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SOFT PMS-ALGEBRAS

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Abstract. In this paper, the concepts of soft PMS-algebras, soft PMS-subalgebras, soft PMS-ideals, and idealistic soft PMS-algebras are introduced, and their properties are studied. The restricted intersection, the extended intersection, union, AND operation, and the cartesian product of soft PMS-algebras, soft PMS-subalgebras, soft PMS-ideals, and idealistic soft PMS-algebras are established. Moreover, the homomorphic image and homomorphic pre-image of soft PMS-algebras are also studied.

1. Introduction

Molodtsov [12] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. Also, he suggested that a variety of applications of soft sets in [12]. At present, work on soft set theory is progressing rapidly. Maji et al. [11] described the applications of soft set theory in a-decision making problems. In theoretical aspects, Maji et al. [10] defined several operations on soft sets, such as intersection, union, "AND" operation, and "OR" operation of soft sets. Hence, M. Irfan Ali [4] established the binary operations known as restricted intersection and extended intersection of soft sets to ensure that the intersection of two soft sets is free from the problems raised in those works.

Aktaş and Çagman [2] studied the basic concepts of soft set theory. They also discussed the notion of soft sets and soft groups in [2]. Thus, the algebraic structure of soft set theories dealing with uncertainties have also been studied by some authors. Some of them are Aktaş and Çagman [2] defined soft groups and obtained some properties of these groups. Jun et al. [7] defined soft sets on BCK/BCI-algebras with their detailed properties. In [8], Y. B. Jun, K. J. Lee, and J. Zhan introduced the notion of soft p-ideals and p-idealistic soft BCI-algebras and investigated their properties. Also, Y. B. Jun and C. H. Park [9] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They discussed the algebraic properties of soft sets in BCK/BCI-algebras. Qiu-Mei Sun et al. [15] defined the concept of soft modules and

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studied their basic properties. Feng et al. [3] defined soft semirings, soft ideals on soft semirings. F. Koyuncu et al [1] defined soft rings, soft ideals on soft rings, and idealistic soft rings.

In 1966, Y. Imai and K. Iseki [6] and in 1980, K. Iseki [5] introduced the two classes of abstract algebras, BCK-algebra and BCI-algebra respectively. In 2016, Sithar Selvam and Nagalakshmi [14] introduced a new algebraic structure called PMS-algebra, which is a generalization of the notions of BCK and BCI-algebras. And also, in 2016, Sithar Selvam and Nagalakshmi [13] studied fuzzy PMS-algebras and fuzzy PMS-ideals with their detailed properties.

This study was initiated to introduce the basic notions of soft PMS algebras, soft PMS subalgebras, soft PMS ideals, and idealistic soft PMS algebras over a PMS algebra, and we deal with the algebraic structure of PMS algebras by applying soft set theory. We discussed the algebraic properties of soft sets in PMS-algebras, which are called soft PMS-subalgebras. A soft PMS-algebra is a parameterized family of PMS-subalgebras. Also, we established the intersection, union, "AND" operation, and "OR" operation of soft PMS-algebras, soft PMS-subalgebras, soft PMS-ideals, and idealistic soft PMS-algebras.

Throughout this paper, $\mathcal{X}$ represents PMS-algebra.

2. Preliminaries

In this section, we will address the concepts and basic properties of PMS-algebras and soft sets that we need for the main results in the next section.

Definition 2.1 ([14]). A PMS-algebra is an algebra $(\mathcal{X},\ *,\ 0)$ of type $(2,\ 0)$ with a constant "0" and a binary operation " $*$ " satisfying the following axioms:

1. $0 * a = a$
2. $(b * a) * (c * a) = c * b$, for all $a,\ b,\ c \in \mathcal{X}$.

Proposition 2.2 ([14]). *In a PMS-algebra $(\mathcal{X},\ *,\ 0)$ the following properties hold for all $a,\ b,\ c\ \in\ \mathcal{X}$.*

1. $a\ *\ a\ =\ 0$
2. $(b * a)\ *\ a\ =\ b$
3. $a\ *\ (b\ *\ a)\ =\ b\ *\ 0$
4. $(b\ *\ a)\ *\ c\ =\ (c\ *\ a)\ *\ b$
5. $(a\ *\ b)\ *\ 0\ =\ b\ *\ a\ =\ (0\ *\ b)\ *\ (0\ *\ a)$.

Definition 2.3 ([14]). Let $\mathcal{S}$ be a nonempty subset of a PMS-algebra $\mathcal{X}$. Then $\mathcal{S}$ is called PMS subalgebra of $\mathcal{X}$ if $a\ *\ b\ \in\ \mathcal{S}$, for all $a,\ b\ \in\ \mathcal{S}$.

Example 2.4 ([14]). Let $\mathcal{X} = \mathbb{Z}$ be the set of integers, and let " $*$ " be a binary operation on $\mathbb{Z}$ defined by $a\ *\ b\ =\ b - a$, for all $a,\ b\ \in\ \mathcal{X}$, where " $-$ " is the usual subtraction. Then $(\mathcal{X},\ *,\ 0)$ is a PMS-algebra, and the set E of even integers is PMS-sub algebra of a PMS-algebra $\mathbb{Z}$.

Definition 2.5 ([14]). Let $\mathcal{X}$ and $\mathcal{Y}$ be PMS-algebras. Then a mapping $f \ : \ \mathcal{X} \longrightarrow \mathcal{Y}$ is called a homomorphism if
$f(a * b) = f(a) * f(b)$, for all $a, \ b \in \mathcal{X}$.
For a homomorphism $f \ : \ \mathcal{X} \longrightarrow \mathcal{Y}$ of PMS-algebras,

1. the kernel of f, denoted by Ker(f), is defined to be the set Ker$(f) = \{a \in \mathcal{X} \mid f(a) = 0\}$.
2. f is a monomorphism if it is injective.
3. f is an epimorphism if it is surjective.

Definition 2.6 ([13]). Let $\mathcal{X}$ be a PMS-algebra. A fuzzy set $\mu \ : \ \mathcal{X} \longrightarrow [0, \ 1]$ is called a fuzzy PMS-subalgebra of $\mathcal{X}$ if $\mu(a * b) \geq min\{\mu(a), \ \ \mu(b)\}$, for all $a, \ b \in \mathcal{X}$.

Let U be an initial universe set, and E be a set of parameters with respect to U. Let $\mathcal{P}(U)$ denotes the power set of U and A be a nonempty subset of E. Then

Definition 2.7 ([12]). A pair $(\mathcal{F}, \ A)$ is called a soft set over a universe set U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} \ : \ A \ \longrightarrow \mathcal{P}(U)$.

Definition 2.8 ([10]). Let $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ be soft sets over a common universe set U. The intersection of $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ is defined to be the soft set $(\mathcal{H}, \ C)$ where $\mathcal{H}$ is a mapping given by $\mathcal{H} : C \longrightarrow \mathcal{P}(U)$ satisfying the following conditions:

1. $C = A \cap B$
2. If C is non-empty and $\mathcal{F}(e) = \mathcal{G}(e)$ for every $e \in C$, then $\mathcal{H}(e) = \mathcal{F}(e)$

In this case, we write $(\mathcal{F}, \ A) \ \widetilde{\cap} \ (\mathcal{G}, \ B) = (\mathcal{H}, \ C)$.

Definition 2.9 ([10]). Let $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ be soft sets over a common universe set U. The union of $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ is defined to be the soft set $(\mathcal{H}, \ C)$, where $\mathcal{H}$ is a mapping giving by $\mathcal{H} : C \longrightarrow \mathcal{P}(U)$ satisfying the following conditions:

1. $C = A \cup B$,
2. for all e $\in C$,

$$\mathcal{H}(e) = \begin{cases} \mathcal{F}(e), \ if \ e \in A - B \\ \mathcal{G}(e), \ if \ e \in B - A \\ \mathcal{F}(e) \ \cup \ \mathcal{G}(e), \ if \ e \in A \cap B \end{cases}$$

In this case, we write $(\mathcal{F}, \ A) \ \widetilde{\cup} \ (\mathcal{G}, \ B) = (\mathcal{H}, \ C)$.

Definition 2.10 ([10]). Let $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ be soft sets over a common universe set U. "$(\mathcal{F}, \ A)$ AND $(\mathcal{G}, \ B)$" denoted by $(\mathcal{F}, \ A) \ \widetilde{\wedge} \ (\mathcal{G}, \ B)$ read as $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ is defined by $(\mathcal{F}, \ A) \ \widetilde{\wedge} \ (\mathcal{G}, \ B) = (\mathcal{H}, \ A \times B)$, where $\mathcal{H}(x, \ y) = \mathcal{F}(x) \cap \mathcal{G}(y)$, for all $(x, \ y) \in A \times B$.

Definition 2.11 ([10]). Let $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ be soft sets over a common universe set U. "$(\mathcal{F}, \ A)$ OR $(\mathcal{G}, \ B)$" denoted by $(\mathcal{F}, \ A) \ \widetilde{\vee} \ (\mathcal{G}, \ B)$ read as $(\mathcal{F}, \ A)$ or $(\mathcal{G}, \ B)$ is defined by $(\mathcal{F}, \ A) \ \widetilde{\vee} \ (\mathcal{G}, \ B) = (\mathcal{H}, \ A \times B)$, where $\mathcal{H}(x, \ y) = \mathcal{F}(x) \cup \mathcal{G}(y)$, for all $(x, \ y) \in A \times B$.

Definition 2.12 ([10]). For soft sets $(\mathcal{F}, \ A)$ and $(\mathcal{G}, \ B)$ over a common universe U, we say that $(\mathcal{F}, \ A)$ is a soft subset of $(\mathcal{G}, \ B)$, denoted by $(\mathcal{F}, \ A) \ \widetilde{\subset} \ (\mathcal{G}, \ B)$ if it satisfies:

1. $A \subset B$,
2. For every $e \in A$, $\mathcal{F}(e) = \mathcal{G}(e)$.

Definition 2.13 ([10]). Let $(\mathcal{F},\ A)$ be a soft set over $\mathcal{X}$ and $(\mathcal{G},\ B)$ be a soft set over $\mathcal{Y}$. The cartesian product of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ is defined as the soft set $(\mathcal{H},\ A\times B) = (\mathcal{F},\ A)\times(\mathcal{G},\ B)$, where $\mathcal{H}(x,\ y) = \mathcal{F}(x)\times\mathcal{G}(y)$, for all $(x,\ y)\in A\times B$.

Definition 2.14 ([4]). The restricted intersection of soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe set U denoted by $(\mathcal{F},\ A)\ \widetilde{\sqcap}\ (\mathcal{G},\ B)$ is defined as the soft set $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \widetilde{\sqcap}\ (\mathcal{G},\ B)$, where $C = A\cap B\neq\emptyset$ and $\mathcal{H}(e) = \mathcal{F}(e)\ \cap\ \mathcal{G}(e)$ for all $e\in C$.

Definition 2.15 ([4]). The extended intersection of soft sets $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ over a common universe set U is the soft set $(\mathcal{H},\ C)$, where $C = A\cup B$, and $\forall\ e\ \in\ C$,

$$\mathcal{H}(e) = \begin{cases} \mathcal{F}(e)\,,\ if\ e\in A-B \\ \mathcal{G}(e)\,,\ if\ e\in B-A \\ \mathcal{F}(e)\cap\mathcal{G}(e)\,,\ if\ e\in A\cap B \end{cases}$$

In this case, we write $(\mathcal{F},\ A)\ \widetilde{\cap_E}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$.

Definition 2.16 ([3]). Let $(\mathcal{F},\ A)$ be a soft set over a universe set U. The set Supp $(\mathcal{F},\ A) = \{x\in A \mid \mathcal{F}(x)\neq\emptyset\}$ is called the support of the soft set $(\mathcal{F},\ A)$.

A soft set is said to be non -null if its support is nonempty.

3. Soft PMS-algebras

Let $\mathcal{X}$ be a PMS-algebra and A be a nonempty set. Let R refers to an arbitrary binary relation between an element of A and an element of $\mathcal{X}$ (i.e., R is a subset of $A\times\mathcal{X}$ unless otherwise specified). A set-valued function $\mathcal{F}:\ A\longrightarrow\mathcal{P}(\mathcal{X})$ can be defined as $\mathcal{F}(x) = \{\ y\in\mathcal{X}\mid x\ R\ y\ \}$ for all $x\ \in\ A$. The pair $(\mathcal{F},\ A)$ is a soft set over $\mathcal{X}$.
For any element x of a PMS-algebra $\mathcal{X}$ the order of x, denoted by $o(x)$, is defined as: $o(x) = \min\{\ n\ \in\ N\mid 0\ *\ x^n\ =\ 0\ \}$, where $x^n = x*x*x*\cdots*x$ in which x appears n times.

Definition 3.1. Let $\mathcal{X}$ be a PMS-algebra and $(\mathcal{F},\ A)$ be a non-null soft set over $\mathcal{X}$. Then $(\mathcal{F},\ A)$ is called a soft PMS-algebra over $\mathcal{X}$ if $\mathcal{F}(x)$ is a PMS subalgebra of $\mathcal{X}$, for all $x\in Supp(\mathcal{F},\ A)$.

Let us illustrate this definition using the following examples.

Example 3.2. Let $\mathcal{X} = \{0,\ 1,\ 2,\ 3\}$ be a set with the following table.

$*$	0	1	2	3
0	0	1	2	3
1	2	0	1	2
2	1	2	0	1
3	3	1	2	0

Then $(\mathcal{X},\ *,\ 0)$ is a PMS-algebra.
Let $(\mathcal{F},\ A)$ be a soft set over $\mathcal{X}$, where $A=\mathcal{X}$ and $\mathcal{F}:\ A\longrightarrow\mathcal{P}(\mathcal{X})$ is a set-valued function given as follows:
Let x$\in A$. Define $\mathcal{F}(x) = \{y\in\mathcal{X}\mid xRy\Leftrightarrow(x*(x*y))\in\{0,\ 1,\ 2\}\}$. Then

$\mathcal{F}(0) = \{0,\ 1,\ 2\}$, $\mathcal{F}(1) = \mathcal{X}$, $\mathcal{F}(2) = \mathcal{X}$ and $\mathcal{F}(3) = \{0,\ 1,\ 2\}$ are PMS-subalgebras of $\mathcal{X}$. Therefore, $(\mathcal{F},\ A)$ is a soft PMS-algebra over $\mathcal{X}$.

Example 3.3. Consider the PMS-algebra $\mathcal{X} = \{0,\ a,\ b,\ c\}$ with the following table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $A = \mathcal{X}$.

1. $\mathcal{F} : A \longrightarrow \mathcal{P}(\mathcal{X})$ be a set-valued function given as follows:
   Let $x \in A$. Define $\mathcal{F}(x) = \{y \in \mathcal{X} \mid x\ R\ y \iff y = x^n,\ n \in N\}$. Then $\mathcal{F}(0) = \{0\}$, $\mathcal{F}(a) = \{0,\ a\}$, $\mathcal{F}(b) = \{0,\ b\}$ and $\mathcal{F}(c) = \{0,\ c\}$ which are PMS-subalgebras of $\mathcal{X}$. Hence $(\mathcal{F},\ A)$ is a soft PMS-algebra.
2. $\mathcal{H} : A \longrightarrow \mathcal{P}(\mathcal{X})$ be a set-valued function given as follows:
   Let $x \in A$. Define $\mathcal{H}(x) = \{y \in X \mid x\ R\ y \iff o(x) = o(y)\}$. Then $\mathcal{H}(0) = \{0\}$ is a PMS-subalgebra of $\mathcal{X}$, but
   $\mathcal{H}(a) = \mathcal{H}(b) = \mathcal{H}(c) = \{a,\ b,\ c\}$ is not a PMS-subalgebra of $\mathcal{X}$. Therefore, $(\mathcal{H},\ A)$ is not a soft PMS-algebra over $\mathcal{X}$.

Let A be a fuzzy PMS-subalgebra of $\mathcal{X}$ with membership function μ_A. Let us consider the family of α -level sets for the function μ_A. Let $\alpha \in [0,1]$, define $\mathcal{F}(\alpha) = \{x \in \mathcal{X} \mid \mu_A(x) \geq \alpha\}$. Then, $\mathcal{F}(\alpha)$ is a PMS-subalgebra of $\mathcal{X}$. We can find the functions $\mu_A(x)$ by means of the following formula:

$$\mu_A(x) = sup\,\{\alpha \in [0,\ 1] \mid x \in \mathcal{F}(\alpha)\}.$$

Thus, every fuzzy PMS-sub algebra A may be considered as the soft PMS algebra $(\mathcal{F},\ [0,\ 1])$.

Theorem 3.4. *Let $(\mathcal{F},\ A)$ be a soft PMS-algebra over $\mathcal{X}$. If B is a subset of A, then $(\mathcal{F}|_B,\ B)$ is a soft PMS-algebra over $\mathcal{X}$.*

Proof. Let $B \subseteq A$ and let $x \in Supp(\mathcal{F}|_B,\ B)$. Then $\mathcal{F}_B(x) \neq \emptyset$. Since $\mathcal{F}(x)$ is PMS-subalgebra of $\mathcal{X}$ for all $x \in A$ and $\mathcal{F}|_B(x) \subseteq \mathcal{F}(x), \forall x \in B$, then $\mathcal{F}|_B(x)$ is PMS -subalgebra of $\mathcal{X}$.
Hence, $(\mathcal{F}|_B,\ B)$ is a soft PMS-algebra. □

The converse of Theorem 3.4 is not true.
The following example shows that there exists a soft set $(\mathcal{F}|_B,\ B)$ over $\mathcal{X}$ such that $(\mathcal{F},\ A)$ is not a soft PMS-algebra over $\mathcal{X}$. There exists a subset B of A such that $(\mathcal{F},\ B)$ is a soft PMS-algebra over $\mathcal{X}$.

Example 3.5. Consider a PMS-algebra $\mathcal{X} = \{0,\ a,\ b,\ c\}$ with the following table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let $(\mathcal{F},\ A)$ be a soft set over $\mathcal{X}$ and $A = \mathcal{X}$ and let $\mathcal{F}:\ A \longrightarrow \mathcal{P}(\mathcal{X})$ be a set-valued function defined as follows:
Let $x \in A$. Define $\mathcal{F}(x) = \ \{y \in X \mid x\ R\ y \Longleftrightarrow y * (y *\ x) \in \{0,\ a,\ b\}\}$.
$(\mathcal{F},\ \ A)$ is not a soft PMS-algebra over $\mathcal{X}$. But if we take $B\ =\ \{a,\ b\}\ \subset\ A$, then $(\mathcal{F},\ B)$ is a soft PMS-algebra over $\mathcal{X}$.

THEOREM 3.6. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be soft PMS-algebras over $\mathcal{X}$. If $A \cap B \neq \emptyset$, then the intersection $(\mathcal{F},\ A)\ \widetilde{\cap}\ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$, where $F(x) = G(x)$ for all $x \in A \cap B$.*

Proof. Let $(\mathcal{F},\ A)\ \widetilde{\sqcap}\ (\mathcal{G},\ B)\ =\ (\mathcal{H},\ C)$. Since $\mathcal{F}(x)$ is a PMS-subalgebra of $\mathcal{X}$ as $(\mathcal{F},\ A)$ is a soft PMS-algebra and $\mathcal{H}(x) = \mathcal{F}(x), \forall x \in A \cap B$. Hence, $\mathcal{H}(x)$ is a PMS-subalgebra of $\mathcal{X}$.
Therefore, $(\mathcal{H},\ A \cap B)$ is a soft PMS-algebra over $\mathcal{X}$. □

THEOREM 3.7. *If $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are soft PMS-algebras over $\mathcal{X}$, then the soft set $(\mathcal{F},\ A)\ \widetilde{\wedge}\ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$.*

Proof. By DEFINITION 2.10, $(\mathcal{F},\ A)\widetilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H}, C)$, where $C = A \times B$ and $\mathcal{H}(x,\ y) = \mathcal{F}(x) \cap \mathcal{G}(y)$ for all $(x,\ y) \in A \times B$. Let $(x,\ y) \in Supp(H,\ C)$. Then $\mathcal{H}(x,\ y) \neq \emptyset$ and $(\mathcal{H},\ C)$ is a non-null soft set over $\mathcal{X}$. If $(x,\ y) \in Supp\,(\mathcal{H}, C)$, then $\mathcal{H}(x, y) = \mathcal{F}(x) \cap \mathcal{G}(y) \neq \emptyset$. It follows that the non-empty sets $\mathcal{F}(x)$and $\mathcal{G}(y)$ are both PMS-subalgebras of $\mathcal{X}$. Hence $H(x, y)$ is a PMS-subalgebra of $\mathcal{X}$ for all $x \in Supp\,(\mathcal{H},\ C)$, and so $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \widetilde{\wedge}\ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$. □

THEOREM 3.8. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be soft PMS-algebras over $\mathcal{X}$. Then the restricted intersection $(\mathcal{F},\ A)\widetilde{\sqcap}\ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$ whenever $A \cap B$ is nonempty.*

Proof. By DEFINITION 2.14, we can write $(\mathcal{F}\ ,\ A)\widetilde{\sqcap}(\mathcal{G}\ ,\ B) = (\mathcal{H}, A \cap B)$, where $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x)$ for all $x \in A \cap B$. Suppose that $(\mathcal{H}, A)$ is a soft set over X. If $x \in Supp\,(\mathcal{H}, (A \cap B))$, then $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x) \neq \emptyset$. It follows that the non-empty sets $\mathcal{F}(x)$and $\mathcal{G}(x)$ are both PMS-subalgebras of X. Hence $\mathcal{H}(x)$ is a PMS-subalgebra of $\mathcal{X}$ for all $x \in Supp(\mathcal{H},\ A)$and
$(\mathcal{F}\ ,\ A)\widetilde{\sqcap}(\mathcal{G},\ B) = (\mathcal{H},\ \ A \cap B)$, is a soft PMS-algebra over $\mathcal{X}$. □

COROLLARY 3.9. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ be soft PMS-algebras over $\mathcal{X}$. Then the restricted intersection $(\mathcal{F},\ A)\widetilde{\sqcap}\ (\mathcal{G},\ A)$ is a soft PMS-algebra over $\mathcal{X}$.*

THEOREM 3.10. *Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be soft PMS-algebras over $\mathcal{X}$. Then the extended intersection $(\mathcal{F},\ A)\ \ \widetilde{\cap_E}\ \ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$.*

Proof. By DEFINITION 2.15, we can write $(\mathcal{H},\ C) = (\mathcal{H},\ A)\widetilde{\cap_E}(\mathcal{G},\ B)$, where $C = A \cup B$ and $\mathcal{H}(x) = \mathcal{F}(x)$ if $x \in A - B$, $\ \mathcal{H}(x) = \mathcal{G}(x)$ if $x \in B - A$ and $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x)$ if $x \in A \cap B$.
Hence, $\mathcal{F}(x)$and $\mathcal{G}(x)$ are PMS-subalgebras of $\mathcal{X}$ because of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are soft PMS-algebras and the intersection of two PMS-subalgebras is again a PMS-subalgebras. Thus $\mathcal{H}(x)$ is PMS-subalgebras of $\mathcal{X}$ and so $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \widetilde{\cap_E}(\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$. □

COROLLARY 3.11. *If $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ are soft PMS-algebras over a PMS-algebra $\mathcal{X}$, then the extended intersection of $(\mathcal{F},\ A)$ and $(\mathcal{G},\ A)$ is a soft PMS-algebra over a PMS-algebra $\mathcal{X}$.*

THEOREM 3.12. *Let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{X}$. *If* $A \cap B = \emptyset$, *then the union of* $(\mathcal{F},\ A)\ \widetilde{\cup}\ (\mathcal{G},\ B)$ *is a soft PMS-algebra over* $\mathcal{X}$.

Proof. Using DEFINITION 2.9, we can write $(\mathcal{H},\ C) = (\mathcal{F},\ A)\widetilde{\cup}(\mathcal{G},\ B)$, where $C = A \cup B$ and for all $x \in C$,

$$\mathcal{H}(x) = \begin{cases} \mathcal{F}(x), & if\ x \in A - B \\ \mathcal{G}(x), & if\ x \in B - A \\ \mathcal{F}(x) \cup \mathcal{G}(x), & if\ x \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, either $x \in A - B$ and $x \in B - A$ for all $x \in C$. If $x \in A - B$, then $\mathcal{F}(x)$ is a PMS-subalgebra of $\mathcal{X}$ because of $(\mathcal{F},\ A)$ is a soft PMS-algebra over $\mathcal{X}$. If $x \in B - A$, then $\mathcal{G}(x)$ is a PMS-subalgebra of $\mathcal{X}$ because of $(\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$. Hence $(\mathcal{H},\ C) = (\mathcal{F},\ A)\ \widetilde{\cup}\ (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X}$. □

THEOREM 3.13. *Let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{X}$ *and* $\mathcal{Y}$, *respectively. Then the cartesian product* $(\mathcal{F},\ A) \times (\mathcal{G},\ B)$ *is a soft PMS-algebra over* $\mathcal{X} \times \mathcal{Y}$.

Proof. $(\mathcal{F},\ A) \times (\mathcal{G},\ B) = (\mathcal{H}, C)$, where $C = A \times B$ and $\mathcal{H}(x,\ y) = \mathcal{F}(x) \times \mathcal{G}(y)$. Let $a = (x,\ y) \in Supp(\mathcal{H},\ C)$. Then $\mathcal{H}(x,\ y) \neq \emptyset$ for all $(x,\ y) \in C$. Since $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ are soft PMS-algebras, then $\mathcal{F}(x)$ and $\mathcal{G}(y)$ are PMS-subalgebras of $\mathcal{X}$ and $\mathcal{Y}$ rspectively for all $x \in A$ and for all $y \in B$. So we have that $\mathcal{F}(x) \times \mathcal{G}(y)$ is a PMS-subalgebra of $\mathcal{X} \times \mathcal{Y}$. Hence, the cartesian product $(\mathcal{F},\ A) \times (\mathcal{G},\ B)$ is a soft PMS-algebra over $\mathcal{X} \times \mathcal{Y}$. □

DEFINITION 3.14. Let $\mathcal{X}$ and $\mathcal{Y}$ be PMS algebras and $f:\ \mathcal{X} \longrightarrow \mathcal{Y}$ be a mapping of PMS-algebras. Let $(\mathcal{F},\ A)$ be a soft set over $\mathcal{X}$ and $(\mathcal{G},\ B)$ be a soft set over $\mathcal{Y}$. Then

1. $(f(\mathcal{F}),\ A)$ is a soft set over $\mathcal{Y}$, where $f(\mathcal{F}):\ A \longrightarrow \mathcal{P}(\mathcal{Y})$ is defined by $f(\mathcal{F})(x) = f(\mathcal{F}(x))$ for all $x \in A$.
2. $(f^{-1}(G),\ B)$ is a soft set over $\mathcal{X}$, where $f^{-1}(G):\ B \longrightarrow \mathcal{P}(\mathcal{X})$ is defined by $f^{-1}(G)(y) = f^{-1}(G(y))$ for all $y \in B$.

THEOREM 3.15. *et* $f:\ \mathcal{X} \longrightarrow \mathcal{Y}$ *be a homomorphism of PMS-algebras.*

1. *If* $(\mathcal{F},\ A)$ *is a soft PMS-algebra over* $\mathcal{X}$, *then*$(f(\mathcal{F}),\ A)$ *is a soft PMS-algebra over* $\mathcal{Y}$.
2. *If* f *is onto and* $(\mathcal{G},\ B)$ *is a soft PMS-algebra over* $\mathcal{Y}$, *then* $(f^{-1}(\mathcal{G}),\ B)$ *is a soft PMS-algebra over* $\mathcal{X}$ *if it is non-null.*

Proof. 1. Since $(\mathcal{F},\ A)$ is a soft PMS-algebra over $\mathcal{X}$, then $(f(\mathcal{F}),\ A)$ is a non-null soft set over $\mathcal{Y}$.
For every $x \in Supp(f(\mathcal{F}),\ A)$, we have $f(\mathcal{F})(x) = f(\mathcal{F}(x)) \neq \emptyset$ is a PMS-subalgebra of $\mathcal{Y}$ since $\mathcal{F}(x)$ is a PMS-sub algebra of $\mathcal{X}$ and its homomorphic image is also a PMS-subalgebra of $\mathcal{Y}$. Hence $(f(\mathcal{F}),\ A)$ is a soft PMS-algebra over $\mathcal{Y}$.

2. It is clear that Supp $(f^{-1}(\ \mathcal{G}),\ B) \subset$ Supp $(\mathcal{G},\ B)$. Let $y \in (f^{-1}(\mathcal{G}),\ B)$. Then $\mathcal{G}(y) \neq \emptyset$. Since the nonempty set $\mathcal{G}(y)$ is PMS-subalgebra of $\mathcal{Y}$. its homomorphic pre-image $f^{-1}(\mathcal{G}(y))$ is also PMS-subalgebra of $\mathcal{X}$. Hence $f^{-1}(\mathcal{G}(y))$ is a PMS-subalgebra of over $\mathcal{X}$ for all $y \in (f^{-1}(\mathcal{G}), B)$.

□

Definition 3.16. A soft PMS-algebra $(\mathcal{F},\ A)$ over a PMS-algebra $\mathcal{X}$ is said to be:

1. trivial soft set if $\mathcal{F}(x) = \{\ 0\ \}$.
2. whole soft set if $\mathcal{F}(x) = \mathcal{X}$, for all $x \in A$.

Example 3.17. Consider a PMS-algebra $\mathcal{X} = \{0,\ a,\ b,\ c\}$ in Example 3.3 (2). For $A = \mathcal{X}$, let $F : A \longrightarrow \mathcal{P}\ (\mathcal{X})$ be a set-valued function defined by $\mathcal{F}\ (x) = \{0\} \cup \{y \in \mathcal{X} \mid x\ R\ y \iff o\ (x) = o(y)\}$ for all $x \in A$. Then, $\mathcal{F}(\ x\ ) = \mathcal{X}$ for all $x \in A$, and so $(\mathcal{F}\ ,\ A)$ is a whole soft PMS-algebra over $\mathcal{X}$.

Theorem 3.18. *Let $f : \mathcal{X} \longrightarrow \mathcal{Y}$ be a homomorphism of PMS-algebras and $(\mathcal{F},\ A)$ be a soft PMS-algebra over $\mathcal{X}$.*

1. *If $\mathcal{F}(x) = ker(f)$ for all $x \in A$, then $(f(\mathcal{F}),\ A)$ is the trivial soft PMS-algebra over $\mathcal{Y}$.*
2. *If f is onto and $(\mathcal{F},\ A)$ is whole, then $(f(\mathcal{F}),\ A)$ is the whole soft PMS-algebra over $\mathcal{Y}$.*
3. *If $\mathcal{G}(y) = f(\mathcal{X})$ for all $y \in B$, then $(f^{-1}(\mathcal{G}),\ B)$ is the whole soft PMS-algebra over $\mathcal{X}$*
4. *If f is injective and $(\mathcal{G}, B)$ is trivial, then $(f^{-1}(\mathcal{G}),\ B)$ the trivial soft PMS-algebra over $\mathcal{X}$.*

Proof. 1. Let $\mathcal{F}\ (x)$ =ker(f) for all $x \in A$. Then $f(\mathcal{F})(x) = f\ (\mathcal{F}\ (x)) = \{0_{\mathcal{Y}}\}$ for all $x \in A$ and hence It follows from Theorem 3.15 and Definition 3.16 that $(f\ (\mathcal{F})\ ,\ A)$ is a trivial soft PMS-algebra over $\mathcal{Y}$.

2. Suppose that f is onto and $(\mathcal{F},\ A)$ is whole. Then, $\mathcal{F}(x) = \mathcal{X}$ for all $x \in A$, and so $f(\mathcal{F})(x) = f(\mathcal{F}(x)) = f(\mathcal{X}) = \mathcal{Y}$ for all $x \in A$. It follows from Theorem 3.15 and Definition 3.16 that $(f(\mathcal{F}),\ A)$ is the whole soft PMS-algebra over $\mathcal{Y}$.
3. Suppose that $\mathcal{G}(y) = f(\mathcal{X})$ for all $y \in B$. Then, $f^{-1}(\mathcal{G}(y)) = f^{-1}((\mathcal{G}(y)) = f^{-1}(f(\mathcal{X})) = \mathcal{X}$ for all $y \in B$. Hence, $(f^{-1}(\mathcal{G}),\ B)$ is the whole soft PMS-algebra over $\mathcal{X}$, by Theorem 3.15 and Definition 3.16.
4. Suppose that f is injective and $(\mathcal{G},\ B)$ is trivial. Then, $\mathcal{G}(y) = \{0\}$ for all $y \in B$, and so $f^{-1}(\mathcal{G}(y)) = f^{-1}((\mathcal{G}(y)) = f^{-1}(\{0\}) = Ker(f) = \{0_{\mathcal{X}}\}$ for all $y \in B$ since f is injective. It follows from Theorem 3.15 and Definition 3.16 that $(f^{-1}(\mathcal{G}),\ B)$ is the trivial soft PMS-algebra over $\mathcal{X}$.

□

Definition 3.19. Let $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ be soft PMS-algebras over $\mathcal{X}$. Then $(\mathcal{F},\ A)$ is called a soft PMS-subalgebra of$(\mathcal{G},\ B)$ if

1. $A\ \subset\ B$
2. $\mathcal{F}(x)$ is a PMS-subalgebra of $\mathcal{G}(x)$ for all $x \in Supp\,(\mathcal{F},\ A)$.

From the above definition, it is clear that if $(\mathcal{F},\ A)$ is a soft PMS-subalgebra of $(\mathcal{G},\ B)$, then $Supp\ (\mathcal{F},\ A)\ \subset\ Supp\ (\mathcal{G},\ B)$.

From the above definition, it is clear that if $(\mathcal{F},\ A)$ is a soft PMS-subalgebra of $(\mathcal{G},\ B)$, then $Supp\ (\mathcal{F},\ A)\ \subset\ Supp\ (\mathcal{G},\ B)$.

Example 3.20. Let $(\mathcal{F},\ A)$ be a soft PMS-algebra over $\mathcal{X}$ given in Example 3.2. Let $B = \{1,\ 2\} \subset A$ and let $\mathcal{H} :\ B \longrightarrow \mathcal{P}(\mathcal{X})$ be a set valued function defined by $\mathcal{H}\,(x) = \ \{y \in \mathcal{X} \mid x\ R\ y \iff y = x^n,\ n \in N\ \}$ for all $x \in B$. Then $\mathcal{H}(1) = \{0,\ 1\}$ and $\mathcal{H}(2) = \{0,\ 2\}$ are PMS-subalgebras of $\mathcal{F}(1)$ and $\mathcal{F}(2)$, respectively. Hence, $(\mathcal{H},\ B)$ is a soft PMS-subalgebra of $(\mathcal{F},\ A)$.

THEOREM 3.21. *Let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ A)$ *be soft PMS-algebras over* $\mathcal{X}$. *If* $\mathcal{F}(x) \subset \mathcal{G}(x)$ *for all* $x \in A$, *then* $(\mathcal{F},\ A)$ *is a soft PMS-subalgebra of* $(\mathcal{G},\ A)$.

Proof. Let $A \cap A = A$ and $\mathcal{F}(x) \subset \mathcal{G}(x)$ for all $x \in A$, and since $(\mathcal{F},\ A)$ is a soft PMS-algebra over $\mathcal{X}$ implies that $\mathcal{F}(x)$ is a PMS-subalgebra of $\mathcal{G}(x)$. Hence, $(\mathcal{F},\ A)$ is a soft PMS-subalgebra of $(\mathcal{G},\ A)$. □

THEOREM 3.22. *Let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{X}$. *If* $(\mathcal{F},\ A) \widetilde{\subset} (\mathcal{G},\ B)$, *then* $(\mathcal{F},\ A)$ *is a soft PMS-subalgebra of* $(\mathcal{G},\ B)$.

Proof. Suppose that $(\mathcal{F},\ A) \widetilde{\subset} (\mathcal{G},\ B)$. Then $A \subset B$ and $\mathcal{F}(x) = \mathcal{G}(x)$ for all $x \in A$, and since $(\mathcal{F},\ A)$ and $(\mathcal{G},\ B)$ are soft PMS-algebras over $\mathcal{X}$. Hence, $(\mathcal{F},\ A)$ is a soft PMS-subalgebra of $(\mathcal{G},\ B)$. □

THEOREM 3.23. *Let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{X}$. *Then the restricted intersection* $(\mathcal{F},\ A) \widetilde{\sqcap} (\mathcal{G},\ B)$ *is a soft PMS-subalgebra of both* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$.

Proof. By DEFINITION 2.14, we can write $(\mathcal{F},\ A) \widetilde{\sqcap} (\mathcal{G}\ ,\ B) = (\mathcal{H}, A)$, where $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x)$ for all $x \in A$. Suppose that $(\mathcal{H},\ A)$ is a non-null soft set over $\mathcal{X}$. If $x \in Supp(\mathcal{H},\ A)$, then $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x) \neq \emptyset$. It follows that the non-empty sets $\mathcal{F}(x)$and $\mathcal{G}(x)$ are both PMS-algebras of over $\mathcal{X}$. Hence $\mathcal{H}(x)$ is a PMS-subalgebra of both $\mathcal{F}(x)$ and $\mathcal{G}(x)$ over $\mathcal{X}$ for all $x \in Supp(\mathcal{H},\ A)$and
$(\mathcal{F}\ ,\ A) \widetilde{\sqcap} (\mathcal{G}\ ,\ A) = (\mathcal{H},\ A)$, is a soft PMS-algebra over $\mathcal{X}$. □

COROLLARY 3.24. *The restricted intersection of* $(\mathcal{F},\ A) \widetilde{\sqcap} (\mathcal{G},\ A)$ *is a soft PMS-sub algebra of both* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ A)$.

THEOREM 3.25. *Let* $f : \mathcal{X} \longrightarrow \mathcal{Y}$ *be a homomorphism of PMS-algebras and let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{X}$. *If* $(\mathcal{F},\ A)$ *is a soft PMS-subalgebra of* $(\mathcal{G},\ B)$, *then* $(f(\mathcal{F}),\ A)$ *is a soft PMS-subalgebra of* $(f(\mathcal{G}),\ B)$.

Proof. Assume that $(\mathcal{F},\ A)$ is a soft PMS-subalgebra of $(\mathcal{G},\ B)$. Then $x \in A$ and $\mathcal{F}(x)$ is a PMS-subalgebra of $\mathcal{G}(x)$. Since f is a homomorphism, then $f(\mathcal{F})(x) = f(\mathcal{F}(x))$ is a PMS-subalgebra of $f(\mathcal{G})(x) = f(\mathcal{G}(x))$ and, hence, $(f(\mathcal{F}), A)$ is a soft PMS-subalgebra of $(f(\mathcal{G}),\ B)$. □

THEOREM 3.26. *Let* $f : \mathcal{X} \longrightarrow \mathcal{Y}$ *be a homomorphism of PMS-algebras and let* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$ *be soft PMS-algebras over* $\mathcal{Y}$. *If* $(\mathcal{G},\ B)$ *is a soft PMS-subalgebra of* $(\mathcal{F},\ A)$, *then* $(f^{-1}(\mathcal{G}),\ B)$ *is a soft PMS-subalgebra of* $(f^{-1}(\mathcal{F}),\ A)$.

Proof. Assume that $(\mathcal{G},\ B)$ is a soft PMS-subalgebra of $(\mathcal{F},\ A)$. Let $y \in Supp(f^{-1}(G),\ B)$. Then $B \subset A$ and $\mathcal{G}(y)$ is a PMS-subalgebra of $\mathcal{F}(y)$ for all $y \in B$. Since f is a homomorphism, $f^{-1}(\mathcal{G})(y) = f^{-1}(\mathcal{G}(y))$ is a PMS-subalgebra of $f^{-1}(\mathcal{F})(y) = f^{-1}(\mathcal{F}(y))$ for all $y \in Supp(f^{-1}(G),\ B)$. Hence, $(f^{-1}(\mathcal{G}),\ B)$ is a soft PMS-subalgebra of $(f^{-1}(\mathcal{F}),\ A)$. □

4. Soft PMS-ideals of a soft PMS algebra

DEFINITION 4.1. Let $(\mathcal{F},\ A)$ be a soft PMS-algebra over $\mathcal{X}$. Then a non-null soft set $(\gamma,\ I)$ is called a soft PMS-ideal of $(\mathcal{F},\ A)$ if

1. $I \subset A$

2. $\gamma(x)$ is a PMS-ideal of $\mathcal{F}(x)$ for all $x \in Supp(\gamma,\ I)$.

THEOREM 4.2. *Let* $(\gamma,\ I)$ *and* $(\sigma\ ,\ J)$ *be soft PMS-ideals of a soft PMS-algebra* $(\ \mathcal{F},\ A)$ *over a PMS-algebra* $\mathcal{X}$. *Then the restricted intersection* $(\gamma,\ I)\,\widetilde{\sqcap}\ (\sigma,\ J)$ *is a soft PMS-ideal of* $(\mathcal{F},\ A)$.

Proof. By DEFINITION 2.14, we can write $(\gamma\ ,\ I)\,\widetilde{\sqcap}\,(\sigma\ ,\ J) = (\delta, K)$, where $K = I \cap J$ and $\delta\,(x) = \gamma(x) \cap \sigma(x)$ for all $x \in K$. Suppose that (δ, K) is a non-null soft set over $\mathcal{X}$. If $x \in Supp\,(\delta, K)$, then $\delta\,(x) = \gamma\,(x) \cap \sigma\,(x) \neq \emptyset$. It follows that the non-empty sets $\gamma\,(x)$ and $\sigma(x)$ are both PMS-ideals of $\mathcal{F}(x)$. Hence $\delta(x)$ is a PMS-ideal of $\mathcal{F}(x)$ for all $x \in Supp(\delta, K)$, and hence
$(\gamma\ ,\ I)\,\widetilde{\sqcap}\,(\sigma\ ,\ J) = (\delta,\ K)$, is a soft PMS-ideal of $(\mathcal{F},\ A)$. □

THEOREM 4.3. *Let* $(\gamma,\ I)$ *and* $(\sigma\ ,\ J)$ *be soft PMS-ideals of* $(\mathcal{F},\ A)$ *and* $(\mathcal{G},\ B)$, *respectively. Then* $(\gamma,\ I)\,\widetilde{\sqcap}\ (\sigma,\ J)$ *is a soft PMS-ideal of* $(\mathcal{F},\ A)\,\widetilde{\sqcap}\ (\mathcal{G},\ B)$.

Proof. By DEFINITION 2.14, we can write $(\gamma,\ I)\,\widetilde{\sqcap}\,(\sigma,\ J) = (\delta,\ K)$, where $K = I \cap J$ and $\delta\,(x) = \gamma(x) \cap \sigma(x)$ for all $x \in K$. Similarly, we have $(\mathcal{F},\ A)\,\widetilde{\sqcap}\ (\mathcal{G},\ B)\ = (\mathcal{H}, C)$ and $C = A \cap B$, where $\mathcal{H}(x) = \mathcal{F}(x) \cap \mathcal{G}(x)$ for all $x \in C$. Since $I \cap J$ is non-null, there exists an $x \in Supp(\delta, K)$ such that $\gamma(x) \cap \sigma(x) \neq \emptyset$. Since $I \cap J \subset A \cap B$, we neew to show that $\gamma(x) \cap \sigma(x) \subset \mathcal{F}(x) \cap \mathcal{G}(x)$, and hence $\delta(x)$ is a PMS-ideal of $\mathcal{H}(x)$ for all $x \in Supp(\delta,\ K)$.
Thus $(\gamma,\ I)\,\widetilde{\sqcap}\ (\sigma,\ J)$ is a soft PMS-ideal of $(\mathcal{F},\ A)\,\widetilde{\sqcap}\ (\mathcal{G},\ B)$. □

THEOREM 4.4. *Let* $(\gamma,\ I)$ *and* $(\sigma\ ,\ J)$ *be soft PMS-ideals of a soft PMS-algebra* $(\ \mathcal{F}, A)$ *over a PMS-algebra* $\mathcal{X}$. *If* I *and* J *are disjoint, then* $(\gamma,\ I)\,\widetilde{\cup}\ (\sigma,\ J)$ *is a soft PMS-ideal of* $(\mathcal{F},\ A)$.

Proof. Assume that $(\gamma,\ I)$ and $(\sigma\ ,\ J)$ are a soft PMS-ideal of $(\mathcal{F},\ A)$. According to DEFINITION 2.8, we can write $(\gamma,\ I)\,\widetilde{\cup}\ (\sigma,\ J)\ = (\delta, K)$, where $K = I \cup J$ and for exery $x \in K$,

$$\delta\,(x) = \begin{cases} \gamma\,(x)\,,\ if\ x \in I - J \\ \sigma\,(x)\,,\ if\ x \in J - I \\ \gamma\,(x) \cup \sigma\,(x)\,,\ if\ x \in I \cap J \end{cases}$$

Clearly, we have $K \subset A$. Suppose $I \cup J = \emptyset$. Then for every $x \in Supp(\delta,\ K)$, we know that either $x \in I\backslash J$ or $x \in J\backslash I$. If $x \in I\backslash J$, then $\delta(x) = \gamma(x) \neq \emptyset$ is a PMS-ideal of $\mathcal{F}(X)$ since $(\gamma,\ I)$ is a soft PMS-ideal of $(\mathcal{F},\ A)$. Similarly, if $x \in J\backslash I$, then $\delta(x) = \sigma(x) \neq \emptyset$ is a PMS-ideal of $\mathcal{F}(x)$ since (σ, J) is a soft PMS-ideal of $(\mathcal{F},\ A)$. Thus we conclude that, $\delta(x)$ is a PMS-ideal of $\mathcal{F}(x)$ for all $x \in Supp(\delta, K)$, and so $(\gamma,\ I)\,\widetilde{\cup}\ (\sigma,\ J)\ = (\delta,\ K)$ is a soft PMS-ideal of $(\mathcal{F},\ A)$. □

THEOREM 4.5. *If* $(\mathcal{F}\ ,\ A)$ *and* $(\mathcal{G},\ B)$ *are soft PMS-ideals over* $\mathcal{X}$, *then the soft set* $(\mathcal{F},\ A)\ \widetilde{\wedge}\ (\mathcal{G},\ B)$ *is a soft PMS-ideal of* $\mathcal{X}$.

Proof. By DEFINITION 2.10, $(\mathcal{F},\ A)\,\widetilde{\wedge}\ (\mathcal{G},\ B) = (\mathcal{H},\ C)$, where $C = A \times B$ and $\mathcal{H}\,(x,\ y) = \mathcal{F}(x) \cap \mathcal{G}(y)$ for all $(x, y) \in A \times B$. Let $(x,\ y) \in Supp(H,\ C)$. Then $\mathcal{H}(x,\ y) \neq \emptyset$ and $(\mathcal{H},\ C)$ is a non-null soft set over $\mathcal{X}$. If $(x,\ y) \in Supp\,(\mathcal{H},\ C)$, then $\mathcal{H}(x,\ y) = \mathcal{F}\,(x) \cap \mathcal{G}\,(y) \neq \emptyset$. It follows that the non-empty sets $\mathcal{F}\,(x)$and $\mathcal{G}(y)$ are both PMS-ideals of $\mathcal{X}$. Hence $H(x, y)$ is PMS-ideal of $\mathcal{X}$ for all $x \in Supp\,(\mathcal{H},\ C)$ and so $(\mathcal{H},\ C) = (\mathcal{F},\ A)\widetilde{\wedge}(\mathcal{G},\ B)$ is a soft PMS-ideal of $\mathcal{X}$. □

5. Idealistic Soft PMS-algebras

Definition 5.1. Let $(\alpha,\ A)$ be a non-null soft set over a PMS-algebra $\mathcal{X}$. Then $(\alpha,\ A)$ is called an idealistic soft PMS-algebra over $\mathcal{X}$ if $\alpha(x)$ is a PMS-ideal of $\mathcal{X}$.

Theorem 5.2. *Let $(\mathcal{F},\ A)$ be a soft set over $\mathcal{X}$ and B is a subset of A. If $(\mathcal{F},\ A)$ is an idealistic soft PMS-algebra over $\mathcal{X}$, then so is $(\mathcal{F}|_B,\ B)$ whenever it is non-null.*

Proof. Let $B \subseteq A$ and let $x \in Supp(\mathcal{F}|_B,\ B)$. Then $\mathcal{F}_B(x) \neq \emptyset$. Since $\mathcal{F}(x)$ is a PMS-ideal of $\mathcal{X}$ for all $x \in A$ and $\mathcal{F}|_B(x) \subseteq \mathcal{F}(x),\ \forall x \in B$, then $\mathcal{F}|_B(x)$ is a PMS -ideal of $\mathcal{X}$.
Hence, $(\mathcal{F}|_B,\ B)$ is an idealistic soft PMS-algebra over $\mathcal{X}$. □

Theorem 5.3. *Let $(\alpha,\ A)$ and $(\beta,\ B)$ be idealistic soft PMS-algebras over $\mathcal{X}$. Then $(\alpha,\ A)\,\widetilde{\sqcap}\,(\beta,\ B)$ is an idealistic soft PMS-algebra over $\mathcal{X}$.*

Proof. By definition 2.14, we can write $(\alpha,\ A)\,\widetilde{\sqcap}\,(\beta,\ B) = (\theta,\ C)$, where $C = A \cap B$ and $\theta(x) = \alpha(x) \cap \beta(x)$ for all $x \in C$. Suppose that $(\theta,\ C)$ is a non-null soft set over $\mathcal{X}$. If $x \in Supp(\theta,\ C)$, then $\theta(x) = \alpha(x) \cap \beta(x) \neq \emptyset$. It follows that the non-empty sets $\alpha(x)$ and $\beta(x)$ are both idealistic soft PMS-algebras over $\mathcal{X}$. Hence $\theta(x)$ is a PMS-ideal of $\mathcal{X}$ for all $x \in Supp(\theta,\ C)$, and hence $(\alpha,\ A)\,\widetilde{\sqcap}\,(\beta,\ B) = (\theta,\ C)$ is an idealistic soft PMS-algebra over $\mathcal{X}$. □

Theorem 5.4. *Let $(\alpha,\ A)$ and $(\beta,\ B)$ be idealistic soft PMS-algebras over $\mathcal{X}$. If A and B are disjoint, then $(\alpha,\ A)\,\widetilde{\cup}\,(\beta,\ B)$ is an idealistic soft PMS-algebra over $\mathcal{X}$.*

Proof. Assume that $(\alpha,\ A)$ and $(\beta,\ B)$ are idealistic soft PMS-algebras over $\mathcal{X}$. According to definition 2.8, we can write $(\alpha,\ A)\,\widetilde{\cup}\,(\beta,\ B) = (\theta,\ C)$, where $C = A \cup B$ and for exery $x \in C$,

$$\theta(x) = \begin{cases} \alpha(x),\ if\ x \in A - B \\ \beta(x),\ if\ x \in B - A \\ \alpha(x) \cup \beta(x),\ if\ x \in A \cap B \end{cases}$$

Since $A \cap B = \emptyset$ and for every $x \in C$, then either $x \in A \backslash B$ or $x \in B \backslash A$. If $x \in A \backslash B$, then $\theta(x) = \alpha(x) \neq \emptyset$ is a PMS-ideal of $\mathcal{X}$ because of (α, A) is an idealistic soft PMS-algebra of $\mathcal{X}$. Similarly, if $x \in B \backslash A$, then $\theta(x) = \beta(x) \neq \emptyset$ is a PMS-ideal of $\mathcal{X}$ because of $(\beta,\ B)$ is an idealistic soft PMS-algebra of $\mathcal{X}$. Thus, we conclude that, $\theta(x)$ is a PMS-ideal of $\mathcal{X}$ for all $x \in Supp(\theta, C)$, and so $(\alpha,\ A)\,\widetilde{\cup}\,(\beta,\ B) = (\theta, C)$ is an idealistic soft PMS-algebra over $\mathcal{X}$. □

Theorem 5.5. *Let $(\alpha,\ A)$ and $(\beta,\ B)$ be idealistic soft PMS-algebras over $\mathcal{X}$. Then $(\alpha,\ A\ )\,\widetilde{\wedge}\,(\beta,\ B\ )$ is an idealistic soft PMS-algebra over $\mathcal{X}$ if it is non-null.*

Proof. By definition 2.10, $(\alpha,\ A)\,\widetilde{\wedge}\,(\beta,\ B) = (\theta,\ C)$, where $C = A \times B$ and $\theta(x,\ y) = \alpha(x) \cap \beta(y)$ for all $(x,\ y) \in A \times B$. Let $(x,\ y) \in Supp(\theta, C)$. Then $\theta(x,\ y) \neq \emptyset$ and $(\theta,\ C)$ is a non-null soft set over $\mathcal{X}$. If $(x,\ y) \in Supp(\theta,\ C)$, then $\theta(x,\ y) = \alpha(x) \cap \beta(y) \neq \emptyset$. It follows that the non-empty sets $\alpha(x)$ and $\beta(y)$ are both PMS-ideals of $\mathcal{X}$. Hence, $\theta(x,\ y)$ is a PMS-ideal of $\mathcal{X}$ for all $x \in Supp(\theta,\ C)$, and so $(\theta,\ C) = (\alpha,\ A)\widetilde{\wedge}(\beta,\ B)$ is an idealistic soft PMS-algebra over $\mathcal{X}$. □

Theorem 5.6. *Let $f : \mathcal{X} \longrightarrow \mathcal{Y}$ be an epimorphism of PMS-algebras. If $(\alpha,\ A)$ be an idealistic soft PMS-algebra over $\mathcal{X}$, then $(f(\alpha),\ A)$ is an idealistic soft PMS-algebra over $\mathcal{Y}$.*

Proof. Since $(\alpha,\ A)$ is a non-null soft set, and $(\alpha,\ A)$ is an idealistic soft PMS-algebra over $\mathcal{X}$, then $(f(\alpha),\ A)$ is a non-null soft set over $\mathcal{Y}$. We see that, for all $x \in Supp(f(\alpha),\ A)$, $f(\alpha)(x) = f(\alpha(x)) \neq \emptyset$. Since the non-empty set $\alpha(x)$ is a PMS-ideal of $\mathcal{X}$, and f is an epimorphism, $f(\alpha(x))$ is a PMS-ideal of $\mathcal{Y}$. Therefore, $f(\alpha(x))$ is a PMS-ideal of $\mathcal{Y}$ for all $x \in Supp(f(\alpha),\ A)$.
Consequently, $(f(\alpha),\ A)$ is an idealistic soft PMS-algebra over $\mathcal{Y}$. □

THEOREM 5.7. *Let $(\alpha,\ A)$ be an idealistic soft PMS-algebra over $\mathcal{X}$ and $f : \mathcal{X} \longrightarrow \mathcal{Y}$ be a homomorphism of PMS-algebras.*

1. *If $\alpha(x) = Ker(f)$, for all $x \in A$, then $(f(\alpha),\ A)$ is the trivial idealistic soft PMS-algebra over $\mathcal{Y}$.*
2. *If $(\alpha,\ A)$ is whole soft set and f is onto, then $(f(\alpha),\ A)$ the whole idealistic soft PMS-algebra over $\mathcal{Y}$.*

Proof. 1. Let $\alpha(x) = \ker(f)$ for all $x \in A$. Then $f(\alpha)(x) = f(\alpha(x)) = \{0_{\mathcal{Y}}\}$ for all $x \in A$ and hence It follows from THEOREM 3.15 and DEFINITION 3.16 that $(f(\alpha),\ A)$ is a trivial idealistic soft PMS-algebra over $\mathcal{Y}$.
2. Suppose that f is onto and $(\alpha,\ A)$ is whole. Then, $\alpha(x) = \mathcal{X}$ for all $x \in A$, and so $f(\alpha)(x) = f(\alpha(x)) = f(\mathcal{X}) = \mathcal{Y}$ for all $x \in A$. It follows from THEOREM 3.15 and DEFINITION 3.16 that $(f(\alpha),\ A)$ is the whole idealistic soft PMS-algebra over $\mathcal{Y}$.

□

6. Conclusion

We applied soft sets to the PMS algebras and discussed the algebraic properties of soft sets in PMS algebras. Also, we established the intersection, union, "AND" operation, restricted intersection, and extended intersection of soft PMS-algebras, soft PMS-subalgebras, soft PMS-ideals, and idealistic soft PMS-algebras. Based on these results, we will apply soft sets to other structures of PMS algebras.
In our ongoing work, we will focus on applying fuzzy soft sets in PMS-algebras and other algebraic structures of PMS-algebras like the G-part of a PMS algebra, the medial of a PMS algebra, the P-radical of a PMS algebra, and so on. So we will focus on the fuzzification of soft PMS algebras and soft PMS ideals.

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