3 indicates a child that makes effort to improve the actual GPA.
- A child will reduce their effort as stress builds up. β_4 represents that a child tends to lower their effort proportionally as stress increases.
- A child will lower effort if they perceive that effort are beyond their tolerance. β_5 represents a child's tendency to diminish effort as their effort reach maximum limits and lead to the dampening of effort. Value of zero indicates no limits in limiting effort (e.g., workaholic).   Positive value indicates he dampens his effort proportionally.
- Parents' influence = amplitude * $sin(2 * \pi * (1/period) * time)$
    - This sinusoidal function represents parents' influence to increase a child's effort. Parents will ask their child to study more and this will make a child increase their effort.
    - Amplitude = 1, Period = 6 (month)
- Initial value of effort = 0.

(4) Stress:

$$\frac{d\ STRESS_t}{dt} = \beta_6 * EFFORT_t - \beta_7 * STRESS_t$$
$$= Stress\ increased\ due\ to\ efforts - dampening\ stress$$

- As a child makes more effort, their stress increases, and then they try to manage the stress. β_6 represents a child's sensitivity in which as effort increases, stress also increase. A value of zero indicates a child's stress does not increase as more effort is exerted. Positive value indicates a child's stress increased as more effort was made. β_7 represents a child's self-control of stress. A value of zero indicates a child does not have any self-control to dampen the stress. Positive values indicate a child has some self-control to dampen the stress.
- Initial value of stress = 0

4.3 Simulations of GSES_C

Various characteristics of a child were simulated with varying parameter values. The case of the perfect child is shown in Figure 3, one who is capable of increasing the actual GPA and exerts self-control of stress and dampens effort; *b1 (perseverance) = 0.00, b2 (ability to change actual GPA) = 0.10, b3 (sensitivity to GPA gap =0.10), b4(sensitivity from stress) = 0.05, b5 (self-control of effort) = 0.05, b6 (stress sensitivity from effort) = 0.05, b7 (self-control of stress) = 0.05*. This child is able to reach the desired GPA level with some effort and stress.
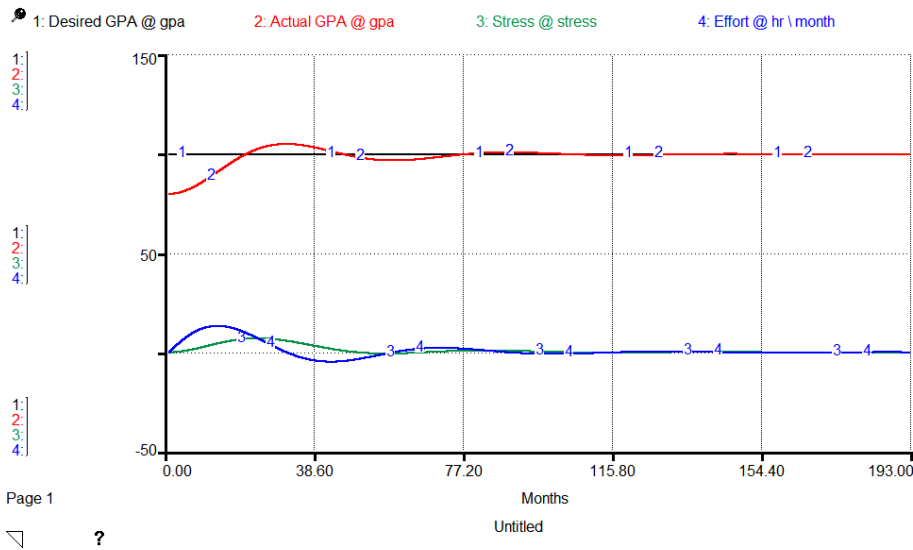
Figure 3. Simulation 1 for a perfect child

The second case (see Figure 4, *b1 = 0.00, b2 = 0.10, b3 = 0.10, b4 = 0.10, b5 = 0.00, b6 = 0.10, b7 = 0.00*) involves a child with a good aptitude to improve the actual GPA but lacking self-control of effort and ability to manage stress, unwilling to lower the desired GPA, and sensitive to GPA gap. A child's desired GPA is achieved and exceeded but the effort and stress continuously oscillate at an elevated level (+/-20).
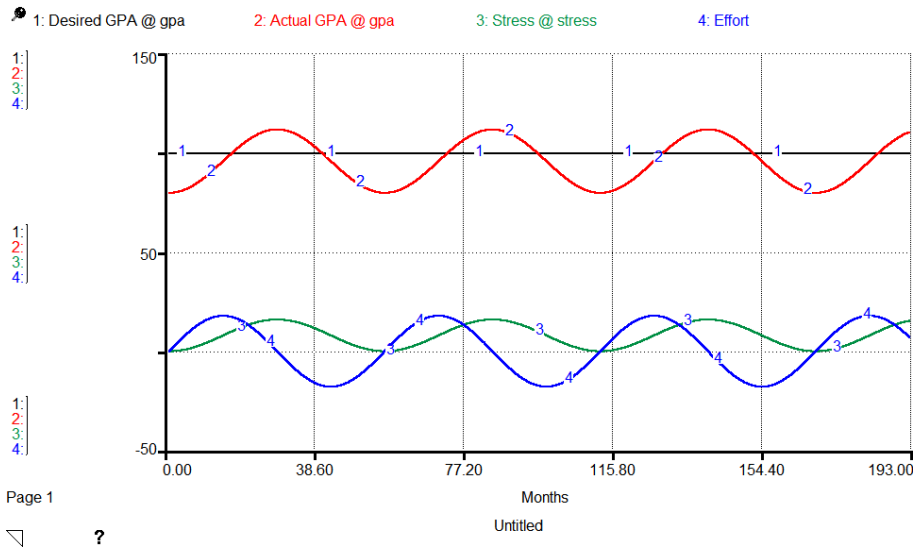

Figure 4. Simulation 2. A child has a good ability to improve the actual GPA but no good self-control of effort and stress, unwilling to lower the desired GPA, and sensitive to GPA gap.

The third case is shown in Figure 5 (*b1 = 0.00, b2 = 0.00, b3 = 0.10, b4 = 0.10, b5 = 0.00, b6 = 0.10, b7 = 0.00*), in which a child does not have the ability to improve actual GPA, doesn't restrict effort, doesn't lower the desired GPA, are sensitive to a GPA gap, and has low self-control of stress. The desired GPA is not

achieved, and stress and effort oscillate forever at a higher elevated level (+/-45).
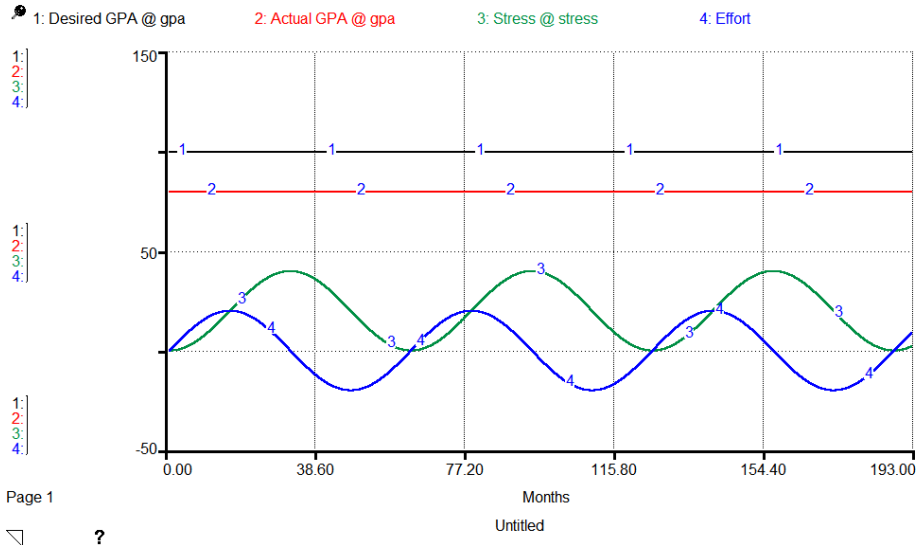

Figure 5. Simulation 3. A child with no ability, unwillingness to lower desired GPA, sensitive to GPA gap and low self-control of stress and do not restrict his effort.

The fourth case of a typical child is shown in Figure 6 *(b1 = 0.05, b2 = 0.05, b3 = 0.05, b4 = 0.10, b5 = 0.05, b6 = 0.10, b7 = 0.05)*, in which a child with mid-ability has some willingness to lower the desired GPA, mid-sensitive to GPA gap and mid-self-control of stress and effort. A child's desired GPA is lowered, and they reach a steady-state with moderate oscillation.
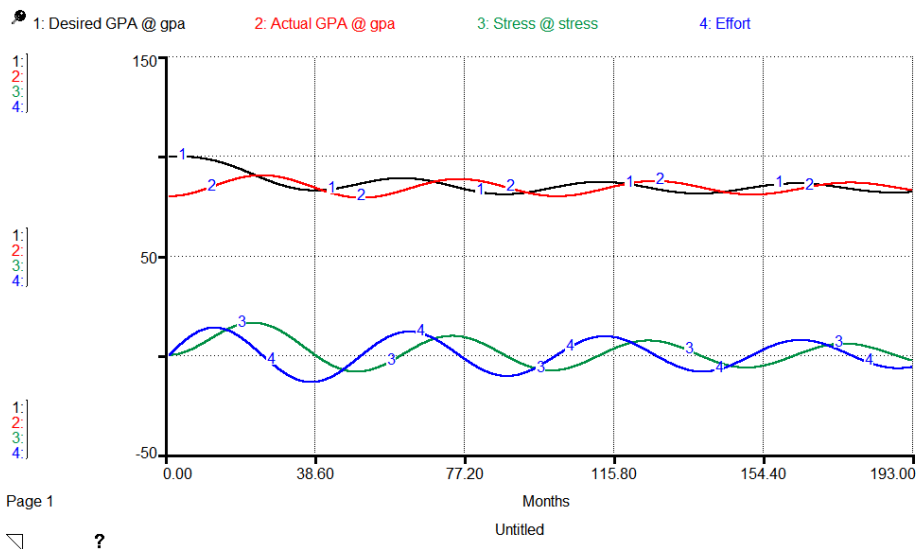

Figure 6. Simulation 4. A child with mid-ability, somewhat willing to lower desired GPA, mid-sensitive to GPA gap and mid-self-control of stress and effort.

The last case of a dangerous situation is shown in Figure 7 *(b1 = 0.10, b2 = 0.05, b3 = 0.10, b4 = 0.10, b5 = 0.00, b6 = 0.10, b7 = 0.05)*, in which a child is quick to adjust the desired GPA level and has no self-control of effort (e.g., workaholic) and stress. A child's actual GPA exceeds the desired GPA but stress, and effort

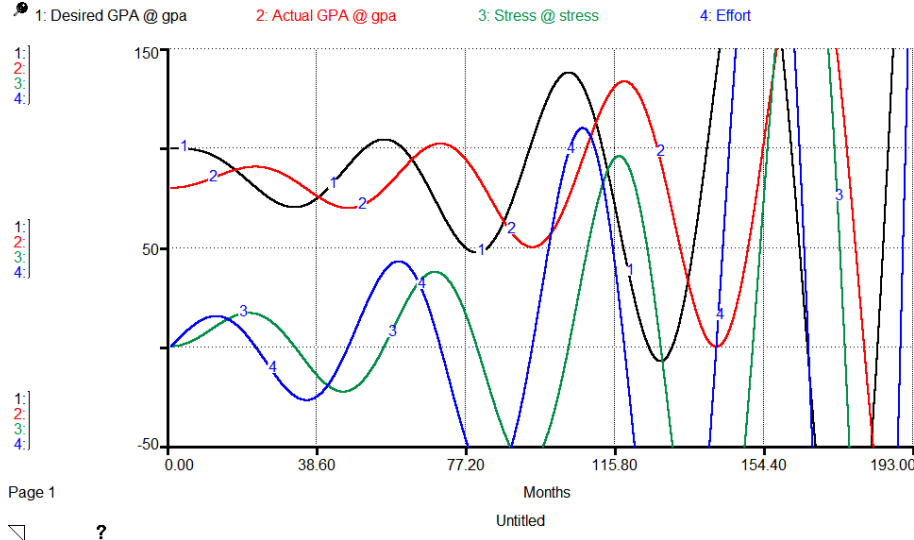oscillate wildly. This case might show a situation in which stress progresses to anxiety and even depression.

Figure 7. Simulation 5. A child is quick to adjust the desired GPA and has no self-control of effort and stress.

In summary, a few observations from simulations are as follows. First, if we define success in academics as achieving a desired GPA level with manageable stress and effort, then success in academics and thus life is the result of combined effects of various factors; (1) Grit or unwilling to lower the desired GPA. (2) Intellectual ability to improve the actual GPA. (3) Self-control of stress. (4) Maintain effort in spite of prolonged effort. (5) self-awareness of the GPA gap. (6) Elevated effort do not increase stress much. (7) Elevated stress does not decrease effort. Children who have all seven characteristics surely achieve their goals. Second, unwilling to change DGPA (i.e., grit parameter (b1) is close to zero) is much more stable than willing to change DGPA. If a child is willing to adjust DGPA quickly, more fluctuations occur from DGPA, AGPA, effort, and stress. This result might suggest a situation being gritty is more important than having more intellectual ability; {grit (b1 = 0.0) & ability (b2 = 0.05)} vs. {grit (b1 = 0.05) & ability (b2 = 0.10)}. The latter case of having more intellectual ability but willing to change DGPA show the four state variables fluctuate wildly. Third, managing stress than managing effort resulted in higher AGPA (i.e., self-control of stress (b7) vs. self-control of effort (b5)).   In other words, more effort and managing stress is more stable than little effort and unmanaging stress. Base case: *b1 (perseverance) = 0.05, b2 (ability to change actual GPA) = 0.10, b3 (sensitivity to GPA gap =0.10), b4(sensitivity from stress) = 0.10, b5 (self-control of effort) = 0.025, b6 (stress sensitivity from effort) = 0.10, b7 (self-control of stress) = 0.025.*

4.4 Parents Influence on a Child's GPA and Stress

Parents want the very best for their children and will do what's needed in order for their children to be successful (e.g., "You have to study hard to enter the best college and to have the best job", private lessons on sports and music, international travel, internship in important company, networking, etc.). However, parents' influence may considerably increase a child's effort and stress levels if the interactions with their children are in synchronization with children's stress (known as resonance frequency in the mass-spring-damper system). The influence of parents who ask a child to make more effort is modeled as a sinusoidal forcing function (i.e., $amplitude * SIN (2 * \pi * (1/period) * time)$).

The result is shown in Figure 8, in which the x-axis is the varying periods by parents' influence and the y-axis is the stress level of the child. The amplitude of the sinusoidal function remains unchanged but only the period (frequency = 1/period) is changed from 1 month to 80 months with an increment of 3 months. As parents make more frequent requests (e.g., nagging) to increase effort, a child's stress is increased considerably from +/- 20 to +/- 80, (dimensionless but 4 times larger in magnitude than typical situation) around the periods of 36 and 46 weeks. Note that the resonance frequency around 40 weeks is an imaginary situation because the parameter values are not calibrated to represent an actual situation. It is observed in simulations that the resonance period becomes shorter (period: 40 → 25 weeks) as a child becomes more sensitive to the GPA gap (e.g., sensitivity: 0.1 → 0.2). The resonance frequency explains a situation in which a child's stress increases considerably and unexpectedly during usual conversations with parents, such as at a dinner table. Another example would be if parents have expressed their concerns habitually, but the child responds hysterically.
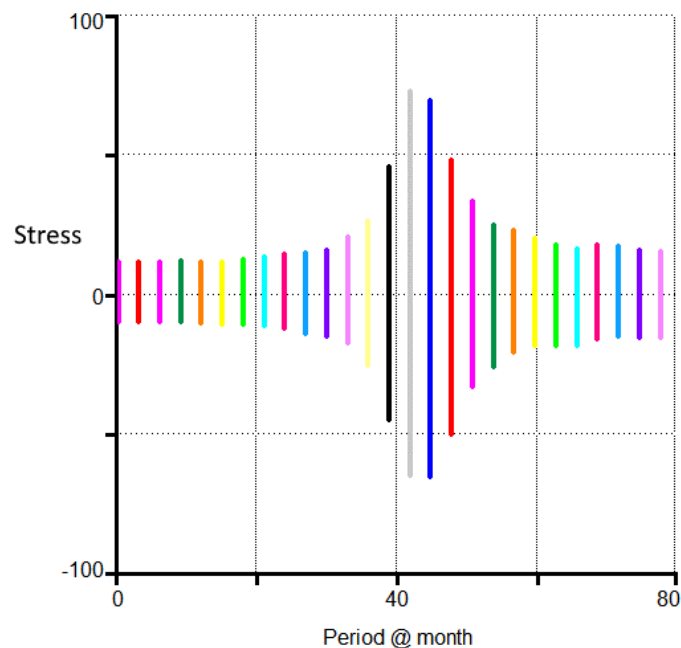

Figure 8. Stress vs. period of parents' influence (x-axis). Around the period of 40 to 50, a child's stress is highly elevated from +/- 20 to +/-80, about 4 times more magnitude.

4.5 Dynamics of Expectation between a Child and Parents (GSES_PC)

The GSES_C model of a child is expanded to include interactions with parents in detail as shown in Figure 9 (GSES_PC). A child has four state variables (a child's desired GPA, the actual GPA, effort and stress) and a parent has two state variables (parents' desired GPA level and stress). In total, 16 parameters are specified; 10 parameters for a child and 6 parameters for parents. Parents have the same structure as a child except that parents do not have to exert effort to improve an actual GPA. Parents expect their child to target the highest GPA, are sensitive to the GPA expectation differences between a parent and their child (e.g., All A+ vs. As and Bs are good enough), have self-control of stress but don't need to make any effort. For various simulations, parents' desired GPA level is 100, while a child thinks that 90 is good enough and the actual GPA is 80. Note that we assume a reinforcing feedback loop between a child's stress and parents' stress. A child's stress will increase parents' stress which will increase a child's stress in turn. Equations for GSES_PC are shown in Table

1.

Figure 9. Stock and flow diagram of GSES_PC between parent and a child

Table 1. Equations for GSES_PC. P stands for parents and K stands for a child or kid

Modeling equations of GSES_PC	No
$$\frac{d\,DGPA_t^K}{dt} = \beta_1 * STRESS_t^P - \beta_2 * STRESS_t^K$$	(5)
$$\frac{d\,DGPA_t^P}{dt} = -\beta_3 * STRESS_t^P - \beta_4 * STRESS_t^K$$	(6)
$$\frac{d\,AGPA_t^K}{dt} = \beta_5 * EFFORTS_t^K$$	(7)
$$\frac{d\,EFFORTS_t^K}{dt} = \beta_6 * (DGPA_t^K - AGPA_t^K) - \beta_7 * EFFORTS_t^K - \beta_{16} * STRESS_t^K$$	(8)
$$\frac{d\,STRESS_t^K}{dt} = \beta_8 * EFFORTS_t^K + \beta_9 * (DGPA_t^P - DGPA_t^K)$$ $$+\beta_{10} * STRESS_t^P - \beta_{11} * STRESS_t^K$$	(9)
$$\frac{d\,STRESS_t^P}{dt} = -\beta_{12} * (DGPA_t^P - DGPA_t^K) + \beta_{13} * (DGPA_t^P - AGPA_t^K)$$ $$+\beta_{14} * STRESS_t^K - \beta_{15} * STRESS_t^P$$	(10)

4.6 Simulations of GSES_PC

   Four different cases consider the characteristics of parents and their child and how they respond to expectations and stress: stubborn or empathetic parents and a child. Simulation results show that empathetic parents who adjust their expectation of GPA is a key in reducing the child's stress even though a child is stubborn or not (Figure 13 and Figure 16).
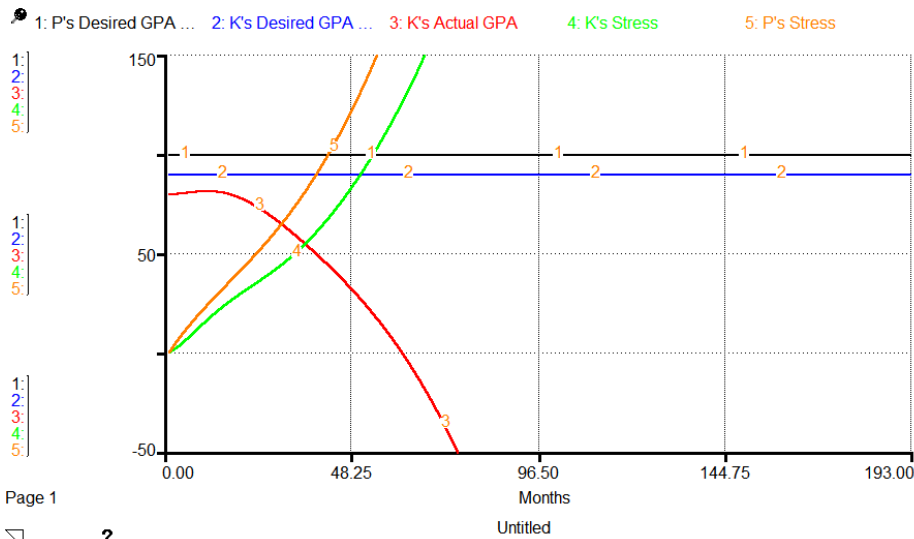

Figure 10. Simulation 1a.   Stubborn parents and a stubborn child.
The parameter values of b1, b2, b3, and b4 are set to zero.

   The first case considers that both parents and a child are unwilling to adjust their desired GPA against each

other's stress. Parents do not lower their expectation of the desired GPA level (i.e., All A+). A child also does not adjust their desired GPA (i.e., A or B are good enough). Parent's desired GPA and a child 's desired GPA remain unchanged during this simulation ($b1 = b2 = b3 = b4 = 0.00$). Simulation results show that their stress explodes without bound and the actual GPA plummeted (Figure 10). Another simulation allows tiny reduction in their stubbornness ($b1 = b2 = b3 = b4 = 0.01$) and both parents' and a child's stress vary considerably to a dangerous level (Figure 11).
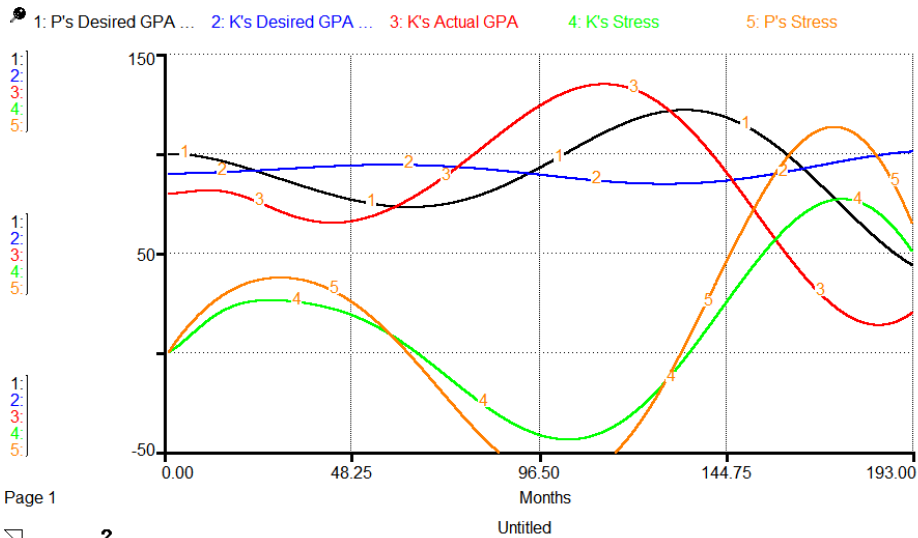

**Figure 11. Simulation 1b.   Stubborn parents and a stubborn child.
The parameter values of b1, b2, b3, and b4 are set to 0.01.**

**Figure 12. Simulation 2a.   Empathetic parents and a stubborn child
(b1 = b2 = b4 = 0, and b3 = 0.1).**

The second case considers two situations for empathetic parents and a stubborn child. Parents adjust their expectation in response to a child's stress, but a child is stubborn and does not lower his desired GPA in response to a parent's stress and expectation. The simulation shows that parents' desired GPA oscillates

moderately and reaches to steady-state slowly. A child's actual GPA of 80 reaches a child's desired GPA of 90 and reaches to steady-state slowly (Figure 12 where *b1 = b2 = b4 = 0, and b3 = 0.1*; Figure 13 where *b1 = b2 = 0* and *b3 = b4 = 0.1*)
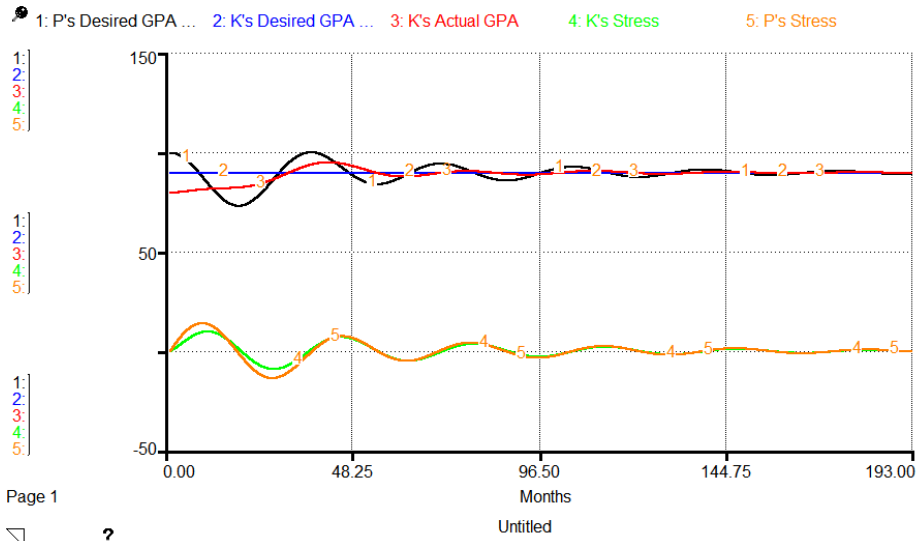

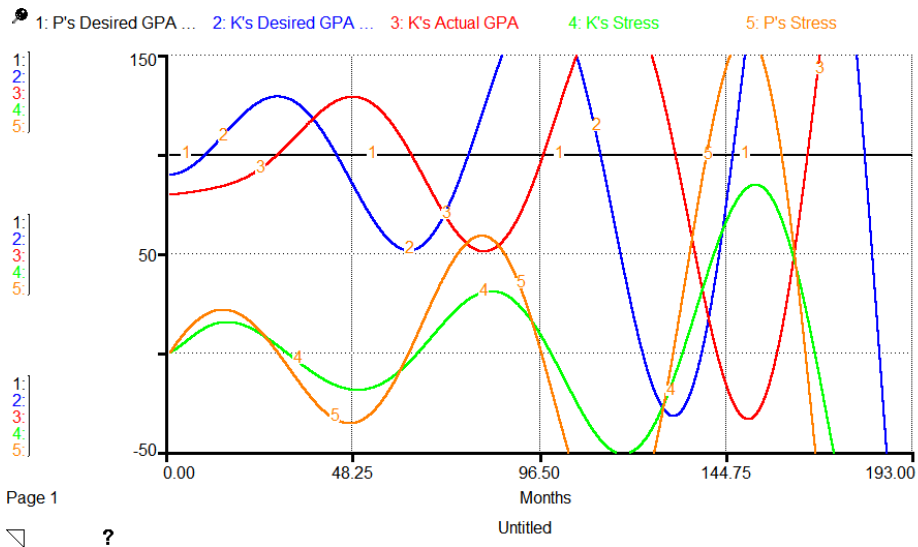**Figure 13. Simulation 2b.   Empathetic parents and a stubborn child (b1 = b2 = 0, and b3 = b4 = 0.1).**

Figure 14. Simulation 3a.   Stubborn parents and an empathetic child who is sensitive to parents' stress but insensitive to his stress (b1 = 0.1, and b2 = b3 = b4 = 0.0).

The third case considers a situation of stubborn parents and an empathetic child. Parents do not adjust their expectation in response to a child's stress, representing authoritarian parenting style. However, a child adjusts his effort in response to a parent's stress and expectation. The stress for both parents and a child as well as actual and desired GPA oscillate to a dangerous level. The simulation results in Figure 14 shows the case where a child adjust his effort in response to parent's stress but ignores his stress (*b1 = 0.1 and b2 = b3 = b4 = 0.0*). The child ignores his stress and it oscillates dangerously. The next simulation shows the case where a child

increases his effort in response to parent's stress and does not ignore his stress (Figure 15, *b1 = b2 = 0.1, b3 = b4 = 0.0*). The stress for both parents and a child oscillates less dangerously but the stress is still very high.

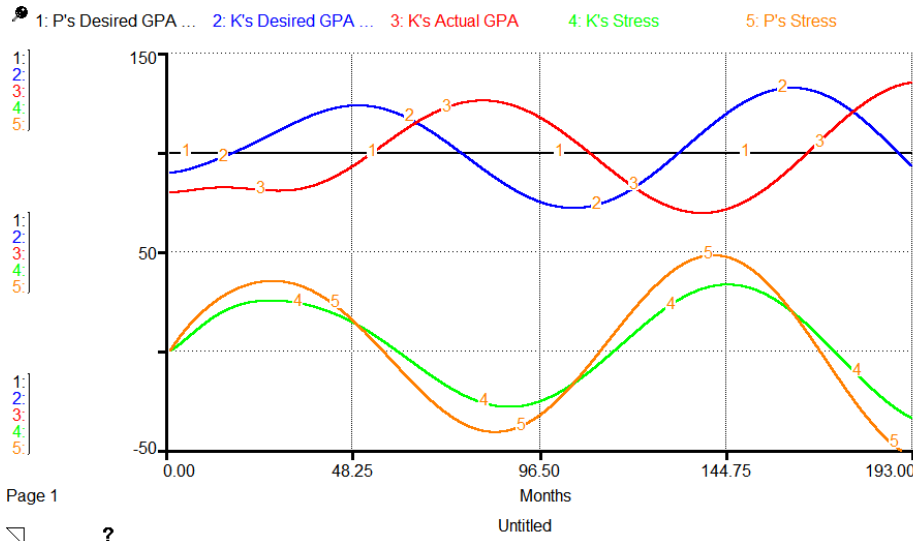**Figure 15.   Simulation 3b.   Stubborn parents and an empathetic child who is sensitive to both parent's and his stress (b1 = b2 = 0.1 and b3 = b4 = 0.0).**

The fourth case considers the situation where both parents and a child are empathetic to each other's stress. Both parents and a child adjust their expectation in response to each other's stress. (Figure 16 with *b1 = b2 = b3 = b4 = 0.1*). The results show that parent's desired level and a child's desired level meet in the middle. Stress and effort oscillate but reach to steady-state quickly.
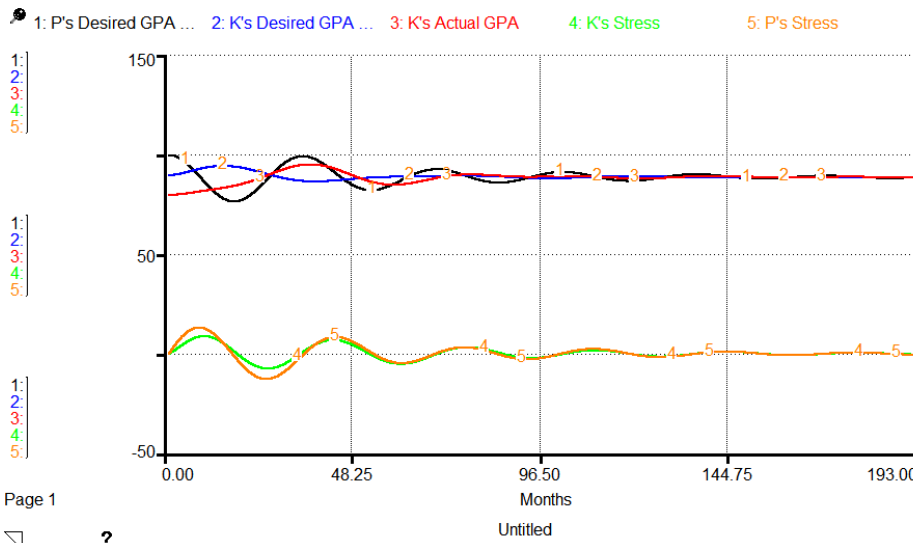

Figure 16. Simulation 4.   Empathetic parents and an empathetic child.

4.7 U-Shaped Happiness Curve caused by the Increasing and then Decreasing Goals

Researchers have claimed to find increasing evidence that happiness through adulthood shows generalized

U-shaped, with the lowest point at around 47 and 48 years old and happier at earlier and later in life [12]. Their analysis was based on surveyed data across 132 countries to determine the relationship between well-being and age. (such as "All things considered, how would you say that you are happy, neutral, or unhappy?"). One possible explanation for the U-shaped happiness curve is that people set goals very high (e.g., accepted to a dream university or job) at earlier ages, pursue their goals, achieve some success, and then may learn to lower their expectations to cope with difficulties and manage the stress.

We modified the GSES model in order to understand the U-shape happiness curve. The desired GPA is set to follow a hill-shape pattern (or triangle shape). Desired GPA increases to the highest value of 130 and then decreases to its initial value of 100.   Happiness is modeled as the reciprocal of the stress.

$$\frac{d \, DGPA_t^K}{dt} = \beta_1 * STRESS_t^K + \ \text{STEP}(0.5, 20) + \ \text{STEP}\,(-1, 80) + \ \text{STEP}(0.5, 140).$$

$$Happiness_t \ = \frac{1}{Max(1, STRESS_t)}$$

Simulation shows that stress from not achieving the desired GPA increases and then falls, showing the U-shape happiness over time as shown in Figure 17. This simulation result may support the hypothesis that the U-shape in life satisfaction is driven by the unmet goals and then people may learn to cope with difficulties and stress by lowering their expectation.
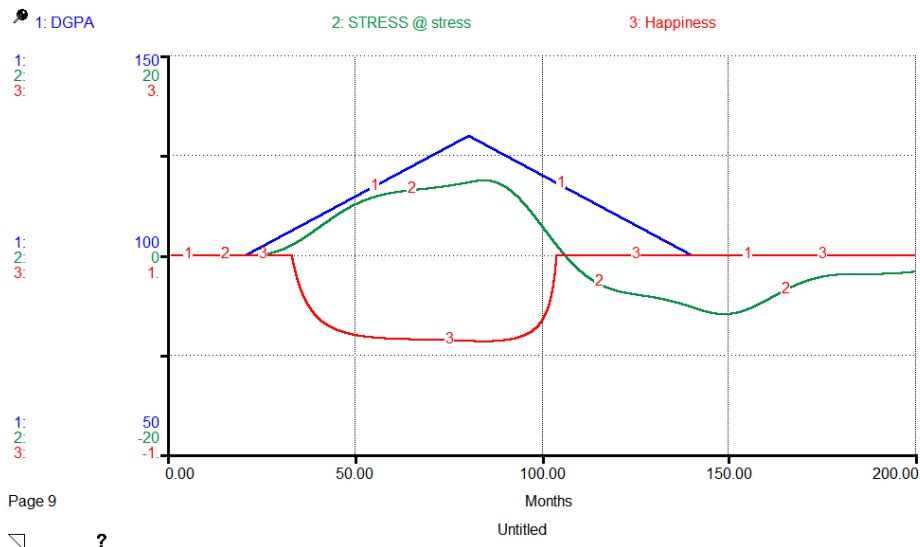

Figure 17.   The increasing and then decreasing goal (i.e., desired GPA or DGPA) caused the U-shaped happiness curve (defined as the reciprocal of stress).

5. DISCUSSION

5.1 Summary of Findings

The simulation results illustrate that success in academics is a combined effect of many factors; a person's ability to improve their current status, perseverance of not lowering the level of ambition, self-control of stress and effort, self-awareness to the gap between a goal and status. The self-control of alleviating stress and dampening effort are important parameters to prevent from exploding stress and effort. The self-control of stress and effort make the GSES models stable with less oscillation in stress and effort. Other parameters except for the self-control of stress and effort make the system oscillate. The GSES model demonstrated that

the U-shape happiness is caused by the unmet goals and stress and then people may have to cope with stress by lowering their expectation.

The four different characteristics of parents and a child reveal that empathetic parents and an empathetic child result in the least amount of stress and effort, followed by empathetic parents with a stubborn child. Stubborn parents with a stubborn child make the stress explode. Stubborn parents and a complying child show a dangerous level of oscillation in stress, which may represent the authoritarian parents and complying child. Empathetic parents who adjust their expectation of GPA resulted in lowering the child's stress even though a child is stubborn or not.

5.2 Weakness

The weakness of the two models is that GSES_C and GESE_PC are linear, in which all parameters are exogenous, constant and the flows are proportional to state variables. Due to the facts that the functional forms are not known in the literature (e.g., how quickly effort increase stress and actual GPA), we assume all the relationships are linear as a first step. Also, there is not much agreement on the feedback structures (e.g., effort increase stress which will decrease effort), and variables types (e.g., whether grit is a stock, a flow, or a parameters). In our analysis, the four variables (goal, status, effort, and stress) are treated as a stock while other factors such as grit and intellectual ability are treated as constant and exogenous parameters. Surely, a child's and parents' ability and perseverance will change over time (i.e., a stock) and become nonlinear. As a child grows, their ability and perseverance may change, as well as their self-control of stress, level of studying and work habits. A child can handle a moderate level of stress but could become very sensitive beyond a certain stress level.

Undoubtedly, the relationships among various factors are nonlinear in life and non-linear models need to be developed in the future. One interesting functional form is to use Cobb-Douglas type production functions widely used in economics [13], $\frac{d\ EFFORT}{dt} = (DESIRED\ GPA - ACTUAL\ GPA)^{\beta_3} * STRESSS^{\beta_4} * EFFORTS^{\beta_5}$. The β_s represent the individual contribution of each state variables to the effort and have the diminishing effect on effort. For example, improving the GPA from 70 to 80 is relatively easy but from 95 to 100 would be very difficult. Another interesting functional form is to use a logistic function both for the effort and the stress; $i.e., \frac{dX}{dt} = rX - \frac{rX^2}{M}$. The logistic functions incorporate the maximum or minimum threshold to exert effort and manage stress by parents and a child. For example, the following two equations might be of interest. The "max effort" and the "max stress" represent a child's threshold to exert effort and manage stress.

$$\frac{d\ EFFORT_t}{dt} = \beta_3 * (DGPA_t - AGPA_t) - \beta_4 * STRESS_t - \beta_5 * \frac{EFFORT_t^2}{MaxEffort}$$

$$\frac{d\ STRESS_t}{dt} = \left\{\beta_6 * EFFORT_t - \beta_7 * \frac{STRESS_t^2}{MaxStress}\right\}$$

Another improvement is to make the parameters in GSES models, especially the grit parameter, endogenous where the parameters are no longer constant and change over time. For example, the grit or perseverance parameter can be modeled as a stock applying the Cobb-Douglas production function. The following may represent how the stress, effort, and the gap between desired and actual GPA influence the level of grit of a child over time.

$$GRIT = f(GPA\ gap, Efforts, Stress) = (GPA\ Gap)^{\beta_1} * EFFORTS^{\beta_2} * STRESSS^{\beta_3}$$

The two models developed in this research do not consider the influence of social and economic situations such as poverty, racism, economic inequality, socio-economic status of parents, political system and other obstacles beyond a person's character. It is highly likely that an individual is unable to overcome such obstacles imposed upon them from a social and economic system. The two models do not consider additional characteristics of individuals such as integrity, empathy, courage, curiosity, responsibility, self-efficacy, etc.

As far as the authors are aware, these two models are the first dynamical models which characterize a child's and parent' personality and ability, in combination with goal, status, effort, and stress. The GSES_C and GSES_PC are generic models that can be applied to various situations. The GSES analyzed the characteristics of individual differences at a personal level and a future model that combines the social-economical system with individual differences may be useful.

6. CONCLUSION

We reviewed the literature on grit, GPA, and stress and found no other developed dynamic models measuring GPA and stress. Therefore, we formulated two linear dynamical models that focus on child and parental characteristics by using the analogy of a child's GPA and applying system dynamics.

The first model analyzes the dynamics of the child, while the second examines the dynamics between the child and the parent. In the first model, academic success is defined as achieving a desired GPA level with manageable stress and effort, and the simulations show that academic success, and by extension, success in life, is a composite of seven factors: perseverance, ability to change actual GPA, sensitivity to GPA gap, sensitivity from stress, self-control of effort, stress sensitivity from effort, and self-control of stress. Simulations of the second model showed that empathetic parents and empathetic children result in the least stress and effort, followed by empathetic parents and assertive children, and that assertive parents and assertive children result in explosive increases in stress, and assertive parents and compliant children show dangerous levels of stress oscillations that may be representative of authoritarian parents and compliant children.

Although the two models developed in this study are limited in that they do not take into account the impact of social and economic circumstances such as poverty, racism, economic inequality, parental socioeconomic status, the political system, and other obstacles beyond individual personality, both models can be used to analyze factors that affect children's academic achievement, as they are the first dynamic models that characterize children's and parents' personalities and abilities along with goal, status, effort, and stress, and are general models that can be applied to a variety of situations.

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