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GROUP S_3 MEAN CORDIAL LABELING FOR STAR RELATED GRAPHS

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Abstract. Let $G = (V, E)$ be a graph. Consider the group S_3. Let $g : V(G) \to S_3$ be a function. For each edge xy assign the label 1 if $\left\lceil \frac{o(g(x))+o(g(y))}{2} \right\rceil$ is odd or 0 otherwise. g is a group S_3 mean cordial labeling if $|v_g(i) - v_g(j)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$, where $v_g(i)$ and $e_g(y)$denote the number of vertices labeled with an element i and number of edges labeled with y $(y = 0, 1)$. The graph G with group S_3 mean cordial labeling is called group S_3 mean cordial graph. In this paper, we discuss group S_3 mean cordial labeling for star related graphs.

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1. Introduction

The concept of labeling was intrdouced by of Rosa [6] in 1967. We follow the basic notation and terminologies as they are found in the text book written by Douglas B. West [7]. In graph labeling we assign integers to the vertices or edges or both subject to some stipulated conditions. Cahit introduced cordial labeling in [1].

Ponraj et al. introduced mean cordial labeling in [5]. Lourdusamy et al. [2] has defined a new labeling called Group S_3 cordial remainder labeling. Motivated by these concepts, Lourdusamy et. al defined a new labeling called group S_3 mean cordial labeling in [3]. Also, they proved ladder and snake related graphs admits group S_3 mean cordial labeling in [4]. Here, we discuss group S_3 mean cordial labeling for star related graphs.

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2. S_3 Mean Cordial Labeling

We denote the elements of symmetric group S_3 by the letters e, a, b, c, d, f where

$$\begin{bmatrix}1 & 2 & 3\\1 & 2 & 3\end{bmatrix} = e, \quad \begin{bmatrix}1 & 2 & 3\\1 & 3 & 2\end{bmatrix} = a, \quad \begin{bmatrix}1 & 2 & 3\\3 & 2 & 1\end{bmatrix} = b,$$

$$\begin{bmatrix}1 & 2 & 3\\2 & 1 & 3\end{bmatrix} = c, \quad \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix} = d, \quad \begin{bmatrix}1 & 2 & 3\\3 & 1 & 2\end{bmatrix} = f,$$

Note that, $o(e) = 1$, $o(a) = o(b) = o(c) = 2$, $o(d) = o(f) = 3$.

Definition 2.1. Let $G = (V, E)$ be a graph. Consider the group S_3. Let $g : V(G) \to S_3$ be a function. For each edge xy assign the label 1 if $\left\lceil \frac{o(g(x))+o(g(y))}{2} \right\rceil$ is odd or 0 otherwise. g is a group S_3 mean cordial labeling if $|v_g(i) - v_g(j)| \leq 1$ and $|e_g(0) - e_g(1)| \leq 1$, where $v_g(i)$ and $e_g(y)$denote the number of vertices labeled with an element i and number of edges labeled with y $(y = 0, 1)$. The graph G with group S_3 mean cordial labeling is called group S_3 mean cordial graph.

3. Main Results

Theorem 3.1. *Star graph $K_{1,n}$ is not group S_3 mean cordial graph.*

Proof. **Case 1.**
Let (V_1, V_2) be the bipartition of $K_{1,n}$ with $V_1 = w$ and $V_2 = \{x_i : 1 \leq i \leq n\}$. Assume that $g(w) = e$. In order to get $|e_g(0) - e_g(1)| \leq 1$, we must have e as the label for $\frac{n}{2}$ vertices. Obviously, it is a contradiction to $|v_g(i) - v_g(j)| \leq 1$.

Case 2.
Without loss of generality $g(w) = a$. To get $|e_g(0) - e_g(1)| \leq 1$, we must have 3 order elements as the label for $\frac{n}{2}$ vertices. This is a contradiction to $|v_g(i) - v_g(j)| \leq 1$.

Case 3.
Without loss of generality $g(w) = d$. To attain $|e_g(0) - e_g(1)| \leq 1$, we must label $\frac{n}{2}$ vertices with e. This is also a contradiction to $|v_g(i) - v_g(j)| \leq 1$. □

Theorem 3.2. *The Bistar $B_{n,n}$ is group S_3 mean cordial graph for every n.*

Proof. Let $V(B_{n,n}) = \{p, q\} \bigcup \{p_i, q_i\}$ and $E(B_{n,n}) = \{pq\} \bigcup \{pp_i, qq_i : 1 \leq i \leq n\}$. Let $g : V(B_{n,n}) \to S_3$ be a function. Assign the label a, e to the vertices p and q respectively.

Case 1. $n \equiv 0$ $(mod$ $3)$ Let $n = 3k$ and$k \geq 1$. Assign the label d, f, a to the vertices $p_{3k-2}, p_{3k-1}, p_{3k}$.For the vertices $q_{3k-2}, q_{3k-1}, q_{3k}$ we assign the label b, c, e.

Case 2. $n \equiv 1\ (mod\ 3)$ Let $n = 3k+1$ and$k \geq 1$.
For $(1 \leq i \leq n-1)$,

$$g(p_i) = \begin{cases} d & \text{if } i \equiv 1\ (mod\ 3) \\ f & \text{if } i \equiv 2\ (mod\ 3) \\ a & \text{if } i \equiv 0\ (mod\ 3) \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i \equiv 1\ (mod\ 3) \\ c & \text{if } i \equiv 2\ (mod\ 3) \\ e & \text{if } i \equiv 0\ (mod\ 3). \end{cases}$$

Then assign the label d and b to the vertices p_n and q_n respectively.

Case 3. $n \equiv 2\ (mod\ 3)$ Let $n = 3k+2$. Assign the label to the vertices p_i, q_i $(1 \leq i \leq n-1)$ as in Case 2. Finally assign the label f and c to the vertices p_n and q_n respectively.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
$3k-2$ and $k \geq 1$	k	k	$k-1$	k	k	$k-1$
$3k-1$ and $k \geq 1$	k	k	k	k	k	k
$3k$ and $k \geq 1$	$k+1$	k	k	k	$k+1$	k

Table 1

Clearly $e_g(0) = n+1$ and $e_g(1) = n$. Therefore $B_{n,n}$ is group S_3 mean cordial graph □

Theorem 3.3. *$S(K_{1,n})$ is group S_3 mean cordial graph.*

Proof. Let vertex set of $S(K_{1,n})$ be $\{p\} \bigcup \{p_i, q_i : 1 \leq i \leq n\}$ and edge set of $S(K_{1,n})$ be $\{pp_i, p_iq_i : 1 \leq i \leq n\}$.
Define $g : V(S(K_{1,n})) \to S_3$ by,
$g(p) = e$;
for $1 \leq i \leq n$,

$$g(p_i) = \begin{cases} a & \text{if } i \equiv 1\ (mod\ 3) \\ b & \text{if } i \equiv 2\ (mod\ 3) \\ e & \text{if } i \equiv 0\ (mod\ 3) \end{cases}$$

and

$$g(q_i) = \begin{cases} d & \text{if } i \equiv 1\ (mod\ 3) \\ f & \text{if } i \equiv 2\ (mod\ 3) \\ c & \text{if } i \equiv 0\ (mod\ 3) \end{cases}$$

Hence $e_g(0) = e_g(1) = n$.

From Table 2, it is easy to verify that $|v_g(i) - v_g(j)| \leq 1$ for all $i, j \in S_3$. Therefore $S(K_{1,n})$ is group S_3 mean cordial graph. □

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
$3k-2$ and $k \geq 1$	k	$k-1$	$k-1$	k	k	$k-1$
$3k-1$ and $k \geq 1$	k	k	$k-1$	k	k	k
$3k$ and $k \geq 1$	k	k	k	k	$k+1$	k

Table 2

Theorem 3.4. *$S(B_{n,n})$ is group S_3 mean cordial graph.*

Proof. Let $V(S(B_{n,n}))$ be $\{p,r,q\}\bigcup\{p_i,p_i^{'},q_i,q_i^{'}:1\leq i\leq n\}$ and $E(S(B_{n,n}))=\{pr,rq\}\bigcup\{pp_i,p_ip_i^{'},qq_i,q_iq_i^{'}:1\leq i\leq n\}$.
Define a mapping $g:V(S(B_{n,n}))\to S_3$ as follows:
$g(p)=a, g(r)=b, g(q)=d$.
For $1\leq i\leq n$,

$$g(p_i)=\begin{cases} e & \text{if } i=3k-2 \text{ and } k\geq 1\\ d & \text{if } i=3k-1 \text{ and } k\geq 1\\ a & \text{if } i=3k \text{ and } k\geq 1\end{cases}$$

$$g(p_i^{'})=\begin{cases} c & \text{if } i=3k-2 \text{ and } k\geq 1\\ b & \text{if } i=3k-1 \text{ and } k\geq 1\\ b & \text{if } i=3k \text{ and } k\geq 1\end{cases}$$

$$g(q_i)=\begin{cases} a & \text{if } i=3k-2 \text{ and } k\geq 1\\ e & \text{if } i=3k-1 \text{ and } k\geq 1\\ d & \text{if } i=3k \text{ and } k\geq 1\end{cases}$$

$$g(q_i^{'})=\begin{cases} f & \text{if } i=3k-2 \text{ and } k\geq 1\\ c & \text{if } i=3k-1 \text{ and } k\geq 1\\ f & \text{if } i=3k \text{ and } k\geq 1\end{cases}$$

We observe that $e_g(0)=e_g(1)=2n+1$. Table 3, given below establishes that the vertex labeling g is a group S_3 mean cordial graph. □

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
$3k-2$ and $k \geq 1$	$k+1$	k	k	k	k	k
$3k-1$ and $k \geq 1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	k
$3k$ and $k \geq 1$	$k+2$	$k+2$	$k+1$	$k+2$	$k+1$	$k+1$

Table 3

Theorem 3.5. *$S^{'}(K_{1,n})$ is group S_3 mean cordial graph.*

Proof. Let $V(S^{'}(K_{1,n})) = \{p,q\} \bigcup \{p_i, q_i : 1 \le i \le n\}$ and $E(S^{'}(K_{1,n})) = \{pp_i, pq_i, qq_i : 1 \le i \le n\}$.
Let $g : V(S^{'}(K_{1,n})) \to S_3$ be a function as defined below, $g(p) = a, g(q) = f, g(p_1) = d, g(p_2) = e, g(q_1) = b, g(q_2) = c$.
For $3 \le i \le n$,

$$g(p_i) = \begin{cases} d & \text{if } i = 6k+3 \text{ and } k \ge 0 \\ b & \text{if } i = 6k+4 \text{ and } k \ge 0 \\ e & \text{if } i = 6k+5 \text{ and } k \ge 0 \\ d & \text{if } i = 6k+6 \text{ and } k \ge 0 \\ f & \text{if } i = 6k+7 \text{ and } k \ge 0 \\ b & \text{if } i = 6k+8 \text{ and } k \ge 0 \end{cases}$$

$$g(q_i) = \begin{cases} a & \text{if } i = 6k+3 \text{ and } k \ge 0 \\ c & \text{if } i = 6k+4 \text{ and } k \ge 0 \\ f & \text{if } i = 6k+5 \text{ and } k \ge 0 \\ e & \text{if } i = 6k+6 \text{ and } k \ge 0 \\ a & \text{if } i = 6k+7 \text{ and } k \ge 0 \\ c & \text{if } i = 6k+8 \text{ and } k \ge 0 \end{cases}$$

Clearly $e_g(0) = \begin{cases} n + \lfloor \frac{n}{2} \rfloor & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$,
$e_g(1) = \begin{cases} n + \lceil \frac{n}{2} \rceil & n \text{ is odd} \\ \frac{3n}{2} & n \text{ is even} \end{cases}$.
We can see that $|e_g(0) - e_g(1)| \le 1$.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	1	1	0	1	0	1
2	2	2	2	2	2	2
$6k-3$ and $k \ge 1$	$k+1$	k	k	$k+1$	k	k
$6k-2$ and $k \ge 1$	$k+1$	$k+1$	$k+1$	$k+1$	k	k
$6k-1$ and $k \ge 1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$
$6k$ and $k \ge 1$	$k+1$	$k+1$	$k+1$	$k+2$	$k+2$	$k+1$
$6k+1$ and $k \ge 1$	$k+2$	$k+1$	$k+1$	$k+2$	$k+2$	$k+2$
$6k+2$ and $k \ge 1$	$k+2$	$k+2$	$k+2$	$k+2$	$k+2$	$k+2$

TABLE 4

Table 4, shows that $|v_g(i)) - v_g(j)| \le 1$ for $i, j \in S_3$. Hence $S^{'}(K_{1,n})$ is group S_3 mean cordial graph. □

Theorem 3.6. *$S^{'}(B_{n,n})$ is group S_3 mean cordial graph.*

Proof. Let $V(S^{'}(B_{n,n})) = \{p, q, p^{'}, q^{'}\} \bigcup \{p_i, q_i, p_i^{'}, q_i^{'} : 1 \leq i \leq n\}$.
Let $E(S^{'}(B_{n,n})) = \{pq, pq^{'}, qp^{'}\} \bigcup \{pp_i, pp_i^{'}, p^{'}p_i, qq_i, qq_i^{'}, q^{'}q_i : 1 \leq i \leq n\}$.
Then $S^{'}(B_{n,n})$ is of order $4n + 4$ and size $6n + 3$. Define $g : V(S^{'}(B_{n,n})) \to S_3$ by ,
$g(p) = e, g(q) = d, g(p^{'}) = a, g(q^{'}) = b, g(p_1) = f, g(p_2) = e, g(q_1) = b, g(q_2) = c, g(p_1^{'}) = c, g(p_2^{'}) = a, g(q_1^{'}) = d, g(q_2^{'}) = f$.
For $3 \leq i \leq n$,

$$g(p_i) = \begin{cases} d & \text{if } i = 3k \text{ and } k \geq 1 \\ e & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ e & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i = 3k \text{ and } k \geq 1 \\ b & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ c & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

$$g(p_i^{'}) = \begin{cases} a & \text{if } i = 3k \text{ and } k \geq 1 \\ c & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ d & \text{if } i = 3k+2 \text{ and } k \geq 1; \end{cases}$$

$$g(q_i^{'}) = \begin{cases} f & \text{if } i = 3k \text{ and } k \geq 1 \\ a & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ f & \text{if } i = 3k+2 \text{ and } k \geq 1. \end{cases}$$

Clearly $e_g(0) = 3n + 1$ and $e_g(1) = 3n + 2$.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	1	2	1	2	1	1
2	2	2	2	2	2	2
$3k$ and $k \geq 1$	$2k+1$	$2k+1$	$2k$	$2k+1$	$2k$	$2k+1$
$3k+1$ and $k \geq 1$	$2k+2$	$2k+2$	$2k+1$	$2k+1$	$2k+1$	$2k+1$
$3k+2$ and $k \geq 1$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$

TABLE 5

It is easy to observe that $|e_g(0) - e_g(1)| \leq 1$ and $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$.
Hence $S^{'}(B_{n,n})$ is group S_3 mean cordial graph. □

Theorem 3.7. *$D_2(K_{1,n})$ is group S_3 mean cordial graph.*

Proof. Let $V(D_2(K_{1,n})) = \{p, q, p_{i,}, q_i : 1 \leq i \leq n\}$ and $E(D_2(K_{1,n})) = \{pp_i, qp_{i,}, pq_i, qq_i : 1 \leq i \leq n\}$ Let $g : V(D_2(K_{1,n})) \to S_3$ be a function defined as follows:

Assign the labels e and d respectively to the vertices p and q.
We let $g(p_1) = a, g(p_2) = f, g(q_1) = b, g(q_2) = c$;
for $3 \leq i \leq n$,

$$g(p_i) = \begin{cases} a & \text{if } i = 3k \text{ and } k \geq 1 \\ e & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ d & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i = 3k \text{ and } k \geq 1 \\ c & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ f & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

The numberof edges labeled with 0 and labeled with 1 are $2n$ each.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	1	1	0	1	1	0
2	1	1	1	1	1	1
$3k$ and $k \geq 1$	$k+1$	$k+1$	k	k	k	k
$3k+1$ and $k \geq 1$	$k+1$	$k+1$	$k+1$	k	$k+1$	k
$3k+2$ and $k \geq 1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$

TABLE 6

Table 6, shows that $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$.
Hence $D_2(K_{1,n})$ is group S_3 mean cordial graph. □

Theorem 3.8. *$D_2(B_{n,n})$ is group S_3 mean cordial graph.*

Proof. Let $V(D_2(B_{n,n})) = \{p, q, p', q', p_i, q_i, p'_i, q'_i : 1 \leq i \leq n\}$.
Let $E(D_2(B_{n,n})) = \{pp', qq', pq', qp', pp_i, qq_i, p'p'_i, q'q'_i, p_iq, pq_i, p'_iq', p'q'_i : 1 \leq i \leq n\}$. Define $g : V(D_2(B_{n,n})) \to S_3$ by
$g(p) = e, g(q) = d, g(p') = e, g(q') = f, g(p_1) = a, g(p_2) = d, g(q_1) = b, g(q_2) = b, g(p'_1) = c, g(p'_2) = f, g(q'_1) = a, g(q'_2) = c$.
For $3 \leq i \leq n$,

$$g(p_i) = \begin{cases} a & \text{if } i = 3k \text{ and } k \geq 1 \\ d & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ e & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i = 3k \text{ and } k \geq 1 \\ f & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ c & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

$$g(p_i^{'}) = \begin{cases} e & \text{if } i = 3k \text{ and } k \geq 1 \\ a & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ d & \text{if } i = 3k+2 \text{ and } k \geq 1; \end{cases}$$

$$g(q_i^{'}) = \begin{cases} c & \text{if } i = 3k \text{ and } k \geq 1 \\ b & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ f & \text{if } i = 3k+2 \text{ and } k \geq 1. \end{cases}$$

Clearly $e_g(0) = 4n+2 = e_g(1)$.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	2	1	1	1	2	1
2	2	2	2	2	2	2
$3k$ and $k \geq 1$	$2k+1$	$2k+1$	$2k+1$	$2k$	$2k+1$	$2k$
$3k+1$ and $k \geq 1$	$2k+2$	$2k+2$	$2k+1$	$2k+1$	$2k+1$	$2k+1$
$3k+2$ and $k \geq 1$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$

TABLE 7

Therefore $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$ (Table 7).
Hence $D_2(B_{n,n})$ is group S_3 mean cordial graph. □

Theorem 3.9. *$B^2_{n,n}$ is group S_3 mean cordial graph.*

Proof. Let the vertex set be $V(B^2_{n,n}) = \{p, q, p_i, q_i : 1 \leq i \leq n\}$ and the edge set be $E(B^2_{n,n}) = \{pq, pp_i, qq_i, qp_i, pq_i : 1 \leq i \leq n\}$ Define a function $g : V(B^2_{n,n}) \rightarrow S_3$ as follows:
$g(p) = e, g(q) = d, g(p_1) = a, g(p_2) = f, g(q_1) = b, g(q_2) = c$;
for $3 \leq i \leq n$,

$$g(p_i) = \begin{cases} a & \text{if } i = 3k\text{and } k \geq 1 \\ e & \text{if } i = 3k+1\text{and } k \geq 1 \\ d & \text{if } i = 3k+2\text{and } k \geq 1 \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i = 3k \text{ and } k \geq 1 \\ c & \text{if } i = 3k+1 \text{ and } k \geq 1 \\ f & \text{if } i = 3k+2 \text{ and } k \geq 1 \end{cases}$$

Note that $e_g(0) = 2n+1$ and $e_g(1) = 2n$.
Table 8 illustrates that $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$.
Thus, $B^2_{n,n}$ is group S_3 mean cordial graph. □

Theorem 3.10. *$< K^{(1)}_{1,n} \Delta K^{(2)}_{1,n} >$ is group S_3 mean cordial graph.*

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
1	1	1	0	1	1	0
2	1	1	1	1	1	1
$3k$ and $k \geq 1$	$k+1$	$k+1$	k	k	k	k
$3k+1$ and $k \geq 1$	$k+1$	$k+1$	$k+1$	k	$k+1$	k
$3k+2$ and $k \geq 1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$	$k+1$

TABLE 8

Proof. Let $V(< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >) = \{p, q, r, p_i, q_i : 1 \leq i \leq n\}$. Then the edge set is $E(< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >) = \{pq, pr, qr, pp_i, qq_i : 1 \leq i \leq n\}$ Define a function $g : V(< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >) \to S_3$ as follows:
$g(r) = d, g(p) = a, g(q) = e$;
for $1 \leq i \leq n$,

$$g(p_i) = \begin{cases} f & \text{if } i = 3k+1 \text{ and } k \geq 0 \\ d & \text{if } i = 3k+2 \text{ and } k \geq 0 \\ a & \text{if } i = 3k \text{ and } k \geq 1 \end{cases}$$

$$g(q_i) = \begin{cases} b & \text{if } i = 3k+1 \text{ and } k \geq 0 \\ c & \text{if } i = 3k+2 \text{ and } k \geq 0 \\ e & \text{if } i = 3k \text{ and } k \geq 1 \end{cases}$$

Note that $e_g(0) = n + 2$ and $e_g(1) = n + 1$.

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$
$3k+1$ and $k \geq 0$	$k+1$	$k+1$	k	$k+1$	$k+1$	k
$3k+2$ and $k \geq 0$	$k+1$	$k+1$	$k+1$	$k+2$	$k+1$	$k+1$
$3k$ and $k \geq 1$	$k+1$	k	k	$k+1$	$k+1$	k

TABLE 9

Here $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.
Thus, g is a group S_3 mean cordial labeling. □

Theorem 3.11. *$K_{1,2} * K_{1,n}$ is group S_3 mean cordial graph.*

Proof. Let $V(K_{1,2} * K_{1,n}) = \{p, q, r, p_i, q_i : 1 \leq i \leq n\}$. Then $E(K_{1,n} * K_{1,n}) = \{pr, qr, pp_i, qq_i : 1 \leq i \leq n\}$ Define $g : V(K_{1,2} * K_{1,n}) \to S_3$ as follows:
as in Theorem 10, let us assign the label to the vertices p, q, r, p_i, q_i for $1 \leq i \leq n$. Here $|v_g(i)) - v_g(j)| \leq 1$ for $i, j \in S_3$. Clearly, $e_g(0) = n + 1 = e_g(1)$. Hence $K_{1,2} * K_{1,n}$ is group S_3 mean cordial graph. □

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