Research Paper
J. Astron. Space Sci. 40(4), 247-258 (2023)
https://doi.org/10.5140/JASS.2023.40.4.247

JASS Journal of Astronomy and Space Sciences

Effect of Radiation Pressure Formed at the Inner Region of the Accretion Disk on the Accretion Flow in the Outer Region

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Studying the accretion phenomena provides a window into understanding most heavenly bodies, from the birth of stars to active galactic nuclei (AGN). We would adopt the effect of the radiation pressure, which reduces accretion rates ($\dot{M}$), on the accretion phenomena. The Shakura-Sunyaev α-disk model of disk accretion is a good candidate theory of advection dominated accretion flow (ADAF). Reduction in the angular velocity leads to the suppression the disk luminosity and surface temperature, essentially indicating the transition of the standard accretion disk model from convection dominated accretion flow (CDAF) to ADAF.

Keywords: advection dominated accretion flow (ADAF), convection dominated accretion flow (CDAF), accretion disk, active galactic nuclei (AGN)

1. INTRODUCTION

The accretion phenomena in astrophysics, namely, the gravitational capture of ambient matter by compact objects [black hole (BH), neutron star (NS), white dwarf (WD), etc.] on a stellar scale or the accretion flow into super massive BHs on a galactic scale are of great interest to both the theoretical astrophysics and observational astronomy communities. Notably, when falling through the steep gravitational potential of a BH or a NS, roughly 10% of the accreted rest-mass energy is converted into radiation. In particular, accretion phenomena involve both the gas fluid inflow and radiation outflow at nearly equal magnitudes. Until now, however, the standard α-disk accretion model of Shakura-Sunyaev (Shakura & Sunyaev 1973) or Novikov-Thorne (Novikov & Thorne 1973) does not involve the effect of radiation on the disk accretion phenomena in a systematic manner; thus, the α-disk accretion model has been proposed, namely, convection dominated accretion flow (CDAF), as the cooling of accreting gas is efficient. Employing the usual hydrodynamics results in the familiar physical observable heavenly bodies which associated with this CDAF model for disk accretion onto various types of systems, such as:

- Young stellar objects (YSOs)
- Cataclysmic variables (CVs)
- X-ray binaries (XBs)
- Active galactic nuclei (AGN)

The CDAF model provides the total radiation flux,

$$D_C(r) = \frac{3GM\dot{M}}{4\pi r^3}\left[1-\beta\left(\frac{r_*}{r}\right)^{1/2}\right], \tag{1}$$

disk luminosity,

$$L_c = \int_{r_1}^{\infty} 4D_c(r)\pi r \mathrm{d}r = \left(\frac{3}{2}-\beta\right)\frac{GM\dot{M}}{r_1} \tag{2}$$

and the surface temperature

Received 30 OCT 2023 **Revised** 13 NOV 2023 **Accepted** 13 NOV 2023
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$$T_s(r) = \frac{3GM\dot{M}}{2\pi b r^3}\left[1 - \beta\left(\frac{r_1}{r}\right)^{1/2}\right]^{1/4}, \qquad (3)$$

where $\dot{M}$ denotes the accretion rate and $|\beta| \le 1$, and r_1 denotes the inner edge of the disk and $2Dc(r) = \frac{1}{2}bT_s^4(r)$. However, recent observations of some of these systems have revealed puzzling anomaly, namely, considerably lower luminosity and surface temperature than those of standard disks associated with the aforementioned CDAF model. To interpret or understand this unexpected anomalous feature of accretion disks, new underlying models have been suggested for accretion flow. Among them, the most notable is the advection dominated accretion flow (ADAF) (Narayan & Yi 1994, 1995a, b; Abramowicz et al. 1995; Abramowicz et al. 1996; Narayan et al. 1996; 1997a, 1997b; Abramowicz et al. 1997; Manmoto et al. 1997; Gammie & Popham 1998; Manmoto 2000), which is briefly summarized as follows. Considerably lower luminosity and surface temperature of some disks can be attributed to the inefficient cooling of the disk, which in turn could be attributed to the transfer of the bulk of the liberated thermal energy by the accreting gas as entropy increase rather than as outgoing radiation. Consequently, the flow becomes even quasi-spherical. However, thus far, this model lacks its own first principle or supporting arguments, despite the active follow-up publications. Therefore, in our study, we propose the first principle underlying the ADAF model. We state that by the systematic inclusion of the radiation pressure on the accretion flow, all the anomalous features of the ADAF can be reproduced.

2. MODIFICATION OF THE "SHAKURA-SUNYAEV α-DISK MODEL" THROUGH THE INCLUSION OF THE RADIATION PRESSURE IN THE ABSENCE OF THE RADIATION FORCE

$$G\frac{m_p M}{r^2} = m_p a_c = m_p r\Omega_c^2, \qquad (4)$$

$$\Omega_c = \left(\frac{GM}{r}\right)^{1/2}. \qquad (5)$$

where G is Newton's gravitational constant; m_p is the mass of the test particle near the central star or BH; M is the mass of the central star or BH; r is the radial distance of m_p and M; a_c is the radial acceleration; and Ω_c is the angular speed of the test particle in the absence of radiation pressure. Conversely, in the presence of the radiation force,

$$G\frac{m_p M}{r^2} - \frac{I}{c}\sigma_T = m_p r\Omega_A^2, \qquad (6)$$

$$G\frac{m_p M}{r^2} - \frac{L}{c4\pi r^2}\sigma_T = m_p r\Omega_A^2, \qquad (7)$$

$$\Omega_A = \left(\frac{1}{r^3}\left\{GM - \frac{\sigma_T}{4\pi m_p c}L\right\}\right)^{1/2}. \qquad (8)$$

where I is the intensity of the central star or BH; c is the speed of light; σ_T is the Thompson scattering cross section; L is the luminosity of the central star or BH; and Ω_A is the angular velocity of the test particle in the presence of radiation force. In particular, the effect of the radiation pressure on accretion flow is the reduction in the angular velocity of accreting matter and eventual suppression of the total radiation flux, disk luminosity, and surface temperature. In particular, in the absence of the radiation force,

$$D_c(r) = \frac{3GM\dot{M}}{4\pi r^3}\left(1 - \left(\frac{r_*}{r}\right)^{1/2}\right) = 2\sigma T_s^4(r), \qquad (9)$$

$$L_c = \int_{r_*}^{\infty} 2\pi r D_c(r)\,dr = \frac{GM\dot{M}}{2r_*}. \qquad (10)$$

representing the total radiation flux and disk Luminosity, respectively.

$$T_s(r) = \left(\frac{3GM\dot{M}}{8\pi G r^3}\left(1 - \left(\frac{r_*}{r}\right)^{1/2}\right)\right)^{1/4}, \qquad (11)$$

where $D_c(r)$ is the total radiation flux; L_c is the disk luminosity; $T_s(r)$ is the surface temperature in the absence of the radiation force; r_* is the inner radius of the disk; and ω is the surface mass density of the disk in the absence of the radiation force.

In the presence of the radiation force,

$$D_A(r) = \frac{3\dot{M}(GM - L\sigma_T)/4\pi m_p c}{4\pi r^3}\left(1 - \left(\frac{r_*}{r}\right)^{1/2}\right) = 2\sigma T_s^4(r) \qquad (12)$$

$$L_A = \int_{r_*}^{\infty} 2\pi r D_A(r) = \dot{M}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right) \qquad (13)$$

$$T_{sA}(r)=\left(\frac{3\dot{M}\left(GM-L\sigma_T\right)/4\pi m_p c}{8\pi\sigma r^2}\right)\left(1-\left(\frac{r_*}{r}\right)\right)^{1/4} \quad (14)$$

where L denotes the luminosity of the central object and all parameters with the subscript A denote the corresponding quantities in the presence of the radiation. In particular, all of them get suppressed by the reduction in the angular velocity Ω(r) of accreting matter; through renormalization,

$$GM \rightarrow G\tilde{M} \equiv GM-\frac{\sigma_T}{4\pi m_p c}L, \quad (15)$$

Notably, as the rotation angular velocity $\Omega_A(r)$, total radiation flux $D_A(r)$, and disk luminosity L_A should be positive and the surface temperature should be real, the luminosity of the central star should be lower than the Eddington luminosity,

$$0<L\lesssim\frac{4\pi GMm_p c}{\sigma_T}=L_{Edd} \quad (16)$$

In the present approach for modifying the standard accretion disk model proposed by Shakura & Sunyaev (1973) and at the Eddington luminosity L_{Edd}, the disk-type accretion process terminates, and the accretion flow transitions from the disk accretion model of Shakura & Sunyaevy (1973) to the spherical, adiabatic, and hydrodynamic accretion proposed by Bondi (1952). In particular, considering the interpretation of the present description as the new candidate theory of the ADAF model, as the luminosity of the central star approaches the Eddington's critical value L_{Edd}, the nature or characteristic of ADAF becomes more "extremal" as both the disk luminosity and surface temperature drop to zero, L_A, $T_A(r) \rightarrow 0$.

In particular, in contrast to the existing conventional ADAF model, our method for the new candidate theory of the ADAF model sensitively responds to the luminosity of the central star. Therefore, this last point can be employed as an "observational probe" to distinguish between our newly proposed candidate theory of ADAF and existing conventional ADAF models, such as that of Narayan & Yi. For example, in existing conventional ADAF models, the inner part of the accretion flow is maintained all the way, provided the central star has low, sub-Eddington luminosity. The essence of our study and its claim is that the present modification of the Shakura-Sunyaev α-disk model of disk accretion can be regarded as a new candidate theory of ADAF: reduction in the angular velocity of accreting matter suppresses the disk luminosity and surface temperature, essentially indicating the transition of the standard accretion disk model from CDAF to ADAF.

3. DETAILED COMPUTATIONS IN TERMS OF HYDRODYNAMICS

3.1 Equations of the Radial Structure

In general, accretion is the process by which compact objects (BH, NS, and WD) or massive stars gravitationally capture ambient matter. Here, we are particularly interested in the disk-type accretion of gas onto compact stars of mass $M\backslash simM_{\odot}$ in BH XBs and onto super massive BHs with $M\simeq10^6\sim10^9M_{\odot}$ in (AGN), which are the most likely sources of energy radiation in the observed rapidly varying emission at high luminosity. Notably, the accretion process onto compact or massive stars is worthy of investigation because when falling through the steep gravitational potential abyss, roughly 10% of the rest-mass energy of accreting matter may be converted into the radiation. In particular, accretion is a process that is considerably more efficient as a cosmic energy source than any other mechanisms in astrophysics, such as nuclear fusion. In practice, we consider the "steady (vanishing partial time derivative)" accretion of ambient fluid into central stars. We also assume that the fluid would be adiabatic in the first approximation, treating the entropy loss owing to radiation as a small perturbation. Subsequently, the fluid is characterized by an equation of state with the adiabatic index Γ; $p = \kappa\rho^{\Gamma}$, and thus, the speed of sound is expressed as

$$c_s^2=\frac{dp}{d\rho}=\Gamma\frac{p}{\rho}. \quad (17)$$

According to our findings, in the absence of the radiation force, the accretion disk is clearly represented by the Keplerian disk, wherein the angular velocity (of the fluid element) is expressed as

$$\Omega_c(r)=\left(\frac{GM}{r^3}\right)^{1/2}, \quad (18)$$

and its angular momentum is expressed as

$$\widetilde{L_e}=r^2\Omega_c=\left(GMr\right)^{1/2}=\frac{J_c}{M}. \quad (19)$$

Meanwhile, in the presence of the radiation force, the accretion disk is still represented by the quasi-Keplerian disk, wherein the angular velocity is expressed as

$$\Omega_A(r) = \left(\frac{1}{3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\right)^{1/2}, \quad (20)$$

whereas its angular momentum is expressed as

$$\widetilde{L_A} = r^2\Omega_A = \left(GMr - \frac{\sigma_T}{4\pi m_p c}Lr\right)^{1/2} = \frac{J_A}{M}, \quad (21)$$

namely, in both cases, the disk comprises shear flows, with velocity gradients as follows

$$\Omega(r) > \Omega(r + dr), \quad (22)$$

$$\tilde{L}(r) < \tilde{L}(r + dr). \quad (23)$$

Therefore, accretion through successive Keplerian orbits in front of the central compact or massive star is only possible if the gas (i.e., the fluid element) constantly loses angular momentum (and is transported outward) owing to some viscous torque.

3.1.1 Viscous Stress and Torque

Therefore, to eventually express this viscous torque, we consider the viscous force (viscous stress, which would directly be proportional to the shear force or shear stress).

$$t_{r\phi} = -2\eta\sigma_{r\phi} \quad (24)$$

where $t_{r\phi}$ and $\sigma_{r\phi}$ denote the viscous stress and shear stress, respectively, and η(g/cm ⋯) denotes the coefficient of dynamic viscosity. As the shear stress originates from the angular velocity gradient, it is expressed as

$$\sigma_{r\phi} = r\frac{d\Omega}{dr} = -\frac{3}{2}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)^{1/2} r^{-3/2} = -\frac{3}{2}\Omega_A(r), \quad (25)$$

and hence,

$$t_{r\phi} = -\eta\sigma_{r\phi} = \frac{3}{2}\eta\left(\frac{1}{r^3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\right)^{1/2}. \quad (26)$$

Namely, viscous stress $t_{r\phi}$ represents the viscous torque per unit area (around the circumference of the disk layer) exerted in the ϕ-direction by fluid elements at *r* on neighboring elements at *r+dr*. Finally, the expression for the viscous torque is as follows

$$\text{viscous torque} = (\text{viscous tress}) \times (\text{area}) \times r \quad (27)$$

namely,

$$\text{viscous torque} = \dot{J}^+ - \dot{J}^- = 4\pi r^2 h t_{r\phi} \quad (28)$$

$$t_{r\phi} 4\pi h r_I^2 = \dot{M}\left[\left(GMr - \frac{\sigma_T}{4\pi m_p c}Lr\right)^{1/2} - \beta\left(GMr_I - \frac{\sigma_T}{4\pi m_p c}r_I\right)^{1/2}\right] \quad (29)$$

and thus,

$$4\pi r_I^2 h t_{r\phi} = (1-\beta)\dot{M}\left(GMr_I - \frac{\sigma_T}{4\pi m_p c}Lr_I\right)^{1/2}, \quad (30)$$

at the inner edge of the disk, $r = r_I$

3.1.1.1 Note 1

Notably, in the standard Shakura-Sunyaev model, (β = 1) represents the “zero-torque” inner boundary condition.

3.1.1.2 Note 2

The practical expression for the viscous stress is as follows

$$t_{r\phi} = \frac{\dot{M}}{4\pi h}\left(\frac{1}{r^3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\right)^{1/2} \times \left(1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right) \quad (31)$$
$$= \frac{\dot{M}\Omega_A}{4\pi h}\left(1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right)$$

Eq. (30) suggests that under a steady-state, the viscous stress $t_{r\phi}$ is only determined by the accretion rate $\dot{M}$, mass M, and luminosity L of the central compact or massive star.

3.1.1.3 Note 3

Furthermore, the derivation for the expression of the aforementioned viscous torque based on the “angular momentum conservation” is described below. As mentioned previously for the case where the effect of radiation pressure on the accretion process is included, the inward rate of angular momentum transport across radius *r* in the disk owing to the inflowing fluid is expressed as

$$\dot{J}^{+} = \dot{M}\tilde{L} = \dot{M}\left(GMr - \frac{\sigma_T}{4\pi m_p c}Lr\right)^{1/2}. \quad (32)$$

Subsequently, if using $\dot{J}^{-}$, namely, the rate at which the angular momentum is deposited into (last down to) the central compact or massive star through the inner edge of the disk, r = r_I is expressed as

$$\dot{J}^{-} = \beta\dot{M}\left(GMr_I - \frac{\sigma_T}{4\pi m_p c}Lr_I\right)^{1/2} \quad (33)$$

for some $|\beta| \le 1$, as the specific angular momentum deposited into the central star cannot exceed the value $\tilde{L}$ at the inner edge of the disk. Lastly, the mass accretion rate $\dot{M}$ based on the "rest mass conservation law" is expressed as follows (i.e., the continuity equation):

$$\frac{\partial\rho_0}{\partial t} + \nabla\cdot(\rho_o\vec{u}) = 0, \quad (34)$$

which upon integration,

$$-\frac{\partial\rho_0}{\partial t}\int d^3x\rho_0 = \int d^3x\nabla\cdot(\rho_0\vec{u}) \quad (35)$$

$$-\frac{dM}{dt} = \int_{r_I}^{\infty}dr\int_0^{2\pi}rd\phi\int_{-h}^{h}dz\nabla\cdot(\rho_o\vec{u}) \quad (36)$$

yields Eq. (36);

$$\dot{M} = 2\pi r v_r \Sigma, \quad (37)$$

where $\Sigma = \int_{-h}^{h} dz\rho_0$ is the surface mass density.

3.1.2 Total Radiation Flux (or Surface Emissivity) and Disk Luminosity

Subsequently, we consider "energy conservation" within the context of standard hydrodynamics. In particular, the heat (or entropy) generated in the accretion disk by viscosity is expressed as (per unit area)

$$\begin{aligned}\varepsilon = 2 = eta\left(\sigma_{ij}\sigma^{ij}\right) &= 2\eta\left(\sigma_{r\phi}\sigma^{r\phi} + \sigma_{\phi r}\sigma^{\phi r}\right)\\ &= 4\eta\sigma_{r\phi}\sigma^{r\phi} = -4\left(t_{r\phi}\sigma_{r\phi}\right).\end{aligned} \quad (38)$$

Therefore, the heat generated by viscosity per unit time is expressed as

$$\begin{aligned}2h\varepsilon = -8ht_{r\phi}\sigma_{r\phi} &= \frac{3\dot{M}}{\pi}\Omega^2\left(1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right)\\ &= \frac{3\dot{M}}{\pi r^3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\left(1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right)\end{aligned} \quad (39)$$

Finally, as the accretion disk possesses two emission surfaces, namely, the top and bottom surfaces, the total radiation flux (or surface emissivity) is expressed as

$$F_A(r) = h\varepsilon = \frac{3\dot{M}}{2\pi r^3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\left(1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right) \equiv Q_e \quad (40)$$

Notably, this is the "classic" relationship between the surface emissivity *Q-e* and accretion rate $\dot{M}$. As the disk is assumed to be "thin," the heat (emission) flows out of the disk vertically, rather than radially. Subsequently, the total power radiated, namely, the total disk luminosity, is expressed as the integral of the total radiation flux over the entire surface of the disk

$$\begin{aligned}L_A &= \int_{r_I}^{\infty}2\pi r\, F_A(r)dr = 3\dot{M}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\times\int_{r_I}^{\infty}\left(\frac{1}{r^2} - \beta\frac{r_I^{1/2}}{r^{5/2}}\right)dr\\ &= \left(\frac{3}{2}-\beta\right)\frac{2\dot{M}}{r_I}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right).\end{aligned} \quad (41)$$

We expect that this total disk luminosity would be the sum of Newtonian gravitational binding energy and rotational kinetic energy extracted from the compact or massive object (perhaps through the "electromagnetic process" proposed by Blandford-Znajek), namely,

$$L = \left(E_{bind} + E_{rot}\right)\dot{M} \quad (42)$$

presumably with

$$E_{bind} = \frac{1}{r_I}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right), \quad (43)$$

$$E_{rot} = 2(1-\beta)\frac{1}{r_I}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right). \quad (44)$$

3.1.3 Surface Temperature

Lastly, assuming that the radiation from the disk is of the "black body" type, the surface temperature of the disk at the radial distance *r* can be expressed as

$$F(r) = \frac{1}{2} b T_s^4(r) \tag{45}$$

$$\begin{aligned} T_s(r) &= \left(\frac{2F(r)}{b}\right)^{1/4} \\ &= \left(\frac{3\dot{M}}{\pi b r^3}\left(GM - \frac{\sigma_T}{4\pi m_p c}L\right)\left(1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right)\right)^{1/4} \end{aligned} \tag{46}$$

Therefore, compared with those of the standard thin disk model of Shakura and Sunyaev or Novikov & Thorne, the important predictions of our new candidate theory of ADAF can be summarized as the suppression of physical characteristics of the accretion disk;

$$\frac{L_A}{L_c} = 1 - \frac{\sigma_T}{4\pi m_p c}\frac{L}{GM} \equiv 1 - f, \tag{47}$$

$$\frac{T_s^A(r)}{T_s^c(r)} = \left(1 - \frac{\sigma_T}{4\pi m_p c}\frac{L}{GM}\right)^{1/4} \equiv (1-f)^{1/4}, \tag{48}$$

and hence more concretely,

$$\begin{aligned} T_s^A(r) \simeq (3\times 10^7 \mathrm{K})\left(\frac{\dot{M}}{10^{-9} M_\odot/\mathrm{year}}\right)^{1/4}\left(\frac{M}{M_\odot}\right)^{-1/2} \\ \left(\frac{GM}{r}\right)^{3/4}\left(1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right)^{1/4}\left(1-\frac{\sigma_T}{4\pi m_p c}\frac{L}{GM}\right)^{1/4}. \end{aligned} \tag{49}$$

3.1.3.1 Note 1

In particular, the total disk luminosity responds sensitively to the underlying disk model and thus becomes noticeably suppressed, whereas the surface temperature responds insensitively and remains nearly the same. This feature of our new candidate theory of ADAF is consistent with the typical characteristic of the standard, existing, and conventional ADAF model of Narayan & Yi, according to whom, the temperature of the accreting gas is nearly virial; conversely, most of the viscously dissipated energy (i.e., the radiation) is trapped or stored as entropy in the accreting gas and is eventually advected into the central star or BH, leading to the low-luminosity, abnormal accretion flow.

3.1.3.2 Note 2

Our new candidate theory of ADAF also enables us to uncover the (possibly) true nature of advection energy (following the terminology of the existing conventional ADAF model) as the fraction of the viscously dissipated energy being radiated is expressed as $(1 - f)L_c$; conversely, the fraction fL_c, which is stored in the accreting gas and is eventually advected into the central star or BH, originates from the recoil of the accreting gas by the scattering of radiation from the central star or BH. In particular, according to our new candidate theory of ADAF, the radiation pressure of the central star or BH (and the recoil of accreting gas because of it) renders part of the viscously dissipated energy to be advected into the central star/BH. Therefore, within the context of our new candidate theory of ADAF, the ADAF limit, $f \to 1$, clearly corresponds to the case when the central object is maximally radiating at the critical Eddington luminosity (in which case the accretion flow essentially becomes quasi-spherical as described below), whereas in contrast, the "little advection" limit $f \to 0$ represents the case when the central object possesses very low luminosity.

3.1.3.3 Note 3

Lastly, we provide the emission spectrum of the accretion disk in some systems of interest predicted by our new candidate of ADAF theory.

- For BH XBs

$$\begin{aligned} T_s(r) \simeq (10^7 \mathrm{K})\left(\frac{\dot{M}}{10^{-8} M_\odot/\mathrm{year}}\right)\left(\frac{M}{3M_\odot}\right)^{-1/2} \\ \left(\frac{10GM}{r}\right)^{3/4}\left(1-\frac{\sigma_T}{4\pi m_p c}\frac{L}{GM}\right)^{1/4} \end{aligned} \tag{50}$$

- For AGN

$$\begin{aligned} T_s(r) \simeq (10^5 \mathrm{K})\left(\frac{\dot{M}}{M_\odot/\mathrm{years}}\right)\left(\frac{M}{10^8 M_\odot}\right)^{-1/2} \\ \left(\frac{10GM}{r}\right)^{3/4}\left(1-\frac{\sigma_T}{4\pi m_p c}\frac{L}{GM}\right)^{1/4}. \end{aligned} \tag{51}$$

3.2 Equation for the Description of Bipolar Outflows/Jets: the Radial Component of the Momentum Equation (Euler Equation)

Subsequently, within the context of our new candidate theory of ADAF, wherein the effect of radiation pressure on the accretion process is included, we aimed to provide a natural explanation for the widespread occurrence of bipolar outflows/jets that are ubiquitous in accreting systems, such as in BH XBs, micro-quasars, and AGN.

Accordingly, the general form of the momentum conservation equation or Euler equation is expressed as

$$\rho\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}=-\nabla p-\rho\nabla\Phi_N+\overrightarrow{F_{rad}}+\eta_v\left(\nabla^2\vec{v}+\frac{1}{3}\nabla(\nabla\cdot\vec{v})\right) \quad (52)$$

where the last term on the right-hand side is the usual viscous force, wherein η_v denotes the (microscopic) kinematic viscosity and

$$F_{rad}=\frac{I}{c}\sigma_T \quad (53)$$

is the radiation force that is systematically included in our new theory of ADAF. In the present treatment below, we neglect the viscous force term and attempt to solve the radial and (possibly) vertical components of this momentum conservation equation to obtain a solution representing the bipolar outflow/jet. To be more concrete, our strategy to achieve this goal is as follows.

To demonstrate the actual occurrence of the bipolar outflow/jet in our new candidate theory of ADAF, we should find a solution to the radial and (possibly) vertical components of this momentum conservation equation that turns the inflowing gas around and renders it to flow outward on a radial trajectory to reach infinity with a net positive energy. In the standard thin disk model of Shakura and Sunyaev or Novikov & Thorne and even in the existing conventional ADAF model, the only force, apart from the viscous force, involved in the hydrodynamics equations (including this momentum conservation equation) is the attractive Newtonian gravity exerted by the central subject. Thus, the generic motion of the accreting gas is apparently radial inflow, which essentially hinders the construction of the outflow mechanism. Conversely, in our new candidate theory of ADAF, in the Euler equation, we used the repulsive radiation force in addition to the attractive Newtonian gravitational force. Therefore, if the repulsive radiation force overwhelms the attractive Newtonian gravity, we can find a solution to the hydrodynamics equations (including this momentum conservation equation), which demonstrates the occurrence of outflows. In particular, the repulsive radiation force clearly plays the key role of scattering/reflecting the incident accreting gas-plasma.

3.3 Equations of the Vertical Structure

In general, as no net motion of the gas/fluid elements occurs in the vertical direction of the accretion disk, momentum conservation along the z-direction is reduced to a "hydrostatic equilibrium" condition.

We now consider the vertical component of the aforementioned momentum conservation equation (Euler equation). The physical meaning and role of this equation is as follows. By equating the component of the (Newtonian) gravitational force plus that of the radiation force of the central compact or massive star along $\widehat{e_z}$ to the vertical pressure gradient in the disk, the thickness of the disk and vertical mass density profile of the accretion disk can be determined.

viz.

$$\rho\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}\cdot\widehat{e_z}=\left(-\nabla p-\rho\nabla\tilde{\Phi}\right)\cdot\widehat{e_z}, \quad (54)$$

where $-\nabla\tilde{\Phi}=-\nabla\Phi_N+\overrightarrow{F_{rad}}$ is the net, effective gravitational force, including the radiation force; namely,

$$\tilde{\Phi}=\frac{1}{r}\left(-GM+\frac{\sigma_T}{4\pi m_p c}L\right). \quad (55)$$

Subsequently,

$$\frac{\partial v_z}{\partial t}+\vec{v}\cdot\nabla v_z=-\frac{1}{\rho}\frac{dp}{dz}-\frac{d}{dz}\left(-\frac{GM}{r}+\frac{\sigma_T}{4\pi m_p cr}L\right), \quad (56)$$

$$\frac{1}{\rho}\frac{dp}{dz}=-\left(\frac{z}{r^3}\right)\left(GM-\frac{\sigma_T}{4\pi m_p c}L\right), \quad (57)$$

and using $c_s^2=\frac{dp}{d\rho}$,

$$\frac{c_s^2}{\rho}\frac{d\rho}{dz}=-z\Omega_z^2, \quad (58)$$

$$\int \frac{d\rho}{\rho} = -\frac{\Omega_A^2}{c_s^2}\int z dz, \tag{59}$$

$$\ln\rho = -\frac{\Omega_A^2}{2c_s^2}z^2 + \text{const}, \tag{60}$$

$$\frac{\rho(z)}{\rho_0} = e^{-\frac{\Omega_a^2}{2c_s^2}z^2}. \tag{61}$$

Thus, the vertical mass density profile of the accretion disk is expressed by the Gaussian function,

$$\rho(z) = \rho(z=0)\exp\left[-\frac{z^2}{2h^2}\right], \tag{62}$$

where the "scale height"

$$h \equiv H/R = c_s/\Omega_A = \left(\frac{p}{\rho}\right)^{1/2}\left(\frac{GM}{r^3} - \frac{\sigma_T}{4\pi m_p c}L\right)^{-1/2} \tag{63}$$

estimates the "half thickness" of the disk. Here the "sound speed" is normally assumed to be

$$c_s = \left(\frac{dp}{d\rho}\right)^{1/2} \tag{64}$$

$$= \frac{p^{gas}}{\rho} = \frac{T}{m_p} \text{ (whole disk)}, \tag{65}$$

$$= \frac{p^{rad}}{\rho} = \frac{1}{3}b\frac{T^4}{\rho} \text{ (inner disk)}. \tag{66}$$

Finally, as the luminosity L of the central object increases, the Keplerian angular velocity of the fluid element decreases, and consequently, the scale height increases; namely, the disk becomes thicker. In particular, when the luminosity of the central compact or massive star approaches the critical Eddington value,

$$L \to L_{Edd} = \frac{4\pi G m_p c_M}{\sigma_T}, \tag{67}$$

the Keplerian angular velocity of the accreting gas drops to zero and the thickness of the accretion disk greatly blows up.

Notably, this point implies that our new candidate model theory of ADAF predicts that if the central object is maximally radiating at the Eddington luminosity, the accretion flow is essentially quasi-spherical. Generically, as the central object becomes more luminous, the accretion flow transitions from the disk-type to the spherical Bondi-type.

4. CLASS OF ADAF MODELS TO ACCOUNT FOR THE LOW-LUMINOSITY ABNORMAL ACCRETION FLOW OBSERVED IN SOME BH XBs AND AGN

① Optically thick ADAF model (Katz 1977; Begelman 1978; Begelman & Meier 1982; Abramowicz et al. 1988): at super Eddington accretion rates/luminosity, the inflowing accreting gas is optically thick and hence captures most of the radiation (i.e., the viscous energy), preventing it from being radiated, and carries it inward or "advects" it onto the central star/BH, inconsistently resulting in the low-luminosity abnormal accretion flow with the standard thin-disk model proposed by Novikov & Thorne, 1973, Shakura & Sunyaev, 1973, and Lynden-Bell & Pringle 1974.

② Optically thin ADAF model (Ichimaru, 1976; Rees et al. 1982; Abramowicz, et al., 1995; Narayan & Yi 1994, 1995a, b): in the opposite limit of low, sub-Eddington accretion rates/luminosity are prevalent. Conversely, the accreting gas has a very low density and is thus optically thin and unable to cool efficiently within the accretion time. The radiation (i.e., the viscous energy) is stored in the inflowing gas as thermal energy and is "advected" into the central star/BH, leading to the low-luminosity abnormal accretion flow that contradicts the standard thin-disk model of Shakura and Sunyaev.

③ Our new candidate theory of ADAF at a generic accretion rate/luminosity; when the accretion rate/luminosity is anywhere between those of the low sub-Eddington and super Eddington accretion rates/luminosity, the low-luminosity abnormal accretion flow can be accounted for even within the context of the standard thin-disk model of Shakura and Sunyaev or Novikov & Thorne, provided that the effect of radiation pressure on the accretion process is included in a simple but straightforward manner.

5. CONCLUDING REMARKS

In conclusion, in our study of the new candidate theory of

ADAF, the inclusion of the effect of radiation pressure on the accretion process is summarized as follows:

① In the case when the central star is a non-luminous BH, regardless of whether it is non-rotating or rotating (possessing the well-known frame dragging effect), the stationary adiabatic accretion flow is most likely disk-type, exhibiting features that can accurately reflect the predictions of the standard thin-disk model of Shakura and Sunyaev or Novikov & Thorne (although the spherical, adiabatic, and hydrodynamical accretion of the Bondi-Hoyle type might be the case in principle).

② The other case is when the central star is highly luminous and exhibits nearly critical or even super Eddington luminosity. According to the results of our study, the accretion flow would exhibit features that significantly deviate (or depart) from the predictions of the standard thin-disk model of Shakura and Sunyaev. In particular, owing to the effect of the radiation pressure on both the radial and vertical structures of the disk, the low-luminosity abnormal accretion flow of the nearly spherical-type is very likely to occur. Additionally, even for the case when the central star possesses a generic accretion rate or luminosity (well below the vertical Eddington value), the low-luminosity abnormal accretion flow of the nearly spherical-type can likely occur, thereby explaining the recent observations in some BH XBs and galactic nuclei.

ACKNOWLEDGMENTS

This study was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (NRF 2022R1A2C1092602).

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APPENDIX

A. An analytical solution based on the self-similar solution of Spruit et al. (1987)

Fortunately, an exact self-consistent solution is available for the hydrodynamics equations (including the Euler equation) within the context of our new candidate theory of ADAF, representing such bipolar outflows/jets. It can be derived from the well-known self-similar solutions discovered first by Spruit et al. (1987) and then elaborated by Narayan & Yi (1994, 1995) in their construction of the optically thin ADAF model.

Therefore, in the following, we shall briefly describe its construction. As a typical approach to study the steady axisymmetric accretion flow in the standard theory of thin accretion disks, we start with the vertical average of the flow to consider a two-dimensional accretion flow in the equatorial $r\phi$ plane described by the following height-integrated hydrodynamics equations comprising the continuity equation, radial and azimuthal components of the momentum conservation equation (i.e., the Euler equation), and energy conservation equation.

$$\frac{d}{dr}(\rho r H v) = 0, \quad \text{(A1)}$$

$$v\frac{dv}{dr} - r\Omega^2 = -r\Omega_K^2 - \frac{1}{\rho}\frac{d}{dr}\left(\rho c_s^2\right), \quad \text{(A2)}$$

$$v\frac{d\left(r^2\Omega\right)}{dr} = \frac{1}{\rho r H}\frac{d}{dr}\left(\frac{\alpha\rho c_s^2 r^3 H}{\Omega_K}\frac{d\Omega}{dr}\right) \quad \text{(A3)}$$

$$\begin{aligned}\rho v H T\frac{ds}{dr} &= q^+ - Q^- = \frac{2\alpha\rho c_s^2 r^2 H}{\Omega_K}\left(\frac{d\Omega}{dr}\right)^2 - Q^- \\ &= f\,\frac{2\alpha\rho c_s^2 r^2 H}{\Omega_K}\left(\frac{d\Omega}{dr}\right)^2,\end{aligned} \quad \text{(A4)}$$

where $\Omega_K = \left(\frac{G\tilde{M}}{r^3}\right)^{1/2}$, and $v_k = r\Omega_K = \left(\frac{G\tilde{M}}{r}\right)^2$ and $c_s = P/\rho$ are the Keplerian angular velocity, and the isothermal sound speed, respectively, and the surface (mass) density of the gas Σ is expressed as $\Sigma = 2\rho H$, wherein H is the scale height, $H = c_s / \Omega_K$. We assumed the α-disk prescription by Shakura & Sunyaev (1973), wherein the kinematic coefficient of viscosity is expressed as

$$v = \alpha c_{stt} = \frac{\alpha c_s^2}{\Omega_K}. \quad \text{(A5)}$$

Subsequently, the parameter f is used to measure the degree to which the flow is advection-dominated, such that the two extreme limits $f = 1$ and $f = 0$ respectively represent pure advection (i.e., no radiative colling) and very efficient colling. For convenience, we defined the parameters $\epsilon \equiv (5/3 - \gamma)/(\gamma - 1)$ and $\epsilon' \equiv \epsilon / f$, wherein γ denotes the ration between the specific heats, and $\epsilon = 0$ in the limit $\gamma = 5/3$ and $\epsilon = 1$ when $\gamma = 4/3$. For simplicity, we assumed that ϵ is independent of r. In the energy conservation equation, Q^+ denotes the energy input per unit area owing to the viscous dissipation, whereas Q^- denotes the energy loss through radiative colling. As is evident, these four height-averaged differential equations provide a self-similar solution (references) of the following form.

$$\rho \propto r^{-3/2}, v \propto r^{-1/2}, \Omega \propto r^{-3/2}, c_s^2 \propto r^{-1}, \quad \text{(A6)}$$

or more precisely,

$$v = -\frac{5+2\epsilon'}{3\alpha}g(\acute{a},\epsilon')v_k \approx -\frac{3\alpha}{5+2\epsilon'}v_K \quad \text{(A7)}$$

$$\Omega = \left(\frac{2\epsilon'(5+2\epsilon')}{9\alpha^2}g(\alpha,\epsilon')\right)^{1/2}\Omega_K \approx \left(\frac{2\epsilon'}{5+2\epsilon'}\right)^{1/2}\Omega_K, \quad \text{(A8)}$$

$$c_s^2 = \frac{2(5+2\epsilon')}{9\alpha^2}g(\alpha,\epsilon')v_K^2 \approx \frac{2}{5+2\epsilon'}v_K^2, \quad \text{(A9)}$$

where

$$g(\alpha,\epsilon') \equiv \left(1 + \frac{18\alpha^2}{(5+2\epsilon)^2}\right)^{1/2} - 1 \quad \text{(A10)}$$

$$\epsilon \equiv \frac{5/3-\gamma}{\gamma-1}, \quad \text{(A11)}$$

$$\epsilon' = \frac{\epsilon}{f} \quad \text{(A12)}$$

$$\frac{4}{3} \lesssim \gamma \leq \frac{5}{3}. \quad \text{(A13)}$$

Notably, the positivity of the Bernoulli constant B_e, and hence b, implies that the viscous stress transfers energy from small to large radii. Thus, whenever b is positive, it implies that if the inflowing gas could somehow be turned around to let it flow outward on a radial trajectory, the gas would reach infinity with net positive energy (Abramowicz & Lasota 1980; Paczynski & Bisnovatyi 1981; Novikov 1997; Jang et al. 2021; Jang & Kim 2023).

Notably, the volume mass density ρ may be obtained in practice from the mass accretion rate, as follows: $\dot{M} = 2\pi r v \Sigma = 4\pi r v H$.

Notably, in the limit of very efficient colling f → 0, ϵ'∞, the solutions provided in equations can be reduced to those of the standard thin-disk model (in this study, we do not consider the case of the thick disk) using v, $c_{s\backslash llvK}$ and $\Omega \to \Omega_K$. Thus, in this study, we mainly focused on the opposite limit of advection-dominated accretion flow, wherein f is a reasonable fraction of unity; namely, where it has a generic value, and thus $\epsilon' \sim \epsilon < 1$.

Moreover, the radial component of the momentum conservation equation indicates that the self-similar solution can satisfy the following:

$$\frac{1}{2}v^2 + r^2\Omega^2 - \Omega_K^2 + \frac{5}{2}c_s^2 = 0; \tag{A14}$$

this in turn allows us to compute the normalized parameter $b = B_e / v_k^2$, where B_e is the Bernoulli constant. In adiabatic flows in the absence of viscosity,

$$\begin{aligned} b &= \frac{1}{v_K^2}\left(\frac{1}{2}v^2 + \frac{1}{2}r^2\Omega^2 - r^2\Omega_K^2 + \frac{\gamma}{\gamma-1}c_s^2\right) \\ &= \frac{r^2\Omega^2}{2v_K^2} + \left(\frac{\gamma}{\gamma-1} - \frac{5}{2}\right)\frac{c_s^2}{v_K^2} = \frac{(3f-1)\epsilon'}{5+2\epsilon'}. \end{aligned} \tag{A15}$$

Some basics in the classical theory of radiation:

a central star is a “black” body;

the accreting object is an atom (with a cross-sectional area of σ_T);

the luminosity is the power of the radiated energy per unit time mostly through an emission of (massless) photons moving at the speed of light c.

$$E = pc; \tag{A16}$$

$\text{power} = L = \frac{\Delta E}{\Delta t} = c\left(\frac{\Delta P}{\Delta t}\right) = cF$. The intensity is the radiated (transferred) energy per unit time per unit area.

$$I = \frac{\Delta E}{A\Delta t} = \frac{1}{A}\left(\frac{\Delta E}{\Delta t}\right) = \frac{Power}{A} = \frac{Power}{4\pi r^2} \tag{A17}$$

Radiation pressure is the radiation stress tensor

$$P_{rad} = \frac{F}{A} = \frac{1}{A}\left(\frac{\Delta P}{\Delta t}\right) = \frac{1}{A\Delta t}\left(\frac{\Delta E}{c}\right) = \frac{I}{c} \tag{A18}$$

For a perfectly absorbing object $(F = \Delta p / \Delta t) P_{rad} = I / c$.

For a perfectly reflecting object $(F = 2\Delta p / \Delta t) P_{rad} = 2I / c$.

Lastly, the “radiation force” on an object with an effective cross-sectional area of σ is

$$F_{rad} = \sigma P_{rad}. \tag{A19}$$

Thus, if the accreting object is the “free-electron,” its (classical) cross section is σ_T, and hence

$$F_{rad} = P_{rad}\sigma_T = \frac{I}{c}\sigma_T. \tag{A20}$$