

## **A Study on Trend Impact Analysis Based of Adaptive Neuro-Fuzzy Inference System**

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### **Abstract**

*Trend Impact Analysis is a prominent hybrid method has been used in future studies with a modified surprise-free forecast. It considers experts' perceptions about how future events may change the surprise-free forecast. It is an advanced forecasting tool used in futures studies for identifying, understanding and analyzing the consequences of unprecedented events on future trends. In this paper, we propose an advanced mechanism to generate more justifiable estimates to the probability of occurrence of an unprecedented event as a function of time with different degrees of severity using adaptive neuro-fuzzy inference system (ANFIS). The key idea of the paper is to enhance the generic process of reasoning with fuzzy logic and neural network by adding the additional step of attributes simulation, as unprecedented events do not occur all of a sudden but rather their occurrence is affected by change in the values of a set of attributes. An ANFIS approach is used to identify the occurrence and severity of an event, depending on the values of its trigger attributes.*

**Keywords:** *trigger attributes, neural network, Fuzzy Logic, ANFIS, Lorenz Model*

### **1. Introduction**

Finding the possibility of occurrence of an event given different values of some other independent variables is done by employing logistic regression [1], decision trees [2], Bayesian classification [3] and other similar models. Fuzzy logic can be used as a reasonable mechanism to generate more justifiable estimates for the occurrence possibility over time. This mechanism may properly and systematically address the problem explained above [4]. However, more and more fuzzy systems have been automatically generated using experiment data, which are not necessarily comprehensible to human beings. In addition, it is a common practice to update the fuzzy systems that are abstracted from experts using different learning methods in order to improve their performance.

Regarding TIA modeling, Agami et al. Attempted a probabilistic approach to account for the occurrence of unexpected future events, depending on the severity of the event occurrences [5]. Their approach provides three levels of shock and probability pairs, a kind of qualitative approach that deduces the severity to low,

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medium and high. The event under consideration should be justified by reference to precedent and potentially a powerful impact [6]. A list of these events can be found in informal agreements among experts, literature studies or Delphi studies [7]. The problem with this probabilistic approach is the independence of events and the uncertainty of the probability of occurrence of events. This can also lead to the loss of interpretability of fuzzy systems. As it is well known, one basic motivation to implement a fuzzy model lies in its transparency. Besides, as one important tool for data mining and knowledge discovery, the importance of interpretability cannot be overemphasized. To conduct a TIA, the architecture and learning procedure underlying ANFIS (adaptive-network-based fuzzy inference system) is presented in this paper, which is a fuzzy inference system implemented in the framework of adaptive networks. By using a hybrid learning procedure, the proposed ANFIS approach can construct an input-output mapping based on both human knowledge (in the form of fuzzy if-then rules) and stipulated input-output data pairs. As there is always a trade-off between interpretability and performance of fuzzy systems, multi-objective learning such as ANFIS is shown to be more powerful [8]. In the simulation, the ANFIS architecture is employed to model nonlinear TIA model, identify nonlinear components in a TIA system, and predict a TIA time series, all yielding desirable results. Recently, Agami et al. Proposed a method of inferring the seriousness of an unexpected event by fuzzy logic to overcome the disadvantage of this probabilistic approach. A more desirable TIA modeling method is to estimate the probability of the occurrence of an unexpected event as a function of time with a significant degree of the event inferred by the fuzzy logic [9]. In this paper, we propose an ANFIS approach, which is an adaptive fuzzy system that can more robustly represent TIA models by fuzzy logic by Agami et al. The core of the proposal is that the unexpected event does not appear suddenly, but the deviation from the base model occurs by touching the attribute values of the base event or by changing some value to reach the threshold of the property. Based on this premise, an adaptive learning process in which attribute values find order is a fundamental mechanism in the real world, so we introduce a neural network learning method for fuzzy logic access to uncertain attribute values and adaptation of related factors. Are used for the comparative experiments of the proposed model and show that they have more desirable properties.

## **2. TIA Framework and Related Models**

TIA combines the impact and event possibilities with results of the base scenario to generate a spanning tree of possible future scenarios. Based on this spanning tree, the median, 5th and 95th percentile scenarios can be computed to indicate three distinctive scenarios. Given different values of some other independent variables, logistic regression can be used to estimate the probability of occurrence of an event.

However, performing extrapolation relies strongly on the regression assumptions. The further the extrapolation goes outside the data, the more room there is for the model to fail due to differences between the assumptions and the sample data or the true values. Decision tree can be used to perform a TIA. But, the reliability of the information in the decision tree depends on feeding the precise internal and external information at the onset. Even a small change in input data can at times, cause large changes in the tree. Changing variables, excluding duplication information, or altering the sequence midway can lead to major changes and might possibly require redrawing the tree. Decision trees are also prone to errors in classification, owing to differences in perceptions and the limitations of applying statistical tools. Bayesian classification can also be used to evaluate a TIA. But, one of the most important disadvantages of this method is that it has strong feature independence assumptions. Also, a subtle issue with Bayes method is that if there is no occurrence of a class label and a certain attribute value together, then the frequency-based probability estimate will be zero. Given Bayes' conditional independence assumption, when all the probabilities are multiplied, we will get zero and this will affect the posterior probability estimate. Hence, the above methods are not adequate to handle unprecedented events. Gordon, who first introduced the TIA concept, has two major steps in TIA analysis. The first step is to find a curve that fits the time series data well, provided that no unexpected future events occur. The second step is the use of expertise to identify a set of possible future events that would impede future predictions through the current time series. For these events, experts estimate the probability of appearance as a function of expectations of time and impact. High-impact events are expected to persist in wavelengths

relatively far in the positive or negative direction [10]. In the case of Monte-Carlo simulations, the TIA algorithm combines the scenario case results from the base case with the probability of an event by impact to create a framework for future possible scenarios. The basic framework of the TIA approach is shown in Figure 2. TIA is usually done by implementing a forecasting method that can either be qualitative such as the Delphi method or quantitative. Quantitative forecasting can generally be classified into two main categories: Time-Series Analysis (Curve Fitting, Exponential Smoothing, etc.) and Explanatory or Causal Analysis (Regression Analysis, Auto Correlation Analysis, etc.). Such methods assume that information contained in historical data can be extracted, analyzed, and reduced to one or more equations that can be used to replicate historical patterns. However, they carry some serious implications: First, all information needed to forecast desired aspects of the future is contained in selective historical data. Second, forces at work in the past will continue to hold in the future. Third, the forecasts generated are surprise-free. However, as Herman Khan who is the founder of the scenario method puts it: "The most surprising future is the one which contains no surprises" TIA study is often made to combine qualitative and quantitative approaches to forecast the future. In particular, a surprise-free forecast is modified to take into account experts' perceptions about how future events may change the surprise-free forecast. Enhanced approach for TIA takes into account not only the impact of unprecedented future events' occurrences, but also the different severity degrees. This study considers the occurrence severity of unprecedented future events when generating future scenarios using TIA. For this reason, we developed an advanced TIA algorithm that generates alternative possible scenarios of the future taking into account different degrees of severity with which such event might occur. Enhanced approach for TIA Take into account not only the impact of unprecedented future events' occurrences, but also the different severity degrees.

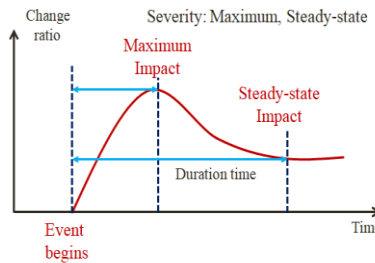


Figure 1. Event impact factors

Fuzzy logic generates more justifiable estimates to the probability of occurrence of an unprecedented event as a function of time with different degrees of severity. The core idea is to customize the generic process of reasoning with Fuzzy Logic by adding the additional step of attributes simulation, as unprecedented events do not occur all of a sudden but rather their occurrence is affected by change in the values of a set of attributes, especially when they reach certain threshold values.

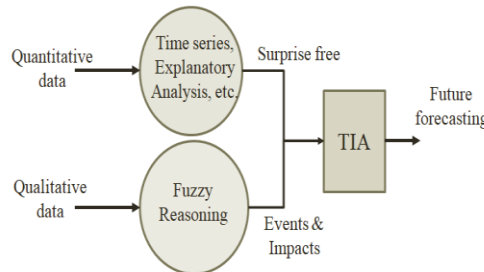


Figure 2. Basic framework of the TIA

Based on this basic framework, most TIA processes are focused on providing, as input, the probability of the occurrence of unexpected events in expert judgment. Given the values of some independent variables, logistic regression [11], decision tree [12], Bayesian classification [13], etc. can be applied to find the probability of occurrence of an event. However, these approaches are inadequate for handling unexpected events because they require historical data for model factor estimation and fitting. A more plausible study is that by Agami et al., Whose work is a qualitative TIA approach that can explain unexpected occurrences of future events, depending on the severity of the occurrences of events [5]. Their approach offers three levels of

shock and probability pairs, a qualitative approach that divides the severity into "low," "moderate," and "high". The basic process can be summarized as follows:

- 1) Generate a random number representing the degree of severity (D).
- 2) When the event and its likelihood are known, the event shock factors are classified into maximum shock, progressive shock, maximum shock point, and progressive shock point. Work on this is done through indexing the association matrix.
- 3) Create a month (M) for the year (Y) in relation to when the event occurred. However, it is assumed that only one event occurs in a given year.
- 4) Calculate the rate change vector using the estimated event impact factor.
- 5) Update the column 'S' corresponding to the current scenario in the scenario matrix.

This approach can improve the prediction process by applying fuzzy logic by Agami et al. Or dynamic prediction through neural networks [14]. In the next chapter, the ANFIS approach, which combines neural networks and fuzzy logic, provides TIA modeling for fuzzy logic that simply sets the severity of events as a function of time, and for generating more reasonable estimates that optimize function parameters into neural network learning. Suggest a way. The temporal patterns were defined by rigid regions that were hard to adjust when there is noise in phase space. It often generated false-positive prediction. It also incurred high computational complexity and lacked stability. Finally, it can not take into account the inter-dependencies between the occurrences of events and how would this affect their estimated values, say, in conducting a cross impact m.

### 3. TIA Algorithm by ANFIS Approach

The TIA process begins by identifying a list of unexpected events. Identify relevant attributes for these events and set a range of fuzzy values for corresponding features for each attribute. You can then proceed to adaptive neural network learning at each point in time to determine the severity of the event. The identification of attribute values uses one stochastic dynamic model. The fuzzy set type for attribute values can be determined based on expert judgment or any threshold value handled in the literature. Table 1 shows the default input for the TIA. The TIA output is a single scenario matrix, with each initial column filled with a base prediction vector (ie, generated by quantitative prediction in an impact-independent scenario).

**Table 1. Input type and Dimension**

Input	Type	Dimension
Number of Scenarios	Scalar	
Number of years	Scalar	
Number of events	Scalar	
Default Prediction Vector	Vector	
maximum impact matrix	Matrix	Number of events×3
steady state impact matrix	Matrix	Number of events×3
Time to Maximum Impact	Matrix	Number of events×3
Time to steady state impact	Matrix	Number of events×3

Assume a Takagi-Sugeno fuzzy system with  $m$  attributes ( $x_1, x_2, \dots, x_m$ ) and one output  $f$ . The fuzzy membership function of the attributes of attribute  $i$  is as follows, and the corresponding ANFIS structure can be represented as shown in Figure 3.

$R_k$ : if  $x_1$  is  $A_{i1}$  and  $x_2$  is  $A_{i2}$  and ... and  $x_m$  is  $A_{im}$  then

$$t_k = p_{k1}x_1 + p_{k2}x_2 + \dots + p_{km}x_m + c_k \quad (1)$$

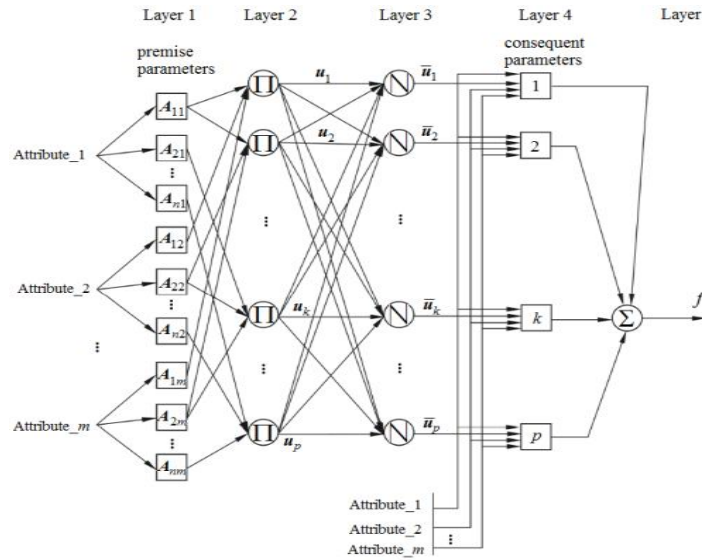


Figure 3.  $m$ -input ANFIS( $p$ 'S rules)

The output for layer 1 in Figure 3 is the degree of fuzzy membership of the input attribute. When using the bell-shaped membership function, the degree of belonging is as follows.

$$\mu_{A_{ij}(x_j)} = \left[ 1 + \left( \frac{x_j - a_{ij}}{c_{ij}} \right)^{2b_{ij}} \right]^{-1}, \quad i = 1, \dots, n; j = 1, \dots, m \quad (2)$$

Where  $a_{ij}$  and  $b_{ij}$  are the arguments for the membership function. All nodes in the second layer are fixed nodes, so the output of a node can be expressed as

$$u_k = \prod_{j=1}^m \mu_{A_{ij}}(x_j) \quad (3)$$

Where  $\Pi$  represents T-norm. The output of each fixed node with the label  $N$  in the third layer is

$$\bar{u}_i = u_i / \sum_{i=1}^p u_i \quad (4)$$

The output of the fourth layer and the output of the last layer are as follows.

$$\bar{u}_i t_i = \bar{u}_i \left( \sum_{j=1}^m p_{ij} x_j + c_i \right) \quad (5)$$

$$f = \sum_{i=1}^p \bar{u}_i t_i = \sum_{i=1}^p u_i \left( \sum_{j=1}^m p_{ij} x_j + c_i \right) / \sum_{i=1}^p u_i \quad (6)$$

With respect to these ANFIS structures, mixed learning algorithms may be applied to renew the factors, conditional factors and adjustments to apply gradient reduction methods, and the renewal of the conclusion factors can be applied with minimum square method. The following are the steps for TIA algorithms using

these AFIS modules.

- 1) Randomly generates the month the event takes place.
- 2) Randomly generate a seed of a probabilistic model (seed is required to generate different random numbers each time using a probabilistic dynamic model).
- 3) The probabilistic dynamic model is performed to determine the attribute value in the event month of a specific year.
- 4) The proposed ANFIS module is used to determine the severity of the event.
- 5) The indexing of the relevant matrix identifies the corresponding event impact factors (maximum impact, time to steady state impact, time to full impact, time to steady state impact).
- 6) Calculate the ratio vector of impact changes.
- 7) In the scenario matrix, update the column corresponding to the current scenario.

#### 4. Simulation

The main advantage of a neural fuzzy network is its ability to model the characteristics of a forecasting problem using a high-level linguistic model instead of low-level complex mathematical expressions. Moreover, the embedded fuzzy system in a neural fuzzy network can self-adjust the parameters of the fuzzy rules using neural network learning algorithms to achieve the desired results.

Our proposal is essentially fuzzy TIA systems with self-tuning capabilities and requires an initial rule base to be specified prior to training. TIA begins usually by identifying the unforeseen event list. then, an adaptive learning is proceeding at each time to determine the event severity and A dynamic model can be used to identify the attributes. Fuzzy sets of the attributes are determined by the expert judgment or certain threshold values based on literatures. For this ANFIS architecture, the parameters are updated by a hybrid learning algorithm. Gradient descent method is applied to the conditional parameters  $a_{ij}$ ,  $b_{ij}$  and  $c_{ij}$ . The gradient descent calculates errors (the derivative of the squared error with respect to each node's output) recursively from the output layer backward to the input nodes. TIA steps using this ANFIS architecture are the following:

- 1) Randomly generate the month on which the event could occur.
- 2) Randomly generate a seed for the stochastic model. Using a stochastic dynamic model, different seed is needed to generate a different sequence of random numbers each time.
- 3) Evaluate the stochastic dynamic model to determine the attribute value at any specific time, and determines the event degree of severity using the ANFIS module.
- 4) By indexing the associated matrices, identify the corresponding event impact parameters such as maximum impact, steady-state impact, time to maximum impact and time to steady-state impact.
- 5) Find the fractional change vector, and updates the current scenario.

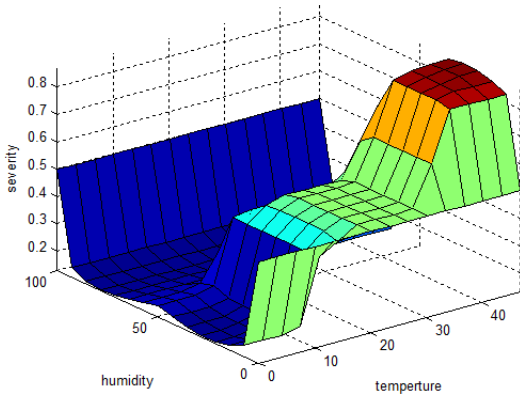
TIA begins usually by identifying the unforeseen event list. It distinguishes the related attributes about these events, and sets the range of fuzzy values for the corresponding features. Then, an adaptive learning is proceeding at each time to determine the event severity. A dynamic model can be used to identify the attributes. Fuzzy sets of the attributes are determined by the expert judgment or certain threshold values based on literatures. Output of TIA is a scenario matrix each whose column is initially filled up basic prediction vector (say, some impacts free scenario by the quantitative prediction methods). We assume that Takagi-Sugeno (TS) fuzzy system has  $m$  attributes,  $x_1, x_2, \dots, x_m$  and an output  $f$ . When fuzzy membership function of attribute  $i$  is  $A_{ji}$ ,  $j=1, 2, \dots, n$ , a TS fuzzy model has the following form of fuzzy rules.

R<sub>k</sub>: if  $x_1$  is  $A_{i1}$  and  $x_2$  is  $A_{i2}$  and... and  $x_m$  is  $A_{im}$  then,  $u_k = p_{k1}x_1 + p_{k2}x_2 + \dots + p_{km}x_m + c_k$

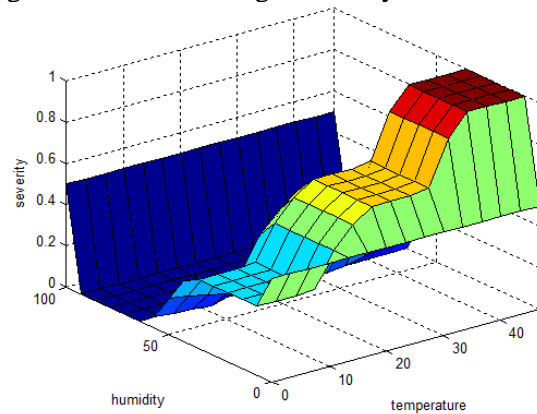
For this ANFIS architecture, gradient descent method is applied to the conditional parameters  $a_{ij}$ ,  $b_{ij}$  and  $c_{ij}$ . Randomly generate the time on which the event could occur and Apply a stochastic dynamic model.

Evaluate the stochastic dynamic model to determine the attribute value at any specific time, and determines the event degree of severity using the ANFIS module. By indexing the associated matrices, identify the corresponding event impact parameters such as maximum impact, steady-state impact, time to maximum impact and time to steady-state impact.

Find the fractional change vector, and updates the current scenario. how would a possible ‘Drought at a River Basin’ (unprecedented event) affect the annual flow of water into a certain lake. In the analysis, 100,000 scenarios are generated and the study is for fifteen years ahead, i.e. from 2010 to 2025. They assumed that surprise-free values for fifteen years ahead, i.e. 180 months, are available. The probabilistic dynamic model associated with the rainfall scenario they applied is the Lorenz model associated with climate forecasting, which represents the highly sensitive butterfly effect of the initial conditions [15]. Figure 4 shows a surface of severity degree using four fuzzy rules based on expert view. The severity is increasing in case of average temperature 30 degree above and average humidity 25.



**Figure 4. Seriousness of the incident surface proposed of the fuzzy inference reasoning**



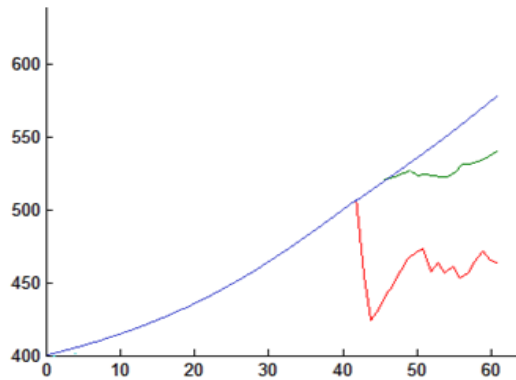
**Figure 5. Inference surface of the approach ANFIS**

Figure 5 shows an estimated surface with the suggested ANFIS method, in which the severity degree of the event represents more detail than simple fuzzy inference.

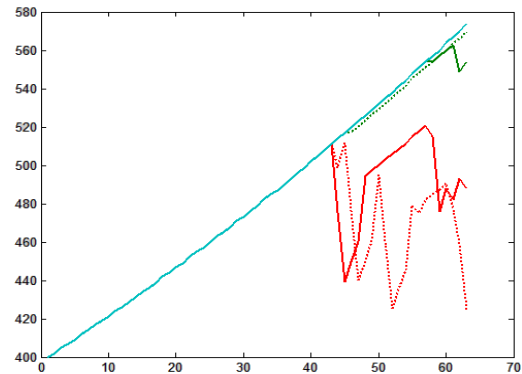
Next figure represents a curve improved on the existing TIA process by allowing the expert to supply some levels of impact and probability pairs.

- 1) IF average temperature is low and average humidity is dry THEN severity is low.
- 2) IF average temperature is medium and average humidity is dry THEN severity is medium.
- 3) IF average humidity is wet THEN severity is NON.
- 4) IF average temperature is high and average humidity is dry THEN severity is high.

The stochastic dynamic model they applied is a demo model inspired by Lorenz's model of weather prediction, which exhibits the sensitive dependence on initial conditions. The probabilistic element introduced to this model is a small perturbation in the initial condition of a certain state variable. These very small perturbations in the initial condition produce large variations in the long term behavior of the model. The results were represented as negative impacts (high -0.1875, medium -0.095, low impact -0.075) in basic prediction vector. The steady-state impact was also negative (high -0.1, medium -0.05, low -0.0375). Time to maximum impact was estimated as high 2 years, medium 4 years and low 6 years. Time to steady-state impact was high 6 years, medium 10 years, and low 12 years. Figure 4 and Figure 5 shows an estimated surface with the suggested ANFIS method, in which the severity degree of the event represents more detail than simple fuzzy inference.



**Figure 6. The quantitative forecasting case**



**Figure 7. Compare the impact of the approach**

Figure 6 and Figure 7 denotes a radical departure from basic prediction vector (sky-blue) because of the unprecedented event incorporated in the analysis. Red dot line is a radical departure in using simple fuzzy logic, and red bold line is a departure due to the suggested ANFIS method. We used 100,000 scenarios which differ based on the event timing and severity of occurrence. We cannot plot all the 100,000 scenarios and thus, we present three representative scenarios: the 90th percentile (5.1 hundred million m<sup>3</sup>) representing the best case, the 50th percentile (4.85 hundred million m<sup>3</sup>) representing the most likely to happen and the 10th percentile (4.7 hundred million m<sup>3</sup>) representing the worst case. In case of applying future scenario in current time 40 (in months), red line shows an impact due to the worst drought. According to simple fuzzy reasoning, there exists a maximum impact in 46 months, and are gradually recovered. However, prolonged draughts give rise to maximum impact again. In applying ANFIS approach, a maximum impact is appeared in 44 months, and gradually recovered. Similarly, prolonged draughts give rise to maximum impact again (in 59 months). However, the impact level is cut in half in comparison with simple fuzzy logic. In case of minimum draughts, both methods are almost indifference, come close to basic prediction vector. But, if normal draughts are prolonged, the impact begins in 56 months, and maximum impact occurs in 60 months in case of ANFIS method. According to this study, severity reasoning by simple fuzzy logic responds sensitively to the worst case, and repeats a periodic property, while ANFIS approach represents more adaptive effect about the future impact

## 5. Conclusions

Trend Impact Analysis (TIA) is a new tool used for identifying, understanding and analyzing the consequences of unprecedented events on future trends. Fuzzy Logic is a good approach for reasoning with uncertainty which is tolerant for imprecise information. However, the disadvantage of fuzzy logic is the lack of self-learning capability. The combination of fuzzy logic and neural network can overcome the disadvantages of fuzzy logic. In particular, ANFIS is combined both the learning capabilities of a neural network and reasoning capabilities of fuzzy logic in order to give enhanced prediction capabilities. In this paper, we improved the TIA algorithm based on simple fuzzy logic by applying the ANFIS. The proposed method generates more adaptive estimates of TIA parameters as a function of the related attributes for occurrence possibility of the events. In particular, there is a need for an approach to adjust the trend curve by quantifying uncertainty values in existing prediction vectors. In this regard, the TIA approach by fuzzy logic is a very desirable approach in that it can quantify inaccurate attribute values. In this study, we improved the TIA analysis method in simple fuzzy logic by introducing the ANFIS approach to generate more adaptive estimates of event occurrence probabilities as projected functions of relevant attribute values. Simulations have shown that the ANFIS approach is more adaptable to future impact predictions.



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