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CONNECTIVITY BETWEEN MATHEMATICS AND SCIENCE CONCEPTS AND A PLAN FOR ORGANIZATION OF EDUCATIONAL PROGRAMS -FOCUSED ON MARGINALIZED LEARNERS†

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Abstract. In this paper, the connection and convergence between mathematics and science (physics) concepts were investigated. In addition, methods to closely analyze the degree of mathematics and science (physics) learning were looked into. Furthermore, methods to express and analyze the learning states of individual learners were investigated and a plan to organize educational programs was sought.

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Key words and phrases : Concept connection, convergence.

1. Introduction

As the polarization of social classes has intensified due to social imbalance, the population of the underprivileged has continued to increase. The intensification of social class polarization as such is a problem that our society must overcome. To resolve the problem of polarization, various welfare support and policies should be implemented. First of all, our society should enable students who are alienated from learning to enjoy the benefits of education. As part of the foregoing, many studies have been conducted to help the education of socially underprivileged students. Nam Sang-il and two others (2017) analyzed 272 papers on science education (125 papers published in journals listed in KCI and candidate journals for listing from 1994 to February 2017 and 147 domestic master's and doctoral dissertations) conducted with underprivileged students. According to the results, 61% of those papers are studies related to dual disabilities, 6% are studies related to North Korean defector students, 20% are studies related

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to underachievers, 5% are studies related to the remaining other underprivileged student groups. In addition, as for the contents of the studies, core terms that were dealt with continuously and intensively were not observed, and studies on various topics and concepts were sporadically conducted ([8]). In the fields of mathematics and science, various studies have been conducted such as studies related to affective and cognitive domains and studies related to educational programs ([1]-[7], [9], [14], [15]), [16]). As for papers targeting the underprivileged in the field of science or mathematics, studies on a certain topic or concept are insufficient, studies were conducted in the short-term, and the development of educational programs intended to provide practical help to the learning of underprivileged students is lacking. Therefore, it is necessary to closely examine the learning situation of students who are alienated from learning mathematics and science and prepare a plan to establish educational programs. In addition, such studies should be conducted from a long-term perspective. Therefore, in this paper, the connectivity between mathematics and science concepts was investigated, and a plan to organize educational programs that can help students who were alienated from learning mathematics and science was sought.

2. Connection of mathematics concepts ([10]-[13])

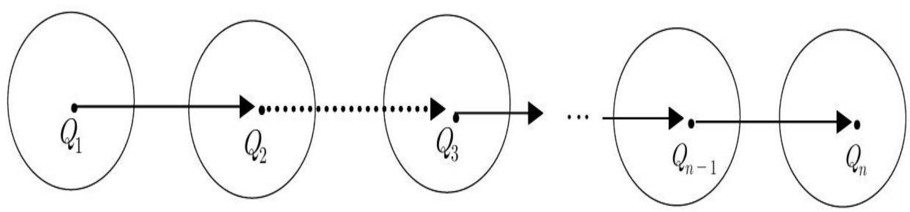
The connections of mathematics concepts based on the results of learner's problem-solving can be expressed on a plane. These expressions are simple when expressing a connection between one or two concepts, but are usually complex or difficult to express when expressing a connection between many concepts. If a sphere is used, expressions on a plane as such will be expressed visually simply, and it will be easy to evaluate how much the learner connects concepts and use the evaluation result. Here, the contents studied already will be examined.

2.1. SINGLE PLANE, CONNECTED SINGLE PLANE. Let us consider the inside of a circle of which the center is p and the radius is r. Let us assume that the inside of the circle is composed of infinite points, and let the center of the circle be p_k (lower level problem) and the radius be r_k. Let a set of infinitely many upper level problems directly related to the p_k inside of the circle as such be $<$ single Plane $P_k >$. In this case, ¡ single plane $p_k >$ means a set of finite upper level problems directly related to the p_k inside the circle, and a set composed of minimum problems directly related to the lower level problem p_k. ¡Plane $P_k >$ refers to the state in which all the concepts on $<$ single Plane $P_k >$ are understood, and ¡plane $p_{ki} >$ means that all 'minimum upper level problems' directly related to the lower level problem p_k have been solved. Since there is no room for confusion about $<$ single plane $>$ and $<$ plane $>$, $<$ single plane $>$ and $<$ plane $>$ are used interchangeably. Here, if the learner has solved the minimum problem inside $<$ plane $p_k >$, he/she will be considered to have solved infinite number of problems inside $<$ Plane $P_k >$.

In general, the state in which all of problem types Q_1 from Q_n have been solved is called ¡connected single plane $>$. $< k-$ connected single plane $>$ means the state in which some of the problem types (k pieces of problem types) out of problem types Q_1 from Q_n have been solved. Here, problem type Q_i means an infinitely many upper level problems (problems of the same type) inside a circle of which the center is $p_i(i = 1, 2, 3, \cdots, n)$.

Let us assume that the learner has partially solved each of Q_i (for each $1 \leq i \leq n$). In this case, the learner is said to be in the $< n-$ partially connected single plane $>$ state ($<$ Figure 1 $>$).

(2) The state in which $k(1 \leq k \leq n-1)$ types of problems have been partially solved out of problem types $Q_i(1 \leq i \leq n)$ is called $< k-$ partially connected single plane¿. For example, if a learner has partially solved the problem laid on Q_2 and Q_3, the learner will be said to be in the $< 1-$ partially connected single plane $>$ state ($<$ Figure 1 $>$). That is, let us assume that a learner is in the state. This means that the learner partially understands the mathematics concepts related to each type of problems among problem types Q_i.

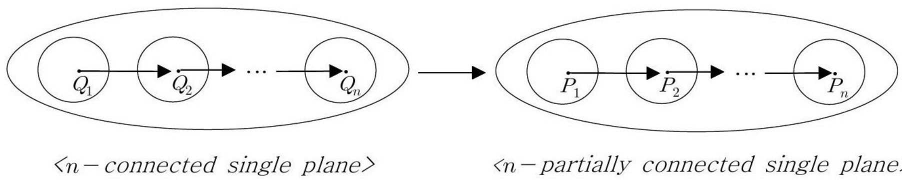


$<$ Figure 1 $>$ $1-$ partially connected single plane

2.2. CONNECTED PLANE AND CONNECTED CYCLE. Let Q (problem type) be a problem type directly related to 'mathematics concept G ' or 'science concept G ', or 'mathematics and science concept G '. That is, $Q = \{Q_i, 1 \leq i \leq n, n : \text{finite}\}$. And let P (problem type) be a problem type directly related to 'mathematics concept C ' or 'science concept C ', or 'mathematics and science concept C ', that is, $P = \{P_i, 1 \leq i \leq n, n : \text{finite}\}$.

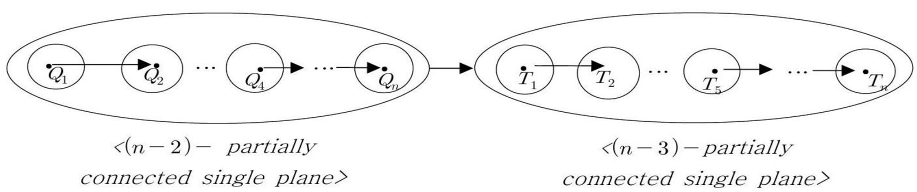
2.2.1. CONNECTED PLANE, K-PART. CONNECTED PLANE. Being in the state means that 'concept G ' and 'concept C ' are connected. In addition, it means that the learner is in the state of $< n$-connected single plane¿. Let us assume that a learner is in the $< n-$ partially connected single plane $>$ state. Then, it means that the learner partially knows 'concept G ' or 'concept C ' and that learning correction is needed.

Let us assume that the learner is in the state of $< n$ - connected single plane $>$ for Q and in the state of $< n$-partially connected single plane¿ for P. Then, the learner is said to be in the $< Q$-connected plane¿ state ($<$ Figure 2 $>$). That is, the learner knows 'concept G ' but partially knows 'concept C '.



< Figure 2 > Q-connected plane

Let us assume that a learner is in the $< k-$ partially connected single plane¿ state. This partially connects Q and P, and is called to be in a $< k-$ partially connected plane¿ or $< k-$ partially connected concept plane¿ state ($k = 1, 2, \cdots n$). In detail, this state is also called a $< (n-2 : Q) - (n-3 : P)-$ partially connected plane¿ state. This means that learners in this state partially understand 'concept G ' and 'concept C '. In addition, a large value of k means that there are many unconnected . This indicates a very poor learning situation in learning. This means that a learner in this state is lacking connection between mathematics concepts, between science concepts, or between mathematics and science concepts. Therefore, this means that learners in this state of learning must be taught learning from the basic concepts related to the lower level problems.



< Figure 3 > k-partially connected plane

2.2.2. CONNECTED CYCLE. How learners connect the concepts they have learned affects their next learning. In the previous section, the connectivity between the learned concepts was explained on a plane. Connections between concepts are often difficult or complicated to map on a plane. Therefore, in , connectivity on a plane can be effectively utilized by expressing it three-dimensionally. That is, it can be seen that if a ¡connected cycle > is formed in , it will become a < connected plane¿.

For example, if a learner is said to be in the learning state of $[S_{q_1} \to S_{q_2}, S_{q_2} \to S_{q_3}]$ and $[S_{t_1} \to S_{t_2}, S_{t_2} \to S_{t_3}]$, it will mean that the learner is forming a relation network by connecting the two concepts. This fact expressed on the $\langle$ plane > is represented visually in the < sphere >. Therefore, since < connected plane > can be expressed as , a lot of information about the learner's learning state can be expressed visually. That is,

$$
\begin{aligned}
&[S_{q_1} \cup S_{q_2} \cup S_{q_3}] \cup [S_{t_1} \cup S_{t_2} \cup S_{t_3}] \\
&\qquad \equiv < \text{3-connected single plane } > \cup < \text{ 3-connected single plane } > \\
&\qquad \equiv < \text{ Plane } Q > \cup < \text{ Plane } T > \\
&\qquad \equiv < \text{ connected plane } > \equiv < \text{ connected cycle } > .
\end{aligned}
$$

3. Connections between mathematics and science concepts

Here, to examine the connectivity between mathematics and science (physics) concepts, convergence and connectivity ([11]-[14]) in the sphere are used after being modified and supplemented. Thereafter, based on the results, learners' learning states will be analyzed and an educational program for guidance will be discussed.

3.1. Definition of terms. Students from the underprivileged and those experiencing difficulties in learning receive insufficient educational benefits compared to students from other classes or students who have little difficulty in learning. This is a social phenomenon. Therefore, it is very important to ensure that underprivileged learners are provided with learning opportunities. In this sense, a student from an underprivileged class or a student experiencing learning difficulties is defined as an underprivileged learner. Hereinafter, a learner or student means a student from the underprivileged or a student experiencing difficulties in learning.

3.2. Organization of problems. In order to examine the connection between mathematics and science (physics) concepts, two problem types, Q and T are considered. Problem type Q was organized around the basic concepts related to the limit, continuity, differentiation, and speed of a function, and problem type T was organized around the basic concepts related to the definition, operation, components of vectors and the decomposition of velocity vectors. In addition, in order to find out how much the learner connects between basic concepts, each of methods $[D_i, E_i, M_i, N_i, V_i, O_i, R_i, F_i (i = 1, 2, 3)]$ was composed of at least 4 items. Type $Q\,(Q_i, i = 1, 2, 3, 4)$ and type $T\,(T_i, i = 1, 2, 3, 4)$ were organized considering the complementary connections and convergence between concepts ($<$ Table 1 $>$, $<$ Table 2 $>$).

3.2.1. Problem type $Q_i (i = 1, 2, 3, 4)$. Each type $(Q_i, i = 1, 2, 3, 4)$ was constructed considering the learning hierarchy, and each type was constructed in three methods. Also, the three methods $(R_i, i = 1, 2, 3)$ as such were constructed considering connections between concepts. The contents of individual types are as follows ($<$ Table 1 $>$).

(1) D_1 : This is a problem of finding the limit value of a linear function. D_2 is a problem of finding the left and right limits of a linear function. D_3 is a problem asking the existence of a limit in a discontinuous function.

(2) E_1 is a problem of intuitionally distinguishing a continuous function from a discontinuous function. E_2 is a problem of finding the function value and the limit value of a rational function. E_3 is a problem of distinguishing between continuous and discontinuous functions according to the definition of the continuity of functions.

(3) M_1 is a problem of finding the differential coefficients of a linear function and a quadratic function using the formula of differentiation. M_2 is a problem of finding the differential coefficient according to the definition of the differential coefficient. M_3 is a problem of finding the derivative according to the definition of the derivative.

(4) N_1 is a problem of finding the horizontal and vertical speeds when time t and the position vector components of point P are given as functions. N_2 is a problem of finding the velocity of point P at time t when the component of the position vector is given as a function. N_3 is a problem of finding the acceleration of point P at time t when the component of the position vector is given as a function.

Problem type	Content	Method	Concept
Q_1	Limit	D_1	Limit value
		D_2	Left-hand limit, right-hand limit
		D_3	Left-hand limit, right-hand limit
Q_2	Continuity of function	E_1	Intuitively continuous and discontinuous
		E_2	Function value, limit value
		E_3	Definition of continuity
Q_3	Differentiation	M_1	Differential coefficient
		M_2	Definition of differential coefficients
		M_3	Definition of derivatives
Q_4	Velocity, acceleration	N_1	Velocity in position vector
		N_2	Velocity
		N_3	Acceleration

< Table 1 > Concept of $Q_i (i = 1, 2, 3, 4)$

3.2.2. Problem type $T_i (i = 1, 2, 3, 4)$. Each type $(T_i, i = 1, 2, 3, 4)$ was constructed considering the learning hierarchy, and each type was constructed in three methods. Also, the three methods $(T_i, i = 1, 2, 3)$ as such were constructed considering connections between concepts. The contents of individual types are as follows (< Table 2 >).

(1) V_1 is a problem of distinguishing between scalars and vectors, and V_2 is a problem of finding the magnitude of a vector when the length is given. V_3 is a problem of finding vectors with the same direction and vectors with the opposite directions when vectors with different magnitudes and directions are given.

(2) O_1 is a problem of expressing the addition and subtraction of vectors on a coordinate plane where there are two vectors, and O_2 is a problem of expressing the addition and subtraction of vectors when multiplied by real numbers. O_3 is a problem of adding and subtracting vectors when vectors are given.

(3) R_1 is a problem of distinguishing the same position vectors. R_2 is a problem of expressing a vector as position vectors of two points and expressing them as components. R_3 is a problem related to the components and magnitude of a vector composed of two points.

(4) F_1 is a problem of finding the resultant force of forces. F_2 is a problem of finding the change amount of the velocity vector and the direction of the acceleration. F_3 is a problem of finding the magnitudes of the components of the velocity vector.

Problem type	Content	Method	Concept
T_1	Definition of vectors	V_1	Scalar, vector
		V_2	Definition of vectors, magnitudes of vectors
		V_3	Vectors according to the direction and magnitude of a vector
T_2	Operations of vectors	O_1	Vector addition and subtraction
		O_2	Real number multiple of a vector
		O_3	Vector addition and subtraction
T_3	Components of a vector	R_1	Position vector
		R_2	Representation of the components of a vector
		R_3	Magnitude of a vector
T_4	Components of a velocity vector	F_1	Resultant force of forces
		F_2	Amount of change in velocity, direction of acceleration
		F_3	Decomposition of a velocity vector

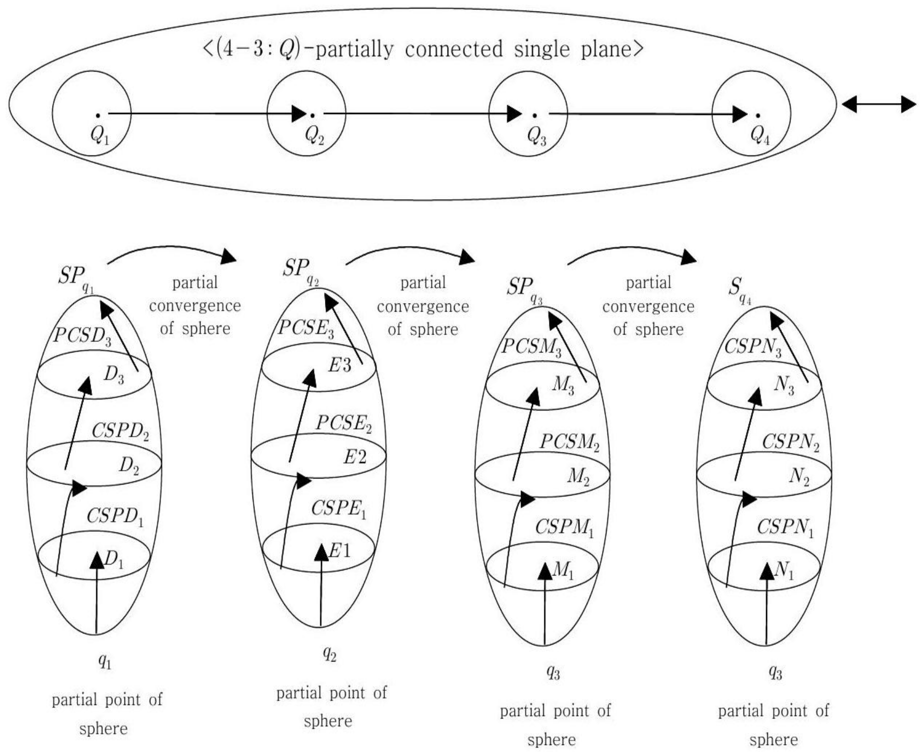
¡ Table 2 > Concept of $T_i(i = 1, 2, 3, 4)$

3.3. Expression of and analysis learning states.

3.3.1. Expression of the learner's learning state. We virtually set the learner's learning results for the problem types Q and P(< table 1 >, < table 2 >) and visually expressed them in the sphere. Thereafter, the learner's learning state was virtually set and analyzed. In the expression, complementary connections and convergence between concepts were considered. The mutually complementary connections and convergence enables understanding the learner's learning state a little more closely. In addition, analyzing how much mutually complementarily the learner connects concepts can be of great help to teaching and learning.

< Fig 1 > is called ' $(4 - 3Q_i, i = 1, 2, 3)$-partially connected cycle.' CSP is called , and means that the concept on the plane is understood. PCS is called , and means that the concept on the plane is partially understood. Since $Q_i(i = 1, 2, 3, 4)$ partially converges on $Q_i(i = 1, 2, 3)$ and totally converges on Q_4, it is called . That is, concepts on $Q_i(i = 1, 2, 3, 4)$ are said to be partially connected.

Plane $D_i(i = 1, 2)$ converges on $D_i(i = 1, 2)$ and partially converges on D_3. That is, $\mathrm{CSPD}_i(i = 1, 2)$ means that the learner understands how to find the limit value on plane $D_i(i = 1, 2)$, and PCSD_3 means that the learner partially understands the concept of the limit on plane D_3. Plane E_1 converges on Q_2, and partially converges on $E_i(i = 2, 3)$. This means that continuous and discontinuous functions on plane E_1 can be intuitively distinguished, and that the function value and limit value of the rational function on plane $E_i(i = 2, 3)$ can be partially obtained, and the concepts of continuity and discontinuity are understood. Plane M_1 converges on Q_3, and partially converges on $M_i(i = 2, 3)$. This means that the differential coefficient on plane M_1 can be obtained using the formula of differentiation, and the concept of the differential coefficient or derivative on $M_i(i = 2, 3)$ is partially understood. Inside Q_4, plane M_1 totally converges. That is, this means that the learner knows how to find the velocity and acceleration at the position vector on plane $N_i(i = 1, 2, 3)(<$ Fig 1 $>)$.

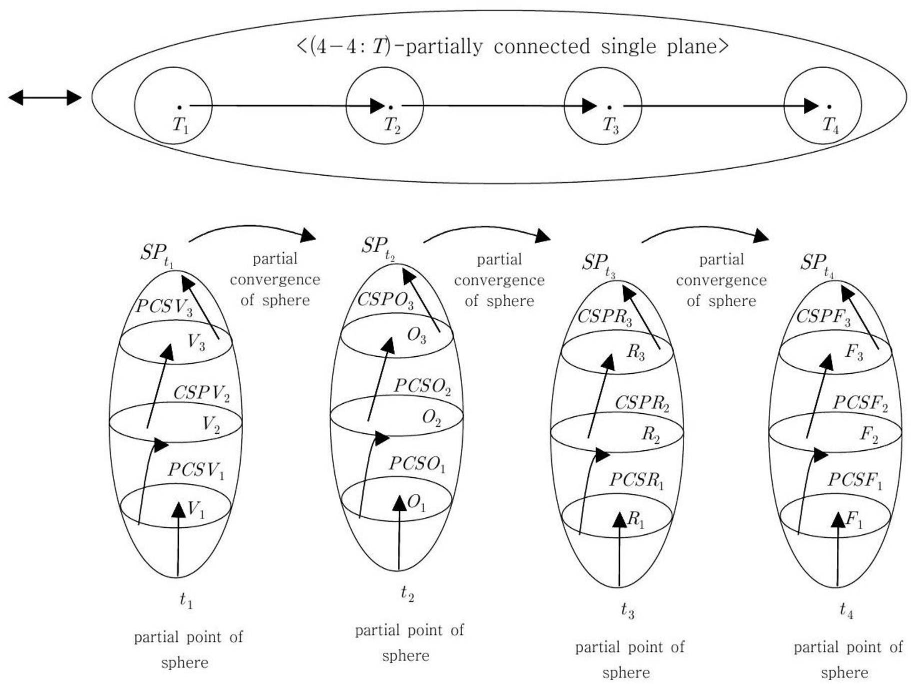


$\langle$ Fig 1$\rangle$ $(4 - 3Q_i, i = 1, 2, 3)$-partially connected cycle

¡Fig 2 $>$ is called ' $(4 - 4T_i, i = 1, 2, 3)$-partially connected cycle.' In addition, since plane M_1 partially converges on $T_i(i = 1, 2, 3, 4)$, it is called as a whole. That is, it means that the learner partially understands the partial connections of the concepts on $T_i(i = 1, 2, 3, 4)$. Therefore, it means that learning correction with this type should be made.

Plane $T_i(i = 1, 3)$ converges on T_1 and partially converges T_2. That is, PCS $V_i(i = 1, 3)$ means that the learner can partially distinguish the scalars and vectors on plane $V_i(i = 1, 3)$, the same vectors and vectors in different directions. $CSPV_2$ means that the learner can find the magnitude of the vector on V_2.

Plane $O_i(i = 1, 2)$ partially converges on T_2, and partially converges on T_2. This indicates that the learner partially understands the addition and subtraction of vectors on the coordinate plane and the addition and subtraction of vectors multiplied by real numbers. On T_3, R_1 partially converges and $R_i(i = 2, 3)$ converges. Therefore, the learner as such experiences difficulties in distinguishing the same position vectors but understands the concepts related to the components and magnitudes of vectors. Since F_3 partially converges and $F_i(i = 1, 2)$ converges on T_4, it is analyzed that the learner knows how to find the resultant force of forces, the amount of change in the velocity vector, and the direction of acceleration (< Fig 2 >).



<Fig2>$(4-4T_i, i=1,2,3,4)$-partially connected cycle

$\langle$ Fig 2$\rangle$ $(4 - 4T_i, i = 1, 2, 3, 4)$-partially connected cycle

3.3.2. Analysis of learning state. Student A is in the learning state of $< S_{q_1}, S_{q_2}, SP_{q_3}, S_{q_4} >$ on $Q\,(Q_i, i = 1, 2, 3, 4)$ and in the learning state of $< SP_{T_1}, S_{t_2}, S_{t_3}, SP_{t_4} >$ on $T\,(T_i, i = 1, 2, 3, 4)$. The means that student A state converges on $Q_i(i = 1, 2, 4)$ and partially converges on Q_3. In addition, the student A converges on $T_i(i = 1, 4)$ and converges on $T_t(i = 2, 3)$. Student B is in the learning state of $< S_{q_1}, SS_{q_2}, SP_{q_3}\ S_{q_4} >$ on $Q\,(Q_i, i = 1,2,3,4)$ and in the learning state of $< SP_{T_1}, S_{t_2}, SS_{t_3}, SP_{t_4} >$ on $T\,(T_i, i = 1,2,3,4)$. This means that student B converges on $Q_i(i = 1,2)$ and partially converges on $Q_i(i = 3,4)$. In addition, student B partially converges on $T_i(i = 1,4)$ and converges on $T_t(i = 2,3)$. Student C is in the state of $< SP_{q_1}, SP_{q_2}, STP_{q_3}, STP_{q_4} >$ on $Q\,(Q_i, i = 1,2,3,4)$ and in the state of $< STP_{T_1}, STP_{t_2}, SP_{t_3}, STP_{t_4} >$ on $T\,(T_i, i = 1,2,3,4)$. This means that student C partially converges on $D_i(i = 1,2)$ and partial converges on $Q_i(i = 3,4)$. In addition, student C totally partially converges on $T_i(i = 1,2,4)$ and partially converges on T_3.

student	form of problem	Q_1	Q_2	Q_3	Q_4
A	method 1	$CSPD_1$	$CSPE_1$	$CSPM_1$	$CSPN_1$
	method 2	$CSPD_2$	$CSPE_2$	$PCSM_2$	$CSPN_2$
	method 3	$CSPD_3$	$CSPE_3$	$PCSM_3$	$CSPN_3$
	evaluation	S_{q_1}	S_{q_2}	SP_{q_3}	S_{q_4}
B	method 1	$CSPD_1$	$CSPE_1$	$CSPM_1$	$CSPN_1$
	method 2	$CSPD_2$	$CSPE_2$	$PCSM_2$	$CSPN_2$
	method 3	$CSPD_3$	$CSPE_3$	$PCSM_3$	$CSPN_3$
	evaluation	S_{q_1}	S_{q_2}	SP_{q_3}	S_{q_4}
C	method 1	$CSPD_1$	$CSPE_1$	$PCSM_1$	$PCSN_1$
	method 2	$PCSD_2$	$PCSE_2$	$PCSM_2$	$PCSN_2$
	method 3	$PCSD_3$	$PCSE_3$	$PCSM_3$	$PCSN_3$
	evaluation	SP_{q_1}	SP_{q_2}	STP_{q_3}	STP_{q_4}
D	method 1	$CSPD_1$	$CSPE_1$	$CSPM_1$	$PCSN_1$
	method 2	$CSPD_2$	$PCSE_2$	$PCSM_2$	$CSPN_2$
	method 3	$PCSD_3$	$PCSE_3$	$PCSM_3$	$PCSN_3$
	evaluation	SP_{q_1}	SP_{q_2}	SP_{q_3}	SP_{q_4}
E	method 1	$CSPD_1$	$CSPE_1$	$CSPM_1$	$CSPN_1$
	method 2	$CSPD_2$	$PCSE_2$	$PCSM_2$	$CSPN_2$
	method 3	$PCSD_3$	$PCSE_3$	$PCSM_3$	$CSPN_3$
	evaluation	SP_{q_1}	SP_{q_2}	SP_{q_3}	S_{q_4}

¡table 3 > Results for problem types related to the limit, continuity, and differentiation of functions

Student D is in the state of $< SP_{q_1}, SP_{q_2}, SP_{q_3}, SP_{q_4} >$ on $Q\,(Q_i, i = 1,2,3,4)$ and in the state of $< STP_{T_1}, SP_{t_2}, SP_{t_3}, STP_{t_4} >$ on $T\,(T_i, i = 1,2,3,4)$. This means that student D partially converges on $D_i(i = 1,2,3,4)$ and partially converges on $Q_i(i = 1,4)$. In addition, student D totally partially converges on $T_i(i = 1,4)$ and partial converges on $T_i(i = 2,3)$. The evaluation of student E is shown to be $SP_{q_1}, SP_{q_2}, SP_{q_3}, S_{q_4}$ on problem type Q and SP_{t_1}, $SP_{t_2}, SP_{t_3}, SP_{t_4}$ on problem type T (< table 3 >, < table 4 >).

student	form of problem	T_1	T_2	T_3	T_4
A	method 1	$PCSV_1$	$CSPO_1$	$CSPR_1$	$PCSF_1$
	method 2	$CSPV_2$	$CSPO_2$	$CSPR_2$	$CSPF_2$
	method 3	$CSPV_3$	$CSPO_3$	$CSPR_3$	$CSPF_3$
	evaluation	SP_{t_1}	S_{t_2}	S_{t_3}	SP_{t_4}
B	method 1	$CSPV_1$	$CSPO_1$	$PCSR_1$	$CSPF_1$
	method 2	$CSPV_2$	$CSPO_2$	$CSPR_2$	$CSPF_2$
	method 3	$CSPV_3$	$CSPO_3$	$CSPR_3$	$CSPF_3$
	evaluation	S_{t_1}	S_{t_2}	SP_{t_3}	SP_{t_4}
C	method 1	$PCSV_1$	$PCSO_1$	$PCSR_1$	$PCSF_1$
	method 2	$PCSV_2$	$PCSO_2$	$PCSR_2$	$PCSF_2$
	method 3	$PCSV_3$	$PCSO_3$	$CSPR_3$	$PCSF_3$
	evaluation	STP_{t_1}	STP_{t_2}	SP_{t_3}	STP_{t_4}
D	method 1	$PCSV_1$	$PCSO_1$	$PCSR_1$	$PCSF_1$
	method 2	$PCSV_2$	$CSPO_2$	$PCSR_2$	$PCSF_2$
	method 3	$PCSV_3$	$CSPO_3$	$CSPR_3$	$PCSF_3$
	evaluation	STP_{t_1}	SP_{t_2}	SP_{t_3}	STP_{t_4}
E	method 1	$PCSV_1$	$PCSO_1$	$PCSR_1$	$PCSF_1$
	method 2	$CSPV_2$	$PCSO_2$	$CSPR_2$	$PCSF_2$
	method 3	$PCSV_3$	$CSPO_3$	$CSPR_3$	$CSPF_3$
	evaluation	SP_{t_1}	SP_{t_2}	SP_{t_3}	SP_{t_4}

< table 4 > Results for types of problems related to vector

4. Educational program development plan and discussion

Educational programs related to science, mathematics, and engineering have been studied and developed extensively. The developed educational programs do not provide practical help to students' learning due to the short-term studies and applications. Such programs should be made to help students' learning in the long run. In addition, the research environment should be improved so that studies related to the development of educational programs can be conducted in tandem. In addition, educational programs should be developed and applied for purposes that consider various factors such as the benefits of education, the effects of education, and the improvement of education, rather than the purpose of research.

Educational programs have been developed for students who experience difficulties in learning mathematics or science subjects. In these studies, the effectiveness of the educational program is proven by investigating and analyzing what difficulties students are experiencing after applying the educational programs. And it is true that these educational programs have helped students who experience difficulties in learning. However, there still remain things that need to be supplemented in the analysis of students' learning situations. That is, most studies are constrained in applying and developing due to social conditions. This is a phenomenon that can be sufficiently overcome if the method of analyzing the learner's learning state is modified and supplemented. Therefore, educational programs considering the learning difficulties of individual learners can be developed by breaking away from various situations. There are many difficulties in learning experienced by individual learners. Although it is impossible to consider everything, methods to analyze learners' learning states should be developed considering the minimum time and economic feasibility.

As shown in $\langle$ table 3 $>$ and $\langle$ table 4 $>$, learner C is experiencing the following difficulties with concepts related to the limit, continuity, and differential of vectors and functions. Learner C can find the limit value of a linear function, but he lacks the understanding of the concept of the left-hand limit and the right-hand limit. In addition, while he can distinguish between continuity and discontinuity by intuition, he experiences difficulties in finding function values and limit values. Furthermore, he lacks the understanding of the continuity and discontinuity of a function according to the definition of continuity. With regard to the concept of differentiation, he is experiencing difficulties in finding the differential coefficient using the formula of differentiation. He partially understands the concepts of differential coefficients and derivatives. He also has difficulties with how to find velocity and acceleration. With regard to the concept of vectors, he has difficulties in finding scalars and vectors, the same vector and different vectors, and the magnitude of vectors. He can partially perform operations (addition, subtraction) of vectors on the coordinate plane. However, he solved well the operation of real number multiples of vectors. With regards to the components of vectors, although he partially understands the indications of the components of position vectors and vectors, he well solve the problem of finding the size of a vector using the formula. With regard to velocity vectors and the components, he partially understands the concepts of the resultant force of forces, the amount of changes in velocity, the direction of acceleration, and the decomposition of velocity vectors. As such, the learning state of learner C is complex and diverse. Therefore, finding the method to simply express in what learning state leaner C is and a teaching plan is very important in education. There may be many things, but educational programs considering the minimum time and economic feasibility should be organized. One method can be analyzed considering the connectivity and convergence between mathematics and scientific concepts described above. That is, learner C is in the state of $< SP_{q_1}, SP_{q_2}, STP_{q_3}, STP_{q_4} >$ on $Q\,(Q_i, i = 1, 2, 3, 4)$ and in the state of $< STP_{T_1}, STP_{t_2}, SP_{t_3}$,$STP_{t_4} >$ on $T\,(T_i, i = 1, 2, 3, 4)$. Therefore, learner C partially converges on $D_i(i = 1, 2)$ and partially converges on T_3. The analysis as such can simply express the learning state of learner C. In addition, if we organize and utilize the education system based on the results, we can easily use it to guide individual learners' learning. However, it takes a lot of time and requires a lot of data to prepare the system at first. However, once the system is constructed, there is an advantage that it can be easily modified and supplemented there after.

Conflicts of interest : The authors declare no conflict of interest.

Data availability : Not applicable

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