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Titius-Bode's Relation in Exoplanetary Systems

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The Titius-Bode's relation has been historically successful in predicting the location of Ceres in the solar system, while its physical basis remains hidden. In this study, we attempt to answer the question of whether the Titius-Bode's relation is universally valid for exoplanetary systems with plural exoplanets. For this purpose, we statistically study the distribution of the ratio of the orbiting periods of two planets in 32 exoplanetary systems hosted by a single star. We only consider the period ratios derived from exoplanets orbiting a single star since celestial objects under investigation are kept as simple as possible and free from uncertainties such as the mass of the host star. We find that the distribution of period ratios of two exoplanets appears inconsistent with that derived from the Titius-Bode's relation using the χ^2 test. We also found that the distance distribution in exoplanetary systems unlikely follows the uniform distribution or the Poisson's distribution. It is noted, however, that more rigorous statistical tests should be carried out to reach a more certain conclusion.

Keywords: solar system, celestial mechanics, general

1. INTRODUCTION

Johann Elert Bode published an astronomical compendium in 1772 in which he mentioned a numerical sequence for reproducing the distances of the planets from the Sun in our solar system proposed by Johann Daniel Titius von Wittenberg in 1766. At that time, only the six planets from Mercury to Saturn were known. On triumphant discovery of the planet Uranus by Frederick William Herschel in 1781, Bode recognized that Titius' relation had correctly predicted the semi-major axis of Uranus and consequently urged his contemporary observers to search for the so-called missing planet at the fifth position between Mars and Jupiter. The subsequent discovery of the dwarf-planet Ceres by Giuseppe Piazzi in 1801, with the semi-major axis predicted by Titius' relation, made people consider this relation as a natural law and called it the Titius-Bode's law or Titius-Bode's rule for some time. The relation was also expected to play a role in exploring the planet Neptune, but its discovery in 1846 was actually led by mathematical calculations for unexpected wobbles in the orbit of Uranus subject to gravitational

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perturbation by a hypothetical planet.

As it embarrassingly breaks down for objects farther than Neptune, numerous modifications were proposed for the Titius-Bode's relation (Blagg 1913; Brodetsky 1914; Wylie 1931; Richardson 1945; Dermott 1968; Nieto 1970; Rawal 1978, 1984, 1986, 1989; Basano & Hughes 1979; Louise 1982; Neuhäuser & Freitzinger 1986; Pletser 1986; Ragnarsson 1995; Dermott 1997; Ortiz et al. 2007; Scardigli 2007; Smirnov 2015). Interestingly enough, similar relations can be found for the satellite systems of giant planets in the solar system (Blagg 1913; Brodetsky 1914; Wylie 1931; Miller 1938; Todd 1938; Dermott 1968; Nieto 1970; Rawal 1984, 1986, 1989; Pletser 1986; Stone & Miner 1986; Li et al. 1995). Historical reviews of the Titius-Bode's relation can be found in Jaki (1972), Nieto (1972), Chapman (2001), and McFadden et al. (1999).

Besides the fact that Neptune's distance is discrepant, the weakest point of the Titius-Bode's relation as a physical law is that there is no evidence for the physical basis to explain such a simple mathematical sequence representing the distance of celestial bodies revolving around a central

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body. Theories of the solar system's formation have been put forward to account for this kind of distance relation. whose underlying physics to produce an ordered structure includes magneto-convection, plasma turbulence or even self-organizing processes. For example, some ideas involving harmonic resonances between their orbits during planetary system formation have been proposed (e.g., Goldreich & Sciama 1965; Patterson 1987; Filippov 1991). Alternative models utilized the self-gravitational instability in thin Keplerian disks (Ruediger & Tschaepe 1988; Rica 1995), and the principle of least action interaction (Ovenden 1972; Patton 1988). In addition, Hayes & Tremaine (1998) employed physical conditions in the initial proto-planetary disk and concluded that the planets in a stable system are not randomly but regularly spaced. Recently, Laskar & Petit (2017) developed a planet formation model based on an angular-momentum-deficit theory. The question of whether the observed patterns are due to stochastic processes was also attempted (e.g., Lecar 1973; Pletser 1988). It turns out, however, that the estimated probability of chance occurrence is sensitive to the restrictions imposed on the initial orbit environments (e.g., Lynch 2003).

Another critical issue on the Titius-Bode's relation is if there indeed exists a simple mathematical relation for the distance interval between planets in a planetary system. This question may be tackled by testing whether the Titius-Bode's relation is valid in exoplanetary systems. If this is the case, it would indirectly demonstrate that the Titius-Bode's relation is more than a pure mathematical relation. More beneficially, it would be greatly useful to make similar predictions for undetected planets in reported multiple exoplanet systems as the case of the discovery of Ceres (e.g., Mousavi-Sadr et al. 2021). In fact, the Titius-Bode's relation has been tried for several exoplanetary systems. The exoplanetary system with five planets, 55 Cancri (HD 75732), was investigated with some form of the Titius-Bode's relation in detail (Chang 2008; Poveda & Lara 2008; Cuntz 2012). Cuntz (2012) claimed that new exoplanet candidates in the 55 Cancri planetary system were predicted through the Titius-Bode's relation. On the other hand, conclusions remain disputed. For example, Chang (2008) found that the distribution of the exoplanets in the 55 Cnc system is apparently inconsistent with that derived from the Titius-Bode's relation. Using data from multi-exoplanetary systems, statistical tests have also been attempted to check if the positions of exoplanets obey a simple relation. Chang (2010) statistically explored the distribution of period ratios of exoplanets for 31 multiple exoplanet systems and found that the adherence of the exoplanets to the Titius-Bode's relation was statistically inconclusive. On the other hand,

Lara et al. (2020) used data from 27 exoplanetary systems with at least 5 planets and showed that the planetary orbital periods in exoplanetary systems were not consistent with a random distribution.

This unsettled position to make a conclusion motivates us to revisit the issue of whether exoplanetary systems satisfy the Titius-Bode's relation with an expanded dataset for improving the statistical significance. Thanks to the space missions such as Kepler and TESS, the number of detected exoplanets has been growing rapidly. As of May 2023, over 5,000 exoplanets have been detected and confirmed. Among these, a total of 856 exoplanetary systems host multiple planets, according to the Extrasolar Planets Encyclopedia (http://exoplanet.eu/catalog). Another merit of the current study is that we can afford to apply the Titius-Bode's relation separately to the planetary systems whose number of detected planets is different. Thus, we examined planetary systems hosting 5, 6, 7, 8 planets, respectively. In particular, we only focus on the exoplanet systems whose host star is a single star since the Titius-Bode's relation was originally proposed for the planetary system hosted by a single star. In other words, the exoplanet systems with multiple host-stars might comply with a different rule than the Titius-Bode's relation, if any. For instance, we exclude the exoplanetary system such as 55 Cancri which is now known as a binary system. Here, as done by Chang (2008, 2010), we investigate the distribution of period ratios of exoplanets detected in 32 multiple exoplanetary systems of a single host star hosting five or more confirmed planets. This paper begins with descriptions of the data of the exoplanetary system that we analyze in this paper in section 2. We present the results of the distribution of period ratios of exoplanets in section 3. Finally, we summarize and conclude our findings in section 4.

2. DATA

The observed parameters for the exoplanetary system are taken from the Extrasolar Planets Encyclopedia and the NASA Exoplanet Archive (https://exoplanetarchive.ipac. caltech.edu/index.html), where recently discovered extrasolar planets are constantly included and updated. In Table 1, we list 32 exoplanetary systems analyzed in this study, in which more than five planets are found to be orbiting around a single host star. In particular, the star KOI-351 (2MASS J18574403+4918185, KIC 11442793, *Kepler-90*, WISE J185744.03+491818.5) is found to have 8 exoplanets. This star is an F-type star whose effective temperature is 6080^{+260}_{-170} K, located about 855 pc from Earth in the constellation of Draco. The planet KOI-351 h with a mass of

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TRAPPIST-1f9.267540.038/91.0451.039TRAPPIST-1g12.324460.0681890.750.258TRAPPIST-1g18.729660.061123.6813.1TRAPPIST-1g12.7570.1283.6813.1HD 1010016.3570.12893.5325.10.17HD 1010048.7400.26995.3921.41.0HD 1010060.4571.4274.101.01.0HD 101002.2053.3819.464.40.1HD 1919392.8650.0043.411010HD 1919392.8330370.21322.9592.866VHD 19193910.120.07523.195866HD 19193910.120.0723.14660.0HD 1919392.2003.21.611.622.7HD 1913952.2003.23.7410.5.7HD 1913952.2003.23.71.611.617HD 1913952.2003.21.6124.74HD 1913952.2073.131.62.7HD 19139448.6590.03771.611.617HD 2191342.2473.111.27108.0622HD 2191342.2473.111.27108.0622HD 2191342.2473.111.271.61HD 2191342.2473.111.271.61HD 2191342.2473.111.271.61HD 219134<	2	TRAPPIST-1 e	6.101013	0.02925	0.92	0.692	M8	
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TRAPPIST IN18.7728660.061890.7550.3264HD 10180 (b6.5370.0264123.6813.13HD 10180 (b49.7480.26995.3925.1HD 10180 (c12.2740.4295.2422.93HD 10180 (c2.0253.3819.464.4HD 10180 (c2.0253.3819.464.4HD 191939 (c2.0253.3819.464.4HD 191939 (c2.037330.17523.1958HD 191939 (c2.040713112.266VHD 191939 (c0.008761.6024.74HD 191939 (c0.008761.6024.74HD 219134 (c6.76480.06331.514.36HD 219134 (c6.76480.06331.514.36HD 219134 (c6.76480.06331.514.36HD 219134 (c6.76480.028771.6116.17HD 219134 (c2.4773.111.27108.0622HD 219134 (c2.4670.17816.425.5HD 219134 (c2.4670.71818.425.5HD 219134 (c2.4670.71818.425.5HD 219134 (c1.7570.21874.29HD 34445 (c1.7870.2874.26HD 219134 (c6.3661.331.20.6HD 219134 (c1.7870.2181.27HD 219134 (c1.7870.28874.26HD 219134 (c1.7870.28874.26 </td <td></td> <td>TRAPPIST-1 g</td> <td>12.352446</td> <td>0.04683</td> <td>1.129</td> <td>1.321</td> <td></td>		TRAPPIST-1 g	12.352446	0.04683	1.129	1.321		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		TRAPPIST-1 h	18.772866	0.06189	0.755	0.326		
Bit DistanceHD 10180 (13.57)0.128593.4511.75HD 10180 (12.27.440.42995.3925.123.9HD 10180 (12.27.440.42935.2423.9HD 10180 (12.27.440.49121.4HD 10180 (12.27.440.4925.2423.9HD 10180 (12.27.453.3819.464.4HD 191939 (12.2650.06043.4110HD 191939 (13.2050.06043.4110HD 191939 (13.2050.06043.4110HD 191939 (13.2050.038761.616.8HD 191939 (10.120.40713112.2HD 191939 (10.120.40713.4660HD 191939 (10.120.038761.6124.74HD 191939 (12.07)0.038761.6124.74HD 191934 (12.77)0.14631.317.3HD 219134 (12.77)0.14631.317.3HD 219134 (12.77)0.14631.317.3HD 219134 (12.77)0.23871.611.61.7HD 219134 (12.77)0.23871.611.61.7HD 219134 (12.77)0.23871.630.6062HD 219134 (11.7870.23871.630.607HD 219134 (12.77)0.23871.611.61.7HD 219134 (12.77)0.4816.00.61HD 219134 (12.77)0.4816.0HD 219134 (12.77)0.4816.0HD 219134 (12.77)0.4816.0HD 219134 (12.77)0.6081.61HD 219134 (12.77) </td <td></td> <td>HD 10180 c</td> <td>5.75969</td> <td>0.06412</td> <td>3.68</td> <td>13.1</td> <td></td> <td></td>		HD 10180 c	5.75969	0.06412	3.68	13.1		
3HD 10100 HD 10100 HD 10100 1010100 HD 101000 22441227440.469295.2425.1 2.3421.4HD 101000 HD 101000 HD 101000 HD 191939 HD 1919390604.671.421.441.4HD 101000 HD 1919390 HD 191939022.0573.3819.4464.4		HD 10180 d	16.357	0.12859	3.45	11.75		1.06
3 HD 10180 f 122.744 0.4929 5.24 23.9 GIV Max HD 10180 h 604.67 1.427 4.91 2.1.4 -		HD 10180 e	49.748	0.2699	5.39	25.1		
HD 10180g HD 10180gh604.671.4274.912.14HD 10180gh2.2053.3819.464.HD 191393c2.85797430.00643.4110HD 191393c2.85797430.17523.1958HD 191393g30.3330370.21322.9952.8HD 191393g0.1120.40713112.2HD 191393g2.2003.213.460HD 191393g2.2003.213.460HD 191393f3.020260.003761.6024.74HD 219134h3.020250.003761.6114.36HD 219134f4.68890.2371.6116.17HD 219134g94.20.37533.2810.80622HD 219134h4.2473.111.27108.662HD 219134h2.2473.111.27108.662HD 34445c41.17.870.48176.0730.7HD 34445c11.7870.02674.261.06HD 34445d11.7870.33351.513.1HD 34445d1.0470.03351.513.1HD 34445d1.0492.0713.9260.6206HD 34445d1.0490.07823.391.26HD 34445d1.05753.127.3.6HD 34445d1.05770.31034.32HD 34445d1.05770.31034.32HD 34445d1.05770.31034.32HD 34445d1.05770.31033.23	3	HD 10180 f	122.744	0.4929	5.24	23.9	G1V	
HD 10180h2,2053,3819.464.4HD 19193928,88032560.08043.4110HD 19193928,530370.21322.9952.8HD 19193938,3530370.21322.9952.8HD 1919392.840.8123.7413.5HD 1919392.2003.213.4660HD 219134b3.0022060.038761.6024.74HD 219134b3.0022060.038761.6024.74HD 219134b2.21770.14631.317.3HD 219134b2.2473.1112.7108.0622HD 219134b2.2473.1112.7108.0622HD 219134b2.2473.1112.7108.0622HD 219134b2.2473.1112.7108.0622HD 34445521.4670.71818.423.55HD 3444551.0492.071.39200.056HD 3444555.7006.361.563.16HD 3444551.0492.071.39200.606HD 3444553.560040.044612.2996.31HD 344553.560040.044612.2996.31HD 344553.560040.044612.2996.31HD 344551.0492.33091.513.1HD 344553.560040.04682.397.92K213864.261660.07823.391.297RK213860.0553.127.3RK213860.055 <td></td> <td>HD 10180 g</td> <td>604.67</td> <td>1.427</td> <td>4.91</td> <td>21.4</td> <td></td>		HD 10180 g	604.67	1.427	4.91	21.4		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		HD 10180 h	2.205	3.381	9.4	64.4		
HD 191939c2852853.1958HD 1913939d3833033370.21322.9952.8HD 1913939c101.120.4071311.2HD 1913939c2.2003.213.4660HD 19139372.2003.213.4660HD 21913455.0029260.038761.6124.36HD 21913455.029260.038761.6124.36HD 21913466.76480.06531.5114.36HD 21913452.27170.14631.317.3HD 21913442.27170.14631.317.3HD 21913452.2473.111.27108.0622HD 21913442.2473.111.27108.0622HD 3444551.17870.26874.2618.8HD 3444561.17870.26874.2618.8HD 3444551.04992.0713.9260.6006HD 3444551.04990.038851.513.1HD 3444551.04990.038851.513.1K2-138c3.560040.04612.296.31K2-138c3.560040.05932.397.92K2-138c3.560070.05141.631.20K2-138c3.560040.05933.127.3K2-138c3.560040.05933.127.3K2-138c3.560040.05933.127.3K2-138c3.560040.05933.127.3K2-138c3.560051		HD 191939 b	8.8803256	0.0804	3.41	10		0.81
4HD 191939038.3530370.21322.9952.80B0BHD 1919391101.120.40713112.20BV0BV0BV0BVHD 19193952.203.240.8123.7413.50BV0BV0BV0BVHD 19193952.203.213.46600BS1.612AFA0BV		HD 191939 c	28.579743	0.1752	3.195	8		
$ \begin{array}{c c c c c c c } & HD 191939e & 101.12 & 0.407 & 13 & 112.2 & G8V & 0.81 \\ & HD 191939g & 244 & 0.812 & 3.74 & 13.5 & & & & \\ & HD 191939f & 2.200 & 3.2 & 13.4 & 660 & & & & & \\ & HD 219134b & 3.092926 & 0.03876 & 1.602 & 4.74 & & & & & & & & & \\ & HD 219134b & 3.092926 & 0.03876 & 1.602 & 4.74 & & & & & & & & & & & & \\ & HD 219134b & 2.2717 & 0.1463 & 1.31 & 7.3 & & & & & & & & & & & & & & & & & & &$		HD 191939 d	38.353037	0.2132	2.995	2.8		
HD 191939g HD 191939f2840.8123.7413.5HD 191939f2.2003.213.4660HD 219134b3.0929260.038761.6024.74HD 219134c6.764580.06531.514.36HD 219134f22.7170.14631.317.3HD 219134d46.8590.2371.6116.17HD 219134h22.7170.14631.317.3HD 219134h22.473.1112.7108.0622HD 219134h2.2473.1112.7108.0622HD 34445c11.7870.26874.2616.8HD 34445c21.4670.71816.425.5HD 34445c11.7870.48176.0730.7HD 34445f676.81.5436.8837.9HD 34445g5,7006.361.36120.6HD 34445g5,7006.361.513.1K2-138b2.33090.033851.513.1K2-138b2.35090.038851.513.1K2-138f1.56040.044612.2996.31K2-138f1.575780.104472.9041.63K2-138f1.362410.1072.872.9Kepler-11b1.030290.0911.81.9Kepler-11c13.92410.1072.732.9Kepler-11f46.68880.252.492Kepler-11g1.83070.01751.211.92Kepler-80f0.9867873 <td< td=""><td>4</td><td>HD 191939 e</td><td>101.12</td><td>0.407</td><td>13</td><td>112.2</td><td rowspan="3">G8V</td></td<>	4	HD 191939 e	101.12	0.407	13	112.2	G8V	
HD 19193972,2003.213.4660HD 219134b3.0929260.038761.6024.74HD 219134c6.764580.066531.5114.36HD 219134f2.2170.14631.317.3HD 219134d46.8590.2371.611617HD 219134g9.4.20.37533.2810.80622HD 219134h2.2473.1112.7108.0622HD 34445649.1750.26874.2616.8HD 344456214.670.71818.4253.5HD 344456117.870.48176.0730.7HD 3444561.0492.0713.9260.6206HD 3444561.0492.0713.6120.6HD 3444565.7006.361.513.1K2-13802.353090.033851.513.1K2-13845.60040.068932.397.92K2-13863.60040.044612.2996.31K2-13861.2757580.104472.9041.63K2-13861.2757580.104472.9041.63K2-13861.2757580.10472.9041.63Kepler-11c13.09410.1072.472.9Kepler-11f13.9960.1953.127.3Kepler-11f4.686880.252.492Kepler-11f4.686880.252.492Kepler-11f4.644890.00751.211.92Kepler-80f0.9967873 <t< td=""><td></td><td>HD 191939 g</td><td>284</td><td>0.812</td><td>3.74</td><td>13.5</td></t<>		HD 191939 g	284	0.812	3.74	13.5		
HD 219134 b 3.092926 0.03876 1.602 4.74 HD 219134 c 6.76458 0.0653 1.511 4.36 HD 219134 d 22.717 0.1463 1.31 7.3 K3V 0.794 HD 219134 d 46.859 0.23753 3.28 10.80622 102.19134 4.6459 0.2687 4.26 16.8 HD 219134 h 2.247 3.11 12.7 108.0622 108.0622 HD 34445 d 117.87 0.4817 6.07 30.7 HD 34445 d 11.787 0.4817 6.07 30.7 HD 34445 d 11.787 0.4817 6.07 30.7 HD 34445 d 1.049 2.07 13.9 260.6206 HD 34445 g 5.700 6.36 13.6 120.6 K2-138 c 3.26104 0.04461 2.299 6.31 K2-138 c 3.26104 0.0782 3.39 12.97 K2-138 f 12.75758 0.10447 2.904 1.63 K2-138 f<		HD 191939 f	2,200	3.2	13.4	660		
Barbon Barbon		HD 219134 b	3.092926	0.03876	1.602	4.74		
5HD 219134f22.7170.14631.317.3KBV0.794HD 219134d46.8590.2371.6116.17KBV0.794HD 219134g94.20.37533.2810.80622HD 219134h2,2473.1112.7108.0622HD 34445e49.1750.26874.2616.8HD 34445c21.4670.71818.4253.560VHD 34445t117.870.48176.0730.7HD 34445t1.0492.0713.9260.6206HD 34445t1.0492.0713.9260.6206HD 34445t1.0492.0713.9260.6206HD 34445t1.0492.0713.9260.6206HD 34445t1.0492.073.131.16K2 138 b2.35000.03851.513.1K2 138 c3.560040.04612.2996.31K2 138 c3.560040.04612.2901.63K2 138 c41.967970.231093.0134.32Kepler-11b10.302410.1072.872.9Kepler-11b13.02410.1072.872.9Kepler-11c13.9960.1953.127.33Kepler-11f46.68880.253.3325Kepler-11f46.68880.0553.127.3Kepler-11f46.68880.0153.127.3Kepler-11f46.68880.0252.143.92Kepler-11g11.83070.		HD 219134 c	6.76458	0.0653	1.511	4.36	K3V	0.794
5 HD 219134 d 46.859 0.237 1.61 16.17 K3V 0.794 HD 219134 g 94.2 0.3753 3.28 10.806622 1 <td></td> <td>HD 219134 f</td> <td>22.717</td> <td>0.1463</td> <td>1.31</td> <td>7.3</td>		HD 219134 f	22.717	0.1463	1.31	7.3		
HD 219134 g94.20.37533.2810.80622HD 219134 h2.2473.1112.7108.0622HD 34445 c49.1750.26874.2616.8HD 34445 d117.870.48176.0730.7HD 34445 d214.670.71818.4253.5HD 34445 f676.81.5436.8837.9HD 34445 f676.81.5436.8837.9HD 34445 f57.006.3613.6120.6HD 34445 g5.7006.3613.6120.6HD 34445 g5.7006.3615.13.1K2-138 b2.353090.033851.513.1K2-138 c3.560040.044612.2996.31K2-138 f12.757580.104472.9041.63K2-138 f12.757580.104472.9041.63K2-138 g41.967970.231093.0134.32Kepler-11 b10.30240.1072.872.9Kepler-11 c13.02410.1072.872.9Kepler-11 g13.02410.1072.872.9Kepler-11 g13.9960.1954.198Kepler-11 g11.830070.4663.3325Kepler-11 g11.830070.4663.3325Kepler-11 g11.830070.4663.3325Sepler-80 f0.98678730.01751.211.92Kepler-80 f0.98678730.06721.536.75Sepler-80	5	HD 219134 d	46.859	0.237	1.61	16.17		
HD 219134h 2,247 3.11 12.7 108.0622 HD 34445e 49.175 0.2687 4.26 16.8 HD 34445d 117.87 0.4817 6.07 30.7 HD 34445d 117.87 0.7181 8.42 53.5 60V 1.07 HD 34445b 676.8 1.543 6.88 37.9 60V 1.07 HD 34445b 1.049 2.07 13.9 260.6206 266.206 HD 34445g 5.700 6.36 13.6 120.6 29 6.31 K2-138b 2.3509 0.03385 1.51 3.1 3.4 3.4 K2-138c 3.56004 0.04461 2.299 6.31 3.1 3.9 2.60.6206 3.9 7.92 K1V 0.93 2.61 3.9 7.92 K1V 0.93 3.13 4.32 3.9 7.92 K1V 0.93 2.61 5.60 1.63 5.60 1.63 5.60 1.63 5.60 1.63 5.60		HD 219134 g	94.2	0.3753	3.28	10.80622		
HD 34445 e 49.175 0.2687 4.26 16.8 HD 34445 d 117.87 0.4817 6.07 30.7 HD 34445 c 214.67 0.7181 8.42 53.5 GOV 1.07 HD 34445 f 676.8 1.543 6.88 37.9 GOV 1.07 HD 34445 g 5.700 6.36 13.6 120.6 10.17 10.17 K2-138 b 2.35309 0.03385 1.51 3.1 1.1		HD 219134 h	2,247	3.11	12.7	108.0622		
HD 3445 d117.870.48176.0730.7HD 3445 c214.670.71818.4253.5GOV1.07HD 3445 f676.81.5436.8837.9GOV1.07HD 3445 f0.76.81.5436.8837.9GOV1.07HD 3445 f5,7006.3613.6120.61.01HD 3445 g5,7006.361.513.1K2-138 b2.353090.033851.513.1K2-138 c3.560040.04612.2996.31K2-138 f5.404790.058932.397.92K2-138 f12.757580.104472.9041.63K2-138 f12.757580.104472.9041.63K2-138 f12.757580.104472.9041.63Kepler-11 c13.02410.0722.872.9Kepler-11 d13.02410.1553.127.3Kepler-11 d2.66450.1553.127.3Kepler-11 f46.68880.252.492Kepler-11 f46.68880.252.492Kepler-11 f46.68880.252.492Kepler-11 g118.38070.0751.211.92Sepler-80 f0.98678730.01751.211.92Sepler-80 f0.98678730.01751.211.92Sepler-80 f0.644490.04911.536.75Sepler-80 b7.052460.06482.6		HD 34445 e	49.175	0.2687	4.26	16.8		-
6HD 34445 c214.670.71818.4253.5OV1.07HD 34445 f676.81.5436.8837.9000.0713.9260.6206HD 3445 g5,7006.3613.6120.6HD 3445 g5,7006.3613.6120.6K2-138 b2.353090.033851.513.1		HD 34445 d	117.87	0.4817	6.07	30.7	G0V K1V	1.07
6HD 34445 f676.81.5436.8837.9G0V1.07HD 34445 b1,0492.0713.9260.6206100100100100HD 34445 g5,7006.3613.6120.6100		HD 34445 c	214.67	0.7181	8.42	53.5		
$ \begin{array}{ c c c c c c } HD 34445 b & 1,049 & 2.07 & 13.9 & 260.6206 \\ \hline HD 34445 g & 5,700 & 6.36 & 13.6 & 120.6 \\ \hline HD 34445 g & 5,700 & 0.03385 & 1.51 & 3.1 \\ \hline K2-138 b & 2.35309 & 0.03385 & 1.51 & 3.1 \\ \hline K2-138 c & 3.56004 & 0.04461 & 2.299 & 6.31 \\ \hline K2-138 d & 5.40479 & 0.05893 & 2.39 & 7.92 \\ \hline K2-138 d & 8.26146 & 0.0782 & 3.39 & 12.97 \\ \hline K2-138 f & 12.75758 & 0.10447 & 2.904 & 1.63 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.0175 & 1.21 \\ \hline K2-16 & 0.99867873 & 0.0175 & 1.21 & 1.92 \\ \hline K2-17 & K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K2-18 & 0.99867873 & 0.0175 & 1.21 \\ \hline K$	6	HD 34445 f	676.8	1.543	6.88	37.9		
$\begin{array}{ c c c c c c } \hline HD 34445 g & 5,700 & 6.36 & 13.6 & 120.6 \\ \hline K2-138 b & 2.35309 & 0.03385 & 1.51 & 3.1 \\ \hline K2-138 c & 3.56004 & 0.04461 & 2.299 & 6.31 \\ \hline K2-138 d & 5.40479 & 0.05893 & 2.39 & 7.92 \\ \hline K2-138 d & 5.40479 & 0.05893 & 2.39 & 12.97 \\ \hline K2-138 f & 12.75758 & 0.10447 & 2.904 & 1.63 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.013 & 4.32 \\ \hline K2-138 g & 41.96797 & 0.23109 & 3.0175 & 1.21 & 1.92 \\ \hline K2-138 g & 4.64489 & 0.0491 & 1.6 & 4.13 & K5 & 0.73 \\ \hline H2-13 g & K2-138 & K2-138 & K2-138 \\ \hline K2-13 g & K2-138 & K2-138 & K2-138 \\ \hline K2-13 g & K2-138 & K2-138 & K2-138 & K2-138 \\ \hline K2-13 g & K2-138 & K2-138 & K2-138 & K2-138 & K2-138 \\ \hline K2-13 g & K2-138 & $		HD 34445 b	1,049	2.07	13.9	260.6206		
K2-138 b 2.35309 0.03385 1.51 3.1 K2-138 c 3.56004 0.04461 2.299 6.31 K2-138 d 5.40479 0.05893 2.39 7.92 K2-138 d 5.40479 0.05893 2.39 7.92 K2-138 e 8.26146 0.0782 3.39 12.97 K2-138 f 12.75758 0.10447 2.904 1.63 K2-138 g 41.96797 0.23109 3.013 4.32 Kepler-11 b 10.3039 0.091 1.8 1.9 Kepler-11 c 13.0241 0.107 2.87 2.9 Kepler-11 d 22.6845 0.155 3.12 7.3 Kepler-11 d 22.6845 0.155 3.12 7.3 Kepler-11 g 118.3807 0.466 3.33 25 Kepler-11 g 118.3807 0.466 3.33 25 Kepler-80 f 0.9867873 0.0175 1.21 1.92 Fepler-80 f 3.0722 0.0372		HD 34445 g	5,700	6.36	13.6	120.6		
K2-138 c3.560040.044612.2996.31K2-138 d5.404790.058932.397.92K2-138 e8.261460.07823.3912.97K2-138 f12.757580.104472.9041.63K2-138 g41.967970.231093.0134.32Kepler-11 b10.30390.0911.81.9Kepler-11 c13.02410.1072.872.90Kepler-11 d22.68450.1553.127.3Kepler-11 f46.68880.252.492Kepler-11 f46.68880.252.492Kepler-11 g118.38070.4663.3325Kepler-80 f0.98678730.01751.211.92Kepler-80 f3.072220.03721.536.759Kepler-80 e4.644890.04911.64.13K50.736Kepler-80 b7.052460.06482.676.93		K2-138 b	2.35309	0.03385	1.51	3.1		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		K2-138 c	3.56004	0.04461	2.299	6.31		
7 K2-138 e 8.26146 0.0782 3.39 12.97 KIV 0.93 K2-138 f 12.75758 0.10447 2.904 1.63		K2-138 d	5.40479	0.05893	2.39	7.92		0.93
K2-138 f 12.75758 0.10447 2.904 1.63 K2-138 g 41.96797 0.23109 3.013 4.32 Kepler-11 b 10.3039 0.091 1.8 1.9 Kepler-11 c 13.0241 0.107 2.87 2.9 Kepler-11 d 22.6845 0.155 3.12 7.3 G 0.96 Kepler-11 e 31.9996 0.195 4.19 8 6 9 9 6 3.33 25 9 9 118.3807 0.466 3.33 25 9 9 118.3807 0.0175 1.21 1.92 1.92 1.93 6.75 1.93 6.75 1.73 6.73 0.73 1.73 1.92 1.93 6.75 1.93 6.75 1.73 6.75 1.73 6.73 0.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73 1.73	7	K2-138 e	8.26146	0.0782	3.39	12.97		
K2-138 g 41.96797 0.23109 3.013 4.32 Kepler-11 b 10.3039 0.091 1.8 1.9 Kepler-11 c 13.0241 0.107 2.87 2.9 Kepler-11 d 22.6845 0.155 3.12 7.3 Kepler-11 e 31.9996 0.195 4.19 8 Kepler-11 f 46.6888 0.25 2.49 2 Kepler-11 g 118.3807 0.466 3.33 25 Kepler-80 f 0.9867873 0.0175 1.21 1.92 Kepler-80 f 0.9867873 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73		K2-138 f	12.75758	0.10447	2.904	1.63		
Kepler-11 b 10.3039 0.091 1.8 1.9 Kepler-11 c 13.0241 0.107 2.87 2.9 Kepler-11 d 22.6845 0.155 3.12 7.3 Kepler-11 e 31.9996 0.195 4.19 8 6 Kepler-11 f 46.6888 0.25 2.49 2 2 Kepler-11 g 118.3807 0.466 3.33 25 25 Kepler-80 f 0.9867873 0.0175 1.21 1.92 Kepler-80 d 3.07222 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 9 Kepler-80 b 7.05246 0.0648 2.67 6.93 6.93		K2-138 g	41.96797	0.23109	3.013	4.32		
Kepler-11 c 13.0241 0.107 2.87 2.9 8 Kepler-11 d 22.6845 0.155 3.12 7.3 Kepler-11 e 31.9996 0.195 4.19 8 6 Kepler-11 f 46.6888 0.25 2.49 2 2 Kepler-11 g 118.3807 0.466 3.33 25 2 Kepler-80 f 0.9867873 0.0175 1.21 1.92 7 Kepler-80 d 3.07222 0.0372 1.53 6.75 7 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 9 Kepler-80 b 7.05246 0.0648 2.67 6.93 7	8	Kepler-11 b	10.3039	0.091	1.8	1.9	G	0.96
Kepler-11 d 22.6845 0.155 3.12 7.3 G 0.96 Kepler-11 e 31.9996 0.195 4.19 8 6 0.96 Kepler-11 f 46.6888 0.25 2.49 2 7		Kepler-11 c	13.0241	0.107	2.87	2.9		
8 4.19 8 G 0.96 Kepler-11 e 31.9996 0.195 4.19 8 0.96 Kepler-11 f 46.6888 0.25 2.49 2 1 Kepler-11 g 118.3807 0.466 3.33 25 1 Kepler-80 f 0.9867873 0.0175 1.21 1.92 1 Kepler-80 d 3.07222 0.0372 1.53 6.75 1 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 kepler-80 b 7.05246 0.0648 2.67 6.93 1 1 1		Kepler-11 d	22.6845	0.155	3.12	7.3		
Kepler-11 f 46.6888 0.25 2.49 2 Kepler-11 g 118.3807 0.466 3.33 25 Kepler-80 f 0.9867873 0.0175 1.21 1.92 Kepler-80 d 3.07222 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 kepler-80 b 7.05246 0.0648 2.67 6.93		Kepler-11 e	31,9996	0.195	4.19	8		
Kepler-11g 118.3807 0.466 3.33 25 Kepler-80 f 0.9867873 0.0175 1.21 1.92 Kepler-80 d 3.07222 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 kepler-80 b 7.05246 0.0648 2.67 6.93 6.93 6.93		Kepler-11 f	46.6888	0.25	2.49	2		
Kepler-80 f 0.9867873 0.0175 1.21 1.92 Kepler-80 d 3.07222 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 Kepler-80 b 7.05246 0.0648 2.67 6.93 6.73		Kepler-11 g	118.3807	0.466	3.33	- 25		
Kepler-80 d 3.07222 0.0372 1.53 6.75 9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 kepler-80 b 7.05246 0.0648 2.67 6.93 6.93 6.93		Kepler-80 f	0.9867873	0.0175	1.21	1.92		
9 Kepler-80 e 4.64489 0.0491 1.6 4.13 K5 0.73 Kepler-80 b 7.05246 0.0648 2.67 6.93 6.93	9	Kepler-80 d	3.07222	0.0372	1.53	6.75		
Kepler-80 b 7.05246 0.0648 2.67 6.93		Kenler-80 e	4 64489	0.0491	1.6	4 13	K5	0.73
10010 00010 00010 000		Kepler-80 h	7.05246	0.0648	2.67	6.93		
Kepler-80 c 9.52355 0.0792 2.74 6.74		Kepler-80 c	9,52355	0.0792	2.74	6.74		

Table 1. Exoplanetary systems analyzed in the present analysis

(Table 1. Continued)

No	Nomo	Planet				Host Star	
INO.	Ivaille	Period (day)	Semi axis (AU)	Radius (R_{\oplus})	Mass (M_{\oplus})	Spec. type	Mass (M _☉)
9	Kepler-80 g	14.64558	1.13	1.51			
	TOI-1136 b	4.17278	-	1.9	4.27		
10	TOI-1136 c	6.25725	-	2.879	8.64		
	TOI-1136 d	12.51937	-	4.627	19.3		
	TOI-1136 e	18.7992	-	2.639	7.46	-	1.022
	TOI-1136 f	26.3162	-	3.88	14.3		
	TOI-1136 g	39.5387	-	2.53	6.94		
	TOI-178 b	1.914558	0.02607	1.152	1.5		
	TOI-178 c	3.23845	0.037	1.669	4.77		
	TOI-178 d	6 5577	0.0592	2 572	3.01		
11	TOI-178 e	9 961881	0.0783	2.312	3.86	-	0.65
	TOI 179 f	15 221015	0.1020	2.207	7.72		
	TOI-1781	20 7095	0.1055	2.207	2.04		
	UD 100226 b	20.7055	0.1275	1.615	2.04		
	HD 100230 D	5.795905	0.04527	1.013	3.24		
10	HD 108230 C	6.203449	0.062	2.071	4.94		0.050
12	HD 108236 d	14.175685	0.1074	2.539	6.98	-	0.853
	HD 108236 e	19.590025	0.1367	3.083	9.71		
	HD 108236 f	29.54115	0.1758	2.017	4.72		
	HD 158259 b	2.178	-	1.285	2.22		
	HD 158259 c	3.432	-	2.23	5.6		
13	HD 158259 d	5.1980814	-	2.18	5.41	G0	1.08
	HD 158259 e	7.951	-	2.34	6.08		
	HD 158259 f	12.028	-	2.35	6.14		
	HD 23472 d	3.97664	0.04298	0.75	0.55		
	HD 23472 e	7.90754	0.068	0.818	0.72		
14	HD 23472 f	12.1621839	0.0906	1.137	0.77	K3.5V	0.75
	HD 23472 b	17.667087	0.1162	2	8.32		
	HD 23472 c	29.79749	0.1646	1.87	3.41		
	HD 40307 b	4.3123	0.0468	1.83	4		
	HD 40307 c	9.6184	0.0799	2.46	6.6		
15	HD 40307 d	20.432	0.1321	3.04	9.5	K2.5V	0.77
	HD 40307 f	51.76	0.247	2.13	5.2		
	HD 40307 g	197.8	0.6	2.56	7.1		
	HIP 41378 b	15.572098	-	2.507	6.83		
	HIP 41378 c	31.70648	-	2.56	7.08		
16	HIP 41378 e	131	-	5.51	26	-	1.15
	HIP 41378 f	1.084	-	10.2	74		
	HIP 41378 d	1.114	_	3.96	14.9		
	K2-268 h	2 151676	0.0308	15	2.86		
	K2-268 d	4 528598	-	1.49	2.83		
17	K2_268 e	6 1312/3	_	1.43	2.03	_	0.84
17	K2-200 C	0.227527	0.0910	1.55	2.55	-	0.04
	K2-200 C	9.527527	0.0019	2.03	1.1 E G		
	K2-2001	20.27057		2.25	5.0		
18	K2-304 U	2.231327	-	1.070	1.20		
	K2-384 C	4.194766	-	1.191	1.82		0.00
	K2-384 d	6.679582	-	1.392	2.52	-	0.33
	K2-384 e	9.715043	-	1.345	2.37		
19	K2-384 t	13.62749	-	2.222	5.57		
	Kepler-102 b	5.28696	0.0552	0.47	4.3		
	Kepler-102 c	7.07142	0.067	0.58	3		
	Kepler-102 d	10.3117	0.0862	1.18	3.8	-	0.81
	Kepler-102 e	16.1457	0.1162	2.22	8.93		
	Kepler-102 f	27.4536	0.1655	0.88	5.2		
	Kepler-122 b	5.766193	0.064	2.34	6.08		
20	Kepler-122 c	12.465988	0.108	5.87	29	-	0.99
	Kepler-122 d	21.587475	0.155	2.2	5.48		

(Table 1. Continued)

No.	Name	Planet				Host Star	
		Period (day)	Semi axis (AU)	Radius (R_{\oplus})	Mass (M_{\oplus})	Spec. type	Mass (M _☉)
20	Kepler-122 e	37.993273	0.227	2.6	27.7		
	Kepler-122 f	56.268	0.287	1.75	36		
21	Kepler-150 b	3.428054	0.044	1.25	2.1		
	Kepler-150 c	7.381998	0.073	3.69	13.2		
	Kepler-150 d	12.56093	0.104	2.79	8.2	-	-
	Kepler-150 e	30.826557	0.189	3.12	9.91		
	Kepler-150 f	637.2093	1.24	3.64	12.9		
	Kepler-154 e	3.93276465	0.047	1.5	2.86		
	Kepler-154 f	9.91935684	0.0871	1.5	2.86		
22	Kenler-154 d	20 54981883	0 1415	3.84	14.1	_	0.89
	Kepler-154 b	33 040532	0.198	2.26	5 73		0.05
	Kepler-154 c	62 303276	0.303	2.25	9.01		
	Kepler-169 b	3 250619	0.00	1.13	1.51		
	Kepler 160 c	6 105460	0.04	1.15	1.02		
22	Kepler 160 d	0.133403	0.002	1.21	1.52		0.96
23	Kepler-169 d	8.348123	0.075	1.25	2.1	-	0.86
	Kepler-169 e	13.767102	0.105	2.2	5.48		
	Kepler-169 f	87.090195	0.359	2.58	7.18		
	Kepler-186 b	3.8867907	0.0343	1.07	1.24		
	Kepler-186 c	7.267302	0.0451	1.25	2.1		
24	Kepler-186 d	13.342996	0.0781	1.4	2.54	M1	0.54
	Kepler-186 e	22.407704	0.11	1.27	2.15		
	Kepler-186 f	129.9441	0.432	1.17	1.71		
	Kepler-238 b	2.090876	0.034	1.73	3.64		
	Kepler-238 c	6.155557	0.069	2.39	6.3		
25	Kepler-238 d	13.233549	0.115	3.07	9.64	-	1.43
	Kepler-238 e	23.654	0.1658	5.6	169.7		
	Kepler-238 f	50.447	0.2747	2	13.5		
	Kepler-292 b	2.580827	0.035	1.32	2.3		
	Kepler-292 c	3.715335	0.045	1.47	2.76		
26	Kepler-292 d	7.055679	0.068	2.23	5.6	-	0.88
	Kepler-292 e	11.97901	0.097	2.67	7.61		
	Kepler-292 f	20.834237	0.141	2.35	6.12		
	Kepler-32 f	0.74296	0.013	0.82	0.477		
	Kepler-32 e	2.896	0.033	1.5	2.86		
27	Kepler-32 b	5.90124	0.05	2.2	1,303	M1V	0.58
	Kepler-32 c	8.7522	0.09	2	158.9		
	Kepler-32 d	22.7802	0.13	2.7	7.75		
	Kepler-33 b	5.66793	0.0677	1.74	3.68		
	Kepler-33 c	13.17562	0.1189	3.2	0.39		
28	Kepler-33 d	21.77596	0.1662	5.35	3.91	-	1.29
	Kepler-33 e	31.7844	0.2138	4.02	5.57		
	Kepler-33 f	41.02902	0.2535	4.46	9.6		
	Kepler-55 d	2.211099	0.029	1.59	3.15		
	Kepler-55 e	4 617534	0.048	1.55	3.02		
29	Kepler-55 f	10 198545	0.081	1.59	3.15		0.62
	Kepler-55 h	27 9/181//19	0.1593	2.43	13		0.02
	Kepler-55 b	42 1516419	0.1555	2.45	43		
30	Kepler-55 C	5 714022	0.0552	1.21	05.0		
	Kepler-02 D	12 4417	0.0000	1.51	3		
	Kepler-62 C	12.4417	0.0929	0.54	4	VOV	0.00
	Kepler-02 u	10.10400	0.12	1.90	14	κZV	0.09
	Kepler-62 e	122.3874	0.427	1.01	30		
	Kepier-62 f	267.291	0.718	1.41	35		
31	Kepler-82 d	2.382961	0.034	1.77	3.78		
	Kepler-82 e	5.902206	0.063	2.47	6.66	-	0.91
	Kepler-82 b	26.44	0.1683	4.07	12.15		
	Kepler-82 c	51.54	0.2626	5.34	13.9		

No.	Name	Planet				Host Star	
		Period (day)	Semi axis (AU)	Radius (R_{\oplus})	Mass (M_{\oplus})	Spec. type	Mass (M _☉)
31	Kepler-82 f	75.732	0.3395	4.84	20.9		
32	Kepler-84 d	4.224537	0.052	1.38	2.48		
	Kepler-84 b	8.726	0.0827	2.23	40		
	Kepler-84 c	12.883	0.1072	2.36	20.2	-	1.022
	Kepler-84 e	27.434389	0.181	2.6	7.27		
	Kepler-84 f	44.552169	0.25	2.2	5.48		

(Table 1. Continued)

 $203\pm5~M_\oplus$ is orbiting at the distance of 1.01 AU which is slightly greater than the inner boundary of the habitable zone (http://www.exoplanetkyoto.org/exohtml/KOI-351. html). The star TRAPPIST-1 (2MASS J23062928-0502285) in the constellation Aquarius with the mass of about 0.0898 \pm 0.0023 M_{\odot} is at the distance of 12.47 pc from Earth and known to have 7 planets, three or four of which are located in the habitable zone (Wilson et al. 2021). Its effective temperature is $2,566 \pm 26K$, making it the coldest star currently known to host exoplanets. In addition, there are currently 9 and 21 exoplanetary systems hosting 6 and 5 exoplanets by a single star, respectively. Even though there are 5 (1) exoplanetary systems with 5 (6) exoplanets in the triple-star (binary-star) systems, respectively, we only concentrate on the exoplanetary system with a single star as stated above.

3. DISTRIBUTION OF RATIO OF ORBITAL PERIODS

In Fig. 1, we show the distribution of the ratio of the periods for two planets resulting from our solar system and from the Titius-Bode's relation with 8 terms. It should be stressed that in this study we consider the ratio of the orbital periods of two planets instead of that of the semi-major axes. As long as exoplanetary systems obey the Titius-Bode's relation, the ratio of the orbiting periods should also satisfy a specific distribution function based on Kepler's third law. Thus, we deal with a quantity almost independent of mass of the planetary system, since the mass is to be canceled out when taking the ratio. By doing so, one might get rid of an unknown role of mass of the host star in such relation to the distributions of the exoplanet's spacing. Furthermore, the period of exoplanets is a primary quantity directly measured by observations while the semi-major axis is secondary information deduced with assumptions such as mass.

To obtain the distribution, we randomly choose two different planets 10⁶ times securing the statistical significance level and take the ratio in a way that the value is always greater than unity. Even if the classical Titius-Bode's relation might appear to be an infinite sequence, it is likely to satisfy only a



Fig. 1. Distribution of the period ratios of two planets. In the upper panel, the distribution from our solar system from Mercury to Uranus with Ceres is shown. In the lower panel, the distribution of the ratio of periods of two planets calculated by the Titius-Bode's relation is shown. To obtain the distribution, we randomly choose two planets and take the ratio of the periods from the outer orbit planet to the inner orbit planet, $\frac{P_{out}}{P_m}$, such that the value is always greater than unity.

finite number of planets. Moreover, the relation rounded-off at Uranus can be exempted from arguments by perfectionism for a moment in the sense that the planets form in a finite size of the protosolar nebula from which the planets were created after a process of accretion of the planetesimals and the number of planets cannot be infinite. It should be mentioned, however, that the number of planets should not be arbitrary since the condition under which regions of the protoplanetary ring of the particles arose should be subject to chemical composition, shape of protoplanetary systems, and so on. In this context, we have presumed that the Titius-Bode's relation holds good up to Uranus. We only include planets up to Uranus while Ceres is involved. The Titius-Bode's relation itself and most of its modifications include Ceres as a representative of a provisional planet in the regular planetary distances. As one may expect, a statistical test by calculating the χ^2 value demonstrates that the observed ratio distribution is consistent with that derived

from the Titius-Bode's relation. That is, the χ^2 value is 6.2 with a probability of 96 %, indicating that two data sets are drawn from the same population. Results of the χ^2 test with different bin sizes also show that they are comparable with high probability of ~90% [see Chang (2010), for test results with different bin size].

In Figs. 2 and 3, we show the distribution of the ratio of periods of two planets resulting from the exoplanetary systems of KOI-351 and TRAPPIST-1, respectively. As for the exoplanetary system of KOI-351, all the planets are distributed inside the orbit of Mars. The semi-major axes of the three planets are less than that of Mercury and the shortest orbiting period is only about a week. Hence, the largest possible ratio of periods of two planets in this system is about 47. On the other hand, for the exoplanetary system of TRAPPIST-1, exoplanets have an orbital period ranging from 1.5 to 19 days, and corresponding semimajor axes are between 0.011 and 0.064 AU. That is, the system is so compact that the distribution of the period ratio is also much narrow. In Fig. 4, the distribution of the ratio of periods resulting from exoplanetary systems hosting 6 exoplanets. Each distribution is also obtained by randomly selecting pairs of exoplanets from all the possible combination in the particular exoplanetary system.

None of the distributions including those from the exoplanetary systems of KOI-351 and TRAPPIST-1 pass the χ^2 test, demonstrating that none of individual exoplanetary system shown in Figs. 2 to 4 obey the Titius-Bode's relation. The same conclusion is drawn from the 21 exoplanetary



Fig. 2. Distribution of the period ratios of two planets resulting from the KOI-351 exoplanetary system hosting 8 exoplanets.



Fig. 3. Distribution of the period ratios of two planets resulting from the TRAPPIST-1 exoplanetary system hosting 7 exoplanets.

systems with 5 exoplanets, even though resulting distributions have not been explicitly included. It turns out that the resulting probability is less than 50% to indicate that the observed distribution is largely inconsistent with that derived from the Titius-Bode's relation. To address any potential modification of the distribution due to the selection effect in detecting exoplanets, we repeatedly calculated the distribution of the ratio of periods of two planets from the Titius-Bode's relation by removing a planet one at a time on purpose. In other words, we simulated the case where only 7 out of 8 planets are observed for some reason. For example, one may imagine the situation where only 7 planets are detected due to an observational restriction although 8 planets are orbiting a host star. The results of the intentional exclusion are 1 of 8 distributions shown in Fig. 5. Note that these distributions are seemingly similar and yet dissimilar to that shown in the lower panel in Fig. 1. Hence, we tentatively conclude that the traditional Titius-Bode's relation cannot be applied to exoplanetary systems, even if the detection of exoplanets in a particular exoplanetary system is incomplete.

For comparison, we show the distribution of the ratio of periods of two planets derived from an artificial planetary system, whose semi-major axis is distributed by the uniform distribution and the Poisson's distribution in Fig. 6, respectively. Our artificial exoplanetary system has 8 planets. To generate the distribution function we generated 10⁶ pairs and the bin size in this particular example is unity. Surprisingly, they appear less inconsistent



Fig. 4. Distribution of the period ratios of two planets resulting from the exoplanetary systems hosting 6 exoplanets. The name of the star is indicated in the right upper corner.

to the observations than the Titius-Bode's relation does. However, results of the χ^2 test suggest that the distributions are unacceptable and dependent on how the test was performed. As a result, one has to rule out the possibility that the distribution of the ratio of the orbiting periods in multiple exoplanet systems is consistent with that derived from the uniform distribution nor the Poisson's distribution.

4. SUMMARY AND CONCLUSION

Since the discovery of Ceres the curious Titius-Bode's

relation has been studied in terms of the physics responsible for this mathematical sequence or the possibility of using as a useful tool for detecting undiscovered celestial bodies. The question we attempted to tackle in this study is if the Titius-Bode's relation is valid universally in exoplanetary systems hosting multiple exoplanets. The answer to this question is important partly because the underlying physics is still under debate and partly because it could greatly help to find out small terrestrial exoplanets in multiple exoplanetary systems or exomoons around exoplanets that would otherwise require hard work in observations and data analysis. In this paper, we have examined whether



Fig. 5. Distribution of the period ratios of two planets calculated by the Titius-Bode's relation. We show the case where only 7 planets are selected and 1 planet is omitted on purpose. The number in each panel indicated the ascending order of the removed exoplanet in distance from the closest to the host star.



Fig. 6. Distribution of the period ratios of two planets in an artificial planetary system. In the upper and lower panels, we show results from the uniform distribution of the semi-major axis and the Poisson's distribution, respectively.

the Titius-Bode's relation is applicable to exoplanetary systems hosting at least 5 exoplanets. For this purpose, we statistically studied the distribution of the orbiting period ratios of two exoplanets. We considered that taking the ratio of the orbiting period has an advantage since the ratio is independent of the mass of the host star.

We find that the distribution of the ratio of the periods of two exoplanets is apparently inconsistent with that derived from the Titius-Bode's relation. As a result of the χ^2 test we carried out, the possibility that the distribution of the ratio of the orbiting periods in multiple exoplanet systems is compatible with that derived from the Titius-Bode's relation should be ruled out. We interpret this conclusion such that the Titius-Bode's relation cannot be used to quantitatively assess the possible existence of missing exoplanets in a multiple exoplanetary systems. Having speculated the uniform distribution and the Poisson's distribution of exoplanets, we tentatively conclude that the distance distribution in exoplanetary systems is also unlikely following

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those distributions. More rigorous statistical tests should be carried out in terms of the properties of parameters generating sub-samples.

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REFERENCES

- Basano L, Hughes DW, A modified Titius-Bode law for planetary orbits, Il Nuovo Cimento C 2, 505-510 (1979). https://doi. org/10.1007/BF02557750
- Blagg MA, On a suggested substitute for Bode's law. (Plate 12.), Mon. Not. R. Astron. Soc. 73, 414-422 (1913). https://doi. org/10.1093/mnras/73.6.414
- Brodetsky S, Some problems of astronomy (XVII Bode's law), Observatory 37, 338-341 (1914).
- Chang HY, Titius-Bode's relation and 55 Cancri, J. Astron. Space Sci. 25, 239-244 (2008). https://doi.org/10.5140/ JASS.2008.25.3.239
- Chang HY, Titius-Bode's relation and distribution of exoplanets, J. Astron. Space Sci. 27, 1-10 (2010). https://doi.org/10.5140/ JASS.2010.27.1.001

Chapman DMF, Reflections: the Titius-Bode rule, part 1: discovering the asteroids, J. R. Astron. Soc. Can. 95, 135-137 (2001).

Cuntz M, Application of the Titius-Bode rule to the 55 Cancri system: tentative prediction of a possibly habitable planet, Publ. Astron. Soc. Jpn. 64, 73 (2012). https://doi.org/10.1093/ pasj/64.4.73

Dermott SF, On the origin of commensurabilities in the solar system—II. the orbital period relation, Mon. Not. R. Astron. Soc. 141, 363-376 (1968). https://doi.org/10.1093/ mnras/141.3.363

- Dermott SF, The Origin of the Solar System (John Wiley & Sons, New York, 1997).
- Filippov AÉ, Collapse of a dusty medium and Titius-Bode law in natural units, JETP Lett. 54, 351 (1991).
- Goldreich P, Sciama DW, An explanation of the frequent occurrence of commensurable mean motions in the solar system, Mon. Not. R. Astron. Soc. 130, 159-181 (1965). https://doi.org/10.1093/mnras/130.3.159
- Hayes W, Tremaine S, Fitting selected random planetary systems to Titius-Bode laws, Icarus 135, 549-557 (1998). https://doi. org/10.1006/icar.1998.5999
- Jaki SL, The original formulation of the Titius-Bode law, J. Hist. Astron. 3, 136-138 (1972). https://doi.org/10.1177/ 002182867200300205
- Lara P, Cordero-Tercero G, Allen C, The reliability of the Titius-Bode relation and its implications for the search for exoplanets, Publ. Astron. Soc. Jpn. 72, 24 (2020). https://doi. org/10.1093/pasj/psz146
- Laskar J, Petit AC, AMD-stability and the classification of planetary systems, Astron. Astrophys. 605, A72 (2017). https://doi.org/10.1051/0004-6361/201630022
- Lecar M, Bode's law, Nature 242, 318-319 (1973). https://doi. org/10.1038/242318a0
- Li XQ, Zhang H, Li QB, Self-similar collapse in nebular disk and the Titius-Bode law, Astron. Astrophys. 304, 617-621 (1995).
- Louise R, A postulate leading to the Titius-Bode law, Moon Planets 26, 93-96 (1982). https://doi.org/10.1007/BF00941371
- Lynch P, On the significance of the Titius-Bode law for the distribution of the planets, Mon. Not. R. Astron. Soc. 341, 1174-1178 (2003). https://doi.org/10.1046/j.1365-8711.2003. 06492.x
- McFadden LA, Weissman PR, Johnson TV, Encyclopedia of the Solar System (Academic Press, New York, 1999).
- Miller J, Bode's law and the systems of the planets and satellites, Nature 142, 670-671 (1938). https://doi.org/10.1038/142670b0
- Mousavi-Sadr M, Gozaliasl G, Jassur DM, Exoplanets prediction in multiplanetary systems, Publ. Astron. Soc. Aust. 38, e015 (2021). https://doi.org/10.1017/pasa.2021.9
- Neuhäuser R, Feitzinger JV, A generalized distance formula for planetary and satellite systems, Astron. Astrophys. 170, 174-178 (1986).
- Nieto MM, Conclusions about the Titius Bode law of planetary distances, Astron. Astrophys 8, 105 (1970).
- Nieto MM, The Titius-Bode Law of Planetary Distances: Its History and Theory (Pergamon Press, Oxford, 1972).
- Ortiz JL, Moreno F, Molina A, Santos Sanz P, Gutiérrez PJ, Possible patterns in the distribution of planetary formation regions, Mon. Not. R. Astron. Soc. 379, 1222-1226 (2007). https://doi. org/10.1111/j.1365-2966.2007.12017.x

Ovenden MW, Physical sciences: Bode's law and the missing

planet, Nature 239, 508-509 (1972). https://doi.org/10.1038/ 239508a0

- Patterson CW, Resonance capture and the evolution of the planets, Icarus 70, 319-333 (1987). https://doi.org/10.1016/0019-1035(87)90138-2
- Patton JM, On the dynamical derivation of the Titius-Bode law, Celest. Mech. 44, 365-391 (1988). https://doi.org/10.1007/ BF01234273
- Pletser V, Lois exponentielles de distance pour les systemes de satellites, Earth Moon Planets 36, 193-210 (1986). https://doi.org/10.1007/BF00055159
- Pletser V, Exponential distance relations in planetary-like systems generated at random, Earth Moon Planets 42, 1-18 (1988). https://doi.org/10.1007/BF00118035
- Poveda A, Lara P, The exo-planetary system of 55 Cancri and the Titius-Bode law, Rev. Mex. Astron. Astrofis. 44, 243-246 (2008). https://doi.org/10.48550/arXiv.0803.2240
- Ragnarsson SI, Planetary distances: a new simplified model, Astron. Astrophys. 301, 609 (1995).
- Rawal JJ, Modified Titius-Bode relation, Bull. Astron. Soc. India 6, 92 (1978).
- Rawal JJ, Contraction of the solar nebula, Earth Moon Planets 31, 175-182 (1984). https://doi.org/10.1007/BF00055528
- Rawal JJ, Further considerations on contracting solar nebula, Earth Moon Planets 34, 93-100 (1986). https://doi.org/ 10.1007/BF00054038

- Rawal JJ, Contractions of subsolar nebulae, Earth Moon Planets 44, 265-274 (1989). https://doi.org/10.1007/BF00054242
- Rica S, Pattern formation through gravitational instability, C. R. Acad. Sci. 320, 489-496 (1995).
- Richardson DE, Distances of planets from the sun and of satellites from their primaries, Popular Astron. 53, 14 (1945).
- Ruediger G, Tschaepe R, The Titius-Bode law in the light of density wave theory, Gerlands Beitr. Geophys. 97, 97-102 (1988).
- Scardigli F, A quantum-like description of the planetary systems, J. Phys. Conf. Ser. 67, 012038 (2007). https://doi.org/10.1088/ 1742-6596/67/1/012038
- Smirnov VA, The physical meaning of the Titius-Bode formula, Odessa Astron. Publ. 28, 62-64 (2015).
- Stone EC, Miner ED, The Voyager 2 encounter with the Uranian system, Science 233, 39-43 (1986). https://doi.org/10.1126/ science.233.4759.39
- Todd GW, Kepler, Newton and Bode, Nature 141, 412 (1938). https://doi.org/10.1038/141412a0
- Wilson DJ, Froning CS, Duvvuri GM, France K, Youngblood A, et al., The Mega-MUSCLES spectral energy distribution of TRAPPIST-1, Astronphys. J. 911, 18 (2021). https://doi. org/10.3847/1538-4357/abe771
- Wylie CC, Bode's and similar empirical laws, Pop. Astron. 39, 75 (1931).