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PRIME-PRODUCING POLYNOMIALS RELATED TO CLASS NUMBER ONE PROBLEM OF NUMBER FIELDS

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Abstract. First, we recall the results for prime-producing polynomials related to class number one problem of quadratic fields. Next, we give the relation between prime-producing cubic polynomials and class number one problem of the simplest cubic fields and then present the conjecture for the relations. Finally, we numerically compare the ratios producing prime values for several polynomials in some interval.

1. Introduction

Euler gave the famous prime-producing polynomial $f(x) = x^2 + x + 41$ which gives prime values for all integers $x = 0, 1, \ldots, 39$. Euler also discovered that for $q = 2, 3, 5, 11, 17$, and 41, the polynomial $f(x) = x^2 + x + q$ gives prime values for all integers $x = 0, 1, \ldots, q-2$. Rabinowitsch [17] proved that for a prime number q, the class number of $\mathbb{Q}(\sqrt{1-4q})$ is equal to 1 if and only if $k^2 + k + q$ is prime for every $k = 0, 1, \ldots, q-2$. For real quadratic fields, many authors [4, 5, 15, 25] considered the connection between prime-producing polynomials and class number one problem. Many authors (cf. [9, 10, 16, 18]) also observed prime-producing cubic polynomials. There is a cubic polynomial of two variables relating class number one problem of the simplest cubic fields (cf. [9, 10]). But we could not find any results giving the relation between prime-producing cubic polynomials of one variable and class number one problem of associated cubic fields. The aim of this paper is to give the relation between prime-producing cubic polynomials of one variable and class number one problem of associated cubic fields. First, we recall known results for some quadratic fields and then give our results for the simplest cubic fields. Now, we remind the result for prime-producing polynomials related to class number one problem of imaginary quadratic fields.

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Theorem 1.1. *Let q be a prime. We put $f_q(x) = x^2 + x + q$ and $K_q = \mathbb{Q}(\sqrt{1-4q})$. Then the following conditions are equivalent*:

(1) $q = 2, 3, 5, 11, 17, 41$.
(2) *$f_q(k)$ is a prime for every $k = 0, 1, \ldots, q-2$.*
(3) *The class number of K_q is one.*

We note that Rabinowitsch [17] proved the equivalence of (1) and (2). Heegner [8] attempted to determine imaginary quadratic fields with class number one. But, there was a gap in Heegner's proof. After that, Stark [22] filled the gap and proved that there exist exactly nine imaginary quadratic fields with class number one. That makes it possible to observe the relation of (1) and (3).

Next, we introduce similar results for real quadratic fields.

Theorem 1.2. *Let q be a positive integer. We put $d = 4q^2 + 1$ and $f_d(x) = -x^2 + x + \frac{d-1}{4}$. Let $K_d = \mathbb{Q}(\sqrt{d})$. Then the following conditions are equivalent*:

(1) *The class number of K_d is one.*
(2) *Every prime $q_0 < q$ remains inert in $K_d/\mathbb{Q}$.*
(3) *$f_d(k)$ is a prime for every integer k such that $2 \le k \le q$.*

Theorem 1.3. *Let q be an odd positive integer. We put $d = q^2 + 4$ and $f_d(x) = -x^2 + x + \frac{d-1}{4}$. Let $K_d = \mathbb{Q}(\sqrt{d})$. Then the following conditions are equivalent*:

(1) *The class number of K_d is one.*
(2) *Every prime $q_0 < q$ remains inert in $K_d/\mathbb{Q}$.*
(3) *$f_d(k)$ is a prime for every integer k such that $1 \le k \le \frac{q-1}{2}$.*

We say that the form $d = 4q^2 + 1$ (resp. $d = q^2 + 4$) is Chowla's type [6] (resp. Yokoi's type [25]) because Chowla (resp. Yokoi) first observed this kinds of real quadratic fields in [6, 25]. Sasaki [20] proved the equivalence relation of (1) and (3) by using continued fraction expansions. On the other hand, Byeon and Kim [4] gave analytic proof for equivalence relation of (1) and (3). Yokoi [25] proved the equivalence relation of (1), (2), and (3). Biró [1, 2] determined this kinds of real quadratic fields with class number one. In fact, there exist exactly six real quadratic fields of Chowla's type (resp. Yokoi's type) with class number one, in that case, q is $1, 2, 3, 5, 7$, and 13 (resp. q is $1, 3, 5, 7, 13$, and 17).

Remark 1. Before Biró's seminal work, there were not any results for determination of some real quadratic fields with class number one. There existed only the results for a characterization of real quadratic fields such as Theorems 1.2 and 1.3. Therefore, Biró's results [1, 2] are considerably meaningful.

2. The simplest cubic fields

Now, we are interested in prime-producing cubic polynomials related to class number one problem. There exist few results giving the relations as Theorems

1.2 and 1.3 in cubic fields compared to quadratic fields. First, we note that every cyclic cubic field can be obtained by adjoining to $\mathbb{Q}$ a root of an irreducible polynomial

$$F_m(x) = x^3 + mx^2 - (m+3)x + 1,$$

where m runs over the set of rational numbers (cf. [11]). The discriminant of the polynomial $F_m(x)$ is D_m^2, where $D_m = m^2+3m+9$. Let α_m be the negative root of $F_m(x)$. Then $K_m = \mathbb{Q}(\alpha_m)$ is a cyclic cubic field which is called the simplest cubic field. The terminology "simplest cubic field" came from a work of Shanks [21]. He studied the arithmetic of a family of cyclic cubic fields in the case that $D_m = m^2+3m+9$ is a prime, where m is an integer. The notion was extended by Washington [23] in which he studied the arithmetic of a family of cyclic cubic fields which corresponds to $m \in \mathbb{Z}$, $m \not\equiv 3 \pmod 9$. Let $m \geq -1$ be a rational integer such that $m \not\equiv 3 \pmod 9$ and $m^2 + 3m + 9$ is square-free or 9 times a square-free integer. Then we have $D_{K_m} = D_m^2 = (m^2 + 3m + 9)^2$ and $\mathcal{O}_{K_m} = \mathbb{Z}[\alpha_m]$, where D_{K_m} is the discriminant of K_m and $\mathcal{O}_{K_m}$ is the ring of integers of K_m [23].

We note that there are infinitely many positive integers m such that $m^2 + 3m + 9$ is square-free [7]. Moreover, we know that $\{1, \alpha_m, \alpha_m'\}$ is an integral basis of $\mathcal{O}_{K_m}$, where α_m' is an algebraic conjugate of α_m. Note that letting $-F_m(-x) = G_m(x) = x^3 - mx^2 - (m+3)x - 1$, the polynomial $G_m(x)$ generates the same field K_m.

In [13], Lettl obtained a lower bound of residues at $s = 1$ of Dedekind zeta functions attached to cyclic cubic fields and, by applying his lower bound to the simplest cubic fields with prime conductors, has shown that there are exactly seven simplest cubic fields of class number one. That is, $m = -1, 1, 2, 4, 7, 8, 10$ gives all the values of m such that the class number of the simplest cubic field is one. Later, Louboutin [14] improved considerably the result of Lettl.

Kim and Hwang [9] applied Siegel's formula for values of the zeta function of a totally real algebraic number field at negative odd integers to the simplest cubic fields. By using the result, they found the following prime-producing polynomial with two variables c, t and one parameter m related to class number one:

$$\begin{aligned}\sigma_m(c,t) = {} & (t^2+(c-1)t)m^2+(-2t^3+(-3c+6)t^2+(-c^2+3c)t \\ & +(-c^2+3c-2))m+(-3t^3+(3c^2-9c+9)t+(c^3-6c^2+9c-3)).\end{aligned}$$

Combining these results, we can give the following equivalence relations.

Theorem 2.1. *Let $m \geq 1$ be an integer such that $m \not\equiv 3 \pmod 9$ and assume $m^2 + 3m + 9$ is square-free or 9 times a square-free integer. Let K_m be the simplest cubic field corresponding to m and h_m the class number of K_m. Then the following conditions are equivalent*:

(1) *h_m is one.*
(2) $m = 1, 2, 4, 7, 8, 10$.

(3) *$\sigma_m(c,t)$ is a prime for any integers c and t in the range $1 \le c \le m+3-3t$ and $1 \le t \le \frac{m+2}{3}$.*

Lettl [13] explicitly determined the simplest cubic fields with class number one by using the lower bounds for $L(1,\chi)\cdot L(1,\bar{\chi})$ for certain cubic character. On the other hand, the equivalence relation of (1) and (3) is proved in Theorem 3.1 of [9]. The cubic polynomial of two variables $\sigma_m(c,t)$ comes from the norm function of some principal ideal. The principal ideals give one to one correspondence with the Siegel Lattice. The range in (3) of Theorem 2.1 implies the representatives of the Siegel Lattice (cf. Theorem 4.4 of [10]).

In particular, by substituting $t = 1$ for $\sigma_m(c,t)$, we have a necessary condition for h_m to be one.

Corollary 2.2. *If h_m is one, then*

$$\sigma_m(k,1) = k^3 - (2m+3)k^2 + m(m+3)k + 2m + 3$$

is a prime for every $k = 1, 2, \ldots, m$.

If we fix one variable in the cubic polynomial $\sigma_m(c,t)$ of two variables, the polynomial $\sigma_m(c,t)$ is closely related to the $F_m(x)$. In fact, since $G_m(x) = (x-1)(x+2)(x-m-1)-(2m+3) = -\sigma_m(m+1-x,1)$, Corollary 2.2 means that if h_m is one, then $-G_m(k)$ is a prime for every $k = 1, 2, \ldots, m$. Noting that $\sigma_m(x,1) = x(x-m)(x-m-3)+2m+3$, we have $\sigma_m(x+m+1,1) = (x+1)(x-2)(x+m+1)+2m+3 = F_m(x)$ and $\sigma_m(m,1) = \sigma_m(0,1) = 2m+3$. Therefore, we have the following result.

Corollary 2.3. *If h_m is one, then $F_m(k)$ is a prime for every integer k in the range $-m-1 \le k \le -1$.*

We will try to give new class number one criterion for the simplest cubic fields by observing that a polynomial $F_m(x)$ producing relatively many primes is related to class number one problem of K_m. Now, we give a necessary condition for the class number of K_m to be one.

Proposition 2.4. *Suppose h_m is one. Then every prime p less than $2m+3$ remains inert in $K_m/\mathbb{Q}$.*

Proof. Suppose h_m is one and a prime p is less than $2m+3$. If a prime p does not remain inert in $K_m/\mathbb{Q}$, since K_m is a cyclic cubic field, $(p) = \mathcal{P}^3$ and $N(\mathcal{P}) = p < 2m+3$, or $(p) = \mathcal{P}_1\mathcal{P}_2\mathcal{P}_3$ and $N(\mathcal{P}_1) = N(\mathcal{P}_2) = N(\mathcal{P}_3) = p < 2m+3$. Here, $\mathcal{P}$, $\mathcal{P}_1$,$\mathcal{P}_2$, $\mathcal{P}_3$ are prime ideals in $\mathcal{O}_{K_m}$ and $N(\mathcal{P})$ is the norm of ideal P. By assumption, $\mathcal{P}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3$ are principal ideals. But, they could not be principal by the result of [12]: For all $\gamma \in \mathcal{O}_K$, $|N_{K_m/\mathbb{Q}}(\gamma)| \ge 2m+3$. It completes the proof. $\square$

From the above results, it is reasonable to conjecture similar equivalent statements as Theorems 1.2 and 1.3 giving the relation between a prime-producing cubic polynomial and class number one problem of the simplest cubic field.

Conjecture 2.5. *Let $m \geq 1$ be an integer such that $m \not\equiv 3 \pmod 9$ and assume m^2+3m+9 is square-free or 9 times a square-free integer. Let K_m be the simplest cubic field corresponding to m. Then the following conditions are equivalent*:

(1) *h_m is one.*
(2) *Every prime p less than $2m+3$ remains inert in $K_m/\mathbb{Q}$.*
(3) *For every prime p less than $2m+3$, the congruence $F_m(x) \equiv 0 \pmod p$ has no solution for x.*
(4) *$F_m(k)$ is a prime for every integer k in the range $-m-1 \leq k \leq -1$.*

Remark 2. Suppose $2m+3$ is a prime. Minkowski's constant for the simplest cubic field is $\frac{2}{9}(m^2+3m+9)$ (cf. p. 166 of [19]). That is, every class of fractional ideals contains an ideal $I \subset \mathcal{O}_{K_m}$ with $N(I) \leq \frac{2}{9}(m^2+3m+9)$. Applying Minkowski's constant for the simplest cubic field, it is easy to check that if for a prime $p \leq \frac{2}{9}(m^2+3m+9)$, every prime ideal dividing a principal ideal $\langle p \rangle$ is principal, then h_m is one. But, in order to show that h_m is one, Minkowski's constant for the simplest cubic field is not useful because a prime $p = 2m+3$ is not inert. In fact, since $F_m(2) = 2m+3$, a prime $p = 2m+3$ is totally ramified or splits completely by Dedekind's theorem (cf. p. 196, Theorem 2 of [19]). Moreover, if $2m+3$ is a prime, then it is well known that a prime $p = 2m+3$ splits completely (cf. Lemma 2.1 of [3]).

By Proposition 2.4, (1) implies (2). Since $\mathcal{O}_{K_m} = \mathbb{Z}[\alpha_m]$, (2) and (3) are equivalent by Dedekind's theorem. On the other hand, (1) implies (4) by Corollary 2.3. In fact, if m is $1, 2, 4, 7, 8, 10$, then we easily see that $f_m(k)$ is a prime for every integer in the range $-m-1 \leq k \leq m+1$ except that $k=0$ and $k=1$, in that case, $F_m(0) = 1$ and $F_m(1) = -1$. By using GP/Pari program, we numerically checked the statements of Conjecture 2.5 in the range $1 \leq m \leq 2 \cdot 10^6$ and also observed m satisfying the condition that $F_m(k)$ is a prime for every integer k in the range $-m-1 \leq k \leq -1$, in that case, m is 1,2,4,7,8,10. It took about 4 days with 2 high-performance laptops to verify the computation.

If h_m is one, it is well known that $2m+3$ is a prime [3, 9]. Therefore, we have the following result.

Proposition 2.6. *Suppose h_m is one. If $F_m(k)$ is a prime for every integer k in the range $-m-1 \leq k \leq m+1$ except that $k=0$ and $k=1$, then every prime p less than $2m+3$ remains inert in $K_m/\mathbb{Q}$.*

Proof. Note that if h_m is one, then $2m+3$ is a prime. It is easy to check that $F_m(k) \geq 2m+3$ for every integer k in the range $-m-1 \leq k \leq m+1$ except that $k=0$ and $k=1$. Since $\frac{p-1}{2} < m+1$, $F_m(k)$ is a prime for every integer k in the range $-\frac{p-1}{2} \leq k \leq \frac{p-1}{2}$ by assumption. That means that for a prime p less than $2m+3$, p does not divide $F_m(k)$ for every integer k in the range $-\frac{p-1}{2} \leq k \leq \frac{p-1}{2}$. That is, the congruence $F_m(x) \equiv 0 \pmod p$ has no solution for x. It completes the proof. $\square$

Remark 3. If we admit $F_m(k)$ to have negative values, we can extend the range of k up to $-2m$. That means that if m is $1, 2, 4, 7, 8, 10$, then $|F_m(k)|$ is a prime for every integer k in the range $-2m \leq k \leq m+1$ except that $k = 0$ and $k = 1$.

Remark 4. It is well known for explicit lower bounds for class numbers of cyclic cubic fields [13, 14]. Therefore, for parametrized families of cyclic cubic fields with known regulators, it is not difficult to determine cyclic cubic fields with class number one. In this paper, we more focus on prime-producing cubic polynomials to class number one problem of cubic fields than determination of cubic fields with class number one.

3. Prime-producing polynomials

One of fascinating problems for producing prime values is whether we can discover a polynomial $f(x)$ with integer coefficients that produces infinitely many prime values. Euclid proved that a polynomial of the form $f(x) = x$ produces infinitely many primes. Dirchlet's theorem on primes in arithmetic progressions extends Euclid's theorem to a polynomial of the form $f(x) = ax + b$, where a and b are relatively prime integers and $a > 0$. Another problem for producing primes is to concern polynomial producing consecutive prime values for all integers x in some interval. Euler discovered that the polynomial $x^2 + x + 41$ produces 40 distinct prime values for all integers $x = 0, 1, \ldots, 39$. Theorem 1.2 says that there exist exactly six polynomials of the form of the polynomial $x^2 + x + q$ producing prime values for all integers $x = 0, 1, \ldots, q-2$ and the largest prime q satisfying that is 41. If we admit that $f(x)$ has negative values and produced primes need not necessarily be distinct, we can extend the range of x producing primes in $f(x)$. For example, the transformed polynomial $(x-40)^2 + (x-40) + 41$ of the polynomial $x^2 + x + 41$ produces prime values for all integers $x = 0, 1, \ldots, 79$. In this case, produced primes are not necessarily distinct. Ruby discovered that for the polynomial $f(x) = 36x^2 - 810 + 2753$, $|f(x)|$ generates distinct prime values for all integers x in the range $[-33, 11]$ (cf. [18]).

On the other hand, one can ask if we can find a polynomial $f(x)$ producing as possible as many primes in some interval. Table 1 gives some quadratic, cubic polynomials that generate (possibly in absolute value) only primes for the first few nonnegative values (cf. [24]). The second column of Table 1 indicates the natural number N where known "good" prime-producing polynomials produces only prime in $[0, N]$ and the third column implies the number of distinct primes that the polynomials produce in the range $x \in [0, N]$. Furthermore, the fourth column of Table 1 lists the percentage P_1 that absolute values of the given polynomials in the range $[0, 2000]$ become prime values and the fifth column implies class number h of number fields obtained by adjoining $\mathbb{Q}$ to a root of the polynomial.

Finally, we compare the ratio of the famous Euler polynomial and a cubic polynomial relating the simplest cubic field produce prime values in the range

TABLE 1.

Polynomials	N	distinct primes	P_1	h
$-66x^3+3845x^2-60897x+251831$	45	46	0.357821	1
$36x^2-810x+2753$	44	45	0.490755	1
$3x^3-183x^2+3318x-18757$	46	43	0.361819	2
$47x^2-1701x+10181$	42	43	0.456272	1
$103x^2-4707x+50383$	42	43	0.463268	5
x^2+x+41	39	40	0.510745	1

TABLE 2.

N	P_2	P_3
100	0.861386	0.594059
1000	0.581419	0.365634
2000	0.510745	0.336332
5000	0.45231	0.309338
10000	0.414859	0.286171
20000	0.380981	0.260937

$x \in [0, N]$. The percentage P_2 (resp. P_3) indicates that x^2+x+41 (resp. $x^3+10x^2-13x+1$) produces prime values in the range $x \in [0, N]$ in Table 2.

Remark 5. There exist numerical results for many prime-producing polynomials related to class number one problem [16, 18]. But, there exist few results giving the relation between prime-producing cubic polynomials and class number one problem of associated cubic fields. In that sense, observation of Conjecture 2.5 will be meaningful.

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