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ASSOCIATED CURVES OF CHARGED PARTICLE MOVING WITH THE EFFECT OF MAGNETIC FIELD

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Abstract. Magnetic curves are the trajectories of charged particals which are influenced by magnetic fields and they satisfy the Lorentz equation. It is important to find relationships between magnetic curves and other special curves. This paper is a study of magnetic curves and this kind of relationships. We give the relationship between β-magnetic curves and Mannheim, Bertrand, involute-evolute curves and we give some geometric properties about them. Then, we study this subject for γ-magnetic curves. Finally, we give an evaluation of what we did.

1. Introduction

The theory of curves in Euclidean 3-space is a fundamental study area in differential geometry. There are many kinds of curves in Euclidean 3-space. For example, helices are one of the most frequently used ones. Associated curves of a given curve are also widely studied. The most popular ones are Bertrand curve pairs, Mannheim curve pairs, involute-evolute curve pairs and spherical indicatrices, [19].

Frenet frame of the curves is widely used to describe the geometric properties of the curves and to classify the curves in Euclidean 3-space. For example, involute-evolute curve pairs, Bertrand curve pairs and Mannheim curve pairs are defined using the Frenet frame of the curves. There are many studies about associated curves, one can see [13, 23, 27, 28].

There is a special kind of curve which is widely studied in differential geometry and physics. It is called magnetic curve. Magnetic curves are the trajectories of charged particles which are under the effect of magnetic fields. They satisfy a certain equation namely the Lorentz equation. This equation generalizes the equation which is satisfied by the geodesics on a manifold. So, one can consider the magnetic curves as a generalization of geodesics on a manifold, [6]. On the other hand, magnetic curves appear also from the variational problem of the Landau-Hall functional. This functional is the kinetic

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energy functional when there is no magnetic field. The geodesics are known as the critical points of the energy functional. This is another way to say that magnetic curves are generalizations of geodesics. So, the geometric properties of magnetic curves give important information of the manifold which contains magnetic curves, [10].

In [11], Izumiya and Takeuchi studied general properties of helices and Bertrand curves and in [20], Masal gave Mannheim B-curve in Euclidean 3-space. In [7], Ekmekci and Ilarslan gave a characterization of Bertrand curves in Lorentzian space, in [21], Matsuda and Yorozu proved that no special Frenet curve in n-dimensional Euclidean space ($n \geq 4$) is a Bertrand curve. Ozyilmaz and Yilmaz studied involute-evolute curve pairs in 4-dimensional Euclidean space in [26], in [8], Fuchs studied evolutes and involutes of space curves, in [2], Bilici and Caliskan studied involutes of the spacelike curve with a timelike binormal in Minkowski 3-space, in [9], Fukunaga and Takahashi studied evolutes and involutes of frontals in the Euclidean plane. In [18], Liu and Wang worked on Mannheim curve pairs in Euclidean 3-space. In [25], Orbay and Kasap studied Mannheim partner curves in $\mathbb{E}^3$ and gave the relationships between the curvatures of the Mannheim curve pairs with respect to each other. In [22], Munteanu studied magnetic curves in Euclidean space. In [6], Druta and Munteanu studied magnetic curves corresponding to Killing magnetic fields in Euclidean 3-space. Magnetic curves in Sasakian manifolds are studied by Druta in [5], magnetic curves on flat para-Kähler manifolds are studied by Jleli and Munteanu in [12]. Inoguchi and Munteanu studied magnetic curves in the real special linear group in [10]. There are many studies about magnetic curves, one can see [14–16].

This study consists of four sections. In Section 1, we mention about the importance of magnetic curves and associated curves and we present some recent works from literature. In Section 2, we present definitions of some special curves which are widely used in differential geometry and we mention about some basic concepts and properties in there. Section 3 includes our main results and has two subsections. In Subsection 3.1 we give the relationship between β-magnetic curves and Mannheim, Bertrand, involute-evolute curves and we give some geometric properties about them. In Subsection 3.2 we study this subject for γ-magnetic curves. In last section, we give an evaluation of what we did.

2. Preliminaries

Assume that (M, g) is a Riemannian manifold. A closed 2-form F on (M, g) is called a magnetic field and the linear endomorphism $\phi : \chi(M) \longrightarrow \chi(M)$ which satisfies

$$F(X, Y) = g(\phi(X), Y), \quad X, Y \in \chi(M) \tag{1}$$

is called the Lorentz force ϕ associated to F.

The magnetic trajectories of F are the magnetic curves ξ in M which satisfy the Lorentz equation

$$\nabla_{\xi'}\xi' = \phi(\xi'), \tag{2}$$

where ∇ is the Levi-Civita connection on M.

Since the equation of geodesics in M is

$$\nabla_{\xi'}\xi' = 0, \tag{3}$$

one can see that the Lorentz equation is a generalization of this equation, so magnetic curves generalize the geodesics, [6].

Let X be a smooth vector field on a manifold M. A smooth curve $\zeta : I \to M$ is said to be an integral curve of X, if for any $t \in I$, [17],

$$\zeta'(t) = X_{\zeta(t)}. \tag{4}$$

Since the Lorentz force is skew-symmetric, we can write

$$\frac{d}{dt}g(\xi', \xi') = 2g(\nabla_{\xi'}\xi', \xi') = 0, \tag{5}$$

so magnetic curves have a constant speed and energy, [6]. A normal magnetic curve is a magnetic curve which is parametrized with arc length. Studying on 3-dimensional Riemannian manifolds, 2-forms and vector fields can be identified. So, magnetic fields mean divergence free vector fields, [22]. A vector field V on M is Killing if and only if for every vector fields Y and Z on M

$$g(\nabla_Y V, Z) + g(\nabla_Z V, Y) = 0 \tag{6}$$

holds. Magnetic fields corresponding to Killing vector fields are known as Killing magnetic fields. Their trajectories are called Killing magnetic curves and are very important since they are related to the Kirchhoff elastic rods, [10].

Let x and x_1 be two space curves which are given with the arc length parameter s and s^*, respectively and $\{\alpha, \beta, \gamma\}$ and $\{\alpha^*, \beta^*, \gamma^*\}$ be their Frenet vector fields, respectively. Then,

(1) x is called a Mannheim curve and (x, x_1) is called a Mannheim curve pair if for every s and s^*, $\{\beta, \gamma^*\}$ is linearly dependent, [25].
(2) (x, x_1) is called a Bertrand curve pair if for every s and s^*, $\{\beta, \beta^*\}$ is linearly dependent, [24].
(3) x_1 is called an involute of x and x is called an evolute of x_1 if for every s and s^*, $\langle \alpha, \alpha^* \rangle = 0$, [24].

Note that evolute of any space curve is defined also as the locus of the centers of curvature of the curve. The original curve is then defined as the involute of the evolute, [1]. It is well-known that the distance between the corresponding points of the Bertrand curve pairs is constant. Every circular helix in Euclidean 3-space is a typical example of Bertrand curves, [21].

3. Results and discussion

In this section, we consider β-magnetic and γ-magnetic curves related to associated curves. We give some geometric properties about them and we give conditions on magnetic curves to be a Mannheim, Bertrand, involute-evolute curve or a straight line.

Let σ be a space curve parametrized with arc length in Euclidean 3-space, $\{\alpha, \beta, \gamma\}$ be its Frenet vector fields and κ, τ be its curvature and torsion, respectively. If there exists a magnetic field V_1 of the curve σ such that

$$V_1 = \tau\alpha - \Omega_1\beta + \kappa\gamma, \tag{7}$$

where $\Omega_1 = g(\phi(\alpha), \gamma)$, then σ is called a β-magnetic trajectory of the magnetic field V_1 and if there exists a magnetic field V_2 of the curve σ such that

$$V_2 = \tau\alpha + \Omega_2\gamma, \tag{8}$$

where $\Omega_2 = g(\phi(\alpha), \beta)$, then σ is called a γ-magnetic trajectory of the magnetic field V_2, [3]. κ and τ are non-zero curvatures in this study.

3.1. Associated curves with $\boldsymbol{\beta}$-magnetic trajectory

Assume that,

$$\eta : I \subset R \to E^3 \tag{9}$$

is a β-magnetic curve parametrized with arc length parameter s, Frenet vector fields of η are $\{\alpha, \beta, \gamma\}$ and Frenet curvatures of η are κ, τ. β-magnetic field of η is

$$W_1 = \frac{1}{\sqrt{\kappa^2 + \Omega_1^2 + \tau^2}}(\tau\alpha - \Omega_1\beta + \kappa\gamma). \tag{10}$$

η_1 is an integral curve of W_1 and the arc length parameter of η_1 is denoted by s^*. Frenet vector fields of η_1 are $\{\alpha^*, \beta^*, \gamma^*\}$ and Frenet curvatures of η_1 are κ^*, τ^*. Then, similar to definitions in [4], η_1 is called W_1-direction curve of η and η is also called W_1-donor curve of η_1.

Theorem 3.1. *β-magnetic curve η is a Mannheim curve.*

Proof. Let η_1 be a Mannheim partner curve of η, so we can write

$$\eta_1(s) = \eta(s) + \lambda_1(s)\beta(s), \tag{11}$$

where λ_1 is a differentiable non-zero function. Let us write $\delta = \sqrt{\kappa^2 + \Omega_1^2 + \tau^2}$. Differentiating both sides of the equation (11) according to arc length parameter s, we get

$$\frac{1}{\delta}(\tau\alpha - \Omega_1\beta + \kappa\gamma)\frac{ds^*}{ds} = (1 - \lambda_1\kappa)\alpha + \lambda'\beta + \lambda\tau\gamma. \tag{12}$$

Therefore, we derive

$$\tau\frac{ds^*}{ds} = (1 - \lambda_1\kappa)\delta, \tag{13}$$

$$-\Omega_1 \frac{ds^*}{ds} = \lambda_1' \delta, \tag{14}$$

$$\kappa \frac{ds^*}{ds} = \lambda_1 \delta \tau. \tag{15}$$

Considering the equations (13) and (15), we obtain

$$\kappa = \lambda_1(\kappa^2 + \tau^2). \tag{16}$$

From [18], η is a Mannheim curve. □

Theorem 3.2. *Let η be a Mannheim curve and (η, η_1) be a Mannheim curve pair. Then,*

$$\tau^*(\lambda_2 + \frac{1}{\kappa}) \sin\theta = \frac{\tau}{\sqrt{\kappa^2 + \Omega_1^2 + \tau^2}}, \tag{17}$$

where θ is the angle between α and α^*.

Proof. Since η is a Mannheim partner curve of η_1, we can write

$$\eta(s^*) = \eta_1(s^*) + \lambda_2(s^*)\gamma^*(s^*), \tag{18}$$

where λ_2 is a differentiable non-zero function. Let us write $\delta = \sqrt{\kappa^2 + \Omega_1^2 + \tau^2}$. Differentiating both sides of the equation (18) according to arc length parameter s^*, we get

$$\frac{d\eta}{ds}\frac{ds}{ds^*} = \frac{\tau}{\delta}\alpha + (\lambda_2' - \frac{\Omega_1}{\delta})\beta + \frac{\kappa}{\delta}\gamma - \lambda_2 \tau^* \beta^*. \tag{19}$$

Taking the inner product of both sides of the equation (19) by β, we get

$$\Omega_1 = \delta \lambda_2'. \tag{20}$$

So equation (19) can be rewritten as

$$\frac{d\eta}{ds}\frac{ds}{ds^*} = \frac{\tau}{\delta}\alpha + \frac{\kappa}{\delta}\gamma - \lambda_2 \tau^* \beta^*. \tag{21}$$

On the other hand, we can write

$$\alpha = \alpha^* \cos\theta + \beta^* \sin\theta. \tag{22}$$

Differentiating both sides of the equation (22) according to s^*, we get

$$\frac{d\alpha}{ds}\frac{ds}{ds^*} = -(\theta' + \kappa^*)\sin\theta\alpha^* + (\theta' + \kappa^*)\cos\theta\beta^* + \tau^* \sin\theta\gamma^*. \tag{23}$$

Then, we derive

$$\theta' = -\kappa^* \quad \text{and} \quad \kappa \frac{ds}{ds^*} = \tau^* \sin\theta. \tag{24}$$

Considering the equations (21), (22), (23) and (24), we obtain

$$\tau^*(\lambda_2 + \frac{1}{\kappa}) \sin\theta = \frac{\tau}{\delta}. \tag{25}$$

□

Corollary 3.3. $\Omega_1 = \lambda_2' \sqrt{\frac{\kappa^2+\tau^2}{1-(\lambda_2')^2}}$.

Proof. The result is obvious from Theorem 3.2. □

Theorem 3.4. *η_1 is a straight line if and only if*

$$\frac{\tau' + \Omega_1\kappa}{\tau} = \frac{\kappa' - \Omega_1\tau}{\kappa} = \frac{\Omega_1'}{\Omega_1}. \tag{26}$$

Proof. The necessary and sufficient condition for η_1 to be a straight line is $\eta_1'' = 0$. Let us write $\delta = \sqrt{\kappa^2 + \Omega_1^2 + \tau^2}$. So, we derive that η_1 is a straight line if and only if

$$\frac{d}{ds}\left[\frac{1}{\delta}(\tau\alpha - \Omega_1\beta + \kappa\gamma)\right] = 0. \tag{27}$$

Then, we compute

$$\frac{d}{ds}\left[\frac{\tau}{\delta}\right] = \frac{-\Omega_1\kappa}{\delta}, \tag{28}$$

$$\frac{d}{ds}\left[\frac{\Omega_1}{\delta}\right] = 0, \tag{29}$$

$$\frac{d}{ds}\left[\frac{\kappa}{\delta}\right] = \frac{\Omega_1\tau}{\delta}. \tag{30}$$

Calculating the derivatives, we get

$$(\tau' + \Omega_1\kappa)\delta = \tau(\kappa\kappa' + \Omega_1\Omega_1' + \tau\tau'), \tag{31}$$

$$\Omega_1'\delta = \Omega_1(\kappa\kappa' + \Omega_1\Omega_1' + \tau\tau'), \tag{32}$$

$$(\kappa' - \Omega_1\tau)\delta = \kappa(\kappa\kappa' + \Omega_1\Omega_1' + \tau\tau'). \tag{33}$$

Considering the equations (31), (32) and (33), we obtain

$$\frac{\tau' + \Omega_1\kappa}{\tau} = \frac{\kappa' - \Omega_1\tau}{\kappa} = \frac{\Omega_1'}{\Omega_1}. \tag{34}$$

□

Corollary 3.5. *Let η_1 be a straight line. If $\Omega_1 = constant$ and $\kappa^2 + \tau^2 = constant$, then β-magnetic curve η is a slant helix.*

Proof. Since η_1 is a straight line,

$$\frac{\tau' + \Omega_1\kappa}{\tau} = \frac{\kappa' - \Omega_1\tau}{\kappa} = \frac{\Omega_1'}{\Omega_1} \tag{35}$$

holds. Then, we derive

$$\Omega_1 = -\frac{\kappa^2}{\kappa^2 + \tau^2} \cdot \left(\frac{\tau}{\kappa}\right)'. \tag{36}$$

Hence,

$$\frac{\kappa^2}{(\kappa^2 + \tau^2)^{\frac{3}{2}}} \cdot \left(\frac{\tau}{\kappa}\right)' = constant. \tag{37}$$

So, η is a slant helix. □

Theorem 3.6. *There is not a β-magnetic curve η such that (η, η_1) is a Bertrand curve pair.*

Proof. Suppose (η, η_1) is a Bertrand curve pair. Then, we can write

$$\eta_1(s) = \eta(s) + \lambda_3(s)\beta(s), \tag{38}$$

where λ_3 is a differentiable non-zero function. Differentiating both sides of equation (38) according to s, we get

$$\alpha^* \frac{ds^*}{ds} = (1 - \lambda_3\kappa)\alpha + \lambda_3'\beta + \lambda_3\tau\gamma. \tag{39}$$

Taking the inner product of both sides of the equation (39) by β, we find

$$\lambda_3' = 0. \tag{40}$$

So, after rearranging equation (39), we derive

$$\tau \frac{ds^*}{ds} = (1 - \lambda_3\kappa)\delta, \tag{41}$$

$$-\Omega_1 \frac{ds^*}{ds} = \lambda_3'\delta, \tag{42}$$

$$\kappa \frac{ds^*}{ds} = \lambda_3\tau\delta, \tag{43}$$

where $\delta = \sqrt{\kappa^2 + \Omega_1^2 + \tau^2}$. Since $\lambda_3' = 0$, from equation (42), we get

$$\Omega_1 \frac{ds^*}{ds} = 0. \tag{44}$$

This is a contradiction, so there is not a β-magnetic curve η such that (η, η_1) is a Bertrand curve pair. □

Theorem 3.7. *There is not a β-magnetic curve η such that η_1 is an involute of η.*

Proof. Suppose η_1 is an involute of η. Then, we can write

$$\eta_1(s) = \eta(s) + \lambda_4(s)\alpha(s), \tag{45}$$

where λ_4 is a differentiable non-zero function. Differentiating both sides of equation (45) according to s, we get

$$\alpha^* \frac{ds^*}{ds} = (1 + \lambda_4')\alpha + \lambda_4\kappa\beta. \tag{46}$$

Taking the inner product of both sides of the equation (46) by α, we find

$$1 + \lambda_4' = 0. \tag{47}$$

So, after rearranging equation (46), we derive

$$-\Omega_1 \frac{ds^*}{ds} = \lambda_4\kappa\delta, \tag{48}$$

$$\tau \frac{ds^*}{ds} = 0, \tag{49}$$

$$\kappa \frac{ds^*}{ds} = 0, \tag{50}$$

where $\delta = \sqrt{\kappa^2 + \Omega_1^2 + \tau^2}$. From equations (49) and (50), we have a contradiction, so there is not a β-magnetic curve η such that η_1 is an involute of η. ☐

3.2. Associated curves with γ-magnetic trajectory

Assume that,

$$\mu : I \subset R \to E^3 \tag{51}$$

is a γ-magnetic curve parametrized with arc length parameter s, Frenet vector fields of μ are $\{\alpha, \beta, \gamma\}$ and Frenet curvatures of μ are κ, τ. γ-magnetic field of μ is

$$W_2 = \frac{1}{\sqrt{\tau^2 + \Omega_2^2}} (\tau\alpha + \Omega_2\gamma). \tag{52}$$

μ_1 is an integral curve of W_2 and the arc length parameter of μ_1 is denoted by s^*. Frenet vector fields of μ_1 are $\{\alpha^*, \beta^*, \gamma^*\}$ and Frenet curvatures of μ_1 are κ^*, τ^*. Then, similar to definitions in [4], μ_1 is called W_2-direction curve of μ and μ is also called W_2-donor curve of μ_1.

Theorem 3.8. *γ-magnetic curve μ is not a Mannheim curve.*

Proof. The result can be seen by similar calculations as in the proof of Theorem 3.1. ☐

Theorem 3.9. *μ_1 is a straight line if and only if*

$$\Omega_2 = \kappa = \tau + c, \tag{53}$$

where c is an arbitrary constant.

Proof. The result can be obtained similar to the proof of Theorem 3.4. ☐

Theorem 3.10. *There is not a γ-magnetic curve μ such that (μ, μ_1) is a Bertrand curve pair.*

Proof. Following the similar method in the proof of Theorem 3.6, one can obtain the result. ☐

Theorem 3.11. *There is not a γ-magnetic curve μ such that μ_1 is an involute of μ.*

Proof. Following the similar method in the proof of Theorem 3.7, one can obtain the result. ☐

4. Conclusion

Magnetic curves are the trajectories of charged particles which are under the effect of magnetic fields. They satisfy a certain equation namely the Lorentz equation. This equation generalizes the equation which is satisfied by the geodesics on a manifold. So, one can consider the magnetic curves as a generalization of geodesics on a manifold. These curves are widely studied in geometry and in their physical practice. This study is a research about the relationship between magnetic curves and associated curves such as Mannheim curves el al. in Euclidean 3-space. So finding the relationships between them is very important.

References

[1] E. As and A. Sarioglugil, *On the Bishop curvatures of involute-evolute curve couple in* E^3, Inter. J. Phys. Sci. **9** (2014), no. 7, 140–145.

[2] M. Bilici and Caliskan, *On the involutes of the spacelike curve with a timelike binormal in Minkowski* 3*-space*, Int. Math. Forum. **4** (2009), no. 31, 1497–1509.

[3] Z. Bozkurt, I. Gök, Y. Yaylı, and F. N. Ekmekci, *A new approach for magnetic curves in* 3*D Riemannian manifolds*, J. Math. Phys. **55** (2014), no. 5, 053501.

[4] J. H. Choi and Y. H. Kim, *Associated curves of a Frenet curve and their applications*, Appl. Math. Comput. **218** (2012), no. 18, 9116–9124.

[5] S. L. Druţă-Romaniuc, J. I. Inoguchi, M. I. Munteanu, and A. I Nistor, *Magnetic curves in Sasakian manifolds*, J. Nonlinear Math. Phys. **22** (2015), no. 3, 428–447.

[6] S. L. Druţă-Romaniuc and M. I. Munteanu, *Magnetic curves corresponding to Killing magnetic fields in* E^3, J. Math. Phys. **52** (2011), no. 11, 113506.

[7] N. Ekmekci and K. Ilarslan, *On Bertrand curves and their characterization*, Differ. Geom. Dyn. Syst. **3** (2001), no. 2, 17–24.

[8] D. Fuchs, *Evolutes and involutes of spatial curves*, Amer. Math. Monthly **120** (2013), no. 3, 217–231. https://doi.org/10.4169/amer.math.monthly.120.03.217

[9] T. Fukunaga and M. Takahashi, *Evolutes and involutes of frontals in the Euclidean plane*, Demonstratio Mathematica **48** (2015), no. 2, 147–166.

[10] J. I. Inoguchi and M. I. Munteanu, *Magnetic curves in the real special linear group*, arXiv preprint arXiv:1811.11993. (2018).

[11] S. Izumiya and N. Takeuchi, *Generic properties of helices and Bertrand curves*, J. Geom. **74** (2002).

[12] M. Jleli and M. I. Munteanu, *Magnetic curves on flat para-K ähler manifolds*, Turkish J. Math. **39** (2015), no. 6, 963–969.

[13] G. U. Kaymanli, C. Ekici, and M. Dede, *On the directional associated curves of timelike space curve*, In Conference Proceedings of Science and Technology. **2** (2018), no. 3, 173–179.

[14] T. Körpinar, *On T-magnetic biharmonic particles with energy and angle in the three dimensional Heisenberg group H*, Adv. Appl. Clifford Algebr. **28** (2018), no. 1, Paper No. 9, 15 pp. https://doi.org/10.1007/s00006-018-0834-2

[15] T. Körpinar, *A new version of normal magnetic force particles in* 3*D Heisenberg space*, Adv. Appl. Clifford Algebr. **28** (2018), no. 4, Paper No. 83, 13 pp. https://doi.org/10.1007/s00006-018-0900-9

[16] T. Körpinar, *A new velocity magnetic particles with flows by spherical frame*, Differ. Equ. Dyn. Syst. (2018), 1–7.

[17] T. Körpinar, M. T. Sariaydin, and E. Turhan, *Associated curves according to bishop frame in Euclidean* 3*-space*, Adv. Model. Optimization **15** (2013), no. 3, 713–717.

[18] H. Liu and F. Wang, *Mannheim partner curves in* 3*-space*, J. Geom. **88** (2008), no. 1–2, 120–126.
[19] N. Macit and M. Duldul, *Some new associated curves of a Frenet curve in E^3 and E^4*, Turkish J. Math. **38** (2014), no. 6, 1023–1037.
[20] M. Masal and A. Z. Azak, *Mannheim B-curve in the Euclidean* 3*-space*, Kuwait J. Sci. **44** (2017), no. 1, 36–41.
[21] H. Matsuda and S. Yorozu, *Notes on Bertrand curves*, ynu.repo.nii.ac.jp. (2003).
[22] M. I. Munteanu, *Magnetic curves in a Euclidean space*: *one example, several approaches*, Publ. Inst. Math. (Beograd) (N.S.) **94(108)** (2013), 141–150. https://doi.org/10.2298/PIM1308141M
[23] S. K. Nurkan, I. A. Guven, and M. K. Karacan, *Characterizations of adjoint curves in Euclidean* 3*-space*, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences **89** (2019), no. 1, 155–161.
[24] B. O'Neill, *Elementary Differential Geometry*, revised second edition, Elsevier/Academic Press, Amsterdam, 2006.
[25] K. Orbay and E. Kasap, *On Mannheim partner curves in E^3*, Int. J. Phys. Sci. **4** (2009), no. 5, 261–264.
[26] E. Ozyilmaz and S. Yilmaz, *Involute-evolute curve couples in the Euclidean* 4*-space*, Int. J. Open Problems Compt. Math. **2** (2009), no. 2, 168–174.
[27] B. Sahiner, *Some associated curves of binormal indicatrix of a curve in Euclidean* 3*-space*, In 16 th International Geometry Symposium, p. 302, 2018.
[28] B. Sahiner and M. Önder, *Slant helices, Darboux helices and similar curves in dual space $D3$*, Mathematica Moravica. **20** (2016), no. 1, 1.

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