논문번호 23-04-04

24

j.inst.Korean.electr.electron.eng.Vol.27,No.4,391~398,December 2023 https://doi.org/10.7471/ikeee.2023.27.4.391

# Regulation control of a dc motor by $\epsilon$ -PID controller based on the frequency response analysis

Alhassan Dodo Adamou Soudeize\*, Ji-Sun Park\*, Ho-Lim Choi\*\*

#### **Abstract**

In this paper, we propose an alternative analysis to regulate DC motors using a PID controller with a gain scaling factor. We start by providing a systematic design method for selecting the PID gains of our proposed controller by seeing the effect of  $\epsilon$  on damping ratio, overshoot and settling time from the frequency response analysis. With the help of matlab (simulink), We proceed to show that the proposed controller provides robust stability against system parameter uncertainty and the effect of the gain scaling factor on steady-state error. The validity of our control method along with the analysis is verified with the simulation results.

Key words: Robust root-locus, Robust steady-state error, Frequency response analysis, Damping ratio, PID controller

#### I. Introduction

PID controllers has been actively studied in the past recent years till date [1-5] in the field of Control engineering for example in most automatic process control applications to regulate flow, temperature, pressure, level, and many other industrial process variables. In [2], they propose a multiobjective approach to design PID controllers organized into three parts. Part one describes the methods for the computation of the complete set of PID controllers stabilizing a given SISO linear timeinvariant plant. Part two proceeds to search for achievable performance for the closed-loop system under PID control. Finally, Part III adresses the optimization-based design of PID controllers for both continuous and discrete-time

systems is considered. In [3], basic frequency analysis and the analysis of feedback systems is explianed. The PID law is often viewed as a simplistic computational control algorithm. However just like all non-convex optimization problems, tuning the PID algorithm for accurate and stable closed-loop control becomes a NP-Hard Problem. This leads to a dilemma, for both users and designers, most especially in practise as seen in [4]. And also, for people to understand the command of PID controllers in different application, the review of PID control in [5] is focused on the classical and modern optimization rules used for PID tuning integrated to intelligent control. Stability is a basic requirement, but beyond that, different systems have different behavior, different applications have different requirements, and requirements

★ Corresponding author

E-mail: hlchoi@dau.ac.kr, Tel: +82-51-200-7734

Acknowledgment

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government (MSIT) (No. 2023R1A2C1002832).

Manuscript received Aug. 31, 2023; revised Sep. 24, 2023; accepted Oct. 12, 2023.

This is an Open-Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License(http://creativecommons.org/licenses/by-nc/3.0) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

<sup>\*</sup> Dept. of Electrical Engineering, Dong-A University

may conflict with one another.

In this paper, the main aim is to achieve a good robust tracking performance of the proposed controller by providing a simplified systematic gain design method using frequency response analysis in the presence of some parameter uncertainties. First we utilize the state-space equation of the DC motors (MAXON DC motors, RE35(11879)) of [6] by finding its Laplace transform. Note that our considered DC motor is used as actuators at the joints of a humanoid robot such as leg, arm joints, etc. Then, with available information of nominal parameters, we obtain a PID controller with a gain-scaling factor  $\epsilon$  > 0. Finally by trial and error, we vary the disturbance value to observe the effect of  $\epsilon > 0$  on steady-state error, damping ratio and overshoot. In summarized simulation results, we show that our control scheme and system analysis agree with the simulation results.

### II. System dynamics of dc motor

The DC motors (MAXON DC motors, RE35(11879)) dynamics from [7] is expressed as

$$0 = \frac{K_m}{rR}v(t) - \frac{J_m \ddot{q}(t)}{r^2} - \frac{B_m + K_b K_m / R}{r^2} \dot{q}(t) - w(t) \quad (1)$$

where q(t) is the position of the link,  $K_m$  is the torque constant,  $J_m$  is the motor inertia,  $B_m$  is the damping coefficient,  $K_b$  is the back emf constant, R is the armature resistance, r is the gear ratio, w(t) is the load torque in voltage which is usually estimated using control schemes which use the additional load torque estimator [8,9] and nonlinear observer [10] instead of using the PID control scheme.

Our control goal is to regulate q(t) to a (piecewise) constant reference signal  $q_d(t)$ . Multiplying (1) above by  $r^2/J_m$ , we have

$$0 = bv(t) - \ddot{q}(t) - a\dot{q}(t) - cw(t)$$
 (2)

where

$$a := \frac{B_m + K_b K_m / R}{J_m}, \qquad b := \frac{r K_m}{J_m R}, \qquad c := \frac{r^2}{J_m}.$$

Practically, we consider that there are some parameter uncertainties in a and b such that  $a=\bar{a}+\delta a$  and  $b=\bar{b}+\delta b$  where  $\bar{a}$  and  $\bar{b}$  denote nominal parts and  $\delta a$  and  $\delta b$  denote any possible uncertain values. We assume that  $\max |\delta a/\bar{a}|$ ,  $|\delta b/\bar{b}| < 0.5$ , that is up to 50% parameter uncertainties are allowed in general. Taking into consideration that w(t) itself is considered as an uncertainty, there is no requirement of setting uncertainty in c. So, we rewrite (2) as

$$0 = (b = \bar{b} + \delta b)v(t) - \ddot{q}(t) - (a = \bar{a} + \delta a)\dot{q}(t) - cw(t)$$
 (3)

Taking the Laplace transform of (3), we have

$$Q(s) = \frac{\overline{b} + \delta b}{s^2 + (\overline{a} + \delta a)s} V(s) - \frac{c}{s^2 + (\overline{a} + \delta a)s} W(s)$$
(4)

which can therefore be expressed in a block diagram along with a feedback controller D(s) as

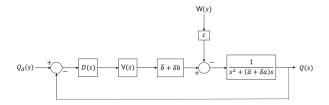


Figure 1. Feedback control scheme of a DC motor.

#### Control objectives

- Propose an ∈-PID controller for DC motors to improve it's settling time and reduce overshoot in the presence of some uncertain parameters.
- 2. Propose the design steps based on system analysis to help us to choose the control parameters  $(k_P, k_I, k_D, \epsilon)$ .
- Prove using both frequency response analysis and simulation that the steady-state error is directly proportional to the gain-scaling factor.

## III. Proposed controller and system analysis based on frequency response analysis

#### 3.1 $\epsilon$ -PID controller

With available information of nominal parameters, the following  $\epsilon$  -PID controller with a gain -scaling factor  $0 < \epsilon < 1$  is proposed in [7]

$$D(s) = k_{P}(\epsilon) + k_{I}(\epsilon)/s + k_{D}(\epsilon)s \tag{5}$$

where

$$k_P(\epsilon) = \frac{k_P}{\bar{b}\epsilon^2},\tag{6}$$

$$k_I(\epsilon) = \frac{k_I}{\bar{b}\epsilon^3},\tag{7}$$

$$k_D(\epsilon) = \frac{k_D}{\bar{b}\epsilon} - \frac{\bar{a}}{\bar{b}}.$$
 (8)

In [7], the robust stability and control performance aspects of (5) are addressed in a state-space framework by using Lyapunov stability analysis. In this paper, we try to explore these aspects by using more classical control methods such as root-locus method. From Fig. 1, we obtain

$$Q(s) = \frac{D(s)(\overline{b} + \delta b)}{D(s)(\overline{b} + \delta b) + s^2 + (\overline{a} + \delta a)s} Q_d(s) + \frac{c}{D(s)(\overline{b} + \delta b) + s^2 + (\overline{a} + \delta a)s} W(s).$$
(9)

Inserting (5) into (9), we have

$$\begin{split} Q(s) &= \\ & \frac{(1+\mu)[\epsilon^2(k_D - \epsilon \bar{a})s^2 + \epsilon k_P s + k_{I]}}{\epsilon^3 s^3 + (1+\mu)[\epsilon^2(k_D + \epsilon (\delta a - \mu \bar{a})/(1+\mu))s^2 + \epsilon k_P s + k_{I}]} \, Q_d(s) \\ &+ \frac{\epsilon^3 c}{\epsilon^3 s^3 + (1+\mu)[\epsilon^2(k_D + \epsilon (\delta a - \mu \bar{a})/(1+\mu))s^2 + \epsilon k_P s + k_{I}]} \, W(s) \end{split}$$

where the disturbance parameter  $\mu = \delta a$ .

From (10) and Fig. 1, the followings are observed:

1. When  $\epsilon$  is selected to be much less than  $k_D$ , the characteristic equation is approximated to be

$$\epsilon^{3}s^{3} + (1+\mu)[\epsilon^{2}(k_{D} + \epsilon(\delta a - \mu \bar{a})/(1+\mu))s^{2} + \epsilon k_{P}s + k_{I}] = 0. \tag{11} \label{eq:11}$$

2. Suppose that  $\epsilon = 1$  for a while. Then, (11) becomes

$$s^{3} + (1+\mu)[k_{D}s^{2} + k_{P}s + k_{I}] = 0.$$
 (12)

3. Using the Routh-array method [3], (12) becomes a Hurwitz polynomial when

$$k_L > 0, \ k_D > 0, \ (1+\mu)k_P k_D > k_L$$
 (13)

The condition (13) can be easily met if  $s^3 + k_D s^2 + k_P s + k_I = 0$  is a Hurwitz polynomial. Now, applying Routh-array method to (11), we can easily see that (11) is a Hurwitz polynomial for all  $\epsilon > 0$  whenever (12) is a Hurwitz polynomial.

4. Assuming that q(t) remains bounded, namely, the controlled system is stable, from (10), we can immediately see that when  $\epsilon$  is selected to be much less than 1, the effect caused by W(s) is significantly reduced by an order of  $\epsilon^3$ . Thus, the robustness against uncertain load torque is achieved by a gain-scaling factor  $\epsilon$ .

## 3.2 Analysis and design of $\epsilon$ -PID controller for DC motor control with parameter uncertainty

In order to check the classical control performance specifications such as overshoot and settling time, the root-locus approach can be used. Taking T(s) from (10) as

$$T(s) = \frac{(1+\mu)[\epsilon^{2}(k_{D}-\epsilon\bar{a})s^{2}+\epsilon k_{P}s+k_{I}]}{\epsilon^{3}s^{3}+(1+\mu)[\epsilon^{2}(k_{D}+\epsilon(\delta a-\mu\bar{a})/(1+\mu))s^{2}+\epsilon k_{P}s+k_{I}]}. \tag{14}$$

The characteristic equation under  $\epsilon \ll k_D$  is

$$\epsilon^3 s^3 + (1+\mu)[\epsilon^2 k_D s^2 + \epsilon k_P s + k_I] = 0.$$
 (15)

Diving (15) by  $\epsilon^3$ , we get

$$s^{3} + \frac{(1+\mu)k_{D}}{\epsilon}s^{2} + \frac{(1+\mu)k_{P}}{\epsilon^{2}}s + \frac{(1+\mu)k_{I}}{\epsilon^{3}} = 0.$$
 (16)

Letting  $\epsilon = 1$  in (16),

$$s^{3} + (1+\mu)k_{D}s^{2} + (1+\mu)k_{P}s + (1+\mu)k_{I} = 0.$$
 (17)

Equation (17) is stable via the Routh-Hurwitz stability criteria if and only if the following is attained

$$\max\left(\frac{k_I}{k_P k_D} - 1, -1\right) < \mu$$

which gives us allowable range of  $\mu$  once  $k_P$ ,  $k_I$ ,  $k_D$  are selected.

When  $\mu = 0$  (no uncertainty) and  $\epsilon = 1$ , we set

$$s^{3} + k_{D}s^{2} + k_{P}s + k_{I} = (s + \alpha)(s^{2} + 2\zeta w_{n}s + w_{n}^{2})$$
 (18)

$$= s^{3} + (2\zeta w_{n} + \alpha)s^{2} + (2\alpha\zeta w_{n} + w_{n}^{2})s + \alpha w_{n}^{2}$$
 (19)

where  $0 < \zeta \le 1$  and  $\alpha \gg \zeta w_n$ .

The effect of the third pole is desired to be negligible since we want to make the behaviour of the proposed controller to be governed by dominant poles. Therefore  $\alpha \gg w_n$ . By comparing both sides of equation (19), we obtain

$$\begin{split} k_P &= 2\alpha \zeta w_n + w_n^2 > 0\,,\\ k_I &= \alpha w_n^2 > 0\,,\\ k_D &= 2\alpha \zeta w_n + \alpha > 0\,, \end{split}$$

thus,  $\zeta$  and  $w_n$  are determined, once  $k_P,\,k_I,\,k_D$  are selected.

However, due to the uncertainty  $\mu$ , the nominal  $\zeta$  and  $w_n$  will be changed when a nonzero  $\mu$  occurs (within the allowable range). In this case, from (17) and (19), we set

$$(1+\mu)k_P = 2\alpha\zeta W_n + w_n^2, (20)$$

$$(1+\mu)k_I = \alpha w_n^2, \tag{21}$$

$$(1+\mu)k_D = 2\alpha\zeta W_n + \alpha. \tag{22}$$

Since  $\alpha \gg \zeta w_n$ ,

$$(1+\mu)k_D = \alpha. (23)$$

Inserting (23) into (20) and we can get

$$w_n = \sqrt{\frac{(1+\mu)k_I}{\alpha}} = \sqrt{\frac{k_I}{k_D}} = \sqrt{\frac{k_I}{\epsilon k_D}}, \qquad (24)$$

$$\zeta = \frac{(1+\mu)k_P - w_n^2}{2\alpha w_n} = \frac{(1+\mu)k_P k_D - k_I}{2(1+\mu)k_D \sqrt{k_D k_I}}.$$
 (25)

Thus, assuming  $\alpha \gg \zeta w_n$ ,  $\mu$  only affects  $\zeta$ .

Consider the table below which shows the robustness against  $\mu$ . The following sets of  $(k_P, k_I, k_D)$  are used for eqn (32) and the table below is obtained.

- Set 1:  $k_P = 8$ ,  $k_I = 6$ ,  $k_D = 7$ ,
- Set 2:  $k_P = 5$ ,  $k_I = 9$ ,  $k_D = 4$ ,
- Set 3:  $k_P = 1$ ,  $k_I = 1$ ,  $k_D = 1$ ,

Table 1. Selected  $\zeta$  for set 1, 2, and 3

|              | Set $1(\zeta)$ | Set 2(ζ) | Set 3(ζ) |
|--------------|----------------|----------|----------|
| $\mu = -0.5$ | 0.48           | 0.397    | -0.5     |
| $\mu = 0$    | 0.55           | 0.451    | 0        |
| $\mu = 0.5$  | 0.57           | 0.469    | 0.17     |

The overshoot  $(M_p(\%))$  is expressed as

$$M_p = 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}. (26)$$

So, in set 1 when  $\mu = -0.5$  for  $\zeta = 0.48$ , its corresponding overshoot is

$$M_n(\zeta = 0.48) = 100e^{\frac{-0.48\pi}{\sqrt{1 - (0.48)^2}}} = 17.9\%.$$

Similarly, in sets 2 and 3 for  $\zeta = 0.397$  and  $\zeta = -0.5$  respectively,

$$M_n(\zeta = 0.397) = 25.7\%, \qquad M_n(\zeta = -0.5) = 612.9\%.$$

The followings are some observations made from the table above

- 1. Set 1 is more robust against  $\mu$ . Even though the difference is not that much, we get less overshoot and less change in damping ratio.
- 2. Set 2 is worse than Set 1 in terms of damping ratio.
- 3. Set 3 is not robust against  $\mu$ . Therefore it should be avoided since it is unstable.

So, from the aforementioned analysis and summarized table, we are led to select a good set of values for  $k_P$ ,  $k_I$ ,  $k_D$  as an initial step.

## IV. Robust effects of the PID controller gains $(k_P,\,k_I,\,k_D)$ and the gain-scaling factor $(\epsilon)$ in the frequency domain

#### 4.1 Effects of $k_P$ , $k_I$ , $k_D$ against disturbance, $\mu$

In order to observe this, the root-locus technique has been used where two cases are considered for two sets of  $[k_P, k_I, k_D]$  values with respect to  $\epsilon$  when  $\mu = 0.5$  in the same scale as shown below.

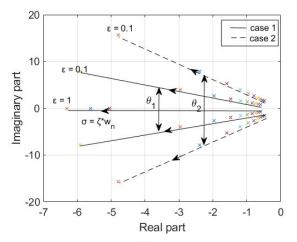


Figure 2. Root-loci of T(s) with (case 1)  $[k_P, k_I, k_D]$ = [8, 6, 7] and (case 2)  $[k_P, k_I, k_D]$ =[5, 9, 4] when  $\mu$  = 0.5 as  $\epsilon$  decreases from 1 to 0.

From Fig. 2,  $\theta_1$  and  $\theta_2$  represent the angles between the pole locations in the real and imaginary sides where

$$\zeta_1 = \sin \theta_1$$
 and  $\zeta_2 = \sin \theta_2$ .

Therefore as the gap-size gets wider, its performance reduces and the damping ratio increases. For this reason, case 1 with smaller gap-size exhibits better performance since it has larger damping ratio and as a result less overshoot.

#### 4.2 Effect of $\epsilon$ on damping-ratio

Here, we are showing the pole locations for

the same values of  $[k_P, k_I, k_D]$  with respect to  $\epsilon$  but by varying  $\mu$  values from -0.5 to 0.5.

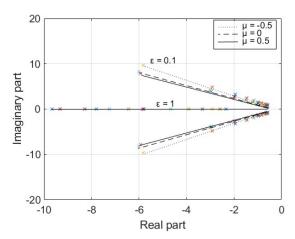


Figure 3. Root-loci of T(s) when  $[k_P, k_I, k_D]$ =[8, 6, 7] as  $\mu$ =[-0.5, 0, 0.5] with respect to  $\epsilon$ .

From Fig. 3 above, as  $\epsilon$  decreases from 1 to 0,  $\sigma$  increases and therefore  $T_s$  decreases when  $\mu$  is incremented irrespective of  $[k_P, k_I, k_D]$  values. Also, it can be observed that  $\epsilon$  has no effect on the damping ratio as a result no effect on overshoot in the presence disturbance  $(\mu)$ .

#### 4.3 Effect of $\epsilon$ on steady-state error

In the unity feedback control diagram with unit-step input for which  $Q_d(s) = \alpha/s$ , the application of the final value theorem to the error formula with respect to  $\epsilon$  is expressed as

$$e_{ss} = \lim_{s \to 0} sE(s). \tag{27}$$

Let W(s) be a lamp function which is  $\beta/s^2$  and (9) becomes

$$E(s) = \left[ \frac{s^2 + (\bar{a} + \delta a)s}{D(s)(\bar{b} + \delta b) + s^2 + (\bar{a} + \delta a)s} \right] \frac{\alpha}{s} + \left[ \frac{c}{D(s)(\bar{b} + \delta b) + s^2 + (\bar{a} + \delta a)s} \right] \frac{\beta}{s^2}.$$
 (28)

Inserting (28) into (27) gives

$$e_{ss} = \lim_{s \to 0} s \left[ \frac{s^2 + (\bar{a} + \delta a)s}{D(s)(\bar{b} + \delta b) + s^2 + (\bar{a} + \delta a)s} \right] \frac{\alpha}{s}$$

$$+ \lim_{s \to 0} s \left[ \frac{c}{D(s)(\bar{b} + \delta b) + s^2 + (\bar{a} + \delta a)s} \right] \frac{\beta}{s^2}.$$
 (29)

Put  $D(s) = (K_P(\epsilon) + k_I(\epsilon)/s + k_D(\epsilon)s)$  into (29) and the first part of the equation is eliminated as  $s \rightarrow 0$ . Then, (29) becomes

$$\begin{split} e_{ss} = &\lim_{s \to 0} \left[ \frac{c}{s^2 + (\bar{a} + \delta a)s + \left[ \frac{k_P}{\bar{b}\epsilon^2} + \frac{k_I}{\bar{b}\epsilon^3 s} + \frac{k_D s}{\bar{b}\epsilon} - \frac{\bar{a}s}{\bar{b}} \right] (\bar{b} + \delta b)} \right] \frac{\beta}{s}, \\ e_{ss} = &\lim_{s \to 0} \left[ \frac{c\beta}{s^3 + (\bar{a} + \delta a)s^2 + \left[ \frac{k_P s}{\bar{b}\epsilon^2} + \frac{k_I}{\bar{b}\epsilon^3} + \frac{k_D s^2}{\bar{b}\epsilon} - \frac{\bar{a}s^2}{\bar{b}} \right] (\bar{b} + \delta b)} \right], \\ e_{ss} = &\frac{c\beta \bar{b}\epsilon^3}{(\bar{b} + \delta b)k_I}. \end{split} \tag{32}$$

As a result, (32) represents the relationship between steady-state error  $(e_{ss})$  and the gain scaling factor  $(\epsilon)$ .

#### 4.3.1 Observation

- The table above shows that  $\epsilon$  is directly proportional to  $e_{ss}$ . As  $\epsilon$  increases,  $e_{ss}$  also increases and vice-versa. So, the disturbance effect caused by the step function W(s) from Fig.1 can be reduced by  $\epsilon$ . Thus, robustness against uncertain load torque is also achieved.
- Also, as  $\epsilon$  decreases, the angular frequency,  $w_n$  increases as a result settling time,  $T_s$  decreases.

#### V. Summary of the analysis and design rule

- 1. Select  $k_P$ ,  $k_I$ ,  $k_D$  such that  $s^3 + k_D + k_P s + k_I = 0$  is a Hurwitz polynomial which provides a way of robust design in the presence of input uncertainty ( $\delta b$  term). Recall that  $|\mu| < 0$ .
- 2. Since  $\epsilon$  is in the range of (0,1) such that  $\epsilon \ll k_D$ . Recall that the disturbance effect caused by W(s) can be reduced in  $O(\epsilon^3)$ . The selection of  $\epsilon$  provides a way to robustly attenuate the parameter uncertainty and the unknown load torque. However, there is a practical limitation as noted in [2] that  $\epsilon$

- should not be smaller than its saturation level.
- 3. Since we want to make the behavior of the proposed controller to be governed by dominant poles, the effect of the third pole should be negligible. So,  $\zeta$  and  $w_n$  are determined once  $k_P, k_I, k_D$  are selected. Recall that  $\zeta$  does not affect the overshoot much whereas it mainly affects  $T_s$  and  $e_{ss}$ .

#### VI. Simulation Results

First, we consider the system parameter values from [10] where  $J_m = 0.0000134 \, \mathrm{Kgm^2}$ ,  $R = 1.16 \omega$ , r = 1,  $B_m = 2.68042 \times 0^{-5} \, \mathrm{Nms}$ ,  $K_m = 0.060438586 \, \mathrm{Nm/A}$  and  $K_b = 0.0603 \, \mathrm{Vs}$ .

## 6.1 Control result when the PID gains $k_P$ , $k_I$ , $k_D$ with no Disturbance

Five cases of the PID gains  $k_P$ ,  $k_I$ ,  $k_D$  have been taken into consideration with  $\epsilon$  equal to unity. The measured output response Q(t) is observed as shown in Fig. 4 below. The reference position  $Q_d(t)$  is set to 50°.

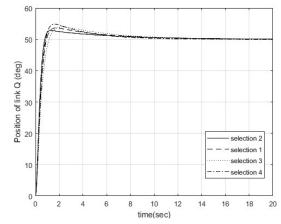


Figure 4. Control results with various values of  $k_P$ ,  $k_I$ ,  $k_D$  when  $\epsilon=1$  with no disturbance.

The table below provides the summarized results.

In Fig. 4, we observe that:

• Selection 2 has the lowest overshoot( $M_p$ ) and

fast response (fast settling time,  $T_s$ ). This is considered to be the most robust system (best case).

- Selections 4 has the highest amount of overshoot with faster responses, T<sub>s</sub> is smaller.
- ullet Lastly, selections 1 and 3 have little amount of overshoot and slower responses,  $T_s$  is larger.

Table 2. Summary of Overshoot and Settling-time for all the selected PID parameters

| Selections | $k_P,k_I,k_D$ | $M_p(\%)$ | $T_s(\mathrm{sec})$ |
|------------|---------------|-----------|---------------------|
| 1          | 24, 5, 10     | 7.23      | 7.6                 |
| 2          | 29, 5, 10     | 5.63      | 6.88                |
| 3          | 24, 5, 12     | 7.88      | 8.54                |
| 4          | 24, 7, 10     | 9.68      | 6.59                |

### 6.2 Control result when considering the Uncertainties $\delta a$ and $\delta b$

In the figure below, we can see the deviations that occur when selecting various sets of PID gains. This observation corresponds to our first controller design rule which explains how to select  $k_P$ ,  $k_I$ ,  $k_D$ . Consider the transfer function in (14), computing the root-locus and we obtain the deviations below for several selections of  $k_P$ ,  $k_I$ ,  $k_D$ .

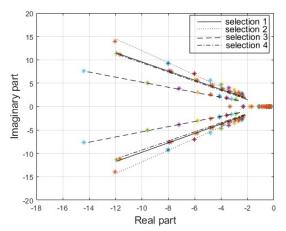


Figure 5. Root loci of T(s) with various selections of  $k_P,\,k_I,\,k_D$  with disturbance when  $0<\epsilon<1$ 

#### 6.3 Control result with respect to Disturbance

In this subsection, we want to prove that  $\epsilon$  is

directly proportional to  $e_{ss}$  even when disturbance is present in our control system. This, therefore, coincides with (32). For our control system to be a robust system, we want no more than 10% of uncertainty. Consider the following control results. As expected,  $e_{ss}$  become smaller as  $\epsilon$  is reduced.

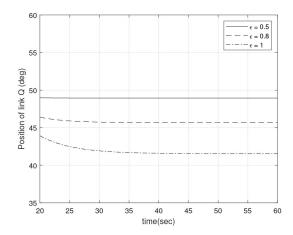


Figure 6. Control results with decreasing values of  $\epsilon$  with  $k_P$ ,  $k_I$ ,  $k_D$ =[29, 5, 10] (selection 2).

Also, from Fig. 6, we can compare our  $\epsilon$ -PID method with traditional PID controller( $\epsilon$ =1) [1]. The results show the validity of  $\epsilon$ -PID method.

#### VII. Conclusion

This paper consists of achieving a good robust tracking performance of the proposed controller by providing a simplified systematic gain design method using frequency response analysis in the presence of some parameter uncertainties. We started by utilizing the state-space equation of the DC motors (MAXON DC motors, RE35(11879)) of [2] by finding its Laplace transform. Then with available information on nominal parameters, we obtain a PID controller with a gain-scaling factor  $\epsilon > 0$ . Finally, we varied the disturbance value to observe the effect of  $\epsilon > 0$  on steady-state error, damping ratio, and overshoot. Notably, our control scheme with analysis and the simulation results are in clear agreement.

#### References

[1] G. F. Franklin, J. D. Powell, and A. Emami-Naeini. *Feedback Control of Dynamic Systems.* Pearson, 2020.

[2] L. H. Keel. "Analytical design of pid controllers [bookshelf]," *IEEE Control Systems Magazine*, vol. 41, pp.80-81, 2021.

DOI: 10.1109/MCS.2020.3032803

[3] H. K Khalil. Nonlinear Systems. 2002.

[4] O. A. Somefun, K. Akingbade, and F. Dahunsi. "The dilemma of pid tunning," *Annual Reviews in Control*, vol.52, pp.65-74, 2021.

https://doi.org/10.1016/j.arcontrol.2021.05.00

[5] R. P Borase, D. K Maghade, S. Y Sondkar, and S. N. Pawar. "A review of pid control, tunning methods and applications," *International Journal of Dynamics Control*, vol.09, pp.818-827, 2021.

DOI: 10.1007/s40435-020-00665-4

[6] H.-L. Choi and J.-T. Lim. "On robust position control of dc motors by  $\epsilon$ -pid controller and its application to humanoid robot arms," *17th IFAC World Congress (IFAC'08)*, vol.41, no.2, pp.14449-14453. 2008.

DOI: 10.3182/20080706-5-KR-1001.1574

[7] H.-L. Choi and J.-T. Lim. "Improved controller design on robust approximate feedback linearization via lmi approach," *IEICE Trans. Fundamentals*, vol.E88-A, no.07, pp.2023-2025, 2005.

DOI: 10.1093/ietfec/e88-a.7.2023

[8] J.-S. Ko, J.-H. Lee, and M.-J. Youn. "Robust digital position control of brushless dc motor with adaptive load torque observer." *IEE Proc.-Electr, Power Appl.*, vol.141, pp.63-70, 1994.

DOI: 10.1049/ip-epa:1994985

[9] H. Lei and W. Lin. "Adaptive regulation of uncertain nonlinear systems by output feedback: A universal control approach," *Systems Control Letters*, vol.56, pp.529-537, 2007.

DOI: 10.1016/j.sysconle.2007.03.002

[10] Z. Qu and Y. Jin. "Robust control of nonlinear systems in the presence of unknown exogenous

dynamics," 40th IEEE Conference on Decision and Control, vol.48, pp.336-343, 2003.

#### **BIOGRAPHY**

#### Alhassan Dodo Adamou Soudeize (Member)



2021: BSC degree in Electrical and Electronic Engineering, Islamic University of Technology (IUT). 2023~present: MS degree in Electrical Engineering, Dong-A University.

#### Ji-Sun Park (Member)



2019: BS degree in Electrical Engineering, Dong-A University. 2021: MS degree in Electrical Engineering, Dong-A University. 2021~present: PhD degree in Electrical Engineering, Dong-A University.

#### Ho-Lim Choi (Member)



Received the B.S.E. degree from the department of electrical engineerting, The Univ. of Iowa, USA in 1996, and M.S. degree in 1999 and Ph.D degree in 2004, from KAIST, respectively. Curtrently, he is a professor at department of electrical engineering, Dong-A university,

Busan. His research interests are in the nonlinear control problems with emphasis on feedback linearization, gain scheduling, singular perturbation, output feedback, time-delay systems, time-optimal control. He is a senior member of IEEE.