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NEW KINDS OF OPEN MAPPINGS VIA FUZZY NANO M-OPEN SETS

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Abstract. In this paper, we introduce the concept of fuzzy nano M open and fuzzy nano M closed mappings in fuzzy nano topological spaces. Also, we study about fuzzy nano M Homeomorphism, almost fuzzy nano M totally mappings, almost fuzzy nano M totally continuous mappings and super fuzzy nano M clopen continuous functions and their properties in fuzzy nano topological spaces. By using these mappings, we can able to extended the relation between normal spaces and regular spaces in fuzzy nano topological spaces.

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1. Introduction

In 1965, Zadeh [18] made his significant theory on fuzzy sets. Later, fuzzy topology was introduced by Chang [1]. Pawlak [8] introduced Rough set theory by handling vagueness and uncertainty. This can be often defined by means of topological operations, interior and closure, called approximations. In 2013, Lellis Thivagar [4] introduced an extension of rough set theory called nano topology and defined its topological spaces in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it.

S. Saha [9] defined δ-open sets in fuzzy topological spaces, nano topological space by Pankajam et al. [7] and neutrosophic topological space by Vadivel et al. [12, 13, 15, 16, 17]. Recently, Lellis Thivagar et al. [5] explored a new concept of neutrosophic nano topology, intuitionistic nano topology and fuzzy nano topology. El-Maghrabi and Al-Juhani [2] proposed the concept of M-open sets in topological spaces in 2011 and examined some of their features. Padma et al. [6] also found M-open sets in nano topological spaces. Thangammal et al.

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[10, 11] introduced fuzzy nano Z-open sets and Kalaiyarasan et al. [3] introduced normal spaces associated with fuzzy nano M-open sets in fuzzy nano topological spaces and their applications.

Research Gap: No investigation on some new mappings such as fuzzy nano M open, fuzzy nano M closed mappings, fuzzy nano M Homeomorphism, almost fuzzy nano M totally mappings, almost fuzzy nano M totally continuous mappings and super fuzzy nano M clopen continuous functions on fuzzy nano topological space has been reported in the fuzzy literature. Also, we can able to extended the relation between normal spaces and regular spaces using these mapping in fuzzy nano topological spaces.

In this paper we introduce fuzzy nano M open and fuzzy nano M closed mappings in fuzzy nano topological spaces. Also, we study about fuzzy nano M Homeomorphism, almost fuzzy nano M totally mappings, almost fuzzy nano M totally continuous mappings and super fuzzy nano M clopen continuous functions and discuss their properties in $\mathcal{F}\mathfrak{Nano}ts$'s.

2. Preliminaries

The basic definitions of fuzzy sets and their properties are defined in [18]. The definitions of fuzzy nano lower approximation (briefly, $\underline{\mathcal{F}\mathfrak{Nano}}(F)$), fuzzy nano upper approximation(briefly, $\overline{\mathcal{F}\mathfrak{Nano}}(F)$), fuzzy nano boundary (briefly, $B_{\mathcal{F}\mathfrak{Nano}}(F)$), fuzzy nano topological space (briefly, $\mathcal{F}\mathfrak{Nano}ts$), fuzzy nano open (briefly, $\mathcal{F}\mathfrak{Nano}o$) sets and fuzzy nano closed (briefly, $\mathcal{F}\mathfrak{Nano}c$) sets are defined in [5].

Definition 2.1. [10, 11, 14] Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathfrak{Nano}ts$ with respect to F where F is a fuzzy subset of U. Then a fuzzy subset S in U is said to be a fuzzy nano

(i) interior of S (briefly, $\mathcal{F}\mathfrak{Nano}int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}int(S) = \bigvee\{I : I \leq S \;\&\; I$ is a $\mathcal{F}\mathfrak{Nano}o$ set in $U\}$.

(ii) closure of S (briefly, $\mathcal{F}\mathfrak{Nano}cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}cl(S) = \bigwedge\{A : S \leq A \;\&\; A$ is a $\mathcal{F}\mathfrak{Nano}c$ set in $U\}$.

(iii) regular open (briefly, $\mathcal{F}\mathfrak{Nano}ro$) set if $S = \mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}cl(S))$.

(iv) regular closed (briefly, $\mathcal{F}\mathfrak{Nano}rc$) set if $S = \mathcal{F}\mathfrak{Nano}cl(\mathcal{F}\mathfrak{Nano}int(S))$.

(v) δ interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\delta int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\delta int(S) = \bigvee\{I : I \leq S \;\&\; I$ is a $\mathcal{F}\mathfrak{Nano}ro$ set in $U\}$.

(vi) δ closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\delta cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\delta cl(S) = \bigwedge\{A : S \leq A \;\&\; A$ is a $\mathcal{F}\mathfrak{Nano}rc$ set in $U\}$.

(vii) semi open (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{S}o$) set if $S \leq \mathcal{F}\mathfrak{Nano}cl(\mathcal{F}\mathfrak{Nano}int(S))$.

(viii) pre open (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{P}o$) set if $S \leq \mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}cl(S))$.

(ix) δ pre open (briefly, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$) set if $S \leq \mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}\delta cl(S))$.

(x) pre interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{P}int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\mathcal{P}int(S) = \bigvee\{I : I \leq S \;\&\; I$ is a $\mathcal{F}\mathfrak{Nano}\mathcal{P}o$ set in $U\}$.

(xi) pre closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{P}cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\mathcal{P}cl(S) = \bigwedge\{A : S \leq A \;\&\; A$ is a $\mathcal{F}\mathfrak{Nano}\mathcal{P}c$ set in $U\}$.

(xii) δ pre interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}int(S) = \bigvee\{I : I \leq S$ & I is a $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ set in $U\}$.

(xiii) δ pre closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}cl(S) = \bigwedge\{A : S \leq A$ & A is a $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}c$ set in $U\}$.

The complement of the respective fuzzy nano open sets are called as fuzzy nano closed sets.

3. Fuzzy nano M open map and fuzzy nano M closed map

In this section, we introduce fuzzy nano M open maps and fuzzy nano M closed maps in $\mathcal{F}\mathfrak{Nano}ts$ and obtain certain characterizations of these classes of maps.

Definition 3.1. [3] Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathfrak{Nano}ts$ with respect to F where F is a fuzzy subset of U. Then a fuzzy subset S in U is said to be a fuzzy nano

(i) θ interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta int(S) = \bigvee\{\mathcal{F}\mathfrak{Nano}int(I) : I \leq S$ & I is a $\mathcal{F}\mathfrak{Nano}c$ set in $U\}$.

(ii) θ closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta cl(S) = \bigwedge\{\mathcal{F}\mathfrak{Nano}cl(A) : S \leq A$ & A is a $\mathcal{F}\mathfrak{Nano}o$ set in $U\}$.

(iii) θ open (briefly, $\mathcal{F}\mathfrak{Nano}\theta o$) set if $S = \mathcal{F}\mathfrak{Nano}\theta int(S)$.

(iv) θ semi open (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$) set if $S \leq \mathcal{F}\mathfrak{Nano}cl(\mathcal{F}\mathfrak{Nano}\theta int(S))$.

(v) θ pre open (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}o$) set if $S \leq \mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}\theta cl(S))$.

(vi) θ semi interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}int(S) = \bigvee\{I : I \leq S$ & I is a $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ set in $U\}$.

(vii) θ semi closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}cl(S) = \bigwedge\{A : S \leq A$ & A is a $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}c$ set in $U\}$.

(viii) θ pre interior of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}int(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}int(S) = \bigvee\{I : I \leq S$ & I is a $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}o$ set in $U\}$.

(ix) θ pre closure of S (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}cl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}cl(S) = \bigwedge\{A : S \leq A$ & A is a $\mathcal{F}\mathfrak{Nano}\theta\mathcal{P}c$ set in $U\}$.

(x) M-open (briefly, $\mathcal{F}\mathfrak{Nano}Mo$) set if $S \leq \mathcal{F}\mathfrak{Nano}cl(\mathcal{F}\mathfrak{Nano}\theta int(S)) \vee \mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}\delta cl(S))$,

(xi) M-closed (briefly, $\mathcal{F}\mathfrak{Nano}Mc$) set if $\mathcal{F}\mathfrak{Nano}int(\mathcal{F}\mathfrak{Nano}\theta cl(S)) \wedge \mathcal{F}\mathfrak{Nano}cl(\mathcal{F}\mathfrak{Nano}\delta int(S)) \leq S$.

(xii) M interior of S (briefly, $\mathcal{F}\mathfrak{Nano}Mint(S)$) is defined by $\mathcal{F}\mathfrak{Nano}Mint(S) = \bigvee\{I : I \leq S$ & I is a $\mathcal{F}\mathfrak{Nano}Mo$ set in $U\}$.

(xiii) M closure of S (briefly, $\mathcal{F}\mathfrak{Nano}Mcl(S)$) is defined by $\mathcal{F}\mathfrak{Nano}Mcl(S) = \bigwedge\{A : S \leq A$ & A is a $\mathcal{F}\mathfrak{Nano}Mc$ set in $U\}$.

The complement of the respective fuzzy nano open sets are called as fuzzy nano closed sets.

The family of all $\mathcal{F}\mathfrak{Nano}Mo$ (resp. $\mathcal{F}\mathfrak{Nano}Mc$) sets of a space $(U, \tau_{\mathcal{F}}(F))$ will be as always denoted by $\mathcal{F}\mathfrak{Nano}MO(U, A)$ (resp. $\mathcal{F}\mathfrak{Nano}MC(U, A)$).

Theorem 3.2. Let S be a fuzzy subset of a space $(U, \tau_{\mathcal{F}}(F))$ Then

(i) S is a $\mathcal{F}\mathfrak{Nano}Mo$ set iff $S = \mathcal{F}\mathfrak{Nano}Mint(S)$,
(ii) S is a $\mathcal{F}\mathfrak{Nano}Mc$ set iff $S = \mathcal{F}\mathfrak{Nano}Mcl(S)$.

Definition 3.3. [3] A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be fuzzy nano

(i) continuous (briefly, $\mathcal{F}\mathfrak{Nano}Cts$) [11], if $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set S of U_2, the set $h^{-1}(S)$ is $\mathcal{F}\mathfrak{Nano}o$ set of U_1.
(ii) θ continuous (briefly, $\mathcal{F}\mathfrak{Nano}\theta Cts$), if $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set S of U_2, the set $h^{-1}(S)$ is $\mathcal{F}\mathfrak{Nano}\theta o$ set of U_1.
(iii) θ semi continuous (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}Cts$), if $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set S of U_2, the set $h^{-1}(S)$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ set of U_1.
(iv) δ pre continuous (briefly, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}Cts$), if $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set S of U_2, the set $h^{-1}(S)$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ set of U_1.
(v) M continuous (briefly, $\mathcal{F}\mathfrak{Nano}MCts$), if $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set S of U_2, the set $h^{-1}(S)$ is $\mathcal{F}\mathfrak{Nano}Mo$ set of U_1.

Theorem 3.4. [3] A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MCts$ iff the inverse image of every $\mathcal{F}\mathfrak{Nano}c$ set in U_2 is $\mathcal{F}\mathfrak{Nano}Mc$ in U_1.

Definition 3.5. [3] A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is called fuzzy nano

(i) irresolute (briefly, $\mathcal{F}\mathfrak{Nano}Irr$) [11] function, if $\forall$ $\mathcal{F}\mathfrak{Nano}\mathcal{S}o$ subset M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\mathfrak{Nano}\mathcal{S}o$ subset of U_1.
(ii) θ semi irresolute (briefly, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}Irr$) function, if $\forall$ $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ subset M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ subset of U_1.
(iii) δ pre irresolute (briefly, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}Irr$) function, if $\forall$ $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ subset M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ subset of U_1.
(iv) M irresolute (briefly, $\mathcal{F}\mathfrak{Nano}MIrr$) function, if $\forall$ $\mathcal{F}\mathfrak{Nano}Mo$ subset M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\mathfrak{Nano}Mo$ subset of U_1.

Definition 3.6. Let $(U_1, \tau_{\mathcal{F}}(F_1))$ and $(U_2, \tau_{\mathcal{F}}(F_2))$ be two $\mathcal{F}\mathfrak{Nano}ts$. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) open map (briefly, $\mathcal{F}\mathfrak{Nano}O$ [11] (resp. $\mathcal{F}\mathfrak{Nano}\theta O$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$ and $\mathcal{F}\mathfrak{Nano}MO$)) if the image of each $\mathcal{F}\mathfrak{Nano}o$ set in U_1 is $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ and $\mathcal{F}\mathfrak{Nano}Mo$) in U_2.

Definition 3.7. Let $(U_1, \tau_{\mathcal{F}}(F_1))$ and $(U_2, \tau_{\mathcal{F}}(F_2))$ be two $\mathcal{F}\mathfrak{Nano}ts$. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) closed map (briefly, $\mathcal{F}\mathfrak{Nano}C$ [11] (resp. $\mathcal{F}\mathfrak{Nano}\theta C$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}C$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}C$ and $\mathcal{F}\mathfrak{Nano}MC$)) if the image of each $\mathcal{F}\mathfrak{Nano}c$ set in U_1 is $\mathcal{F}\mathfrak{Nano}c$ (resp. $\mathcal{F}\mathfrak{Nano}\theta c$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}c$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}c$ and $\mathcal{F}\mathfrak{Nano}Mc$) in U_2.

Theorem 3.8. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be a mapping. Then every

(i) $\mathcal{F}\mathfrak{Nano}\theta O$ is $\mathcal{F}\mathfrak{Nano}O$.
(ii) $\mathcal{F}\mathfrak{Nano}\theta O$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$.
(iii) $\mathcal{F}\mathfrak{Nano}O$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$.

(iv) $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$ is $\mathcal{F}\mathfrak{Nano}MO$.
(v) $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$ is $\mathcal{F}\mathfrak{Nano}MO$.

Proof. (i) Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}\theta O$ and L is a $\mathcal{F}\mathfrak{Nano}o$ in U_1. Then $h(L)$ is $\mathcal{F}\mathfrak{Nano}\theta o$ in U_2. Since every $\mathcal{F}\mathfrak{Nano}\theta o$ is $\mathcal{F}\mathfrak{Nano}o$, $h(L)$ is $\mathcal{F}\mathfrak{Nano}o$ in U_2. Therefore h is $\mathcal{F}\mathfrak{Nano}O$.

(ii) Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}\theta O$ and L is a $\mathcal{F}\mathfrak{Nano}o$ in U_1. Then $h(L)$ is $\mathcal{F}\mathfrak{Nano}\theta o$ in U_2. Since every $\mathcal{F}\mathfrak{Nano}\theta o$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $h(L)$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ in U_2. Therefore h is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$.

(iii) Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}O$ and L is a $\mathcal{F}\mathfrak{Nano}o$ in U_1. Then $h(L)$ is $\mathcal{F}\mathfrak{Nano}o$ in U_2. Since every $\mathcal{F}\mathfrak{Nano}o$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$, $h(L)$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ in U_2. Therefore h is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$.

(iv) Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$ and L is a $\mathcal{F}\mathfrak{Nano}o$ in U_1. Then $h(L)$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ in U_2. Since every $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ is $\mathcal{F}\mathfrak{Nano}Mo$, $h(L)$ is $\mathcal{F}\mathfrak{Nano}Mo$ in U_2. Therefore h is $\mathcal{F}\mathfrak{Nano}MO$.

(v) Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$ and L is a $\mathcal{F}\mathfrak{Nano}o$ in U_1. Then $h(L)$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ in U_2. Since every $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ is $\mathcal{F}\mathfrak{Nano}Mo$, $h(L)$ is $\mathcal{F}\mathfrak{Nano}Mo$ in U_2. Therefore h is $\mathcal{F}\mathfrak{Nano}MO$. □

The converse of the Theorem 3.8 need not be true.

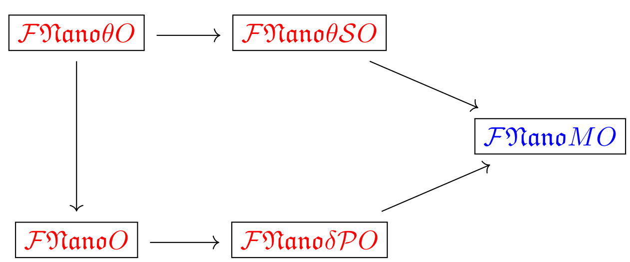

FIGURE 1. $\mathcal{F}\mathfrak{Nano}MO$ mapping's in $\mathcal{F}\mathfrak{Nano}ts$.

Example 3.9. Assume $U = \{s_1, s_2, s_3, s_4\}$ and $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$. Let $S = \left\{\left\langle\frac{s_1}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle, \left\langle\frac{s_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of U.

$$\underline{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.1}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\},$$
$$\overline{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\},$$
$$B_{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\}.$$

Thus $\tau_{\mathcal{F}}(S) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(S), \overline{\mathcal{F}\mathfrak{Nano}}(S) = B_{\mathcal{F}\mathfrak{Nano}}(S)\}$.

Then $h : (U, \tau_{\mathcal{F}}(F)) \to (U, \tau_{\mathcal{F}}(F))$ is an identity function, the set $A = \left\{\left\langle\frac{s_1, s_4}{0.1}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\}$ is $\mathcal{F}\mathfrak{Nano}O$ but not $\mathcal{F}\mathfrak{Nano}\theta O$. Since, A is a $\mathcal{F}\mathfrak{Nano}o$ set in U but $h(A)$ is not $\mathcal{F}\mathfrak{Nano}\theta o$ set in U.

Example 3.10. Assume $U=\{s_1,s_2,s_3,s_4\}$ and $U/R=\{\{s_1,s_4\},\{s_2\},\{s_3\}\}$.
Let $S=\left\{\left\langle\frac{s_1}{0.1}\right\rangle,\left\langle\frac{s_2}{0.1}\right\rangle,\left\langle\frac{s_3}{0.4}\right\rangle,\left\langle\frac{s_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of U.

$$\underline{\mathcal{F}\mathfrak{Nano}}(S)=\left\{\left\langle\frac{s_1,s_4}{0.1}\right\rangle,\left\langle\frac{s_2}{0.1}\right\rangle,\left\langle\frac{s_3}{0.4}\right\rangle\right\}=\overline{\mathcal{F}\mathfrak{Nano}}(S)=B_{\mathcal{F}\mathfrak{Nano}}(S).$$

Thus $\sigma_{\mathcal{F}}(S)=\{0_{\mathcal{F}},1_{\mathcal{F}},\underline{\mathcal{F}\mathfrak{Nano}}(S)=\overline{\mathcal{F}\mathfrak{Nano}}(S)=B_{\mathcal{F}\mathfrak{Nano}}(S)\}$.
Also, $V=\{t_1,t_2,t_3,t_4\}$ and $V/R=\{\{t_1,t_4\},\{t_2\},\{t_3\}\}$.
Let $T=\left\{\left\langle\frac{t_1}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle,\left\langle\frac{t_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of V.

$$\underline{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.1}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\},$$
$$\overline{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\},$$
$$B_{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\}.$$

Thus $\tau_{\mathcal{F}}(T)=\{0_{\mathcal{F}},1_{\mathcal{F}},\underline{\mathcal{F}\mathfrak{Nano}}(T),\overline{\mathcal{F}\mathfrak{Nano}}(T)=B_{\mathcal{F}\mathfrak{Nano}}(T)\}$.
Then $h:(U,\tau_{\mathcal{F}}(F))\to(V,\sigma_{\mathcal{F}}(F))$ is an identity function, the set $A=\left\{\left\langle\frac{t_1,t_4}{0.1}\right\rangle,\left\langle\frac{t_2}{0.1}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\}$ is $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$ (resp. $\mathcal{F}\mathfrak{Nano}MO$) but not $\mathcal{F}\mathfrak{Nano}O$ (resp. $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$). Since, A is a $\mathcal{F}\mathfrak{Nano}o$ set in U but $h(A)$ is not $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$) set in V.

Example 3.11. Assume $U=\{s_1,s_2,s_3,s_4\}$ and $U/R=\{\{s_1,s_4\},\{s_2\},\{s_3\}\}$. Let $S=\left\{\left\langle\frac{s_1}{0.8}\right\rangle,\left\langle\frac{s_2}{0.7}\right\rangle,\left\langle\frac{s_3}{0.6}\right\rangle,\left\langle\frac{s_4}{0.8}\right\rangle\right\}$ be a $\mathcal{F}subs$ of U.

$$\underline{\mathcal{F}\mathfrak{N}}(S)=\left\{\left\langle\frac{s_1,s_4}{0.8}\right\rangle,\left\langle\frac{s_2}{0.7}\right\rangle,\left\langle\frac{s_3}{0.6}\right\rangle\right\}=\overline{\mathcal{F}\mathfrak{N}}(S),$$
$$B_{\mathcal{F}\mathfrak{N}}(S)=\left\{\left\langle\frac{s_1,s_4}{0.2}\right\rangle,\left\langle\frac{s_2}{0.3}\right\rangle,\left\langle\frac{s_3}{0.4}\right\rangle\right\}.$$

Thus $\sigma_{\mathcal{F}}(S)=\{0_{\mathcal{F}},1_{\mathcal{F}},\underline{\mathcal{F}\mathfrak{N}}(S)=\overline{\mathcal{F}\mathfrak{N}}(S),B_{\mathcal{F}\mathfrak{N}}(S)\}$.
Also, $V=\{t_1,t_2,t_3,t_4\}$ and $V/R=\{\{t_1,t_4\},\{t_2\},\{t_3\}\}$.
Let $T=\left\{\left\langle\frac{t_1}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle,\left\langle\frac{t_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of V.

$$\underline{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.1}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\},$$
$$\overline{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\},$$
$$B_{\mathcal{F}\mathfrak{Nano}}(T)=\left\{\left\langle\frac{t_1,t_4}{0.2}\right\rangle,\left\langle\frac{t_2}{0.3}\right\rangle,\left\langle\frac{t_3}{0.4}\right\rangle\right\}.$$

Thus $\tau_{\mathcal{F}}(T)=\{0_{\mathcal{F}},1_{\mathcal{F}},\underline{\mathcal{F}\mathfrak{Nano}}(T),\overline{\mathcal{F}\mathfrak{Nano}}(T)=B_{\mathcal{F}\mathfrak{Nano}}(T)\}$.
Then $h:(U,\tau_{\mathcal{F}}(F))\to(V,\sigma_{\mathcal{F}}(F))$ is an identity function, the set $B=\left\{\left\langle\frac{t_1,t_4}{0.8}\right\rangle,\left\langle\frac{t_2}{0.7}\right\rangle,\left\langle\frac{t_3}{0.6}\right\rangle\right\}$ is $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}O$ (resp. $\mathcal{F}\mathfrak{Nano}MO$) but not $\mathcal{F}\mathfrak{Nano}\theta O$ (resp. $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$). Since, B is a $\mathcal{F}\mathfrak{Nano}o$ set in U but $h(B)$ is not $\mathcal{F}\mathfrak{Nano}\theta o$ (resp. $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$) set in V.

Theorem 3.12. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MC$ mapping if and only if $\mathcal{F}\mathfrak{Nano}Mcl(h(A)) \leq h(\mathcal{F}\mathfrak{Nano}cl(A))$ for every fuzzy set A of U_1.

Proof. Suppose $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is a $\mathcal{F}\mathfrak{Nano}MC$ function and A is any fuzzy set in U_1. Then $\mathcal{F}\mathfrak{Nano}cl(A)$ is a $\mathcal{F}\mathfrak{Nano}c$ set in U_1. Since h is $\mathcal{F}\mathfrak{Nano}Mc$, $h(\mathcal{F}\mathfrak{Nano}cl(A))$ is a $\mathcal{F}\mathfrak{Nano}Mc$ set in U_2. Then by Theorem 3.2 (ii), $\mathcal{F}\mathfrak{Nano}Mcl(h(\mathcal{F}\mathfrak{Nano}cl(A))) = h(\mathcal{F}\mathfrak{Nano}cl(A))$. Therefore $\mathcal{F}\mathfrak{Nano}Mcl(h(A)) \leq \mathcal{F}\mathfrak{Nano}Mcl(h(\mathcal{F}\mathfrak{Nano}cl(A))) = h(\mathcal{F}\mathfrak{Nano}cl(A))$. Hence $\mathcal{F}\mathfrak{Nano}Mcl(h(A)) \leq h(\mathcal{F}\mathfrak{Nano}cl(A))$.

Conversely, let S be a $\mathcal{F}\mathfrak{Nano}c$ set in U_1. Then $\mathcal{F}\mathfrak{Nano}cl(S) = S$ and so $h(S) = h(\mathcal{F}\mathfrak{Nano}cl(S))$. By our assumption $\mathcal{F}\mathfrak{Nano}Mcl(h(S)) \leq h(S)$. But $h(S) \leq \mathcal{F}\mathfrak{Nano}Mcl(h(S))$. Hence $\mathcal{F}\mathfrak{Nano}Mcl(h(S)) = h(S)$ and therefore by Theorem 3.2 (ii), $h(S)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_2. Thus h is a $\mathcal{F}\mathfrak{Nano}MC$ map. ☐

Theorem 3.13. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MC$ mapping iff $\forall$ fuzzy set S of U_2 and $\forall$ $\mathcal{F}\mathfrak{Nano}o$ set U of U_1 containing $h^{-1}(S)$ there exists a $\mathcal{F}\mathfrak{Nano}Mo$ set V of $U_2 \ni S \leq V$ and $h^{-1}(V) \leq U$.

Remark 3.1. The composition of two $\mathcal{F}\mathfrak{Nano}MO$ maps need not be a $\mathcal{F}\mathfrak{Nano}$ MO map, which is shown in the following example.

Example 3.14. Assume $U = \{s_1, s_2, s_3, s_4\}$ and $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$. Let $S = \left\{\left\langle\frac{s_1}{0.1}\right\rangle, \left\langle\frac{s_2}{0.0}\right\rangle, \left\langle\frac{s_3}{0.6}\right\rangle, \left\langle\frac{s_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of U.

$$\underline{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.1}\right\rangle, \left\langle\frac{s_2}{0.0}\right\rangle, \left\langle\frac{s_3}{0.6}\right\rangle\right\} = \overline{\mathcal{F}\mathfrak{Nano}}(S) = B_{\mathcal{F}\mathfrak{Nano}}(S).$$

Thus $\tau_{\mathcal{F}}(S) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(S) = \overline{\mathcal{F}\mathfrak{Nano}}(S) = B_{\mathcal{F}\mathfrak{Nano}}(S)\}$.
Also, $V = \{t_1, t_2, t_3, t_4\}$ and $V/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$.
Let $T = \left\{\left\langle\frac{t_1}{0.1}\right\rangle, \left\langle\frac{t_2}{0.2}\right\rangle, \left\langle\frac{t_3}{0.1}\right\rangle, \left\langle\frac{t_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of V.

$$\underline{\mathcal{F}\mathfrak{Nano}}(T) = \left\{\left\langle\frac{t_1, t_4}{0.1}\right\rangle, \left\langle\frac{t_2}{0.2}\right\rangle, \left\langle\frac{t_3}{0.1}\right\rangle\right\} = \overline{\mathcal{F}\mathfrak{Nano}}(T) = B_{\mathcal{F}\mathfrak{Nano}}(T).$$

Thus $\sigma_{\mathcal{F}}(T) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(T) = \overline{\mathcal{F}\mathfrak{Nano}}(T) = B_{\mathcal{F}\mathfrak{Nano}}(T)\}$.
Also, $W = \{r_1, r_2, r_3, r_4\}$ and $W/R = \{\{r_1, r_4\}, \{r_2\}, \{r_3\}\}$. Let $R = \left\{\left\langle\frac{r_1}{0.2}\right\rangle, \left\langle\frac{r_2}{0.3}\right\rangle, \left\langle\frac{r_3}{0.4}\right\rangle, \left\langle\frac{r_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of W.

$$\underline{\mathcal{F}\mathfrak{Nano}}(R) = \left\{\left\langle\frac{r_1, r_4}{0.1}\right\rangle, \left\langle\frac{r_2}{0.3}\right\rangle, \left\langle\frac{r_3}{0.4}\right\rangle\right\},$$
$$\overline{\mathcal{F}\mathfrak{Nano}}(R) = \left\{\left\langle\frac{r_1, r_4}{0.2}\right\rangle, \left\langle\frac{r_2}{0.3}\right\rangle, \left\langle\frac{r_3}{0.4}\right\rangle\right\},$$
$$B_{\mathcal{F}\mathfrak{Nano}}(R) = \left\{\left\langle\frac{r_1, r_4}{0.2}\right\rangle, \left\langle\frac{r_2}{0.3}\right\rangle, \left\langle\frac{r_3}{0.4}\right\rangle\right\}.$$

Thus $\rho_{\mathcal{F}}(R) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(R), \overline{\mathcal{F}\mathfrak{Nano}}(R) = B_{\mathcal{F}\mathfrak{Nano}}(R)\}$.
Then $h : (U, \tau_{\mathcal{F}}(F)) \to (V, \sigma_{\mathcal{F}}(F))$ and $g : (V, \sigma_{\mathcal{F}}(F)) \to (W, \rho_{\mathcal{F}}(F))$ are $\mathcal{F}\mathfrak{Nano}MO$ but $(g \circ h)$ is not $\mathcal{F}\mathfrak{Nano}MO$.

Since, $B = \left\{\left\langle\frac{r_1, r_4}{0.1}\right\rangle, \left\langle\frac{r_2}{0.0}\right\rangle, \left\langle\frac{r_3}{0.6}\right\rangle\right\}$ is $\mathcal{F}\mathfrak{Nano}o$ set in U but $(g \circ h)(B)$ is not $\mathcal{F}\mathfrak{Nano}Mo$ set in W.

Theorem 3.15. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be a $\mathcal{F}\mathfrak{Nano}C$ map and $g : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ be a $\mathcal{F}\mathfrak{Nano}MC$ map. Then their composition $g \circ h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ is $\mathcal{F}\mathfrak{Nano}MC$.

Theorem 3.16. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ and $g : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ be two mappings such that their composition $g \circ h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ is $\mathcal{F}\mathfrak{Nano}MC$ map. Then the followings are true.

(i) If h is $\mathcal{F}\mathfrak{Nano}Cts$ and surjective, then g is $\mathcal{F}\mathfrak{Nano}MC$ map.
(ii) If g is $\mathcal{F}\mathfrak{Nano}MIrr$ and injective, then h is $\mathcal{F}\mathfrak{Nano}MC$ map.

Proof. (i) Let A be a $\mathcal{F}\mathfrak{Nano}c$ set of U_2. Since h is $\mathcal{F}\mathfrak{Nano}Cts$ map, $h^{-1}(A)$ is $\mathcal{F}\mathfrak{Nano}c$ in U_1. Since $g \circ h$ is $\mathcal{F}\mathfrak{Nano}MC$ map, $(g \circ h)(h^{-1}(A))$ is $\mathcal{F}\mathfrak{Nano}Mc$ in M. Since h is surjective, $g(A)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_3. Hence g is $\mathcal{F}\mathfrak{Nano}MC$ map.

(ii) Let B be any $\mathcal{F}\mathfrak{Nano}c$ set of U_1. Since $g \circ h$ is $\mathcal{F}\mathfrak{Nano}MC$ map, $(g \circ h)(B)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_3. Since g is $\mathcal{F}\mathfrak{Nano}MIrr$, $g^{-1}(g \circ h(B))$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_2. Since g is injective, $h(B)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_2. Hence h is $\mathcal{F}\mathfrak{Nano}MC$ map. □

Theorem 3.17. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{F}\mathfrak{Nano}MC$ map.

(i) If A is $\mathcal{F}\mathfrak{Nano}c$ set of U_1, then the restriction $h_A : (U_A, \tau_{\mathcal{F}}(F_A)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MC$ map.
(ii) If $A = h^{-1}(B)$ for some $\mathcal{F}\mathfrak{Nano}c$ set B of U_2, then the restriction $h_A : (U_A, \tau_{\mathcal{F}}(F_A)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MC$ map.

Proof. (i) Let B be any $\mathcal{F}\mathfrak{Nano}c$ set of A. Then $B = A \wedge L$ for some $\mathcal{F}\mathfrak{Nano}c$ set L of U_1 and so B is $\mathcal{F}\mathfrak{Nano}c$ in U_1. By hypothesis, $h(B)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_2. But $h(B) = h_A(B)$, therefore h_A is a $\mathcal{F}\mathfrak{Nano}MC$ map.

(ii) Let D be a $\mathcal{F}\mathfrak{Nano}c$ set of A. Then $D = A \wedge H$, for some $\mathcal{F}\mathfrak{Nano}c$ set H in U_1. Now, $h_A(D) = h(D) = h(A \wedge H) = h(h^{-1}(B) \wedge H) = B \wedge h(H)$. Since h is $\mathcal{F}\mathfrak{Nano}MC$, $h(H)$ is $\mathcal{F}\mathfrak{Nano}Mc$ in U_2. Hence h_A is a $\mathcal{F}\mathfrak{Nano}MC$ map. □

Theorem 3.18. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{F}\mathfrak{Nano}MO$ map if and only if $h(\mathcal{F}\mathfrak{Nano}int(A)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$, for every fuzzy set A of U_1.

Proof. Suppose $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is a $\mathcal{F}\mathfrak{Nano}MO$ function and A is any fuzzy set in U_1. Then $\mathcal{F}\mathfrak{Nano}int(A)$ is a $\mathcal{F}\mathfrak{Nano}o$ set in U_1. Since h is $\mathcal{F}\mathfrak{Nano}MO$, $h(\mathcal{F}\mathfrak{Nano}int(A))$ is a $\mathcal{F}\mathfrak{Nano}Mo$ set. Since $\mathcal{F}\mathfrak{Nano}Mint(h(\mathcal{F}\mathfrak{Nano}int(A))) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$, $h(\mathcal{F}\mathfrak{Nano}int(A)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$.

Conversely, $h(\mathcal{F}\mathfrak{Nano}int(A)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$ for every fuzzy set A in U_1. Let U be a $\mathcal{F}\mathfrak{Nano}o$ set in U_1. Then $\mathcal{F}\mathfrak{Nano}int(U) = U$ and by hypothesis, $h(U) \leq \mathcal{F}\mathfrak{Nano}Mint(h(U))$. But $\mathcal{F}\mathfrak{Nano}Mint(h(U)) \leq h(U)$. Therefore, $h(U) = \mathcal{F}\mathfrak{Nano}Mint(h(U))$. Then by Theorem 3.2 (i), $h(U)$ is $\mathcal{F}\mathfrak{Nano}Mo$. Hence h is a $\mathcal{F}\mathfrak{Nano}MO$ map. □

Definition 3.19. Let A and B be any two fuzzy subsets of a $\mathcal{F}\mathfrak{Nano}ts$'s. Then A is fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) q-neighbourhood (briefly, $\mathcal{F}\mathfrak{Nano}q$-$nbhd$ [11] (resp. $\mathcal{F}\mathfrak{Nano}\theta q$-$nbhd$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}q$-$nbhd$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}q$-$nbhd$ and $\mathcal{F}\mathfrak{Nano}Mq$-$nbhd$)) with B if there exists a $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ and $\mathcal{F}\mathfrak{Nano}Mo$) set O with $AqO \leq B$.

Theorem 3.20. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be a mapping. Then the following statements are equivalent.

(i) h is a $\mathcal{F}\mathfrak{Nano}MO$ mapping,

(ii) For a subset A of U_1, $h(\mathcal{F}\mathfrak{Nano}int(A)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$.

(iii) For each $x_\alpha \in U_1$ and for each $\mathcal{F}\mathfrak{Nano}q$-$nbhd$ U of x_α in U_1, there exists A $\mathcal{F}\mathfrak{Nano}Mq$-$nbhd$ W of $h(x_\alpha)$ in U_2 such that $W \leq h(U)$.

Proof. (i) $\Rightarrow$ (ii): Suppose $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is a $\mathcal{F}\mathfrak{Nano}MO$ function and $A \leq U_1$. Then $\mathcal{F}\mathfrak{Nano}int(A)$ is a $\mathcal{F}\mathfrak{Nano}o$ set in U_1. Since h is $\mathcal{F}\mathfrak{Nano}MO$ map, $h(\mathcal{F}\mathfrak{Nano}int(A))$ is a $\mathcal{F}\mathfrak{Nano}Mo$ set. Since $\mathcal{F}\mathfrak{Nano}Mint(h(\mathcal{F}\mathfrak{Nano}int(A))) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$, $h(\mathcal{F}\mathfrak{Nano}int(A)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(A))$. This proves (ii).

(ii) $\Rightarrow$ (iii): Let $x_\alpha \in U_1$ and U be any arbitrary $\mathcal{F}\mathfrak{Nano}q$-$nbhd$ of x_α in U_1. Then there exists a $\mathcal{F}\mathfrak{Nano}o$ set G such that $x_\alpha \in G \leq U$. By (ii), $h(G) = h(\mathcal{F}\mathfrak{Nano}int(G)) \leq \mathcal{F}\mathfrak{Nano}Mint(h(G))$. But, $\mathcal{F}\mathfrak{Nano}Mint(h(G)) \leq h(G)$. Therefore, $\mathcal{F}\mathfrak{Nano}Mint(h(G)) = h(G)$ and hence $h(G)$ is $\mathcal{F}\mathfrak{Nano}Mo$ in U_2. Since $x_\alpha \in G \leq U$, $h(x_\alpha) \in h(G) \leq h(U)$ and so (iii) holds, by taking $W = h(G)$.

(iii) $\Rightarrow$ (i): Let U be any $\mathcal{F}\mathfrak{Nano}o$ set in U_1. Let $x_\alpha \in U$ and $h(x_\alpha) = y_\beta$. Then for each $x_\alpha \in U$, $y \in h(U)$, by assumption there exists a $\mathcal{F}\mathfrak{Nano}qM$-$nbhd$ $W(y_\beta)$ of y_β in U_2 such that $W(y_\beta) \leq h(U)$. Since $W(y_\beta)$ is a $\mathcal{F}\mathfrak{Nano}qM$-$nbhd$ of y_β, there exists a $\mathcal{F}\mathfrak{Nano}Mo$ set $V(y_\beta)$ in U_2 such that $y_\beta \in V(y_\beta) \leq W(y_\beta)$. Therefore, $h(U) = \vee\{V(y_\beta) | y_\beta \in h(U)\}$. Since the union of $\mathcal{F}\mathfrak{Nano}Mo$ sets is $\mathcal{F}\mathfrak{Nano}Mo$, $h(U)$ is a $\mathcal{F}\mathfrak{Nano}Mo$ set in U_2. Thus, h is a $\mathcal{F}\mathfrak{Nano}MO$ map. □

Theorem 3.21. For any bijective map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ the following statements are equivalent:

(i) $h^{-1} : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_1, \tau_{\mathcal{F}}(F_1))$ is $\mathcal{F}\mathfrak{Nano}MCts$.

(ii) h is $\mathcal{F}\mathfrak{Nano}MO$ map.

(iii) h is $\mathcal{F}\mathfrak{Nano}MC$ map.

Remark 3.2. Theorems 3.12 to 3.21 and Remark 3.1 are holds for $\mathcal{F}\mathfrak{Nano}o$, $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ & $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ sets.

4. Fuzzy nano M homeomorphism

The purpose of this section is to introduces the idea of fuzzy nano M homeomorphism in $\mathcal{F}\mathfrak{Nano}ts$ and establish some of their attributes.

Definition 4.1. Let U_1 and U_2 be $\mathcal{F}\mathfrak{Nano}ts$. A mapping $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be a fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) homeomorphism (briefly, $\mathcal{F}\mathfrak{Nano}Hom$ (resp. $\mathcal{F}\mathfrak{Nano}\theta Hom$, $\mathcal{F}\mathfrak{Nano}\theta SHom$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}Hom$ and $\mathcal{F}\mathfrak{Nano}MHom$)) if h is bijective, $\mathcal{F}\mathfrak{Nano}Cts$ (resp. $\mathcal{F}\mathfrak{Nano}\theta Cts$, $\mathcal{F}\mathfrak{Nano}\theta SCts$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}Cts$ and $\mathcal{F}\mathfrak{Nano}MCts$) function and $\mathcal{F}\mathfrak{Nano}O$ (resp. $\mathcal{F}\mathfrak{Nano}\theta O$, $\mathcal{F}\mathfrak{Nano}\theta SO$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}O$ and $\mathcal{F}\mathfrak{Nano}MO$) mapping.

Example 4.2. Assume $U = \{s_1, s_2, s_3, s_4\}$ and $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$. Let $S = \left\{\left\langle\frac{s_1}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle, \left\langle\frac{s_4}{0.1}\right\rangle\right\}$ be a $\mathcal{F}subs$ of U.

$$\underline{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.1}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\},$$
$$\overline{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\},$$
$$B_{\mathcal{F}\mathfrak{Nano}}(S) = \left\{\left\langle\frac{s_1, s_4}{0.2}\right\rangle, \left\langle\frac{s_2}{0.3}\right\rangle, \left\langle\frac{s_3}{0.4}\right\rangle\right\}.$$

Thus $\tau_{\mathcal{F}}(S) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(S), \overline{\mathcal{F}\mathfrak{Nano}}(S) = B_{\mathcal{F}\mathfrak{Nano}}(S)\}$.

Also, $V = \{t_1, t_2, t_3, t_4\}$ and $V/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$.

Let $T = \left\{\left\langle\frac{t_1}{0.2}\right\rangle, \left\langle\frac{t_2}{0.3}\right\rangle, \left\langle\frac{t_3}{0.3}\right\rangle, \left\langle\frac{t_4}{0.2}\right\rangle\right\}$ be a $\mathcal{F}subs$ of V.

$$\underline{\mathcal{F}\mathfrak{Nano}}(T) = \left\{\left\langle\frac{t_1, t_4}{0.2}\right\rangle, \left\langle\frac{t_2}{0.3}\right\rangle, \left\langle\frac{t_3}{0.3}\right\rangle\right\} = \overline{\mathcal{F}\mathfrak{Nano}}(T) = B_{\mathcal{F}\mathfrak{Nano}}(T).$$

Thus $\sigma_{\mathcal{F}}(T) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathfrak{Nano}}(T) = \overline{\mathcal{F}\mathfrak{Nano}}(T) = B_{\mathcal{F}\mathfrak{Nano}}(T)\}$.

Then $h : (U, \tau_{\mathcal{F}}(F)) \to (V, \sigma_{\mathcal{F}}(F))$ is an identity function, the set $B = \left\{\left\langle\frac{t_1,t_4}{0.2}\right\rangle, \left\langle\frac{t_2}{0.3}\right\rangle, \left\langle\frac{t_3}{0.3}\right\rangle\right\}$ is $\mathcal{F}\mathfrak{Nano}MHom$.

Theorem 4.3. Let $(U_1, \tau_{\mathcal{F}}(F_1))$ and $(U_2, \tau_{\mathcal{F}}(F_2))$ be two $\mathcal{F}\mathfrak{Nano}ts$ and $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be a bijective function. Then h is a $\mathcal{F}\mathfrak{Nano}MHom$ if and only if h is a $\mathcal{F}\mathfrak{Nano}MCts$ function and $\mathcal{F}\mathfrak{Nano}MC$ mapping.

Proof. Let h be a $\mathcal{F}\mathfrak{Nano}MHom$ homeomorphism. From Definition 4.1 h is a $\mathcal{F}\mathfrak{Nano}MCts$ function. From Theorem 3.21, we have h^{-1} is a $\mathcal{F}\mathfrak{Nano}MC$ function. So, $(h^{-1})^{-1} = f$ is a $\mathcal{F}\mathfrak{Nano}MC$ function. □

Theorem 4.4. Let $g : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be a bijective mapping. If g is $\mathcal{F}\mathfrak{Nano}MCts$, the following statements are identical in this case:

(a) g is a $\mathcal{F}\mathfrak{Nano}MC$ mapping.
(b) g is a $\mathcal{F}\mathfrak{Nano}MO$ mapping.
(c) g^{-1} is a $\mathcal{F}\mathfrak{Nano}MHom$.

Proof. (a) ⇒ (b) Let us assume that g is a bijective mapping and a $\mathcal{F}\mathfrak{Nano}MC$ mapping. Hence, g^{-1} is a $\mathcal{F}\mathfrak{Nano}MCts$ mapping. Since each $\mathcal{F}\mathfrak{Nano}o$ set is a $\mathcal{F}\mathfrak{Nano}Mo$ set, g is a $\mathcal{F}\mathfrak{Nano}MO$ mapping.

(b) ⇒ (c) Let g be a bijective and $\mathcal{F}\mathfrak{Nano}MO$ mapping. Furthermore, g^{-1} is a $\mathcal{F}\mathfrak{Nano}MCts$ mapping. Hence, g and g^{-1} are $\mathcal{F}\mathfrak{Nano}MCts$. Therefore, g is a $\mathcal{F}\mathfrak{Nano}MHom$.

(c) ⇒ (a) Let g be a $\mathcal{F}\mathfrak{Nano}MHom$. Then g and g^{-1} are $\mathcal{F}\mathfrak{Nano}MCts$. Since each $\mathcal{F}\mathfrak{Nano}c$ set in U_1 is a $\mathcal{F}\mathfrak{Nano}Mc$ set in U_2, hence g is a $\mathcal{F}\mathfrak{Nano}MC$ mapping. □

Remark 4.1. Theorems 4.3 and 4.4 are holds for $\mathcal{F}\mathfrak{Nano}o$, $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ & $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ sets.

5. Almost fuzzy nano M totally mappings

In this section, we introduce almost fuzzy nano M totally mappings and we discuss some basic properties.

Definition 5.1. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be

(i) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) open map (briefly, $\mathcal{AF}\mathfrak{Nano}O$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta O$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}O$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}O$ and $\mathcal{AF}\mathfrak{Nano}MO$)) if the image of each $\mathcal{F}\mathfrak{Nano}ro$ set in U_1 is $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ and $\mathcal{F}\mathfrak{Nano}Mo$) in U_2.

(ii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) closed map (briefly, $\mathcal{AF}\mathfrak{Nano}C$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta C$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}C$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}C$ and $\mathcal{AF}\mathfrak{Nano}MC$)) if the image of each $\mathcal{F}\mathfrak{Nano}rc$ set in U_1 is $\mathcal{F}\mathfrak{Nano}c$ (resp. $\mathcal{F}\mathfrak{Nano}\theta c$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}c$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}c$ and $\mathcal{F}\mathfrak{Nano}Mc$) in U_2.

(iii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen map (briefly, $\mathcal{AF}\mathfrak{Nano}clO$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta clO$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}clO$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}clO$ and $\mathcal{AF}\mathfrak{Nano}MclO$)) if the image of each $\mathcal{F}\mathfrak{Nano}rclo$ set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

(iv) fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally open map (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{T}O$ (resp. $\mathcal{F}\mathfrak{Nano}\theta\mathcal{T}O$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}O$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}O$ and $\mathcal{F}\mathfrak{Nano}M\mathcal{T}O$)) if the image of each $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ and $\mathcal{F}\mathfrak{Nano}Mo$) set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

(v) fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally closed map (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{T}C$ (resp. $\mathcal{F}\mathfrak{Nano}\theta\mathcal{T}C$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}C$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}C$ and $\mathcal{F}\mathfrak{Nano}M\mathcal{T}C$)) if the image of each $\mathcal{F}\mathfrak{Nano}c$ (resp. $\mathcal{F}\mathfrak{Nano}\theta c$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}c$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}c$ and $\mathcal{F}\mathfrak{Nano}Mc$) set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

(vi) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally open map (briefly, $\mathcal{AF}\mathfrak{Nano}\mathcal{T}O$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{T}O$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}O$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}O$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$)) if the image of each $\mathcal{F}\mathfrak{Nano}ro$ set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

(vii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally closed map (briefly, $\mathcal{AF}\mathfrak{Nano}\mathcal{T}C$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{T}C$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}C$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}C$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$)) if the image of each $\mathcal{F}\mathfrak{Nano}rc$ set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

(viii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally clopen map (briefly, $\mathcal{AF}\mathfrak{Nano}\mathcal{T}clO$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{T}clO$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}clO$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}clO$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}clO$)) if the image of each $\mathcal{F}\mathfrak{Nano}rclo$ set in U_1 is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_2.

Theorem 5.2. Every $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ map is $\mathcal{AF}\mathfrak{Nano}MC$.

Proof. Let U_1 and U_2 be $\mathcal{F}\mathfrak{Nano}ts$. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be an $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ mapping. To prove h is $\mathcal{AF}\mathfrak{Nano}MC$, let H be any $\mathcal{F}\mathfrak{Nano}rc$ subset of U_1. Since h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ mapping, $h(H)$ is $\mathcal{AF}\mathfrak{Nano}Mclo$ in U_2. This implies that $h(H)$ is $\mathcal{F}\mathfrak{Nano}c$ in U_2. Therefore h is $\mathcal{AF}\mathfrak{Nano}MC$. □

Corollary 5.3. Every $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$ map is $\mathcal{AF}\mathfrak{Nano}MO$.

Theorem 5.4. If a bijective function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$, then the image of each $\mathcal{F}\mathfrak{Nano}rc$ set in U_1 is $\mathcal{AF}\mathfrak{Nano}Mclo$ in U_2.

Theorem 5.5. A surjective function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$ iff $\forall$ subset B of U_2 and for each $\mathcal{F}\mathfrak{Nano}ro$ set U containing $h^{-1}(B)$, there is a $\mathcal{F}\mathfrak{Nano}Mclo$ set V of $U_2 \ni B \leq V$ & $h^{-1}(V) \leq U$.

Theorem 5.6. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$ iff if $\forall$ subset A of U_2 and each $\mathcal{F}\mathfrak{Nano}rc$ set U containing $h^{-1}(A)$ there is a $\mathcal{F}\mathfrak{Nano}Mclo$ set V of $U_2 \ni A \leq V$ & $h^{-1}(V) \leq U$.

Corollary 5.7. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ iff for each subset A of U_2 and each $\mathcal{F}\mathfrak{Nano}ro$ set U containing $h^{-1}(A)$, there is a $\mathcal{F}\mathfrak{Nano}Mclo$ set V of $U_2 \ni A \leq V$ & $h^{-1}(V) \leq U$.

Theorem 5.8. If $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ and A is $\mathcal{F}\mathfrak{Nano}rc$ subset of U_1 then $h_A : (U_A, \tau_{\mathcal{F}}(F_A)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$.

Proof. Consider the function $h_A : (U_A, \tau_{\mathcal{F}}(F_A)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ and let V be any $\mathcal{F}\mathfrak{Nano}Mclo$ set in U_2. Since h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$, $h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}rc$ subset of U_1. Since A is $\mathcal{F}\mathfrak{Nano}rc$ subset of U_1 and $h_A^{-1}(V) = A \wedge h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}rc$ in A, it follows $h_A^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}rc$ in A. Hence h_A is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$. □

Remark 5.1. *$\mathcal{AF}\mathfrak{Nano}M\mathcal{T}clO$ mapping is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ map.*

Remark 5.2. Theorems 5.2 to 5.8, Corollaries 5.3 and 5.7 and Remark 5.1 are holds for $\mathcal{F}\mathfrak{Nano}o$, $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ & $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ sets.

6. Almost fuzzy nano M totally continuous functions

Definition 6.1. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be

(i) fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally continuous (briefly, $\mathcal{F}\mathfrak{Nano}\mathcal{T}Cts$ (resp. $\mathcal{F}\mathfrak{Nano}\theta\mathcal{T}Cts$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}Cts$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}Cts$ and $\mathcal{F}\mathfrak{Nano}M\mathcal{T}Cts$)) if $h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_1 for each $\mathcal{F}\mathfrak{Nano}o$ (resp. $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ and $\mathcal{F}\mathfrak{Nano}Mo$) set V in U_2.

(ii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally continuous (briefly, $\mathcal{AF}\mathfrak{Nano}\mathcal{T}Cts$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{T}Cts$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}Cts$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}Cts$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$)) if $h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_1 for each $\mathcal{F}\mathfrak{Nano}ro$ set V in U_2.

(iii) Almost fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) totally clopen continuous (briefly, $\mathcal{AF}\mathfrak{Nano}\mathcal{T}cloCts$ (resp. $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{T}cloCts$, $\mathcal{AF}\mathfrak{Nano}\theta\mathcal{S}\mathcal{T}cloCts$, $\mathcal{AF}\mathfrak{Nano}\delta\mathcal{P}\mathcal{T}cloCts$ and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}cloCts$)) if $h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}$ clo (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_1 for each $\mathcal{F}\mathfrak{Nano}rclo$ set V in U_2.

Theorem 6.2. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$ function if the inverse image of every $\mathcal{F}\mathfrak{Nano}rc$ set of U_2 is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_1.

Proof. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$ and F be any $\mathcal{F}\mathfrak{Nano}rc$ set in U_2. Then F^c is $\mathcal{F}\mathfrak{Nano}ro$ set in U_2. Since h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$, $h^{-1}(F^c)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_1. That is $(h^{-1}(F))^c$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_1. This implies that $h^{-1}(F)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_1. □

Theorem 6.3. A function $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$ is an $\mathcal{AF}\mathfrak{Nano}MCts$ function.

Proof. Suppose $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$ and U is any $\mathcal{F}\mathfrak{Nano}ro$ subset of U_2. Since h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$, $h^{-1}(U)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_1. This implies that $h^{-1}(U)$ is $\mathcal{F}\mathfrak{Nano}Mo$ in U_1. Therefore the function h is $\mathcal{AF}\mathfrak{Nano}MCts$. □

Theorem 6.4. For any bijective map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ the following statements are equivalent:

(i) $h^{-1} : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_1, \tau_{\mathcal{F}}(F_1))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}Cts$.
(ii) h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$.
(iii) h is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$.

Remark 6.1. Theorems 6.2 to 6.4 are holds for $\mathcal{F}\mathfrak{Nano}o$, $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ & $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ sets.

7. Super fuzzy nano M clopen continuous functions

Definition 7.1. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be super fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen continuous (briefly, $\mathcal{SUF}\mathfrak{Nano}cloCts$ (resp. $\mathcal{SUF}\mathfrak{Nano}\theta cloCts$, $\mathcal{SUF}\mathfrak{Nano}\theta\mathcal{S}cloCts$, $\mathcal{SUF}\mathfrak{Nano}\delta\mathcal{P}cloCts$ and $\mathcal{SUF}\mathfrak{Nano}M$ $cloCts$)) if for each $x_\alpha \in U_1$ and for each $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}$ $\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) set V containing $h(x_\alpha)$ in U_2, there exist a $\mathcal{F}\mathfrak{Nano}ro$ set U containing x_α such that $h(U) \leq V$.

Theorem 7.2. Let $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ be $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$. Then h is $\mathcal{SUF}\mathfrak{Nano}McloCts$ if $h(x_\alpha)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_2.

Proof. Let G be $\mathcal{F}\mathfrak{Nano}Mclo$ in U_2. Now $h^{-1}(G)$ is $\mathcal{F}\mathfrak{Nano}ro$ in U_1. Since the intersection of $\mathcal{F}\mathfrak{Nano}Mclo$ set is $\mathcal{F}\mathfrak{Nano}Mclo$ in $Y, h(h^{-1}(G)) = G \wedge h(x_\alpha)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_2. Therefore, $h^{-1}(G)$ is $\mathcal{F}\mathfrak{Nano}ro$ in U_1. Hence h is $\mathcal{SUF}\mathfrak{Nano}McloCts$ function. □

Theorem 7.3. If $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is surjective and $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}O$, then h is $\mathcal{SUF}\mathfrak{Nano}MgloCts$.

Definition 7.4. A map $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be fuzzy nano (resp. θ, $\theta\mathcal{S}$, $\delta\mathcal{P}$ and M) clopen irresolute function (briefly, $\mathcal{F}\mathfrak{Nano}cloIrr$ (resp. $\mathcal{F}\mathfrak{Nano}\theta cloIrr$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}cloIrr$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}cloIrr$ and $\mathcal{F}\mathfrak{Nano}McloIrr$)) if $h^{-1}(V)$ is $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) in U_1 for each $\mathcal{F}\mathfrak{Nano}clo$ (resp. $\mathcal{F}\mathfrak{Nano}\theta clo$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}clo$, $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}clo$ and $\mathcal{F}\mathfrak{Nano}Mclo$) set V in U_2.

Theorem 7.5. Let $(U_1, \tau_{\mathcal{F}}(F_1))$, $(U_2, \tau_{\mathcal{F}}(F_2))$ and $(U_3, \tau_{\mathcal{F}}(F_3))$ be $\mathcal{F}\mathfrak{Nano}ts$. Then the composition $g \circ h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ is $\mathcal{SUF}\mathfrak{Nano}MCloCts$ function where $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ is $\mathcal{SUF}\mathfrak{Nano}MCloCts$ function and $g : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ is $\mathcal{F}\mathfrak{Nano}MCloIrr$ function.

Proof. Let A be a $\mathcal{F}\mathfrak{Nano}rc$ set of U_1. Since h is $\mathcal{SUF}\mathfrak{Nano}MCloCts$, $h(A)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ in U_2. Then by hypothesis, $h(A)$ is $\mathcal{F}\mathfrak{Nano}Mclo$ set. Since g is $\mathcal{F}\mathfrak{Nano}MCloIrr$, $g(h(A)) = (g \circ h)(A)$. Therefore $g \circ f$ is $\mathcal{SUF}\mathfrak{Nano}MCloCts$. □

Theorem 7.6. If $h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_2, \tau_{\mathcal{F}}(F_2))$ and $g : (U_2, \tau_{\mathcal{F}}(F_2)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ are two mappings such that their composition $g \circ h : (U_1, \tau_{\mathcal{F}}(F_1)) \to (U_3, \tau_{\mathcal{F}}(F_3))$ is $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ mapping then the following statements are true.

(i) If h is $\mathcal{SUF}\mathfrak{Nano}MCloCts$ and surjective, then g is a $\mathcal{F}\mathfrak{Nano}MCloIrr$ function.

(ii) If g is $\mathcal{F}\mathfrak{Nano}MCloIrr$ function and injective, then h is an $\mathcal{AF}\mathfrak{Nano}M\mathcal{T}C$ function.

Remark 7.1. Theorems 7.2 to 7.6 are holds for $\mathcal{F}\mathfrak{Nano}o$, $\mathcal{F}\mathfrak{Nano}\theta o$, $\mathcal{F}\mathfrak{Nano}\theta\mathcal{S}o$ & $\mathcal{F}\mathfrak{Nano}\delta\mathcal{P}o$ sets.

8. Conclusion

In this paper, we have continued to study the properties of fuzzy nano M open and fuzzy nano M closed mappings in fuzzy nano topological spaces. Also, we study about fuzzy nano M Homeomorphism, almost fuzzy nano M totally mappings, almost fuzzy nano M totally continuous mappings and super fuzzy nano M clopen continuous functions and established the relations between them we obtain some new characterizations of these mappings in fuzzy nano topological spaces.

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