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RESEARCH ARTICLE

Exploring the Epistemic Actions in Pre-service Teachers' Tasks

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Abstract

This study analyzes the tasks selected and implemented by pre-service mathematics teachers to support students' development of epistemic actions. Data was collected from 20 students who participated in a mathematics education curriculum theory course during one semester, and multiple data sources were used to gather information about the microteaching sessions. The study focused on the tasks selected and demonstrated during microteaching by pre-service teachers. The results suggest that providing students with a variety of learning opportunities that engage them in different combinations of abductive and deductive epistemic actions is important. The tasks selected by pre-service teachers primarily focused on understanding concepts, calculation, and reasoning. However, the use of engineering tools may present challenges as it requires students to engage in two epistemic actions simultaneously. The study's findings can inform the development of more effective approaches to mathematics education and can guide the development of teacher training programs.

Keywords: pre-service teachers, mathematics education, epistemic actions, tasks, microteaching, engineering tools.

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I. INTRODUCTION

Mathematics learning and achievement are closely linked to students' ability to engage in a variety of epistemic actions, such as finding patterns and analyzing materials. Hwang et al. (2023) emphasized the significance of providing middle school students with opportunities to experience these epistemic actions in mathematics classes, as this is crucial for their mathematical development. Epistemic actions refer to the observable behaviors that result from the use of intellectual resources by the learner (Bailin, 2002), and they are considered essential for participating in mathematical practices (Hershkowitz et al., 2001). These actions are an integral part of mathematical problem-solving, proving, and understanding, and are critical for students to develop a deep and meaningful understanding of mathematical concepts.

Epistemic actions can be categorized into various elements, such as understanding and calculating concepts, making inferences, representing data mathematically, transforming mathematical expressions, and formulating and executing problem-solving strategies (Bailin, 2002; I. Kim et al., 2022). These actions are seen as essential for mathematical practices and are key to students' development of mathematical skills and understanding. Engaging in these epistemic actions requires students to use their intellectual resources, such as reasoning and problem-solving abilities, and to actively participate in mathematical practices. According to Hwang et al. (2023), *selecting* and *analyzing* were two of the most important epistemic actions for mathematics learning. They emphasized that engaging students in these actions can help initiate problem solving process with abductive inferences. These actions are seen as essential for mathematical practices and are critical for students to develop a deep and meaningful understanding of mathematical concepts.

Despite the importance of these epistemic actions for mathematics learning, it is not yet clear how mathematics teachers choose and provide tasks that allow students to engage in these actions. aims to address. It is important for mathematics teachers to provide students with a variety of learning opportunities that engage them in abductive and deductive epistemic actions (Hwang et al., 2023). For example, teachers can create opportunities for students to analyze mathematical patterns and relationships, to work with mathematical representations and models, and to engage in mathematical discourse with their peers (Hwang et al., 2017). By providing these opportunities, teachers can support students in developing a deeper understanding of mathematical concepts and in engaging in the mathematical practices. This, in turn, can lead to greater mathematics learning and achievement for students. Thus, this lack of understanding about epistemic actions in classroom tasks is a gap in the literature that this study

This study will focus on the tasks used by pre-service mathematics teachers. The goal of the study is to analyze the tasks selected and implemented by pre-service teachers in order to understand how they support students in developing their epistemic actions. To achieve this goal, the study will answer the following research questions: What tasks do pre-service teachers choose to use in their mathematics classes? How are these tasks implemented to support students in abductive and deductive epistemic actions? To answer these research questions, the study will analyze the tasks in the textbook used by pre-service teachers, the tasks selected for the class, and the tasks demonstrated during a microteaching. The results of this study will provide valuable insights into the opportunities that are currently provided to pre-service teachers and can inform the development of more effective approaches to mathematics education. This information can be used to guide the development of teacher training programs and to improve the preparation of pre-service teachers for their future careers as mathematics educators.

II. LITERATURE REVIEW

Mathematical learning is a dynamic and interactive process that requires students to engage in mathematical practices to deepen their understanding of mathematical concepts and knowledge. One crucial aspect of this process is the use of epistemic actions, which are observable behaviors that result from the use of intellectual resources by the learner (Hwang et al., 2023; Hershkowitz et al., 2001). Epistemic actions are considered essential for participating in mathematical practices and are key to students' development of mathematical skills and understanding. Epistemic actions can be seen as bridging between both a cognitive and a sociocultural perspective (Cobb, 1994). From a cognitive perspective, epistemic actions are viewed as application of learner's intellectual resources for reasoning and solving problems (Bailin, 2002). From a sociocultural perspective, epistemic actions are seen as important for participating in mathematical practices and engaging in mathematical discourse with others. Both perspectives emphasize the importance of epistemic actions for mathematical learning and achievement (Hwang et al., 2023).

This study builds on existing research (I. Kim et al., 2022; Hwang et al., 2023) by considering the following elements as key epistemic actions in mathematics learning: understanding and calculating concepts, making inferences, representing data mathematically, transforming mathematical expressions, and formulating and executing problem-solving strategies (see Table 1; I. Kim et al., 2022). These actions have been identified as essential for participating in mathematical practices (Hwang et al., 2023) and for students' mathematics achievement. These five epistemic actions provide a comprehensive framework for understanding the key behaviors involved in mathematical learning and can be used to guide teacher preparation and instruction. While these elements were originally considered to be cognitive in nature, recent research has shown that they can also be seen as representing the actions required by the task, rather than simply describing the type of reasoning involved (Bailin, 2002). These actions are considered epistemic because they accompany the understanding of certain mathematical knowledge and are essential for students' participation in mathematical practices.

Table 1. Epistemic actions suggested by I. Kim et al. (2022)

Epistemic Actions	Description
Understanding concepts and Calculation	Understand the basic concepts of mathematics and calculate simple equations (e.g. set operations, solving equations, solving inequalities, factoring, etc.).
Reasoning	Students are able to deduce mathematical facts or statements given to them or distinguish whether they are true or false, and can logically describe the entire or part of the problem-solving process.
Representing	The student can represent or interpret real-life data or information presented in a problem using symbols, equations, graphs, etc. and use the results.
Transforming Mathematical Representation	A student can convert a given mathematical expression into a different mathematical representation, such as symbols, equations, graphs, etc., according to the purpose and use it.
Formulating and Implementing Problem-Solving Strategies	Identifying the relationship between mathematical concepts and areas, and formulating and implementing problem-solving strategies.

Characteristics of Epistemic Actions

Epistemic actions can be divided into abductive actions and deductive actions, considering their relationship with logical thinking (Kirwan, 1995). Abductive actions are described as the process of searching for reasons and formulating hypotheses in uncertain situations (Kirwan, 1995). These actions are characterized by their exploratory and data-driven nature, and are valuable for discovering new relationships and patterns in mathematical concepts. On the other hand, deductive actions provide reasons and have a more structured and organized approach to determining the truth of a fact from another. Deductive actions are based on well-established mathematical principles and procedures, and are used to verify the validity of the results generated by abductive actions (Kirwan, 1995). This distinction between abductive and deductive actions highlights the complementary nature of these two types of reasoning in mathematical learning (Hanna & Jahnke, 2007). Abductive and deductive actions reinforce each other and work together to deepen students' understanding of mathematical concepts (Choi et al., 2020). By engaging in both abductive and deductive actions, students can develop a well-rounded understanding of mathematical concepts and skills, and improve their mathematical achievement.

When it comes to the five epistemic actions, formulating and implementing problem-solving strategies are known to have a strong abductive nature (Choi et al., 2020; Hwang et al., 2023). This means that selecting and formulating hypotheses depends on experience and is not always easily observable. Similarly, making judgments based on observation also has an abductive nature. On the other hand, the epistemic actions of understanding and calculating mathematical concepts, reasoning, and transforming mathematical representation are primarily deductive in nature (Choi et al., 2020; Hwang et al., 2023). For example, calculations may involve some abductive elements, such as choosing an efficient calculation method, but they are primarily based on the laws and conventions defined by numbers and operations. Similarly, transforming mathematical expressions also has a deductive nature, as rules are applied to change expressions based on certain conditions. Representing has a balance of both abductive and deductive natures (National Council of Teachers of Mathematics [NCTM], 2000). The process of choosing means and methods to express data and formulating strategies has abductive elements, but transforming expressions involves deductive elements once these choices have been made.

III. METHODS

I collected data from 20 students who participated in a mathematics education curriculum theory course during one semester in 2022. The aim of the course was designed to provide students with a better understanding of the mathematics curriculum in various mathematical areas such as geometry, calculus, probability, and statistics. The course included a series of microteaching sessions, which are simulated teaching sessions designed to help teachers develop and refine their teaching skills, such as lesson planning, classroom management, and instructional delivery.

During the microteaching sessions, five groups of four pre-service teachers were formed based on content areas (see Table 2). Each group had one pre-service teacher provided a 10-minute lesson plan explanation to the class before each microteaching session. Subsequently, another pre-service teacher who act as the instructor, conducted a 20-minute microteaching session, during which they taught a mathematical concept to the class. The remaining students in the class were instructed to behave like high school students.

Table 2. Content areas and lesson topics of the five groups

	Content Area	Lesson Topic
Group A	Calculus	The equation of a tangent line
Group B	Calculus	The local maximum and local minimum of a function
Group C	Geometry	Ellipse
Group D	Geometry	Parabola
Group E	Statistics and Probability	Permutation and Combination

The study used multiple data sources to collect information about the microteaching sessions, including the lesson plans, class observation notes, videotaped lessons, and all lesson materials. After all microteaching sessions were completed, I collected the submitted lesson plans, lesson materials, and students' reflections on their activities. In addition, all classes were video recorded and transcribed for further analysis. The lesson materials used in the microteaching sessions would have included handouts, worksheets, and other instructional materials designed to support student learning. The lesson plan would have outlined the course objectives and the topics covered in each microteaching session and included details on the specific lesson materials used. The class observation notes would have captured the behaviors, interactions, and teaching strategies of the instructor and students during the microteaching sessions, including how the lesson materials were used to facilitate student learning. The videotaped lessons would have provided a visual record of the sessions, allowing researchers to analyze various aspects of the teaching and learning process, including how the lesson materials were used to support student understanding. By using these different data sources, the researchers were likely able to gain a more comprehensive understanding of the opportunities for epistemic actions in tasks and how these tasks were implemented.

The researcher examined the students' lesson materials and transcribed videos to identify main tasks (there was one tasks implemented in each microteaching session) and scenes that displayed evidence of the five epistemic actions. Then, the data was analyzed with constant comparative technique and using the codes from framework shown in Figure 1. Through the analysis, I addressed the main epistemic actions required in the tasks in each group and compare whether there are similarities and differences among the groups.

IV. RESULTS

The results of the study indicate that the common epistemic actions displayed by both groups that dealt with multivariable calculus were reasoning with mathematical concepts or statements from a given graph. Group A, which dealt with the equation of tangents, focused on drawing a tangent line of a quadratic function and finding the conditions that must be met for a tangent to exist (see Figure 1). This activity required students to represent a tangent line, but also reason why a red vertical line cannot be a tangent line of a given quadratic function. In other words, students engaged in the epistemic action of expressing and to use their previously learned knowledge of tangents to explore new mathematical concepts.

When students engaged in the right task in Figure 1, one student discussed his thinking about the definition of a tangent line.

> Student: I thought that the one on the left wasn't a tangent line because when you zoom in on the point, it creates a line like this [he is pointing in a 45-degree inclined direction] when we repeat zooming in many times. But it becomes a straight line like this, and this red part is perpendicular to the x-axis, so they're not on the same line.
>
> Teacher: So, if you zoom in on the key point, it would create a line like this [while drawing a vertical line with their hand], and it's not the same as this line [pointing to the quadratic function], so it's not a tangent line. Then, what about this one?

Student: I think the one on the right would create a straight line that looks like this, and it's the same as the blue reference line, so I think it's a tangent line.

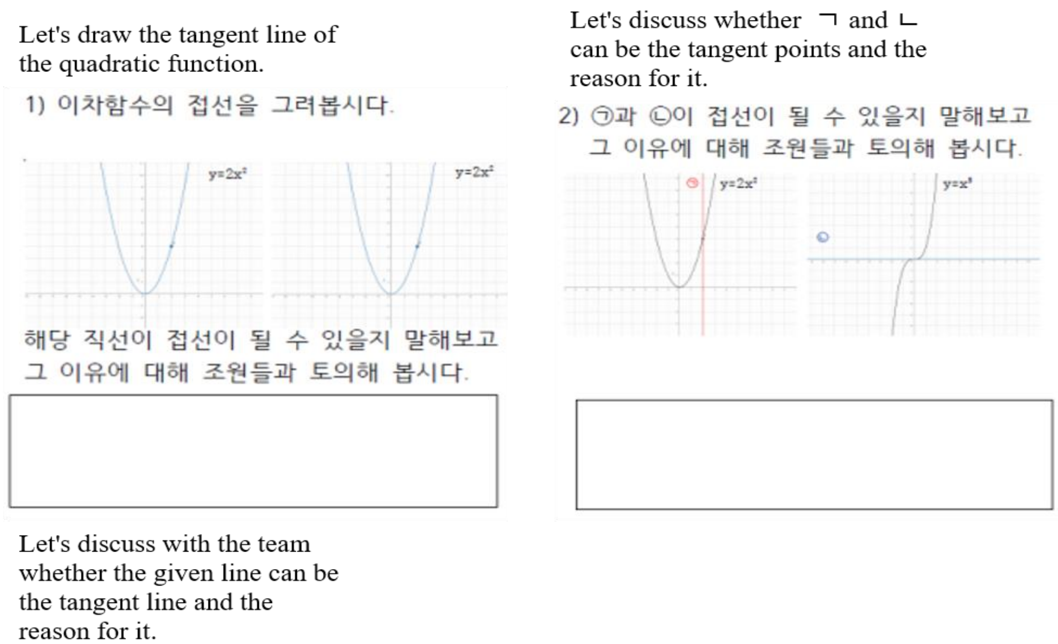

Figure 1. Tasks in the class on the equation of a tangent line

The line intersects the quadratic function at one point, but the students explain that a tangent line should closely follow the curve when observed at a small interval around the point. It's interesting to note that the students did not rely on specific mathematical terminology, but rather graphical representations to define a tangent line. The students conclude that the red line in Figure 1 is not a tangent line because it differs significantly from the curve around the intersection point when zoomed in.

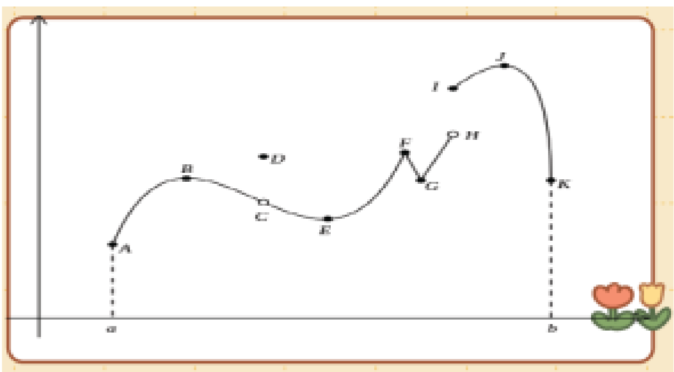

Figure 2. Graph used in the task in the class on the local minimum and local maximum of a function

Group B, which dealt with the maxima and minima of functions, found the local maxima and local minima in the graph, and explained them. This was also an activity that required the epistemic action of inferring. The students were asked to come up with definitions for maxima and minima, which provided them with an opportunity to engage in higher-level mathematical thinking. However, the results suggest that finding the maxima and minima of a function may not be possible without prior explanation of the concept. Therefore, before students were allowed to find the maximums and minimums themselves, an activity was performed to help them understand the concept and perform the calculation.

> Instructor: So here, there's a mixture of local minimums and maximums. So now, let's take a look at the activity sheet. While looking at the activity sheet, try to mark where you think the local minimums are, and then let's think about it. There are rules, and we'll have a time to find and share our own reasons for why this is a local minimum or maximum.
>
> Student 1: I'm not sure about this one [pointing to point D], and I'm not sure about these two either [pointing to points I and H]. But it seems like the maximums are the bigger ones. It seems like this one is the biggest, so it seems like it's the maximum, and the smallest one seems like the smallest, so it seems like it's the minimum.
>
> Instructor: So, for now, you've found them. So now, let's come up with our own definition for the activity, let's not overthink it and just remind what you learned. Even if you get it wrong, it doesn't matter because we'll write it down and then compare it later to check what's wrong and what's right.

In this conversation during the implementation of the task, the instructor explained that there are local minimums and maximums and asked the students to mark them on the activity sheet. One student was unsure about a few points, but e think that the maximums are the bigger ones, and the smallest one seems like the minimum. This can be evidence for students' abduction, which is related to generating hypothesis. This clearly showed students' action of reasoning. In addition, the instructor acknowledges the student's findings and encourages the students to come up with their own definition of the activity. The instructor also emphasizes that it's okay to get it wrong, as they will write it down. This enabled students represent their own thinking of what local maximum and local minimum are.

In Groups A and B, the microteaching sessions offered different opportunities for students to engage in epistemic actions. The first group's simulated lesson used the previously learned concept of tangents to explore the mathematical conditions that must be met for a tangent to exist, while the second group's simulated lesson focused on finding the maxima and minima of a function. Both groups helped students reason abductively and deductively. However, the students created graphical representation of a tangent line first and write texts to explain students' own ideas in the microteaching of Group A. In the microteaching of Group B, the graph was given, and students needed to find the definition of the mathematics concepts based on the observation. Although Groups A and B covered the same mathematics content area, these differences in the microteaching sessions highlight the importance of providing students with a variety of learning opportunities to engage in different epistemic actions and to build a deeper understanding of mathematical concepts.

The two groups that dealt with geometry required different epistemic actions depending on the use of engineering tools. In the class dealing with ellipses (Group C), reasoning was used as a means of explaining the definition of ellipses without the use of engineering tools. For example, students were asked to find the equation of a parabola based on their understanding and calculation, and then to prove the properties of ellipses using the definition given in the textbook. Then, the instructor questioned "if it is possible to prove this [the statement that the sum of the distances from a point on an ellipse to the two foci is always constant], then is it the definition of an ellipse?" Answering this question required a level of geometric thinking development at least at the fourth stage of Van Hiele theory, and raises questions about whether this is feasible for high school students. This question was answered in class with visual representations, which is provided by only the instructor as seen in Figure 3.

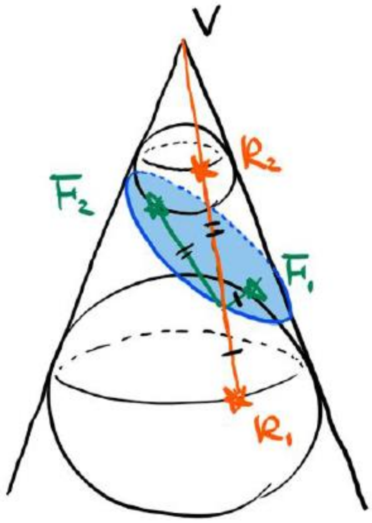

Figure 3. The visual representations used to prove the textbook definition of an ellipse

In Group D, which dealt with parabolas, the main activity was writing the visual representation of parabolas shown using engineering tools. Students were encouraged to observe the activities, write down their thoughts, and express any difficulties or questions they had. However, all activities were limited to expressing and transforming expressions. Some opinions suggested that learning was difficult because two types of epistemic actions – reasoning and representing – had to occur simultaneously. For example, when the instructor implemented the task in Figure 4, he said the following using the GeoGebra:

> It's amazing to see that only at point (0,1), the distance between the quadratic function and the straight line is equal to the distance between the point and a point on the y-axis. In fact, there are two points on the graph where this occurs. One is here and the other is there. But, this point (0,1) can also change. The changing order of this point is different (Microteaching of Group D, October 12, 2022).

The instructor had to explicitly explain each manipulation for the students to effectively engage in the class. Even with accurate explanations, it would be challenging for students to write down their thoughts that come to mind while understanding the teacher's program usage.

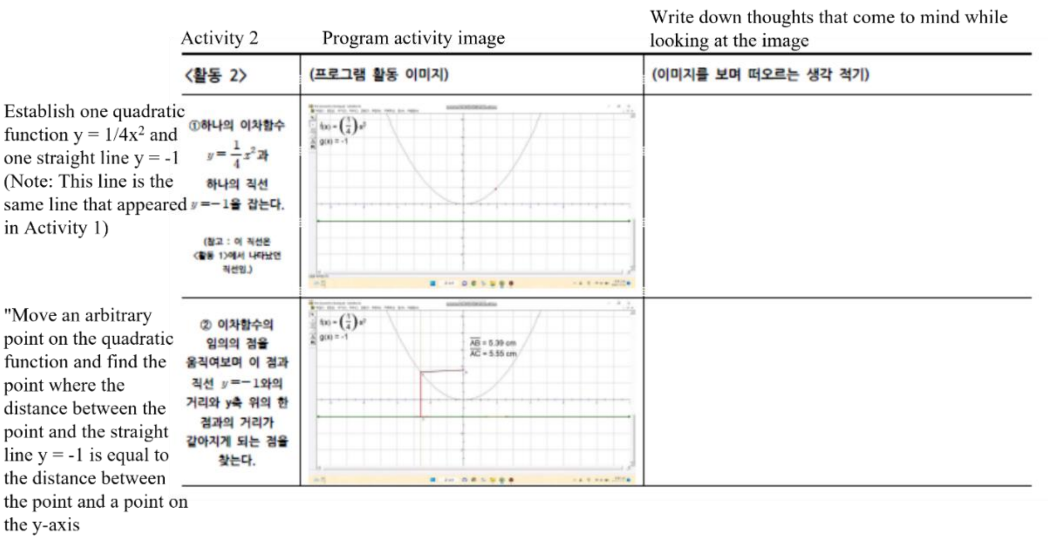

Figure 4. Activity 2 in the task given by Group D

1. In 2050, the Korean Teacher University Mathematics Education and Academic Conference will offer you the opportunity to experience 3 geometry programs and 4 case number programs. How many opportunities do you have to experience either a geometry program or a case number program? [5 more similar questions follow]
6. Answer the following questions: Are the methods for finding problems 1, 2 and problems 3, 4, 5 the same? If not, what, how, and why are 1, 2 and 3, 4, 5 the same and different?

Figure 5. Translated task for permutation and combination

Group E, which dealt with permutation and combination, mainly performed activities (see in Figure 5) that required understanding concepts, calculation, and inference. Students were asked to solve problems, compare their solutions, and identify similarities and differences, which required them to engage in reasoning actions. However, reasoning was required after solving five equivalent tasks applying the formulas of permutation and combinations.

V. DISCUSSION AND CONCLUSION

Overall, the study's results suggest that offering students various learning opportunities that involve different combinations of abductive and deductive epistemic actions can be effective. Pre-service teachers tended to select tasks that focused primarily on understanding concepts, calculation, and reasoning. This finding is consistent with earlier research conducted by D. Kim et al. (2021), which revealed that many teachers place a significant emphasis on problem-solving that involves calculation, understanding concepts, and reasoning as key mathematical competencies outlined in the national curriculum. However, using technology tools may pose challenges for students, as it requires them to engage in two epistemic actions simultaneously. While combining epistemic actions is not always challenging, it is important to investigate why these two actions may conflict.

While it is important to provide opportunities for understanding concepts and calculations, the results of this study also emphasize the need for pre-service teachers to incorporate tasks that foster the formulation and execution of problem-solving strategies. As the highest epistemic action in the hierarchical structure of the five epistemic actions (as illustrated in Figure III-3-3 by I. Kim et al., 2022 on p. 96), this action is a critical aspect of the problem-solving process and can significantly impact student achievement.

Moreover, the findings suggest that the tasks selected by pre-service teachers may lack diversity, particularly in the case of calculus where tasks that allow for visual representation and transformation of functions can effectively emphasize the relationship between the visual representation of a graph and the algebraic representation of a function. Despite this, the study results indicate that the tasks proposed by pre-service teachers in the microteaching sessions may be overly focused on understanding concepts and reasoning, suggesting the need for more diverse and varied task selection. Additionally, all tasks in the study required students to perform at most two epistemic actions either sequentially or simultaneously, highlighting a potential area for further exploration in future studies.

In this study, it was found that pre-service teachers selected and implemented tasks that promote multiple epistemic actions. However, this study has several limitations. While the framework used in this study was designed to analyze assessment results, it may not be appropriate for examining microteaching. It is important to note that the list of epistemic actions provided in the study is not exhaustive. Therefore, it is essential to evaluate the opportunities for epistemic actions offered to students during tasks and how they engage with the tasks in an exploratory manner.

One of the pathways for future research is to connect the cognitive demands of tasks (Boston & Smith, 2009) with the epistemic actions required to complete the tasks. While this study highlights the importance of teaching tasks that promote epistemic actions, further research can show how doing mathematics is related to specific epistemic actions. By understanding this relationship, teachers can design learning environments that promote the use of a set of epistemic actions. Eventually, quipping pre-service teachers with the necessary tools and strategies can help educators prepare their students for success in mathematics and beyond.

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