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$SIRVQ$ train models for the first outbreak of the Omicron variant in Korea

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We will suggest a train model to explain the weekly periodic character of Covid19 in South Korea on the first half of 2022, the period of the Omicron variant outbreak. In the model, the daily new infected individuals board a train, divided into 4 compartments. The train moves at night by the length of one compartment. Then the infected are quarantined during the daytime after their compartment reaches the quarantine area. Then it remains empty on the 5-th night after boarding. The parameters of the model are fitted with the daily measured quarantine populations and generate the simulated quarantine populations that hit the real weekly and global peaks.

Keywords: Train model, Omicron variant, weekly periodic.

MSC: 37N35

1 Introduction

Coronavirus disease 2019 (COVID-19), which occurred at the end of 2019, continues to threaten people around the world until 2022. The virus has reduced people's social activities such as travel and exchanges, and has had a negative impact on education and production activities. As a specific case of damage, as of February 10, 2022, 404,753,568 confirmed infections and 5,796,448 deaths have been reported worldwide [1]. The World Health Organization (WHO) declared a global public health emergency [2] against COVID-19 on January 3, 2020, and declared a pandemic on March 12, 2020. After that, many institutions focused on the development of a vaccine against COVID-19, and in March 2021, inoculation of the vaccine that received emergency use authorization (EUA) and approval from the U.S. Food and

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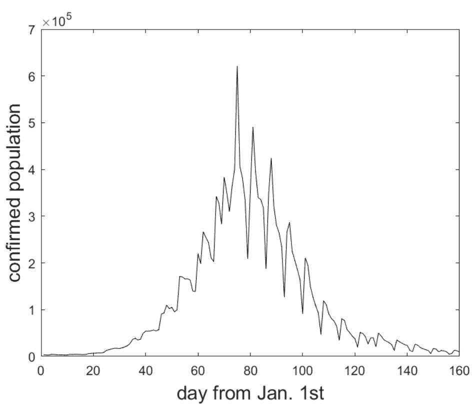
Drug Administration (FDA) started. As the vaccination progressed, the number of infections worldwide decreased in the middle of 2021, but the number of infected people increased again due to the epidemic of the Delta variant. In addition, the highly contagious Omicron variant appeared at the end of 2021, and the number of infections is increasing explosively in early 2022.

In the first half of 2022, there was the first outbreak of the Omicron variant in South Korea [6, 3]. The government gathered daily the number of confirmed populations diagnosed as having the COVID-19 virus. As shown in Figure 1, those daily confirmed populations have some interesting characters:

- weekly periodic with large fluctuation,
- most weekly peaks occur 2 days after holidays, typically on Tuesday,
- the global peak occurs 3 days after holiday, on Wednesday, Mar 16.



(a) daily confirmed populations

(b) ★: weekly peaks, ★: global peak

Figure 1. Weekly periodic fluctuations and peaks of the daily confirmed populations

The typical SIR based models using ordinary differential equations would explain little of these weekly characters. It might be due to that they focus on a trend of varying populations and how the given populations determine those at future instance.

However, the weekly radical peak of the confirmed population should be one of the most targets to control the outbreak of illness. If we could predict the peak, it would provide alerts us to keep social distance in time.

In this paper, we will suggest a new model to explain these weekly peaks of the confirmed populations. In our model, we add the vaccinated group V and quarantined group Q to the SIR model, named $SIRVQ$ train model, then reflect the time delay between infection and confirmed. It is effective to trace the weekly peaks since the most infection comes on weekend, while the most confirmed does on weekday.

The main feature of the suggested model is an imaginary train that all the newly infected must board. Divided into several compartments, they head for the quarantine area so that the infected on the n-th compartment are quarantined n days after boarding. Before confirmed, some ratio of them propagate the illness on a day-by-day basis.

To chase population dynamics, the suggested approach uses only recurrence relations of sequences instead of continuous differential equations. Thus, it would be educated and developed more easily, and even in high school math class upon a knowledge of sequences.

The paper is organized as follows. In the next section, we introduce a few models by which the infected train moves, the transmission rate is weighted, the vaccine effect is attenuated and lastly the quarantined enter into the recovered group. Based on those models, we state the daily dynamics of populations in Section 3. The simulation detail and result are given in the last two sections.

2 $SIRVQ$ train model

Let $N = 52$ million be the total population in South Korea. We divide it into the following disjoint 5 groups:

$S =$ Susceptible including the vaccinated whose effect are terminated

$I =$ Infected

$R =$ Recovered or immuned or isolated from the infected

$V =$ Vaccinated whose effect are valid

$Q =$ Quarantined or confirmed

The confirmed is identified with the quarantined, since the confirmed group is well quarantined immediately after confirmed in South Korea.

Throughout the paper, we use the following notation:

- I_n: the newly infected on the n-th day,
- V_n: the population who get vaccinated on the n-th day [6],
- $\mathcal{P}(V_n)$: the population among V_n who remain not infected.

2.1 Train model for the daily new infected

We model that the daily new infected I_n board a train of 4 compartments at night, divided by the ratio γ_k for the k-th compartment as in Figure 2.

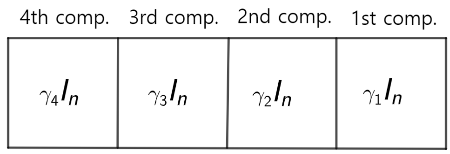


Figure 2. A train of 4 compartments for the daily new infected I_n, where $\gamma_1+\gamma_2+\gamma_3+\gamma_4=1$

The head of the train faces to the border of the quarantine area as in Figure 3, where it is the long vertical dashed line. The train moves toward the quarantine area, done by the length of one compartment before dawn.

Then during daytime on the next day, the population of the two compartments facing the border propagate the illness. The entire of the left among them join the propagation, while α portion of the right do. The left one is orange and the right one is gray in Figure 3. Before sunset, the entire of the compartment crossing the border are quarantined.

After sunset, the train repeats the above until the train remains empty at the 5-th night after boarding.

Thus, on the $(n+1)$-th day, the population who propagate the illness is

$$\gamma_2 I_n + \gamma_3 I_{n-1} + \gamma_4 I_{n-2} + \alpha\left(\gamma_1 I_n + \gamma_2 I_{n-1} + \gamma_3 I_{n-2} + \gamma_4 I_{n-3}\right),$$

and the confirmed population is

$$\gamma_1 I_n + \gamma_2 I_{n-1} + \gamma_3 I_{n-2} + \gamma_4 I_{n-3}.$$

2.2 Weight model for transmission rate

The transmission rate would be weighted along the amount of interactions between people. It might be reflected by the highway traffic we can collected [5]. We assume that its effect on people interaction is reduced on weekdays, since it contains an amount of traffic for working. Denote

- β: constant transmission rate for susceptible S,
- β_n: weighted transmission rate for susceptible S on n-th day,
- m_n : the amonunt of interactions between people on n-th day,
- t_n : total highway traffic over the country on n-th day,
- δ : constant traffic for working on weekdays, reducing the effect of t_n on m_n.

Then we set

$$\beta_n = \beta m_n, \quad m_n = \begin{cases} t_n & \text{on weekend or holiday,} \\ t_n - \delta & \text{on weekday.} \end{cases} \tag{1}$$

The collected daily highway traffic t_n is depicted in Figure 4.

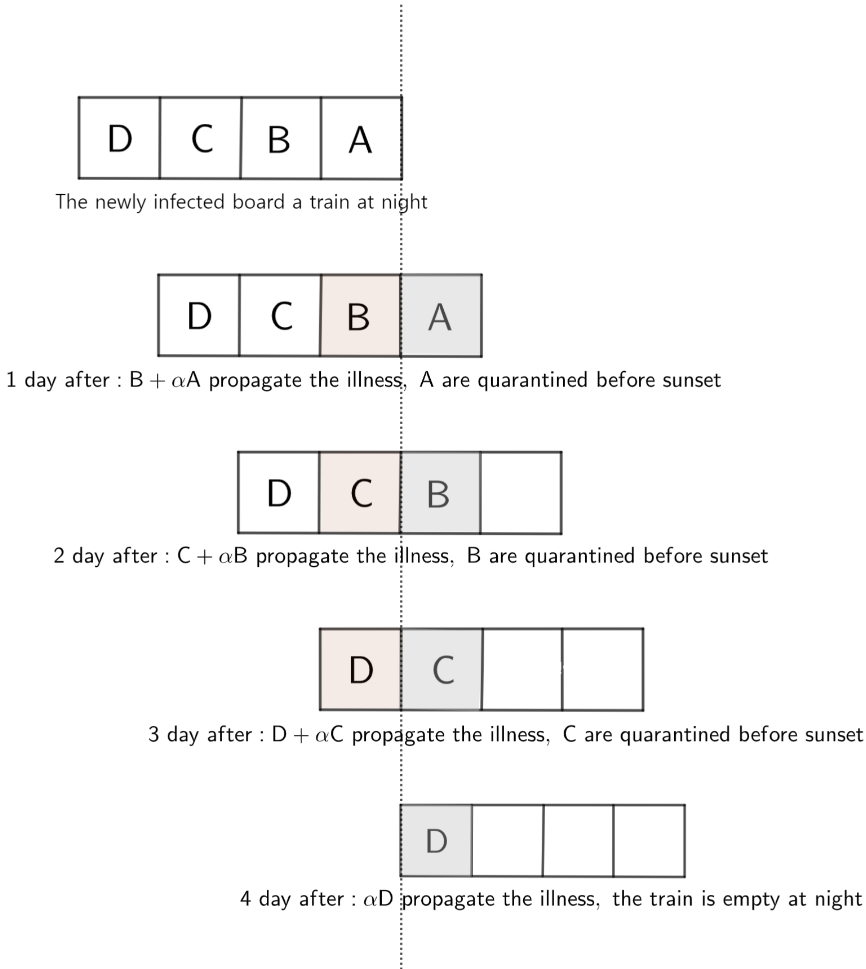


Figure 3. Moving the train for the infected, the colored compartments propagate the illness

2.3 Attenuation model for vaccine effect

We assume that the vaccine effect is attenuated linearly from μ to 1 during 90 days. That is, 1 day after vaccination, the transmission rate for V_n is

$$\mu\beta_{n+1},$$

where β_{n+1} is the one for the susceptible in (1). Then, the rate for V_n increases day by day as

$$\begin{array}{rc}
\text{2 days after :} & \left(\mu+\frac{1}{90}(1-\mu)\right)\beta_{n+2} \\
\text{3 days after :} & \left(\mu+\frac{2}{90}(1-\mu)\right)\beta_{n+3} \\
\vdots & \vdots \\
\text{90 days after:} & \left(\mu+\frac{89}{90}(1-\mu)\right)\beta_{n+90} \\
\text{91 days after:} & \beta_{n+91}
\end{array} \tag{2}$$

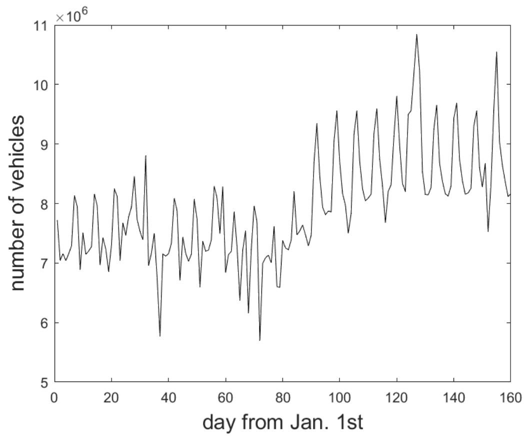


Figure 4. daily highway traffic t_n

2.4 Train model for the quarantined

In South Korea, the confirmed population is well controlled to be quarantined for 7 days. Thus, abusing the notations, we can represent the total quarantined as a train of 7 compartments as

$$Q = [Q(1),\ Q(2), \cdots, Q(7)].$$

On the next day, all the quarantined move to the right compartment. Then $Q(7)$ leaves the train and enter into the recovered population R. The leftmost vacant compartment is occupied by the newly confirmed C. That is, the train results in

$$[C,\ Q(1), \cdots, Q(6)].$$

3 Daily dynamics

In this section, we state the daily dynamics for the population of each S, I, R, V, Q group, based on the models in the previous section. Assume the following parameters in our model are given:

$$\alpha,\ \beta,\ \gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4,\ \delta,\ \mu. \tag{3}$$

At the n-th day night, we have that

- daily infected populations for the past 4 days

$$I_n,\ I_{n-1},\ I_{n-2},\ I_{n-3},$$

- remaining vaccinated populations for the past 90 days

$$\mathcal{P}(V_n),\ \mathcal{P}(V_{n-1}), P(V_{n-2}), \cdots, \mathcal{P}(V_{n-89}),$$

- total recovered population R and the quarantine train Q

$$Q = [Q(1),\ Q(2), \cdots, Q(7)].$$

Then, our model determines the population of each group on the next $(n+1)$-th day as

- the total infected
$$I = I_n + (\gamma_2 + \gamma_3 + \gamma_4)I_{n-1} + (\gamma_3 + \gamma_4)I_{n-2} + \gamma_4 I_{n-3},$$
- the infected propagating the illness
$$I_p = \gamma_2 I_n + \gamma_3 I_{n-1} + \gamma_4 I_{n-2} + \alpha\left(\gamma_1 I_n + \gamma_2 I_{n-1} + \gamma_3 I_{n-2} + \gamma_4 I_{n-3}\right),$$
- the susceptible
$$S = N - I - R - \sum_{k=0}^{89} \mathcal{P}(V_{n-k}) - \sum_{k=1}^{7} Q(k),$$
- the new infected from S
$$I_S = \frac{1}{N} m_{n+1} \beta S \cdot I_p,$$
- the new infected from the remaining vaccinated $\mathcal{P}(V_{n-k})$
$$I_{V_{n-k}} = \frac{1}{N} m_{n+1} \left(\mu + \frac{k}{90}(1-\mu)\right) \beta \mathcal{P}(V_{n-k}) \cdot I_p, \quad k = 0, 1, 2, \cdots, 89,$$
- the new infected
$$I_{n+1} = I_S + I_{V_n} + I_{V_{n-1}} + \cdots + I_{V_{n-89}},$$
- the new confirmed
$$C_{n+1} = \gamma_1 I_n + \gamma_2 I_{n-1} + \gamma_3 I_{n-2} + \gamma_4 I_{n-3}, \tag{4}$$
- the total recovered and new quarantine train
$$R \longleftarrow R + Q(7), \quad Q \longleftarrow [C_{n+1}, Q(1), Q(2), \cdots, Q(6)],$$
- the remaining vaccinated
$$\mathcal{P}(V_{n-k}) \longleftarrow \mathcal{P}(V_{n-k}) - I_{V_{n-k}}, \quad k = 0, 1, 2, \cdots, 89.$$

4 Method

We have C_n^m, the measured daily confirmed population on the n-th day [6]. The unknown parameters in (3) and initial populations
$$R,\ Q,\ I_0,\ I_{-1},\ I_{-2},\ I_{-3},\ \mathcal{P}(V_0),\ \mathcal{P}(V_{-1}), \cdots, \mathcal{P}(V_{-89}),$$
would be fixed so that they produce the daily confirmed population C_n in (4) minimizing
$$(C_1 - C_1^m)^2 + (C_2 - C_2^m)^2 + \cdots + (C_{160} - C_{160}^m)^2. \tag{5}$$
To reduce the unknowns heuristically, we set some parameters:
$$\gamma_1 = 0,\ \gamma_2 = \gamma_3 = \gamma_4 = 1/3,\ \mu = 2/5,$$
and some initial populations:
$$I_0 = 6000,\ I_{-1} = 4500,\ I_{-2} = 3000,\ I_{-3} = 1500,\ Q = [5000, 5000, \cdots, 5000],$$
and simplify the remaining vaccinated as $\mathcal{P}(V_k) = V_k,\ k = 0, -1, -2, \cdots, -89$.

Eventually, we have fixed α of the train, β, δ of the transmission and initial total recovered R to minimize (5) with an aid of Matlab command `lsqcurvefit`.

5 Result

We have fixed

$$\alpha = 0.1575,\ \beta = 25.9,\ \delta = 3.1593 * 10^6,\ R = 1.8235 * 10^7,$$

which simulated the daily confirmed populations as depicted in Figure 5. It shows an aspect weekly periodic as the measured.

Our $SIRVQ$ train model hits the global peak on Wednesday and several weekly peaks on Tuesday marked with black circles in Figure 5. The periodic valleys also appear along to those of the measured. Although the peaks are not as sharp as the measured, it is enough to be prospective for the future study.

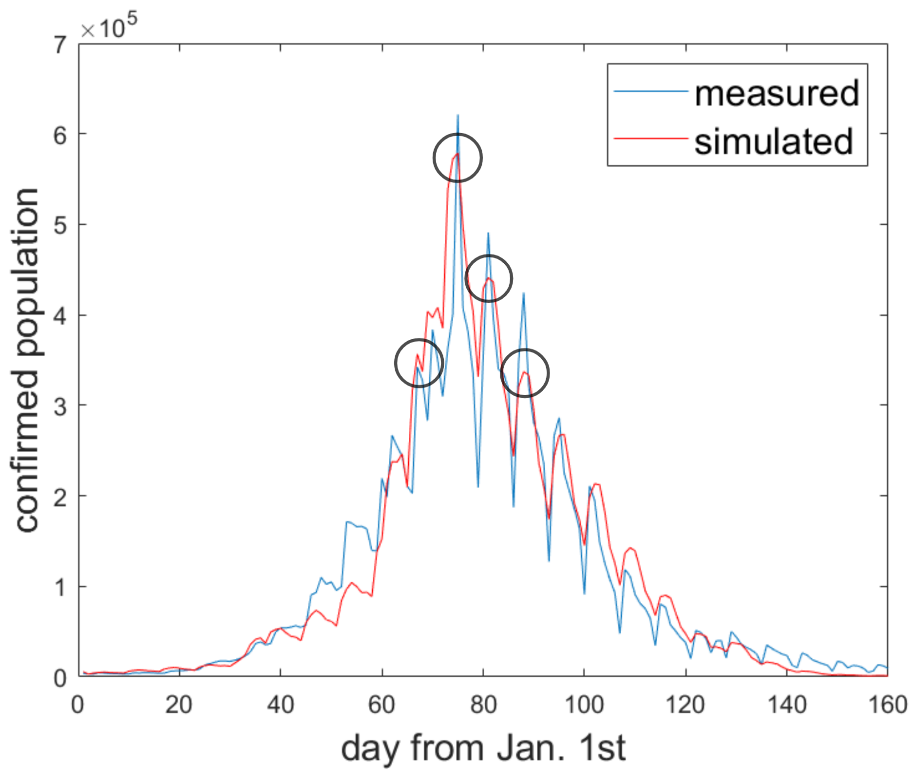


Figure 5. Comparison of the measured and simulated confirmed populations

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