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APPLICATIONS OF QUASI POWER INCREASING SEQUENCES TO INFINITE SERIES

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Abstract. In the present paper, two theorems on absolute matrix summability of infinite series are generalized for the $\varphi-|A,p_n|_k$ summability method using quasi β-power increasing sequences instead of almost increasing sequences.

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1. Introduction

Let $\sum a_n$ be a given infinite series with the partial sums (s_n). Let (p_n) be a sequence of positive numbers such that

$$P_n=\sum_{v=0}^{n} p_v \to \infty \quad as \quad n\to\infty, \quad (P_{-i}=p_{-i}=0, \quad i\geq 1).$$

Let $A=(a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of nonzero diagonal entries and let (φ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $\varphi-|A,p_n|_k$, $k\geq 1$, if (see [11])

$$\sum_{n=1}^{\infty} \varphi_n^{k-1}|A_n(s)-A_{n-1}(s)|^k<\infty,$$

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where

$$A_n(s)=\sum_{v=0}^{n} a_{nv}s_v, \quad n=0,1,...$$

For $\varphi_n = \frac{P_n}{p_n}$, $\varphi - |A,p_n|_k$ summability reduces to $|A,p_n|_k$ summability (see [21]). For $\varphi_n = n$ for all n, $\varphi - |A,p_n|_k$ summability is the same as $|A|_k$ summability (see [22]). Also, for $\varphi_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$, we get $|\bar{N},p_n|_k$ summability (see [2]).

A positive sequence (b_n) is said to be almost increasing if there exist a positive increasing sequence (c_n) and two positive constants K and M such that $Kc_n \leq b_n \leq Mc_n$ (see [1]). Obviously, every increasing sequence is almost increasing but the converse need not be true as can be seen from the example $b_n = 3^{(-1)^n} n^3$. There are some different studies on almost increasing sequences, for more details, see [4, 8, 9, 10, 13, 14, 16, 17, 18, 19]. Furthermore, in [20], the following theorems on almost increasing sequences have been obtained.

Theorem 1.1. *Let (X_n) be an almost increasing sequence, and let there be sequences (γ_n) and (λ_n) such that*

$$|\Delta\lambda_n| \leq \gamma_n, \tag{1}$$

$$\gamma_n \to 0 \quad as \quad n \to \infty, \tag{2}$$

$$\sum_{n=1}^{\infty} n|\Delta\gamma_n|X_n < \infty, \tag{3}$$

$$|\lambda_n|X_n = O(1) \quad as \qquad n \to \infty. \tag{4}$$

Let $A=(a_{nv})$ be a positive normal matrix such that

$$\bar{a}_{n0} = 1, \quad n=0,1,..., \tag{5}$$

$$a_{n-1,v} \geq a_{nv} \quad for \quad n \geq v+1, \tag{6}$$

$$a_{nn} = O\left(\frac{p_n}{P_n}\right). \tag{7}$$

If

$$\sum_{v=1}^{n} \frac{|s_v|^k}{v} = O(X_n) \quad as \quad n \to \infty, \tag{8}$$

$$\sum_{v=1}^{n} \frac{p_v}{P_v}|s_v|^k = O(X_n) \quad as \quad n \to \infty, \tag{9}$$

then the series $\sum a_n\lambda_n$ is summable $|A,p_n|_k$, $k \geq 1$.

Theorem 1.2. *Let (X_n) be an almost increasing sequence, and let the conditions (1)-(4), (9) be satisfied. Let $A = (a_{nv})$ be a positive normal matrix as in Theorem 1.1. If the conditions*

$$\sum_{v=1}^{\infty} P_v |\Delta\gamma_v| X_v < \infty, \tag{10}$$

$$\sum_{v=1}^{n} \frac{|s_v|^k}{P_v} = O(X_n) \quad as \quad n \to \infty \tag{11}$$

are satisfied, then the series $\sum a_n \lambda_n$ is summable $|A, p_n|_k$, $k \geq 1$.

2. Main Results

A positive sequence $X = (X_n)$ is said to be a quasi β-power increasing sequence if there exists a constant $K = K(\beta, X) \geq 1$ such that $Kn^\beta X_n \geq m^\beta X_m$ holds for all $n \geq m \geq 1$ (see [6]). Every almost increasing sequence is a quasi β-power increasing sequence for any nonnegative β, but the converse need not be true as can be seen by taking an example, say $X_n = n^{-\beta}$ for $\beta > 0$. A sequence (λ_n) is said to be of bounded variation, denoted by $(\lambda_n) \in \mathcal{BV}$, if $\sum_{n=1}^{\infty} |\Delta\lambda_n| = \sum_{n=1}^{\infty} |\lambda_n - \lambda_{n+1}| < \infty$. For some studies on quasi power increasing sequences, we refer to [3, 5, 12, 15]. The aim of this paper is to generalize Theorem 1.1 and Theorem 1.2 to $\varphi - |A, p_n|_k$ summability method using quasi β-power increasing sequences. Before stating our theorems, we must first introduce some further notations. Given a normal matrix $A = (a_{nv})$, two lower semimatrices $\bar{A} = (\bar{a}_{nv})$ and $\hat{A} = (\hat{a}_{nv})$ are defined as follows:

$$\bar{a}_{nv} = \sum_{i=v}^{n} a_{ni}, \quad n, v = 0, 1, ... \tag{12}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}, \quad \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}, \quad n = 1, 2, ... \tag{13}$$

and

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v \tag{14}$$

$$\bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v. \tag{15}$$

Theorem 2.1. *Let $(\lambda_n) \in \mathcal{BV}$ and $A = (a_{nv})$ be a positive normal matrix as in Theorem 1.1. Let (X_n) be a quasi β-power increasing sequence for some $0 < \beta < 1$ and $\left(\frac{\varphi_n p_n}{P_n}\right)$ be a non-increasing sequence. If the conditions (1)-(4) and*

$$\sum_{v=1}^{n} \left(\frac{\varphi_v p_v}{P_v}\right)^{k-1} \frac{|s_v|^k}{v} = O(X_n) \quad as \quad n \to \infty, \tag{16}$$

$$\sum_{v=1}^{n}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^k|s_v|^k=O(X_n)\quad as\quad n\to\infty, \tag{17}$$

are satisfied, then the series $\sum a_n\lambda_n$ *is summable* $\varphi-|A,p_n|_k$, $k\geq 1$.

Theorem 2.2. *Let* $(\lambda_n)\in\mathcal{BV}$ *and* $A=(a_{nv})$ *be a positive normal matrix as in Theorem 1.1. Let* (X_n) *be a quasi* β*-power increasing sequence for some* $0<\beta<1$ *and* $\left(\frac{\varphi_n p_n}{P_n}\right)$ *be a non-increasing sequence. If the conditions (1)-(4), (10), (17) and*

$$\sum_{v=1}^{n}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}\frac{|s_v|^k}{P_v}=O(X_n)\quad as\quad n\to\infty \tag{18}$$

are satisfied, then the series $\sum a_n\lambda_n$ *is summable* $\varphi-|A,p_n|_k$, $k\geq 1$.

Lemma 2.3. ([6]) *Let* (X_n) *be a quasi* β*-power increasing sequence for some* $0<\beta<1$. *If the conditions (2) and (3) are satisfied, then*

$$nX_n\gamma_n \;=\; O(1)\quad as\quad n\to\infty, \tag{19}$$

$$\sum_{n=1}^{\infty}X_n\gamma_n<\infty. \tag{20}$$

Lemma 2.4. ([3]) *Let* (X_n) *be a quasi* β*-power increasing sequence for some* $0<\beta<1$. *If the conditions (2) and (10) are satisfied, then*

$$P_nX_n\gamma_n \;=\; O(1)\quad as\quad n\to\infty, \tag{21}$$

$$\sum_{n=1}^{\infty}p_nX_n\gamma_n<\infty. \tag{22}$$

3. Proof of Theorem 2.1

Let (M_n) denotes A-transform of the series $\sum a_n\lambda_n$. Then, by (14) and (15), we have

$$\bar{\Delta}M_n \;=\; \sum_{v=1}^{n}\hat{a}_{nv}\lambda_v a_v.$$

By Abel's transformation, we have

$$\begin{aligned}\bar{\Delta}M_n &= \sum_{v=1}^{n-1}\Delta_v(\hat{a}_{nv}\lambda_v)\sum_{k=1}^{v}a_k+\hat{a}_{nn}\lambda_n\sum_{v=1}^{n}a_v\\ &= \sum_{v=1}^{n-1}(\hat{a}_{nv}\lambda_v-\hat{a}_{n,v+1}\lambda_{v+1}-\hat{a}_{n,v+1}\lambda_v+\hat{a}_{n,v+1}\lambda_v)s_v+a_{nn}\lambda_n s_n\\ &= \sum_{v=1}^{n-1}\Delta_v(\hat{a}_{nv})\lambda_v s_v+\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\Delta\lambda_v s_v+a_{nn}\lambda_n s_n=M_{n,1}+M_{n,2}+M_{n,3}.\end{aligned}$$

To prove Theorem 2.1, by Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty}\varphi_n^{k-1}\left|M_{n,r}\right|^k<\infty \quad for \quad r=1,2,3.$$

From Hölder's inequality, we obtain

$$\begin{aligned}
&\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid M_{n,1}\mid^k\\
\leq\ &\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|\left|s_v\right|\right)^k\\
\leq\ &\sum_{n=2}^{m+1}\varphi_n^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|^k\left|s_v\right|^k\right)\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\right)^{k-1}.
\end{aligned}$$

Here, by (13) and (12), we have

$$\begin{aligned}
\Delta_v(\hat{a}_{nv}) &= \hat{a}_{nv}-\hat{a}_{n,v+1}\\
&= \bar{a}_{nv}-\bar{a}_{n-1,v}-\bar{a}_{n,v+1}+\bar{a}_{n-1,v+1}\\
&= a_{nv}-a_{n-1,v}. \qquad (23)
\end{aligned}$$

Then, by (6), (12) and (5), we get

$$\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right| = \sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv})\leq a_{nn}. \qquad (24)$$

By using (24), (7), we have

$$\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid M_{n,1}\mid^k &= O(1)\sum_{n=2}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left(\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right|\left|\lambda_v\right|^k\left|s_v\right|^k\right)\\
&= O(1)\sum_{v=1}^{m}|\lambda_v||s_v|^k\sum_{n=v+1}^{m+1}\left(\frac{\varphi_n p_n}{P_n}\right)^{k-1}\left|\Delta_v(\hat{a}_{nv})\right|\\
&= O(1)\sum_{v=1}^{m}\left(\frac{\varphi_v p_v}{P_v}\right)^{k-1}|\lambda_v||s_v|^k\sum_{n=v+1}^{m+1}\left|\Delta_v(\hat{a}_{nv})\right|.
\end{aligned}$$

We get $\sum_{n=v+1}^{m+1}\left|\Delta_v(\hat{a}_{nv})\right|\leq a_{vv}$ by (23) and (6), then

$$\begin{aligned}
\sum_{n=2}^{m+1}\varphi_n^{k-1}\mid M_{n,1}\mid^k &= O(1)\sum_{v=1}^{m}\varphi_v^{k-1}\left(\frac{p_v}{P_v}\right)^k\left|\lambda_v\right|\left|s_v\right|^k\\
&= O(1)\sum_{v=1}^{m-1}\Delta|\lambda_v|\sum_{r=1}^{v}\varphi_r^{k-1}\left(\frac{p_r}{P_r}\right)^k\left|s_r\right|^k
\end{aligned}$$

$$
\begin{aligned}
&+ \quad O(1)|\lambda_m| \sum_{r=1}^{m} \varphi_r^{k-1} \left(\frac{p_r}{P_r}\right)^k |s_r|^k \\
&= \quad O(1) \sum_{v=1}^{m-1} \gamma_v X_v + O(1)|\lambda_m| X_m \\
&= \quad O(1) \quad as \quad m \to \infty,
\end{aligned}
$$

by Abel's transformation, and using the conditions (1), (17), (20) and (4).

Again, using Hölder's inequality, we obtain

$$
\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k &\leq \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\Delta\lambda_v||s_v| \right)^k \\
&\leq \sum_{n=2}^{m+1} \varphi_n^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\Delta\lambda_v||s_v|^k \right) \\
&\times \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}||\Delta\lambda_v| \right)^{k-1}.
\end{aligned}
$$

Now, by (13), (12), (6), we get

$$
|\hat{a}_{n,v+1}| = \bar{a}_{n,v+1} - \bar{a}_{n-1,v+1} = a_{nn} + \sum_{i=v+1}^{n-1} (a_{ni} - a_{n-1,i}) \leq a_{nn}.
$$

Then, using (7) and (1), we have

$$
\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k &= O(1) \sum_{n=2}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}|\gamma_v|s_v|^k \right) \\
&\times \left(\sum_{v=1}^{n-1} |\Delta\lambda_v| \right)^{k-1}.
\end{aligned}
$$

Here, using the fact that $(\lambda_n) \in \mathcal{BV}$, we get

$$
\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k &= O(1) \sum_{n=2}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} \left(\sum_{v=1}^{n-1} |\hat{a}_{n,v+1}|\gamma_v|s_v|^k \right) \\
&= O(1) \sum_{v=1}^{m} \gamma_v |s_v|^k \sum_{n=v+1}^{m+1} \left(\frac{\varphi_n p_n}{P_n}\right)^{k-1} |\hat{a}_{n,v+1}|.
\end{aligned}
$$

We have $\sum_{n=v+1}^{m+1} |\hat{a}_{n,v+1}| \leq 1$ by (13), (12), (5) and (6), then

$$\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k = O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v} \right)^{k-1} v\gamma_v \frac{|s_v|^k}{v}.$$

Thus,

$$\begin{aligned}
\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k &= O(1) \sum_{v=1}^{m-1} \Delta(v\gamma_v) \sum_{r=1}^{v} \left(\frac{\varphi_r p_r}{P_r} \right)^{k-1} \frac{|s_r|^k}{r} \\
& \quad + O(1) m\gamma_m \sum_{r=1}^{m} \left(\frac{\varphi_r p_r}{P_r} \right)^{k-1} \frac{|s_r|^k}{r} \\
&= O(1) \sum_{v=1}^{m-1} v|\Delta\gamma_v| X_v + O(1) \sum_{v=1}^{m-1} \gamma_{v+1} X_{v+1} \\
&+ O(1) m\gamma_m X_m = O(1) \quad as \quad m \to \infty,
\end{aligned}$$

by using Abel's transformation, and using the conditions (16), (3), (20) and (19).

Finally, we have

$$\begin{aligned}
\sum_{n=1}^{m} \varphi_n^{k-1} \mid M_{n,3} \mid^k &= \sum_{n=1}^{m} \varphi_n^{k-1} a_{nn}^k |\lambda_n|^k |s_n|^k \\
&= O(1) \sum_{n=1}^{m} \varphi_n^{k-1} \left(\frac{p_n}{P_n} \right)^k |\lambda_n| \, |s_n|^k \\
&= O(1) \quad as \quad m \to \infty,
\end{aligned}$$

as in $M_{n,1}$, thus the proof of Theorem 2.1 is completed.

4. Proof of Theorem 2.2

For the proof of Theorem 2.2, it is sufficient to show

$$\sum_{n=1}^{\infty} \varphi_n^{k-1} \mid M_{n,r} \mid^k < \infty \quad for \quad r = 2.$$

First, as in the proof of Theorem 2.1, we have

$$\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k = O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v} \right)^{k-1} \gamma_v |s_v|^k.$$

Then, by Abel's transformation, and using the conditions (18), (10), (22) and (21), we get

$$\sum_{n=2}^{m+1} \varphi_n^{k-1} \mid M_{n,2} \mid^k = O(1) \sum_{v=1}^{m} \left(\frac{\varphi_v p_v}{P_v} \right)^{k-1} P_v \gamma_v \frac{|s_v|^k}{P_v}$$

$$
\begin{aligned}
&= O(1)\sum_{v=1}^{m-1}\Delta(P_v\gamma_v)\sum_{r=1}^{v}\left(\frac{\varphi_r p_r}{P_r}\right)^{k-1}\frac{|s_r|^k}{P_r}\\
&+ O(1)P_m\gamma_m\sum_{r=1}^{m}\left(\frac{\varphi_r p_r}{P_r}\right)^{k-1}\frac{|s_r|^k}{P_r}\\
&= O(1)\sum_{v=1}^{m-1}P_v|\Delta\gamma_v|X_v + O(1)\sum_{v=1}^{m-1}p_{v+1}\gamma_{v+1}X_{v+1}\\
&+ O(1)P_m\gamma_m X_m = O(1) \quad as \quad m\to\infty.
\end{aligned}
$$

This completes the proof of Theorem 2.2.

5. Conclusions

If we take (X_n) as an almost increasing sequence and $\varphi_n = \frac{P_n}{p_n}$ in Theorem 2.1 and Theorem 2.2, the conditions (16), (17) reduce to the conditions (8), (9) and the condition (18) reduces to (11), then we get Theorem 1.1 and Theorem 1.2. Also, if we take (X_n) as an almost increasing sequence, $\varphi_n = \frac{P_n}{p_n}$ and $a_{nv} = \frac{p_v}{P_n}$ in Theorem 2.1 and Theorem 2.2, then we get two known theorems on $|\bar{N}, p_n|_k$ summability method (see [7]). In these cases, the condition $(\lambda_n) \in \mathcal{BV}$ is not needed.

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