

Research on the Application of Fractal Geometry in Digital Arts

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Abstract

Fractal geometry, a relatively new branch of mathematics, was first introduced by Benoit Mandelbrot in 1975. Since then, its applications have expanded into various fields of natural science. In fact, it has been recognized as one of the three significant scientific discoveries of the mid-20th century, along with the Dissipative System and Chaos Theory. With the help of fractal geometry, designers can create intricate and expressive artistic patterns, using the concept of self-similarity found in nature. The impact of fractal geometry on the digital art world is significant and its exploration could lead to new avenues for creativity and expression. This paper aims to explore and analyze the development and applications of fractal geometry in digital art design. It also aims to showcase the benefits of applying fractal geometry in art creation and paves the way for future research on sacred geometry.

Keywords: *Fractals, Fractals Geometry, Digital Arts, Graphic Design, Pattern Art, Islamic Geometric Pattern.*

1. Introduction

The application history of geometry in the field of art can be traced back to thousands of years ago, but in today's society, fractal geometry is more and more applied to pattern design, and its frequency and scope are increasing. For example, fractal geometry theory is widely used in digital art and design, such as pattern modeling, book frames, decorative patterns, and anti-counterfeiting logos. Since digital art design is not a subject with a long history of development, the application and development of fractal geometry in digital art design is also a relatively new field. So there is a relatively large space to study the development of fractal geometry in art creation. Therefore, this paper aims to explore the value of fractal geometry in art by analyzing the application cases of fractal geometry in the field of digital art as a preliminary study.

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2. Theory

British mathematician Lewis Fry Richardson discovered the mystery of fractals for the first time while studying the British coastline in the early 20th century. He found that the length of a coastline is not absolute, it depends on what ruler you use to measure it. Considering the irregular patterns formed by the erosion of the coastline, the finer the measuring instruments you use, the longer the coastline will be. Therefore, Lewis proposed the concept of "The area is limited, but the circumference is unlimited." It was also the earliest fractal theory.

Fractal geometry is a geometry that focuses on irregular geometric forms, mainly studying structures or shapes that is infinitely complex but have certain meanings, and they usually having a fractal dimension strictly exceeding the topological dimension. Many fractals appear similar at various scales, as illustrated in successive magnifications of the Mandelbrot set [2]. Simply put, fractal geometry refers to a geometric shape that can be divided into multiple parts, each of which is the same or similar to the reduced shape of the entire. It usually has characteristics like infinity, precision, and complexity, and it has a certain symbolic meaning [3]. Because the research objects of fractal geometry commonly exist in nature, such as well-known snowflakes, leaves, mountains, clouds, waves, etc., fractal geometry is also known as the "geometry of nature". As shown in Figure 1, it is a hidden fractal geometry in nature.



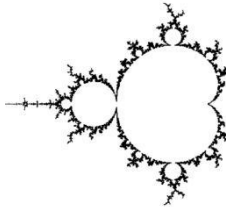
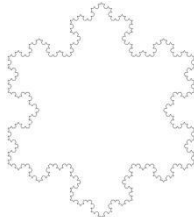
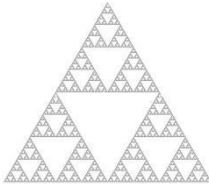

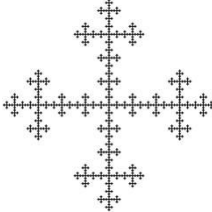
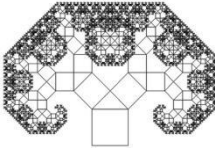
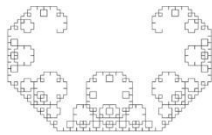
Figure 1. The Hidden Fractal Geometry in Nature

3. Application and Development

3.1 The Early Basic Forms

The history of fractals traces a path from chiefly theoretical studies to modern applications in computer graphics, with several notable people contributing canonical fractal forms along the way[4]. These include the classic Mandelbrot set, Koch Snowflake, H-Tree, Sierpiński triangle, Vicsek fractal, Lévy C curve, Pythagoras tree, Hexaflake, etc.. With the help of computer graphics, scientists and mathematicians have transformed complex and profound fractal principles into visually pleasing shapes, making people to begin to understand the aesthetic value of fractal geometry. Table 1 shows some forms of fractal geometry. Scanning the QR code shows the process of fractal iteration.

Table 1. Main Forms of Fractal Geometry

Fractal Form	Introduction	Representative Shape	Fractal Iteration
Mandelbrot set	Mandelbrot set exhibit an infinitely complicated boundary that reveals progressively ever-finer recursive detail at increasing magnifications; mathematically, one would say that the boundary of the Mandelbrot set is a fractal curve.		
Koch Snowflake	Koch Snowflake is a fractal structure proposed by Swedish mathematician Heilig von Koch in 1904. And it is a fractal curve and one of the earliest fractals to have been described.		
H-Tree	H-Tree is a fractal tree structure constructed from perpendicular line segments, because its repeating pattern is similar to the letter "H" and is an example of a fractal tree crown, it is named H-Tree.		
Sierpiński triangle	The Sierpiński triangle is a fractal attractive fixed set with the overall shape of an equilateral triangle, subdivided recursively into smaller equilateral triangles.		 scan me !
Vicsek fractal	Vicsek fractal, also known as Vicsek snowflake or box fractal. It's a fractal arising from a construction similar to that of the Sierpinski carpet, proposed by Tamás Vicsek.		
Pythagoras tree	The Pythagoras tree is a plane fractal constructed from squares, and invented by the Dutch mathematics teacher Albert E. Bosman in 1942.		
Lévy C curve	In mathematics, the Lévy C curve is a self-similar fractal curve. It is a special case of a period-doubling curve, a de Rham curve.		

3.2 Traditional Decorative Patterns

After fractal appeared in people's eyes in geometric form, architects and artists began to try to combine fractal principles with works of art, among which the more classic is Islamic geometric patterns.

In Islamic art, decorative patterns are the soul, and their geometric patterns follow certain rules. They are used in a complex manner within a plane, forming a continuous, symmetrical, and infinitely extended decorative feature. All geometric patterns and shapes are constantly evolving through points, lines, and surfaces, forming "infinite" patterns. Islamic geometric patterns are presented in the following forms in both plane and space: particleization, fractal, and Muqarnas, which is a process of upward transformation from plane to space.

Islamic architectural artists extract key elements from the essence of classical art and explain them in the Islamic language. Under a specific theme, the artist strictly abides by the rules of composition and draws a basic "unit figure" from numerous complex figures formed by pentagrams, interlaced equilateral triangles, squares, and regular hexagons. Then he repeatedly uses this figure alone or adds the shape of vines. Its infinitely extended lines and symmetry form a regular pattern of tiling. Between these endless lines, in an instant, infinite variations emerged, highlighting the importance of unity and order, which is the Islamic geometric decorative pattern. In Islamic dome architecture, fractal structures are fully interpreted, and an infinite number of levels of patterns can be seen in the dome. Regardless of which level of fractal, there can always be a finer level of existence. Figure 2 shows the geometric fractal decoration in Islamic dome architecture.

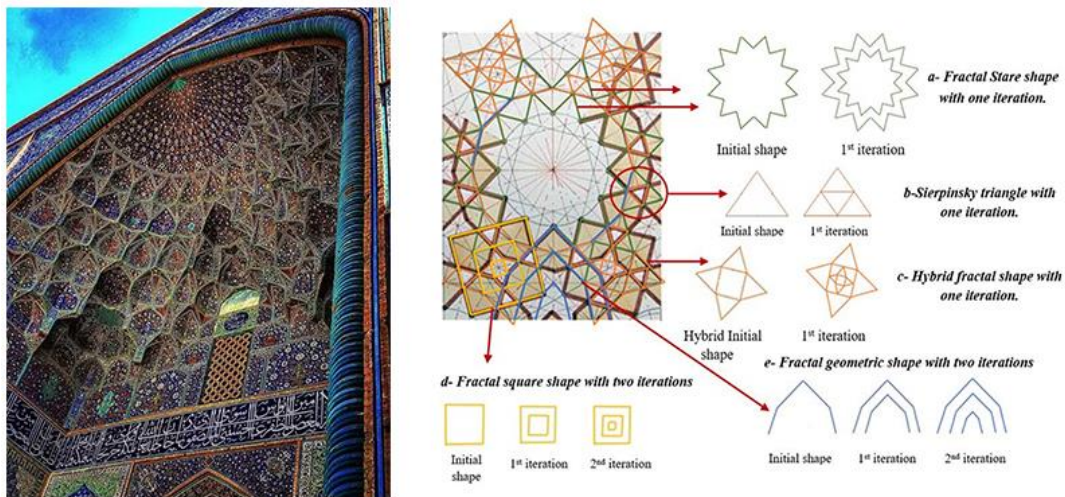


Figure 2. The fractal in Islamic dome

Figure Source: Assist. Prof. Dr. Doaa Ismail Ismail Attia, "The Fractal shapes in Islamic design & its effects on the occupiers of the interior environment", 2020

3.3 Creative Digital Art Works

In addition to traditional decorative patterns, artists are now using digital art techniques to draw inspiration from fractals in nature, resulting in the creation of stunning works of fractal art. Figure 3-a showcases artwork that imitates the appearance and basic characteristics of natural creatures. This work focuses on the exploration

of the fundamental laws and principles of fractal geometry, with minimal visual effects added. In contrast, the works in Figure 3-b demonstrate a significant leap in artistic color and creative form, while still adhering to the principles of fractal geometry. The vibrant colors and complex forms make digital fractal art more appealing to the general public. The works in Figure 3-c represent the transformation of fractal art from two-dimensional to three-dimensional spatial form. They also reflect another characteristic of fractal art - nesting - making them even more intriguing and engaging.

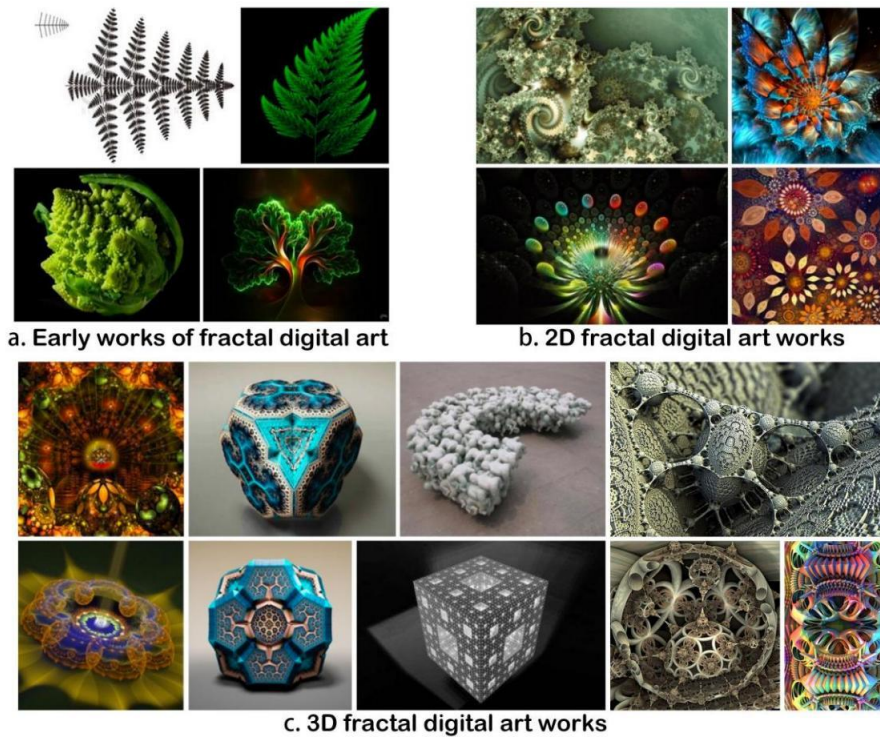


Figure 3. The Fractal in Digital Art Works

Table 2. The Advantages of Fractal Geometry in Digital Art

Advantages	Introduction
Sense of Balance, Harmony, and Infinite	Integrating fractal geometry into digital art can make artists have a sense of unity and coherence in their works of art, and this infinite sense of extension can bring infinite possibilities to their works.
Intricate and Detailed Designs	Digital tools allow for precise lines, shapes, and colors, making it easier to create complex fractal art designs.
Potential for Innovation	Using fractal geometry in digital arts provides opportunities for experimentation and creative expression.

3.4 The Advantages of Fractal Arts

According to the above summary of the development of fractal geometry in the art field and the analysis of actual cases, this paper summarizes three creative advantages of fractal geometry in digital art, refer to Table 2 for details [3].

4. Conclusion

The emergence of fractal geometry has piqued the interest of scholars and designers alike. Its unique concepts have opened up new avenues for artistic creation and its potential is becoming increasingly evident. Through this paper, we have explored the possibility of applying fractal geometry in digital art, and have identified its significant value in creating innovative and complex patterns. The process of combining fractal geometry with artistic creation is a constantly evolving area of study, and there is much to be learned and explored in this field. We hope that this preliminary exploration will pave the way for future research on fractal art, enabling digital art creators to continue pushing the boundaries of what is possible in the realm of artistic expression. The potential of fractal geometry is immense, and we look forward to seeing the further integration of this fascinating concept in digital art and beyond.

Acknowledgement

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