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RESEARCH ARTICLE

Intervening in Mathematics Group Work in the Middle Grades

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Abstract

Over the last three decades, there has been an increasingly strong emphasis on group-centered approaches to mathematics teaching. One primary responsibility for teachers who use group-centered instruction is to “check in”, or intervene, with groups to monitor group learning and provide mathematical support when necessary. While prior research has contributed valuable insight for successful teacher interventions in mathematics group work, there is a need for more fine-grained analyses of interactions between teachers and students. In this study, we co-conducted research with an exemplary middle grade teacher (Ms. Green) to learn about fine-grained details of her intervention practices, hoping to generate knowledge about successful teacher interventions that can be expanded, replicated, and/or contradicted in other contexts. Analyzing Ms. Green’s practices as an exemplary case, we found that she used exceptionally short interventions (35 seconds on average), provided space for student dialogue, and applied four distinct strategies to support groups to make mathematical progress: (1) observing/listening before speaking; (2) using a combination of social and analytic scaffolds; (3) redirecting students to task instructions; (4) abruptly walking away. These findings imply that successful interventions may be characterized by brevity, shared dialogue between the teacher and students, and distinct (and sometimes unnatural) teaching moves.

Keywords Collaborative learning; Mathematics; Group work; Teacher intervention

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I. INTRODUCTION

Over the last three decades, there has been an increasingly strong emphasis on group-centered approaches to mathematics teaching (Baxter & Williams, 2010; Gillies, 2019; Kotsopoulos, 2014; Langer-Osuna, 2017). In group-centered mathematics instruction, students are active participants; rather than passively listening to the teacher, students work collaboratively in groups to construct knowledge by solving problems and discussing ideas with their peers. The teachers' role in such classrooms is important—they must create the conditions by which students can learn with and from their peers.

One primary responsibility for teachers who use group-centered instruction is to "check in", or intervene, with groups to monitor group learning and provide mathematical support when necessary (Dekker & Elshout-Mohr, 2004; Ding et al., 2007; Hofmann & Mercer, 2016). Intervening is an intricate instructional practice, as teachers need to carefully consider how they should support groups. When teachers provide too much mathematical support, it can lead to decreased challenge (and less learning); conversely, when teachers provide too little support, it can lead to decreased group effort (Egbert, 2003; Liljedahl, 2020). Adding further complexity to these intricacies is that teachers must make in-the-moment decisions for how to respond to students during interventions to advance students' mathematical thinking (Campbell & Yeo, 2021; Campbell & Yeo, 2022; Jacobs & Empson, 2016). For these reasons, it is important to examine how teachers can successfully intervene in group work in mathematics.

There has been a moderate amount of research exploring how teachers should intervene in group work to support groups' mathematical progress. Research suggests that teachers can invite learners to speak, listen silently to group discussion before speaking, probe student thinking, and coordinate learners' ideas during group interventions (e.g., Campbell & Yeo, 2021; Hofmann & Mercer, 2016; Webb et al., 2019). Such exploratory research is vital to the success of group-centered instruction.

While prior research has contributed valuable insight for successful teacher interventions in mathematics group work, there is a need for more fine-grained analyses. Prior research mostly examines teaching moves during group interventions (e.g. see Baxter & Williams, 2010; Dekker & Elshout-Mohr, 2004; Hofmann & Mercer, 2016); however, this research does not consider important details of intervening, such as how much time an intervention should last or how much the teacher should talk in comparison to students during an intervention. Further, prior research rarely uses a systematic process for describing the effects of specific teaching moves (e.g. do some teaching moves lead to more or less mathematical progress?). This study attends to the limitations of prior research.

In this study, we explore strategies for intervening in mathematics group work in middle school classrooms using a fine-grained scale of analysis. We analyze video data from an exemplary middle grade teacher (Ms. Green), exploring minute details of her group interventions to inform the field regarding the length of successful interventions (in seconds), how much teachers might talk in comparison to students, and the types of teaching moves that lead to mathematical progress. While case study analysis may not generalize to other settings, our research generates knowledge that can be expanded, replicated, and/or contradicted.

II. THEORETICAL BACKGROUND

Group-centered mathematics instruction is endorsed by policy experts and panels in several geographic regions (e.g., National Council of Teachers of Mathematics, 2000; Ontario Ministry of Education and Training, 2020). These experts and scholars believe that mathematics instruction should be characterized by collaboration, problem-solving, and productive struggle (e.g., Boaler, 2015; National Council of Teachers of Mathematics, 2014). In such classrooms, students actively co-contribute/construct knowledge in collaboration with peers, rather than acquire knowledge by listening to their teachers (e.g., Hunter, 2008; Munter et al., 2015). The teachers' role is to facilitate learning rather than direct it (Stein et al., 2008).

Teachers who use group-centered approaches to mathematics instruction often structure their classes in three phases: before, during, and after (e.g., Stein et al., 2008; Van de Walle et al., 2013). In the before phase, teachers launch a mathematics task by describing the task to students and providing any necessary instructions or scaffolds for getting started. The task is generally non-routine, requiring students to problem-solve and "get stuck" before arriving at a solution (Jackson et al., 2011; Yeo et al., 2022). Following the launch, teachers place students in small groups. In the during phase, teachers walk around the classroom to monitor groups by listening to group strategies, providing support, and offering extensions. Teachers support learners to construct knowledge related to the learning objectives without directly telling them the appropriate procedures. Finally, in the after phase, teachers lead a whole class discussion where groups share different strategies for completing the mathematics task. The teacher tries to meaningfully connect learners' ideas to promote understanding of the learning objectives.

In this study, we examine the during, or monitoring, phase of group-centered instruction. The during phase is perhaps the most complex phase of instruction because teachers need to make in-the-moment decisions about how to respond to learners (Campbell & Yeo, 2022; Jacobs et al., 2010; Pak, 2022). Teachers need to examine student work, interpret their understanding, and decide how to respond in a moment of time (Campbell & Yeo, 2022; Jacobs et al., 2010; Jacobs & Empson, 2016). Scholars have studied ways to reduce the complexity of the monitoring phase before the point of intervention (i.e., before teachers interact with a group of learners). This research suggests that teachers can anticipate student responses, physically organize the classroom to support equitable spatial privilege, and create ground rules for group engagement (Langer-Osuna, 2016; Mercer et al., 1999; Stein et al., 2008). These supports can lessen the demand of the monitoring phase before teachers intervene.

Another line of research examines what teachers should say and do at the point of intervention. The teacher's goal is to offer just enough support to help groups make mathematical progress without over-scaffolding and lowering the challenge of the task (Liljedahl, 2020). Research suggests that teachers can listen before responding to learners

(Hofmann & Mercer, 2016), explore the details of groups' thinking by examining their work and listening to their ideas (Campbell & Yeo, 2021; Jacobs & Empson, 2016), revoice students' ideas (Hofmann & Mercer, 2016; O'Connor & Michaels, 2019), ask students to explain other group members' ideas (Webb et al., 2019), and press students to engage with a mathematical idea through questioning (Franke et al., 2015). These teaching moves facilitate student discovery rather than offering direct mathematical support. Research also suggests that sometimes teachers need to provide direct help, or hints, to learners when they get stuck (Baxter & Williams, 2010; Liljedahl, 2020). Hints may include providing students with a strategy or focusing students' attention on an important mathematical idea. Whether teachers facilitate discovery or offer direct support, the goal is to help learners think for themselves and construct knowledge with peers.

While prior research identifies successful teaching moves at the point of intervention, there is a need for more detailed analyses of teacher interventions. Several questions remain: How long should teachers spend intervening with a group of students? How much should teachers talk in comparison to learners? What types of teaching moves lead to mathematical progress? In this study, we intricately analyze Ms. Green's interventions to consider these variables.

III. METHODS

We use case study analysis (Merriam, 1998) to examine how an exemplary middle grade teacher (Ms. Green) intervenes with groups. A case study is "an intensive, holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit" (Merriam, 1998, p. xiii). We chose Ms. Green as our case because she is a true exemplar in relation to group-centered mathematics instruction. Her teaching is consistent with research on successful group-centered instruction, and her students excel because of her teaching practices. As evidence of her success, students in Ms. Green's class completed a standardized assessment (NWEA MAP Growth Assessment[1]) at the beginning and end of the 2021-2022 school year. At the beginning of the school year, the mean score for Ms. Green's students was below the national mean grade level average. However, at the end of the school year, the mean score for Ms. Green's students was well above the national mean grade level average, indicating that Ms. Green's students outperformed most other students at the same grade level nationally despite starting below the grade level average at the start of the year. Furthermore, one of Ms. Green's students placed first in "critical thinking" at a regional mathematics competition in March 2022 (toward the end of the school year). Ms. Green's teaching was clearly influential to her students.

The purpose of this case study analysis is to explore Ms. Green's practice and generate findings that may support other practitioners to successfully intervene in mathematics group work. In what follows, we provide details regarding the participants and setting, procedures, and data analysis.

[1] Details for this assessment may be found here: https://www.nwea.org/map-growth/

Participants and Setting

The study took place at a rural K-12 school in the Midwestern United States during the 2021-2022 school year. The school serves 127 students from two small towns with a combined population of approximately 420 people. Approximately 90% of students are white, and approximately 40% of students qualify for free and reduced lunch. The participants for the study were Ms. Green, an elementary school teacher, and her sixth-grade students. Thirteen out of the 16 students in Ms. Green's class consented to participate in the study[2].

Ms. Green's Classroom. Ms. Green's teaching practices align with the transformational approach to mathematics teaching described by Peter Liljedahl in his book, *Building Thinking Classrooms in Mathematics*, Grades K-12 (Liljedahl, 2020). In this section, we describe how her "thinking classroom" (Liljedahl, 2020) operates.

As students enter the classroom each day, they are placed in random groups of three with each group having a vertical non-permanent surface, such as a whiteboard, where they work collaboratively. Each group is given a single marker to share. Using the Illustrative Mathematics Grade 6 Curriculum, students are given a warmup question to solve with their group that uses prior knowledge from previous lessons that may be needed in today's task. Once the warmup task is completed by most students, Ms. Green asks students to gather at one of the whiteboards to discuss the solution and how that group found their answer.

After consolidating the warm-up, Ms. Green spends about 5 minutes giving students the instructions for their first new task of the day (usually from the Illustrative Mathematics Curriculum). Students are sent with their groups back to their whiteboards to solve this task in any way that makes sense to them. A timer is set to beep at 2-minute increments, and students trade the marker with another group member each time the timer beeps. As students work, they are encouraged to look around the classroom at the work other students are doing on their whiteboards for ideas or to check with another group to see if their answers match. Ms. Green circulates throughout the room while students are working and observes the groups' work, intervening where needed. She may intervene by asking a group a leading question (either to clarify the work or to help students see their next steps), asking a group member to explain the groups' thinking, giving the group a "hint" or short mini lesson on a point they have misunderstood, or asking the group to check in with another group. She carries a marker of a different color than the students are using so that she can circle or annotate student work, or add to whiteboards more information that students should not erase. These interactions are kept deliberately short with a hope of giving the students just enough information to allow them to continue working on their own.

As groups complete each task, Ms. Green presents a more challenging task. As groups finish a task and feel confident with their answer, they check with other groups to see if their answers are similar or look for the next task to complete. Once all groups have completed the minimum level Ms. Green wanted to reach for the day (or it is close to the end of class), she again gathers all students at one or more of the whiteboards for a consolidation time. Using group work as examples, the students and Ms. Green discuss the different methods used to solve each task and the most efficient strategies. Following this, students are given time to create “notes to their future selves” in a notebook. They may write anything they wish but are encouraged to include vocabulary, worked examples, procedures, and any points which were new to them today or that they feel will be needed in further work. These notes can be used at any time in class as a reference.

Data Collection

To capture group interventions, Ms. Green set up video recorders across her classroom for four days during the Spring semester (January 2022-May 2022). The four days were spread out across the semester to capture a variety of mathematical content. Ms. Green used three video recorders: two recorders to capture group interactions for two focus groups and one recorder to capture the whole class setting. For the purposes of this study, we use video recordings of group interactions. In total, there were seven video recordings of different groups working together on mathematics tasks across the four days of instruction[2]. Each of the seven video recordings were approximately 35 minutes in length.

Across the seven video recordings of groups (as a unit of analysis), we identified 25 teacher interventions. We counted a teacher intervention as any stretch wherein Ms. Green spoke with a group. There were instances when Ms. Green arrived at a group and observed their work but did not speak to the group. These instances were not coded as interventions. Ms. Green’s talk was transcribed for each intervention.

Data Analysis

All authors, including Ms. Green, participated in the data analysis process. Ms. Green’s insight was vital for helping the research team interpret the data accurately. Each of the authors collaborated and discussed analytic procedures to improve the rigor of the analysis.

To analyze the data, we coded the interventions according to several indicators. First, we coded how long each intervention lasted (in seconds) to understand how much time Ms. Green spent monitoring the group. The intervention started when Ms. Green arrived at the group and ended when Ms. Green left the group. Then, we tracked the distribution of talk within the intervention using a timetable to understand how often Ms. Green spoke in relation to students during the intervention (see Figure 1). For instance, the intervention revealed in Figure 1 shows that the intervention started with a four second pause (i.e., no students were talking, and Ms. Green observed student work without speaking). Within the intervention, Ms. Green talked for a total of 10 seconds, Student 1 talked for 15 seconds, Student 2 talked for seven seconds, and five seconds were recorded as pauses.

Next, we examined groups’ conversations and written work before and after Ms. Green intervened to understand whether Ms. Green’s interventions resulted in mathematical progress. In the phase before Ms. Green’s intervention, we coded groups’ mathematical strategies according to three codes: (1) *appropriate*—appropriate strategy for the given task as defined by the Illustrative Mathematics Curriculum; (2) *inappropriate*—

[2] We lost one video recording due to technical difficulties.

inappropriate strategy for the given task as defined by the Illustrative Mathematics Curriculum; (3) *N/A*—the strategy is neither appropriate nor inappropriate (e.g. the group has not started on the task). Following Ms. Green's intervention, we examined the groups' progress on the task and coded their work according to three codes: (1) *appropriate*—appropriate strategy for the given task as defined by the Illustrative Mathematics Curriculum; (2) *approaching*—the strategy is nearing an appropriate strategy (closer than before the intervention) but is still inappropriate; (3) *inappropriate and not approaching*—the group is no closer to an appropriate strategy than they were before Ms. Green intervened. We provide an example of how we coded one group's work before and after an intervention in Table 1. See the details of the task here:
https://curriculum.illustrativemathematics.org/MS/students/1/3/12/index.html

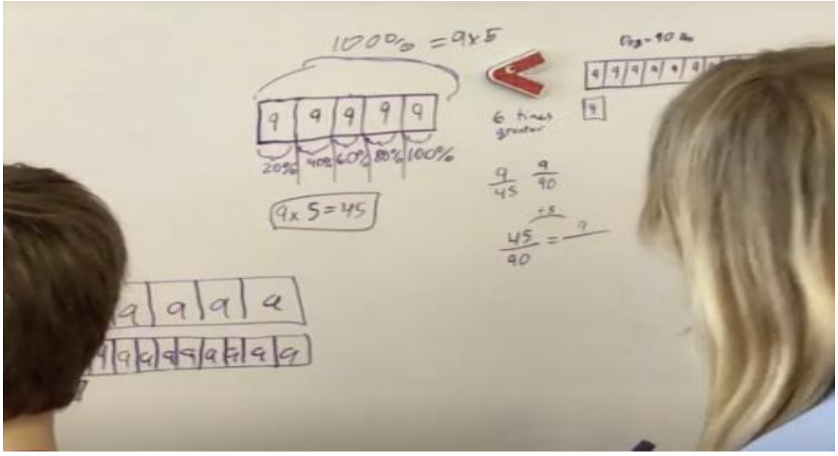
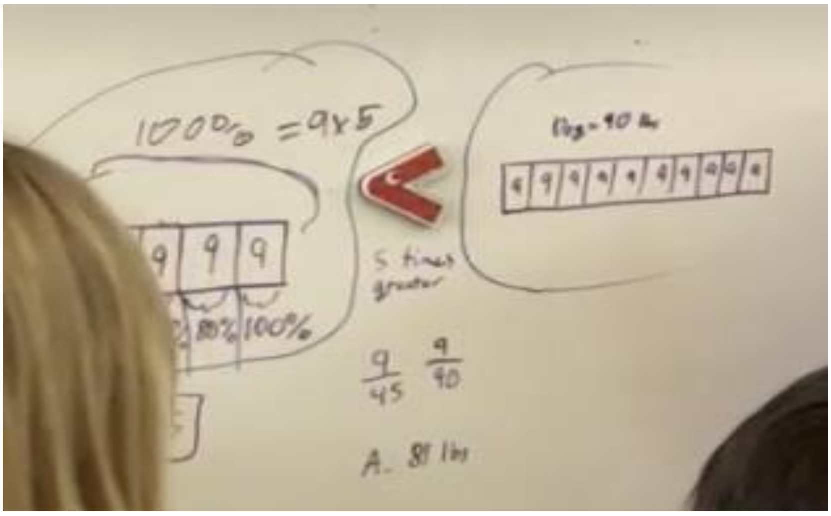
Time	Speaker
4 s	pause
3 s	Student 1
3 s	Ms. Green
3 s	Student 1
2 s	Ms. Green
9 s	Student 1
5 s	Ms. Green
7 s	Student 2
1 s	pause

Figure 1. Distribution of talk timetable.

Next, we coded Ms. Green's talk during each intervention using an open coding process to understand how she maintained the challenge of the task and simultaneously offered support to learners. We examined codes that were similar and collapsed those codes into themes, following the process of thematic analysis (Nowell et al., 2017). Themes and codes were iteratively revisited and defined.

After coding the data, we compared interventions coded as inappropriate (before intervention) with interventions coded as appropriate (before intervention) to determine whether there were differences in how Ms. Green intervened when groups used inappropriate versus appropriate strategies. We compared several indicators, including the length of intervention, distribution of talk, and proportion of students who talked during the intervention. Then, we compared interventions where groups made progress before and after the intervention (i.e., inappropriate→approaching/appropriate) with interventions where groups made no progress before and after the intervention (i.e., inappropriate→inappropriate) to determine whether there were differences in interventions when groups did and did not make progress. This comparative analysis allowed us to determine characteristics of Ms. Green's interventions that may have supported groups to achieve mathematical progress.

Table 1. Example of strategy coding scheme

Before intervention	
Code	Inappropriate
Group Written Work	
Summary of Group Interaction	The group is trying to make a multiplicative comparison between the weight of a puppy and a dog (see task in above link). They are comparing the wrong diagrams before Ms. Green's intervention. As a result, their solution (6 times greater) is incorrect (as defined by the Illustrative Mathematics Curriculum).
After Intervention	
Code	Approaching
Group Written Work	
Summary of Group Interaction	After the intervention, the group understands the task directions, and they start comparing the correct diagrams. However, their answer is still not correct (as defined by the Illustrative Mathematics curriculum). They have the correct understanding of the additive comparison, but they still have not made the correct multiplicative comparison. Therefore, their strategy is approaching.

IV. FINDINGS

In this section, we explore the characteristics of Ms. Green's interventions through quantitative and qualitative data. We start by exploring quantitative characteristics of Ms. Green's interventions. Then, we explore both quantitative and qualitative findings to examine the aspects of Ms. Green's teaching that supported groups to make mathematical progress.

Quantitative Characteristics of Ms. Green's Interventions

Ms. Green engaged in 25 distinct teacher interventions across the seven video recordings. Table 2 reveals that nine interventions were coded as "appropriate" (i.e., the group constructed an appropriate strategy before intervention), 12 were coded as "inappropriate" (i.e., the group constructed an inappropriate strategy before intervention), and 4 were coded as "N/A" (i.e., the group did not yet construct a strategy before intervention, etc.). Table 2 shows the mean and standard deviation of three characteristics of Ms. Green's interventions across the coded interventions: (1) duration—how long Ms. Green spent intervening with the group; (2) Teacher proportion (T proportion)—the proportion of talk time by Ms. Green during the intervention in comparison to the group of students; (3) Participation—a calculation of the proportion of group members who spoke at least once during the intervention.

Table 2. Means and standard deviations of Ms. Green's intervening (initial response)

	Appropriate (n=9)	Inappropriate (n=12)	N/A (n=4)	Total (n=25)
Duration (s)	25.78 (11.93)	42.18 (31.49)	34.75 (26.23)	34.16 (25.16)
T proportion (%)	55 (0.25)	66 (0.20)	67 (0.25)	61 (0.23)
Participation (%)	67 (0.34)	42 (0.22)	42 (0.17)	51 (0.27)

Note. Four groups revealed strategies coded as N/A before the intervention.

As illustrated in Table 2, Ms. Green spent very little time intervening with groups, especially if the group constructed an appropriate strategy. On average, Ms. Green spent 34.16 seconds intervening with groups across all 25 interventions. She spent just 25.78 seconds on average intervening with groups who constructed an appropriate strategy. She spent nearly twice as much time intervening with groups who constructed an inappropriate strategy (42.18 seconds), though her interventions were still short.

In regard to the proportion of talk during interventions, Ms. Green's talk contributed to 61% of the total talk time on average (meaning students talked for 39% of the time). When groups constructed appropriate strategies, Ms. Green's talk contributed to 55% of the total talk time on average. When groups constructed inappropriate strategies, Ms. Green's talk contributed to 66% of the total talk time on average. This indicates that Ms. Green talked less during an intervention when groups constructed appropriate strategies. Still, she provided room for students to talk regardless of whether their strategies were appropriate or inappropriate.

There was variation in relation to how many students participated during an intervention. On average, about half of the students (51%) in a group talked at least once during an intervention. As students usually worked in groups of three, this indicates that there was variation in relation to how many students participated. When groups constructed appropriate strategies, more students participated in the intervention (67%) than when they constructed inappropriate strategies (42%).

In summary, Ms. Green's interventions were noticeably short, and she generally spoke less and provided more room for student participation when students constructed appropriate strategies. These findings are noteworthy, providing specific implications in relation to how long teachers might intervene and how much they should talk in comparison to students.

Characteristics that Support Mathematical Progress

In this section, we start by examining quantitative findings, followed by examining thematic findings of Ms. Green's interventions. Table 3 reveals quantitative characteristics of Ms. Green's interventions depending on whether groups did or did not make mathematical progress (Inappropriate to Approaching or Appropriate vs. Inappropriate to Inappropriate). As shown in the table, nearly all groups made mathematical progress after Ms. Green intervened, with nine interventions resulting in advanced mathematical progress after the intervention ("Inappropriate" to "Approaching or Appropriate") compared to only three interventions resulting in no advancement of mathematical progress after the intervention ("Inappropriate" to "Inappropriate"). The 13 interventions coded as "Appropriate" or "N/A" before the intervention are not included in this table because they do not reveal information regarding groups' propensity to make mathematical progress after an intervention.

Table 3. Means and standard deviations of intervening of transitioning responses

	Inappropriate to Inappropriate (*n*=3)	Inappropriate to Approaching or Appropriate (*n*=9)
Duration (s)	29.67 (9.43)	43.78 (34.46)
T proportion (%)	79 (0.11)	59 (0.21)
Participation (%)	44 (0.16)	41 (0.22)

On average, Ms. Green spent longer intervening with groups who made mathematical progress (43.78 s) compared to groups who failed to make mathematical progress (29.67 s). Interestingly, Ms. Green assumed a smaller proportion of the talk time when groups made mathematical progress (0.59) compared to when they did not make mathematical progress (0.79). This might indicate Ms. Green's interventions were longer for groups who made mathematical progress because she provided more room for students to talk. The number of students who spoke at least once during an intervention (Participation) was roughly the same regardless of whether groups made mathematical progress. These findings might indicate that longer interventions that allow students more talk time supported groups to make mathematical progress.

Thematic Findings

In this section, we explore the nine interventions that led to mathematical progress ("Inappropriate to Approaching or Appropriate") to determine themes of Ms. Green's interventions that may have supported groups to make mathematical progress. These nine interventions are significant because they can provide insight into successful teaching moves during group interventions. We found four distinct strategies that Ms. Green used within interventions where groups made mathematical progress: (1) observing/listening before speaking; (2) using a combination of social and analytic scaffolds; (3) redirecting students to task instructions; (4) abruptly walking away.

The first strategy, observing/listening before speaking, refers to Ms. Green's propensity to observe student work and/or listen to the group's ideas before speaking. We noticed, when constructing the timetables, that there was often a gap before Ms. Green's first talking turn. This gap was filled by either a pause or another student speaking. Even when Ms. Green spoke first, she often opened the conversational floor for students to speak, using language such as "Talk to me." This practice aligns with other research indicating that teachers should analyze student work and interpret their understanding before responding (e.g., Campbell & Yeo, 2021; Jacobs et al., 2010; Jacobs & Empson, 2016).

The second strategy, using a combination of social and analytic scaffolding, refers to Ms. Green's propensity to use two types of teaching moves within a single intervention: moves that supported groups to collaborate productively and moves that scaffolded the mathematical content. Baxter and Williams (2010) defined social scaffolding as "support the teacher provides that helps students to learn to work with each other (p. 11) and analytic scaffolding as "support offered to students by materials, teachers, or one another, in building mathematical understanding." Ms. Green's propensity to combine these scaffolds is illustrated in the following exchange. Ms. Green intervened during an argument where students were comparing the weight of a 90-pound dog with a 9-pound dog. The task asked students to determine how many times greater the 90-pound dog was compared to the 9-pound dog.

> [Students engaged in argument]
> **Student 1:** Nine. That's nine times more greater because if it weighs nine pounds for one thing, that's 90. Nine times nine is 81.
> **Ms. Green:** OK. My next question to you would be does that match the diagram? Does that match the diagram they drew?
> **Student 2:** No.
> **Ms. Green:** OK, well Student 2 you gotta explain why it doesn't match the diagram.
> **Student 1:** Because it has the full number of how much they weigh.
> **Student 2:** Because there are ten in that… [crosstalk]… Yes, but it's 1/10 of the big dog.
> **Ms. Green:** Do you agree with her that it's 1/10 of the big dog?
> **Student 1:** But if we're comparing to the big dog side, that's 9 out of 10.

Ms. Green: It is, but they didn't ask you how many pounds greater.
They asked you how many times greater
[Ms. Green walks away, students continue arguing]

In this interaction, Student 1 reasoned that the solution was 9, while Student 2 reasoned that the answer was 10. We see in this exchange that Ms. Green provided social scaffolding twice. First, she explicitly told Student 2 to explain her reasoning to the group: "OK, well Student 2 you gotta explain why it doesn't match the diagram." Second, after Student 2's explanation, Ms. Green asked Student 1, "Do you agree with her that it's 1/10 of the big dog?". We also see in this exchange that Ms. Green provided analytic scaffolding two times through questioning and explicit prompts. First, Ms. Green asked "Does that match the diagram they drew?" to support Student 1 to understand the multiplicative comparison. Second, Ms. Green explicitly told Student 1, "It is, but they didn't ask you how many pounds greater. They asked you how many times greater." After providing this explicit prompt, Ms. Green walked away from the group. The group needed support to communicate their ideas and move forward in their mathematical understanding. Without Ms. Green's social and analytic scaffolding, the group may have become frustrated and been unable to make mathematical progress.

The third strategy, redirecting students to task instructions, refers to Ms. Green's propensity to attune students' attention to the task instructions. Oftentimes, the only support groups needed to achieve mathematical progress was to better understand the task instructions. She used prompts such as "What does the question say?" or "OK, but the question is asking you about…". These prompts provided a gentle redirection to help groups focus on the primary components of the task.

The last strategy, abruptly walking away, refers to Ms. Green's propensity to walk away from groups at what seemed to be an unnatural part of the conversation. Ms. Green's intervention closures broke the norms of polite conversation. She often simply walked away from groups after her last comment, without giving students an opportunity to respond. To an outside observer, this may appear awkward, but her students were accustomed to this norm. Liljedahl (2020) discusses "walking away" as an intervention practice that helps teachers refrain from providing too much help and thereby making the task too easy for learners. This strategy may have supported Ms. Green to refrain from providing too much help and thereby supporting learners to make mathematical progress on their own.

V. DISCUSSION

In this study, we explored how an exemplary teacher (Ms. Green) intervened in mathematics group work with middle grade students. We examined the length of her interventions, how much she talked in comparison to group members, and the strategies she used to support groups to make mathematical progress. We found that Ms. Green used very short interventions (approximately 35 seconds on average) and that she provided space for students to talk during interventions. We also found four strategies that Ms. Green used to support students to make mathematical progress: (1) observing/listening before speaking; (2) using a combination of social and analytic scaffolds; (3) redirecting students to task instructions; (4) abruptly walking away. We discuss our findings and provide implications for research and practice.

The first finding relates to the length of Ms. Green's interventions. Ms. Green intervened for just 34 seconds on average across all interventions. Her interventions were especially short for groups that constructed appropriate strategies (26 seconds). However, when groups constructed inappropriate strategies, longer interventions seemed to support groups to make mathematical progress more readily than shorter interventions. By, keeping her interventions deliberately short, Ms. Green may have been able to refrain from giving students too much support and making the task too easy. Furthermore, Ms. Green may have used longer interventions (and more support) when groups constructed inappropriate strategies to encourage them to keep exerting effort.

The second significant finding of this study relates to the space Ms. Green provided students to talk during interventions. On average, Ms. Green used 61% of the total talk time during an intervention, and about half of the students within a group spoke during each intervention on average. Furthermore, when groups made mathematical progress, Ms. Green only used 59% of the total talk time. This suggests that Ms. Green allowed group members to talk during interventions rather than dominating conversation. This finding aligns with other research suggesting that teachers need to provide space for learners to talk during an intervention (Campbell & Yeo, 2021; Hofmann & Mercer, 2016; Webb et al., 2019). Hofmann and Mercer (2016) suggested that teachers need to invite students to speak and listen silently to their explanations. Ms. Green's propensity to provide students room to share their reasoning aligns with this expectation.

The last significant finding relates to the four strategies Ms. Green used to support groups to make mathematical progress during interventions: (1) observing/listening before speaking; (2) using a combination of social and analytic scaffolds; (3) redirecting students to task instructions; (4) abruptly walking away. First, Ms. Green's propensity to observe and listen before speaking directly aligns with Hofmann and Mercer's (2016) suggestion that teachers should listen silently to students before speaking. This can allow the teacher to interpret learners' mathematical understandings (e.g. Jacobs et al., 2010). Second, Ms. Green's propensity to use social and analytic scaffolding is a point of contention for scholars. Some scholars who use group-centered instruction seem to believe that teachers should provide social scaffolds rather than using mathematical scaffolding. However, other scholars believe that sometimes it is necessary to provide analytic support (e.g. Baxter & Williams, 2010; Liljedahl, 2020). If Ms. Green had not offered specific mathematical support, groups may have stopped trying because the task became too difficult. The third practice, redirecting students to task instructions, was a unique finding that, to our knowledge, is not replicated in prior research. Oftentimes, Ms. Green only needed to redirect students to the task instructions to help them make mathematical progress. Lastly, the fourth strategy Ms. Green used (abruptly walking away) aligns with Liljedahl's (2020) suggestion that teachers walk away after offering a question to students during an intervention. This practice seemed awkward from an outsider's perspective, but it may have helped Ms. Green refrain from lingering too long with groups and removing the challenge from the task.

Implications

The findings of this study reveal several specific implications for practice. First, it may be beneficial for teachers to use seemingly short interventions (approximately 35 seconds) to support groups. When groups construct inappropriate strategies, teachers may need to marginally exceed this time limit (approximately 45 seconds). Second, it may be beneficial for teachers to provide room for students to talk during an intervention, though teachers may need to take up more of the conversational floor when groups construct inappropriate strategies. Lastly, teachers can support groups to make mathematical progress by observing/listening to their ideas before speaking, using social and analytic scaffolds, redirecting students to task instructions, and abruptly walking away. We emphasize that teachers may need to provide explicit mathematical support (analytic scaffolding) to help students achieve mathematical progress.

In relation to research, our study provides a detailed analysis of how one teacher intervened with middle grade groups. Our intricate scale of analysis allowed us to provide insight into the length of successful interventions, among other indicators. Nevertheless, more research needs to attend to the intricate details of successful interventions to better understand how teachers can navigate the complexities of intervening.

Limitations and Conclusion

There are several limitations to our study. First, we analyzed the practices of one middle grade teacher to inform the field about how teachers might successfully intervene. Our study generates theory; by definition, case study analysis cannot extrapolate to other settings. However, our study provides the grounds by which other researchers can iteratively modify, extend, replicate, and/or contradict the findings of this study. We hope future researchers take up this worthy cause. Furthermore, researchers need to explore intervention practices at different grade levels and under different conditions than were examined in this study.

This study contributes to the growing body of research on group-centered instruction. Intervening is an important component of group-centered instruction, and there is still much research to be done to learn how teachers can intervene. We hope this research compels scholars and practitioners toward further pursuit of successful intervention practices in mathematics education.

References

Baxter, J. A., & Williams, S. (2010). Social and analytic scaffolding in middle school mathematics: Managing the dilemma of telling. *Journal of Mathematics Teacher Education, 13*(1), 7-26. https://doi.org/10.1007/s10857-009-9121-4

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* John Wiley & Sons.

Campbell, T. G., & Yeo, S. (2022). Professional noticing of coordinated mathematical thinking. *British Educational Research Journal, 48*(3), 488-503. https://doi.org/10.1002/berj.3778

Campbell, T. G., Yeo, S. (2023). Exploring in-the-moment teaching moves that support sociomathematical and general social norms in dialogic instruction. *International Journal of Science and Mathematics Education, 21*, 1-23. https://doi.org/10.1007/s10763-021-10234-6

Dekker, R., & Elshout-Mohr, M. (2004). Teacher interventions aimed at mathematical level raising during collaborative learning. *Educational Studies in Mathematics, 56*(1), 39-65. https://doi.org/10.1023/B:EDUC.0000028402.10122.ff

Ding, M., Li, X., Piccolo, D., & Kulm, G. (2007). Teacher interventions in cooperative-learning mathematics classes. *The Journal of Educational Research, 100*(3), 162-175. https://doi.org/10.3200/JOER.100.3.162-175

Egbert, J. (2003). A study of flow theory in the foreign language classroom. *Modern Language Journal, 87*, 499-518. https://doi.org/10.3138/cmlr.60.5.549

Franke, M. L., Turrou, A. C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with others' mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal, 116*(1), 126-148. https://doi.org/10.1086/683174

Gillies, R. M. (2019). Promoting academically productive student dialogue during collaborative learning. *International Journal of Educational Research, 97*, 200-209. https://doi.org/10.1016/j.ijer.2017.07.014

Hofmann, R., & Mercer, N. (2016). Teacher interventions in small group work in secondary mathematics and science lessons. *Language and Education, 30*(5), 400-416. https://doi.org/10.1080/09500782.2015.1125363

Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions: Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 31– 39). Brisbane, Australia: Mathematics Education Research Group of Australasia.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L. & Shahan, E. (2011). *Investigating how setting up cognitively demanding tasks is related to opportunities to learn in middle-grades mathematics classrooms* [Paper presentation]. Research Presession of the National Council of Teachers of Mathematics Annual Meeting, Indianapolis, IN, United States.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*, 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Jacobs, V. R., & Empson, S. B. (2016). Responding to children's mathematical thinking in the moment: an emerging framework of teaching moves. *ZDM, 48*(1-2), 185-197. https://doi.org/10.1007/s11858-015-0717-0

Kotsopoulos, D. (2014). The case of Mitchell's cube: Interactive and reflexive positioning during collaborative learning in mathematics. *Mind, Culture, and Activity, 21*(1), 34-52. https://doi.org/10.1080/10749039.2013.790905

Langer-Osuna, J. M. (2016). The social construction of authority among peers and its implications for collaborative mathematics problem solving. *Mathematical Thinking and Learning, 18*(2), 107–124. https://doi.org/10.1080/10986065.2016.1148529

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics Education*, *48*(3), 237-247. https://doi.org/10.5951/jresematheduc.48.3.0237

Liljedahl, P. (2020). *Building thinking classrooms in mathematics, Grades K-12: 14 teaching practices for enhancing learning*. Corwin.

Mercer, N., Wegerif, R., & Dawes, L. (1999). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal*, *25*(1), 95–111. https://doi.org/10.1080/0141192990250107

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. Jossey-Bass.

Munter, C., Stein, M. K., & Smith, M. A. (2015). Dialogic and direct instruction: Two distinct models of mathematics instruction and the debate(s) surrounding them. *Teachers College Record, 117*(11), 1-32. https://doi.org/10.1177/016146811511701

National Council of Teachers of Mathematics/NCTM. (2000). *Principles and standards for school mathematics.* Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.* Author.

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods, 16*(1), 1-13. https://doi.org/10.1177/160940691773384

O'Connor, C., & Michaels, S. (2019). Supporting teachers in taking up productive talk moves: The long road to professional learning at scale. *International Journal of Education Research, 97*, 166-175. https://doi.org/10.1016/j.ijer.2017.11.003

Ontario Ministry of Education and Training. (2020). The Ontario curriculum Grades 1–8 mathematics, Available from https://www.dcp.edu.gov.on.ca/en/curriculum/elementary-mathematics.

Pak, B. (2022). Examining how teacher identities explain their interactions with students in small groups. *Research in mathematical Education*, *25*(2), 117-133. https://doi.org/10.7468/jksmed.2022.25.2.117

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313-340. https://doi.org/10.1080/10986060802229675

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8^{th} ed.). Pearson.

Webb, N. M., Franke, M. L., Ing, M., Turrou, A. C., Johnson, N. C., & Zimmerman, J. (2019). Teacher practices that promote productive dialogue and learning in mathematics classrooms. *International Journal of Educational Research, 97,* 176-186. https://doi.org/10.1016/j.ijer.2017.07.009

Yeo, S., Colen, J., Kwon, N. Y., Cho, H., Kim, J., & Lim, W. (2022). Development of mathematical task analytic framework: Proactive and reactive features. *Research in mathematical Education*, *25*(4), 285-309. https://doi.org/10.7468/jksmed.2022.25.4.285